Error sources and data limitations for the prediction of surface gravity: a case study using benchmarks

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Gravity-based heights require gravity values at levelled benchmarks (BMs), which sometimes have to be predicted from surrounding observations. We use EGM2008 and the Australian National Gravity Database (ANGD) as examples of model and terrestrial observed data respectively to predict gravity at Australian national levelling network (ANLN) BMs. The aim is to quantify errors that may propagate into the predicted BM gravity values and then into gravimetric height corrections (HCs). Our results indicate that an approximate ±1 arc-minute horizontal position error of the BMs causes maximum errors in EGM2008 BM gravity of ~ 22 mGal (~55 mm in the HC at ~2200 m elevation) and ~18 mGal for ANGD BM gravity because the values are not computed at the true location of the BM. We use RTM (residual terrain modelling) techniques to show that ~50% of EGM2008 BM gravity error in a moderately mountainous region can be accounted for by signal omission. Non-representative sampling of ANGD gravity in this region may cause errors of up to 50 mGals (~120 mm for the Helmert orthometric correction at ~2200 m elevation). For modelled gravity at BMs to be viable, levelling networks need horizontal BM positions accurate to a few metres, while RTM techniques can be used to reduce signal omission error. Unrepresentative gravity sampling in mountains can be remedied by denser and more representative re-surveys, and/or gravity can be forward modelled into regions of sparser gravity.

Key words: gravity, levelling, height corrections, RTM, Helmert orthometric heights

1. INTRODUCTION

Physically meaningful heights are governed by the difference between gravity potential at the point of interest (W_P) and gravity potential on the geoid (W_0), scaled to dimensions of length by some value with dimensions of acceleration. Ideally, this should be the integral mean value of gravity along the plumblines, but this is very difficult to realise in practice, so various different approximations, including constant values, have been used (summarised below).

Levelling networks provide the geometric height difference (Δn) relative to the local equipotential surface, but these need to be converted into potential differences that account for the non-parallelism of the equipotential surfaces and to ensure holonomity (e.g., *Sansó and Vaníček, 2006*). Gravity potential cannot be measured directly, so surface gravity observations (g_{obs}) are combined with levelling to obtain (e.g., *Heiskanen and Moritz, 1967, p. 162*)

$$W_0 - W_P = C = \int_0^P g \, \mathrm{dn} \tag{1}$$

where C is the geopotential number, which is independent of the levelling route taken between the geoid and P.

Orthometric (H^O), dynamic (H^D) and normal (H^N) heights are all based on C and thus require g_{obs} in their realisation (see *Heiskanen and Moritz, 1967, Chapter 4; Jekeli, 2000, Molodensky et al., 1962*). In contrast, normal-orthometric heights (H^{N-O}; e.g., *Bomford, 1971, p. 230*) use only normal gravity (γ), while H^N use γ and g_{obs} (e.g., *Heiskanen and Moritz 1967, p. 170*). H^O and H^D are both measured relative to the geoid; H^N is measured relative to the quasigeoid, which is not an equipotential surface; H^{N-O} is measured relative to an indeterminate non-equipotential surface (*Filmer et al., 2010*). H^D are not suitable for national vertical datums, so are not discussed further.

All height systems are practically implemented through the application of height corrections (HCs) to Δn between benchmarks (BMs) in the levelling network, which is then typically least-squares adjusted to realise heights at BMs. Alternatively, the C-based method can be used (e.g., *Marti and Schlatter, 2002*). Either method realises heights at BMs in one of the aforementioned height systems, which is fundamental for any physically meaningful vertical datum. Both methods require a gravity value at each BM (g_{BM}), but for this study, we use the HC method. For HC formulas, the reader is referred to, e.g., *Heiskanen and Moritz (1967, Chapter 4*).

Because many national vertical datums were realised when insufficient gravity observations were available for gravimetric HCs to be applied (e.g., *Roelse et al., 1971; Christie, 1994; de Freitas et al., 2002*), normal gravity was used to compute normal-orthometric corrections (NOCs) to realise H^{N-O} as an approximation of H^{O}. Differences between H^{N-O} and H^{O} or H^{N} can be at the decimetre level (*Filmer et al., 2010*) so redefinition of older levelling-based vertical datums using H^{N-O} should consider the implementation of a gravimetric height system; i.e., H^{O} or H^{N}. This can be difficult if g_{obs} are not taken directly on BMs, making it necessary to predict the required gravity values.

Our motivation is to assess the suitability of two possible methods for predicting g_{BM} (Section 2.2) for gravimetric height systems to be used in redefined vertical datums. We use modelled gravity from Earth Gravitational Model 2008 (EGM2008; *Pavlis et al., 2012*) and terrestrial g_{obs} from the 2007 release of the Australian National Gravity Database (ANGD). We identify and empirically quantify g_{BM} errors from the prediction methods and gravity sources used, and relate their effect on HCs so that these problems can be considered prior to any redefinition of a vertical datum. Throughout, we use g_{BM} to denote all methods of interpolation/computation used to predict gravity at a BM.

A particular problem that we have encountered is the effect of uncertainty in horizontal BM positions at which the gravity predictions are to be made and their effect on HCs computed from g_{BM}. This does not appear to have been addressed previously, although *Tscherning (1980)* points out the need for accurate horizontal BM positions. In addition, we consider errors at g_{BM} from barometric height errors at terrestrial g_{obs} (e.g., *Bellamy and Lodwick, 1968*), the effect of inconsistent datums (e.g., *Featherstone, 1995*), and signal omission error when using EGM2008-modelled gravity (e.g., *Hirt et al., 2010a*).

Various methods of gravity prediction at BMs have been presented by others, including least-squares collocation *(Tscherning, 1980)*, least-squares surface fitting, and weighted means (e.g., *Kassim, 1980)*. The US National Geodetic Survey provides surface gravity predictions (http://www.ngs.noaa.gov/TOOLS/Gravity/gravcon.html) using the multiquadratic-biharmonic method of *Hardy and Nelson (1986)*, although *Jekeli (1994)* raises doubts over the legitimacy of this method. These methods all use terrestrial gravity data to predict g_{BM}, with various levels of precision that are dependent on the local terrain, type of gravity anomaly, but particularly on the quality of the source data. Our primary purpose is to show that the accuracy of g_{BM} is limited by BM coordinate uncertainty, the resolution of modelled gravity, and the spatial sampling of g_{obs}. No comparison is made among the [very many] options for interpolation and prediction.

The state of terrestrial gravity and levelling databases is a contributing factor in the accurate prediction of g_{BM}. Since many gravity surveys are not undertaken solely for computing HCs to differential levelling observations, but for national mapping programs, resource exploration and military purposes, the spatial density of the gravity observations are often unsuitable for predicting g_{BM} (e.g., *Mitchell, 1973; de Freitas et al., 2002*). The ANGD has been developed primarily for resource exploration, and geodetic uses have been generally focussed on regional quasi/geoid computation (e.g., *Featherstone et al., 2011*).

Satellite gravity missions have allowed the long-wavelength gravity field to be realised more accurately than before, but terrestrial gravity is still required to provide the high-frequency contributions. Global gravitational models, such as EGM2008, have been developed from a combination of digital elevation, GRACE (*Tapley et al., 2004*), satellite altimetry and terrestrial gravity data (*Pavlis et al., 2012*), and can be used to derive surface gravity values at BMs (e.g., *Filmer et al., 2010*). Thus, modelled gravity can be used for HCs in place of the traditional terrestrial gravity database, but possible errors do need to be considered, as is the case here.

2. DATA USED AND GRAVITY PREDICTION METHOD

2.1 Data

The ANLN (provided by Geoscience Australia; *G. Johnston, 2007, pers. comm.*) is Australia's official national levelling network, containing 87,951 BMs in the 2007 version. ANLN data used for this study comprise Australian Height Datum (AHD; Roelse et al., 1975) H^{N-O} and the horizontal positions of the BMs (latitude; ϕ_{BM} and longitude; λ_{BM}, referred to as Ω_{BM}^{ANLN}). A truncated version of the *Rapp (1961)* NOC using Geodetic Reference System 1967 (GRS67; *IAG, 1971*) parameters was applied to the ANLN levelling sections prior to the national adjustment. The effects of truncating the *Rapp (1961)* formula and using GRS67 (rather than GRS80) parameters were shown in *Filmer et al., (2010)* to be negligible. The ANLN consists of mostly pre-1971 levelling, which was used to realise the AHD in 1971, but has received some updates over the years (e.g., *Wellman and Tracey, 1987; Morgan, 1992*). *Filmer and Featherstone (2009)* discuss the current status of and problems in the ANLN.

The tide-free release of EGM2008 was used for modelled g_{BM}. EGM2008 used 905,483 Australian land gravity observations (*Factor, 2008,* pers. comm.) of which 156,269 are not held in the ANGD and located mostly around Darwin in northern Australia. The ANGD has been compiled over the past ~60 years (e.g., *Murray, 1997*). The 2007 release (*Tracey et al., 2007*) is used for this study because it contains ANLN BM names that are not available in subsequent releases, but are necessary to identify g_{obs} co-located with ANLN BMs (g_{obs}^{BM}). 9,527 g_{obs}^{BM} (out of 87,951) were identified in the 2007 release of the ANGD (*Filmer et al., 2010*) and are used as control data for some tests in this paper.

The ANGD 2007 release contains 1,246,613 land-based and marine g_{obs} which are tied to the Australian Absolute Gravity Datum 2007 (AAGD07; *Tracey et al., 2007*), which is 0.078 mGal less than IGSN71 (*Morelli et al., 1973*).

The version 4 digital elevation model (DEM) of *Jarvis et al., (2008)*, derived from the Shuttle Radar Topography Mission (SRTM) (*Farr et al., 2007*) is used to approximate EGM2008 signal omission error in south east Australia using the residual terrain model (RTM) technique (*Forsberg, 1984*; see Section 3.4). Following *Hirt (2010)*, RTM data are constructed as the difference between the version-4 SRTM elevation data set and the high-degree spherical harmonic DTM2006.0 (*Pavlis et al.,* *2007*) topography expanded to degree 2160, which is compatible with EGM2008 to degree 2190 (cf. *Hirt et al., 2010b*).

2.2 Prediction and interpolation of g_{BM}

The process used to compute EGM2008-predicted g_{BM} (referred to as g_{BM}^{EGM}) using the EGM2008 gravity disturbance (δg^{EGM}) and GRS80 (*Moritz, 1980*) γ is

$$g_{BM}^{EGM} = \delta g_{BM}^{EGM} + \gamma_{BM} \tag{2}$$

where δg_{BM}^{EGM} and γ_{BM} are δg^{EGM} and GRS80 γ computed at the BM respectively. These values were all computed using derived ellipsoid heights (accurate to ~1 m or better) from the AHD H^{N-O} (available at the ANLN BM) and the EGM2008 height anomaly. Full details of this computation (Eq. (2)) can be found in Section 3.1 of *Filmer et al. (2010),* so will not be repeated here. One alternative (and equivalent) method of obtaining g_{BM}^{EGM} from EGM2008 is to compute the gravitational potential via the gradient of the potential and then add the centrifugal component. This could be done using e.g., harmonic_synth.f (*Holmes and Pavlis, 2008*), but would require three runs just to get the gradients, so that this method provides no computational advantage.

g_{BM} predicted from the ANGD (referred to as g_{BM}^{ANGD}) is computed in several steps. Firstly, simple planar Bouguer gravity anomalies (Δg_{SPB}) are computed at ANGD g_{obs} as

$$\Delta g_{SPB} = g_{obs} - \delta g_{BP} + \delta g_{AC} + \delta g_{F2} - \gamma \tag{3}$$

where δg_{BP} is the Bouguer plate (BP) attraction where $\delta g_{BP} = 2\pi\rho G H^{O}$, G is the universal gravitational constant (6.67259^{-11} m^3 kg^{-1} s^{-2} (*Cohen and Taylor 1995)* used here, but changing estimates of G are insignificant in relation to the constant density approximation), ρ is the mass-density of the BP (usually 2670 kg m^{-3}, taken as a constant), δg_{AC} is the atmospheric correction (8.71-1.03 x 10$^{-3}H^O$ μm sec^{-2}; e.g., *Featherstone and Dentith, 1997*) and δg_{F2} the second-order free air correction (see *Hackney and Featherstone, 2006*). As an approximation of H^O, ANGD g_{obs} use heights that are mostly observed using barometers at g_{obs} and are only accurate to ~±5 m (*Fraser et al., 1976*; see Section 3.1 in this paper). Δg_{SPB} are used because they are suitable for gridding in Australia (*Goos et al., 2003*), but this may not be the case in more topographically rugged areas (e.g. *Janák and Vaníček, 2005*). *Zhang and Featherstone (2004)* showed that there was no advantage in using isostatic gravity anomalies in place of Δg_{SPB} over Australia.

Δg_{SPB} were then interpolated onto a 2 arc-minute grid using the tensioned spline algorithm of *Smith and Wessel (1990)*, using T = 0.25. The grid resolution of 2 arc-minutes (~4 km) was selected on the basis that the distance between BMs is typically ~5 km and the observational spacings of ANGD g_{obs} is ~11 km (~7 km in South Australia) (*Fraser et al., 1976; Murray, 1997*). *Angus-Leppan (1982)* summarises that the spacings of g_{obs} to predict g_{BM} for HCs should be <2 km (at BMs along the levelling; cf. *Papp et al., 2009*), albeit to keep the effect of the HC error on the levelling to <0.1$\sqrt{d}$ mm (d is the distance in km along the levelling route) as suggested by *Ramsayer (1965)*. Our grid and the original ANGD resolution are larger than this (and we do generally not have g_{obs} along levelling lines), and although many regions of the ANGD have now been infilled by denser gravity surveys, the spacing of g_{obs} in some regions remains an obstacle. Notwithstanding, we consider the requirement for g_{BM} error (εg_{BM}) to effect HC error by <0.1$\sqrt{d}$ mm to be unrealistic in Australia, where the ANLN is a mostly third-order network with a misclosure tolerance of 12$\sqrt{d}$ mm.

The gridded Δg_{SPB} were then bi-cubically interpolated from the 2 arc-minute grid to ANLN BMs. The 'reverse' BP reduction at the ANLN BM location

$$g_{BM}^{ANGD} = \Delta g_{SPB} + \gamma - \delta g_{F2} + \delta g_{BP} - \delta g_{AC} \tag{4}$$

was used to realise g_{BM}^{ANGD}. The AHD H^{N-O} available at the BM (accurate to <1 m) are used to compute the values in Eq. (4)

The use of double interpolation, although not ideal, is an enforced practicality because our software requires the input data to be gridded. *Barlow (1977)* suggests an interpolation error of ~2 mGal for g_{obs} spaced at 11 km, so that simple linear propagation gives a double interpolation error of ~2.8 mGal. A validation of g_{BM}^{ANGD} using the 9,527 g_{obs}^{BM} gave an RMS for g_{BM}^{ANGD} minus g_{obs}^{BM} (cf. Eq.(10)) of 2.1 mGal (*Filmer, 2010, Chapter 6*). This comprises all error components (see Section 3) which would be expected to 'swamp' the interpolation error, suggesting that actual interpolation error is somewhat less than 2.8 mGal. Thus, we assume that the effects of the second interpolation only marginally add to the interpolation error compared to if direct interpolation had been used.

3. ERRORS IN g_{BM}^{EGM} AND g_{BM}^{ANGD}

There are numerous ways that errors can propagate into g_{BM}^{EGM} and g_{BM}^{ANGD}. We first need to establish a threshold below which we consider εg_{BM} to be negligible. *Filmer and Featherstone (2011)* conducted an error analysis to determine the sensitivity of the Helmert orthometric correction (HOC; *Helmert, 1890*) and the normal correction (NC) to input values, including g_{BM}, using test areas in Australia. We draw upon these results throughout this paper to quantify the height (or HC) error resulting from εg_{BM}. Thus, following *Filmer and Featherstone (2011)*, εg_{BM} of ~3 mGal will cause an error in the HOC; of ~10 mm at the highest point in Australia (Mt Kosciosko; 2228 m), ~5 mm at 1,000 m, and decreasing below this elevation. εg_{BM} of 3 mGal will affect the NC by <1 mm, because it is much less sensitive to εg_{BM} than the HOC (*Filmer and Featherstone, 2011*).

Because most ANLN BMs are <1,000 m in AHD height, we consider εg_{BM} <3 mGal to be acceptable and apply this threshold throughout. It is difficult to determine a realistic 'tolerance' for εg_{BM}, because the significance of the resultant HC error is dependent on the length of the levelling section between BMs to which it is applied (e.g., *Ramsayer, 1965*). However, we consider <3 mGal, which limits maximum HOC error in Australia to <10 mm, to be a pragmatic threshold when the quality of Australian levelling is considered (cf. *Filmer and Featherstone, 2009*). In countries with higher mountains and a larger proportion of their BMs at higher elevations than in Australia, this 3 mGal threshold will need to be lowered accordingly, although this is dependent on whether the HOC or NC is to be used.

εg_{BM} can result from the original data, or the methods used to compute g_{BM} from the source data. We discuss some of the possible errors in the original data used here (e.g., barometric height errors at terrestrial g_{obs} and omission errors in EGM2008), but our focus is on how these data errors propagate through the prediction methods and into g_{BM}^{EGM} and g_{BM}^{ANGD}, and whether εg_{BM} from these errors are >3 mGal and thus cause potentially large errors in gravimetric HCs.

3.1 Barometer height errors at terrestrial gravity observations

Most ANGD g_{obs} heights were determined by barometer (H_b; *Fraser et al., 1976; Barlow, 1977; Bellamy and Lodwick, 1968; Murray, 1997*), unless the gravity observation was taken directly on a BM with a levelled AHD height (as recommended by *Angus-Leppan, 1982*). H_b at ANGD g_{obs} are estimated to have a relative accuracy of ±5 m within each survey (conducted by helicopter) but this can increase to ±10 m for ties among separate helicopter surveys (*Fraser et al., 1976; Barlow, 1977*). *Bellamy and Lodwick (1968)* suggest that this may be occasionally exceeded in mountainous terrain, but testing g_{BM}^{ANGD} against some g_{obs}^{BM} did not indicate larger errors in mountainous regions (*Filmer 2010, Chapter 6*).

The significance of H_b errors (δH_b) at g_{obs} on g_{BM}^{ANGD} is determined by their effect on the Δg_{SPB} (Section 2.2). *Barlow (1977)* suggests that Δg_{SPB} errors ($\varepsilon\Delta g_{SPB}$) resulting from δH_b are likely to be between 1.5 mGal and 2.5 mGal (cf. *Fraser et al., 1976)*, including interpolation errors, although this is dependent on the spacing between observations, horizontal gravity gradients and ruggedness of the terrain. This is just below our 'threshold' of 3 mGal; so, despite δH_b making an error contribution to the ANGD, their propagation into g_{BM}^{ANGD} and the resultant gravimetric HCs is not >10mm. In an extreme example, δH_b of ~15 m would be required to cause $\varepsilon\Delta g_{SPB}$ (and hence εg_{BM}) of 3 mGal (using 0.1967 mGal m^{-1}; *Heiskanen and Moritz, 1967, p. 131*). Here, we clarify that for terrestrial g_{obs}, Eq. (3) uses the ANGD height at g_{obs}, which are mostly H_b and thus contain errors of ~±5 m. In the 'reverse' Δg_{SPB} Eq. (4) uses AHD H^{N-O} at the BM which is accurate to <1m (mostly <0.5 m).

The effect of δH_b on g_{BM}^{EGM} is less direct, but quantification is problematic because δH_b at g_{obs} affect the mean gravity anomalies used in EGM2008 (see below), but all values in Eq. (2), including δg_{BM}^{EGM} are computed directly at the ANLN BM using AHD H^{N-O}. Thus, simply estimating the effect of δH_b at a discrete point is not comparable, as follows. *Pavlis et al. (2012)* describe how the LSC algorithm used to estimate each 5 arc-minute area mean gravity anomaly in the EGM2008 computation use overlapping point value data within a 1 degree by 1 degree cell (~110 km by 110 km). This is likely to 'smooth' any effects of large δH_b within this large area, so that any δH_b error propagating into the EGM2008 coefficients, then δg_{BM}^{EGM}, and finally into g_{BM}^{EGM} will be small. *Pavlis et al. (2012)* (their Fig.12a) indicate a commission error implied height anomaly error of 50-100 mm in EGM2008 over Australia. A 100 mm error in height anomaly is equivalent to 0.03 mGal in free-air which maps approximately into δg_{BM}^{EGM} (cf. *Heiskanen and Moritz, 1967, p.85*) (and hence g_{BM}^{EGM}), suggesting that δH_b are negligible for g_{BM}^{EGM} when used to compute gravimetric HCs.

3.2 Inconsistent horizontal geodetic datums

Uncertainty in Ω_{BM}^{ANLN} can propagate into g_{BM}^{EGM} through the computation of δg_{BM}^{EGM}, but also into g_{BM}^{ANGD} because Δg_{SPB} are interpolated from the location of the gravity station ($\Omega_{g_{obs}}^{ANGD}$) to Ω_{BM}^{ANLN}. This causes g_{BM} to be predicted at a different location to the actual position of the BM (cf. *Heck, 1990; Featherstone, 1995*). As with many other terrestrial datasets (e.g., *Hinze et al., 2005*), the ANLN and ANGD have been compiled over long time periods, during which Australia has changed from a non-geocentric geodetic datum (Australian Geodetic Datum 1966; AGD66) to a geocentric geodetic datum (Geocentric Datum of Australia 1994; GDA94). This is likely to be a problem in other countries, which may be compounded by a lack of adequate metadata.

It is assumed that Ω_{BM}^{ANLN} are in AGD66 (*G. Holloway, 2009, pers. comm.*), although there may be some doubt as to BMs levelled prior to 1966, when AGD66 was realised, as the ANLN includes levelling dating back to the 1950s (*Roelse et al., 1975*). However, EGM2008 is geocentric, causing a difference between (ϕ, λ) where δg^{EGM} is computed and Ω_{BM}^{ANLN} where g_{BM}^{EGM} is realised. $\Omega_{g_{obs}}^{ANGD}$ have been transformed to GDA94 (although some older positions may be unreliable), so a similar discrepancy exists for g_{BM}^{ANGD}. Possible errors in δg^{EGM} resulting from using Ω_{BM}^{ANLN}, rather than transformed GDA94 Ω_{BM}^{ANLN} (Ω_{BM}^{GDA}) are estimated, as follows.

Grid transformation software GDAit (Department of Geomatics, University of Melbourne; http://www.geom.unimelb.edu.au/gda94/) was used to transform all 87,591 Ω_{BM}^{ANLN} to Ω_{BM}^{GDA}. δg^{EGM} were computed at Ω_{BM}^{ANLN} (δg_{BM}^{EGM}) and Ω_{BM}^{GDA} (δg_{GDA}^{EGM}) for the same BM (cf. *Featherstone, 1995*), so that

$$\varepsilon\delta g_{BM-GDA} = \delta g_{BM}^{EGM} - \delta g_{GDA}^{EGM} \tag{5}$$

where $\varepsilon\delta g_{BM-GDA}$ is the δg_{BM}^{EGM} error caused by horizontal datum inconsistency (Fig. 1(a)).

FIGURE 1 TO BE PLACED HERE

A similar procedure was conducted for the ANGD, where Δg_{SPB} were predicted at Ω_{BM}^{ANLN} and at Ω_{BM}^{GDA} (Δg_{SPB}^{GDA}) so that

$$\varepsilon\Delta g_{BM-GDA} = \Delta g_{SPB} - \Delta g_{SPB}^{GDA} \tag{6}$$

where $\varepsilon\Delta g_{BM-GDA}$ is the Δg_{SPB} error at the BM (Fig. 1(b)). $\varepsilon\delta g_{BM-GDA}$ and $\varepsilon\Delta g_{BM-GDA}$ maxima are 2.3 and 2.2 mGal in magnitude respectively (RMS are both ~±0.2 mGal) but <3 mGal so that the effect on gravimetric HCs is small. It appears that, although $\varepsilon\delta g_{BM-GDA}$ and $\varepsilon\Delta g_{BM-GDA}$ statistics are similar, there is a difference between their spatial distributions. δg^{EGM} is more sensitive to inconsistent horizontal datums at high elevations along the east coast (e.g., ~37°S, 148°E; Fig. 1(a)) because

δg^{EGM} are rougher in this region, but this is not the case for ANGD Δg_{SPB} (Fig. 1 (b)), where Δg_{SPB} are smoother (cf. *Goos et al., 2003*, *Zhang and Featherstone, 2004*).

3.3 BM positional uncertainty

Although the systematic effect of inconsistent geodetic datums in Australia propagating into predicted g_{BM} appears to have a small effect on gravimetric HCs, BM locations also contain other uncertainties. Ω_{BM}^{ANLN} were scaled from 1:250,000 topographic maps to the nearest arc-minute (*Roelse et al., 1975*), which causes an uncertainty of ±30 arc-seconds (~900 m) in both ϕ_{BM} and λ_{BM}, which gives a maximum error of ~1270 m. Approximate plotting on maps and other approximations/errors may result in the real BM positional uncertainty being larger than this, perhaps with a maximum error of 1 arc-minute or ~1.8 km. This is not unrealistic because *Filmer (2010, Chapter 6)* found extreme cases of differences between $\Omega_{g_{obs}}^{ANGD}$ and Ω_{BM}^{ANLN} reaching ~2.5 km.

3.3.1 Effect on normal gravity

ϕ_{BM} is used to compute γ on the GRS80 ellipsoid which is required for g_{BM}^{EGM} (Eq.(2)) and ANGD Δg_{SPB} (Eq.(3)). Hence, an error in ϕ_{BM} ($\delta\phi_{BM}$) will propagate into γ and thus g_{BM}^{EGM} and g_{BM}^{ANGD} (cf. *Heck, 1990; Featherstone, 1995*).

We re-computed γ using a simulated 1 arc-minute maximum error in ϕ_{BM} for the 87,951 ANLN BMs. The largest difference was 1.51 mGal, which when propagated into g_{BM}^{EGM} or g_{BM}^{ANGD} will have a negligible effect on gravimetric HCs.

3.3.2 Effect on g_{BM}^{EGM} *and* g_{BM}^{ANGD}

An uncertainty in Ω_{BM}^{ANLN} causes g_{BM}^{EGM} and g_{BM}^{ANGD} to be computed at a different location to the true position of the BM.

A subset of 2,453 ANLN BMs in the Australian Alps was used as a test area (Fig. 2) where maximum errors in δg_{BM}^{EGM} are located in this moderately rugged terrain (cf. *Claessens et al., 2009; Filmer et al., 2010*). δg^{EGM} was synthesised to degree 2190 in 1 arc-minute NE, SE, SW, and NW directions from the given Ω_{BM}^{ANLN} in the test area (δg_1^{EGM}), and the maximum-magnitude value taken as a worst-case scenario. The simulated error ($\varepsilon\delta g_1^{EGM}$) for δg_{BM}^{EGM} is

$$\varepsilon\delta g_1^{EGM} = \delta g_{BM}^{EGM} - \delta g_1^{EGM} \tag{7}$$

The simulation was repeated for ANGD Δg_{SPB} so that

$$\varepsilon\Delta g_1 = \Delta g_{SPB} - \Delta g_1 \tag{8}$$

where $\varepsilon\Delta g_1$ is the error resulting from computing Δg_{SPB} 1 arc-minute NE, SE, SW and NW from Ω_{BM}^{ANLN} (Δg_1). Maximum $\varepsilon\delta g_1^{EGM}$ was 21.9 mGal, which would map directly into g_{BM}^{EGM}, which, based on the sensitivity analysis of *Filmer and Feathestone* (2011), would cause a HOC error of ~55 mm at ~2,200 m elevation, while maximum $\varepsilon\Delta g_1$ was 4.7 mGal, and would cause an ~15 mm error in the HOC at ~2,200 m.

The use of the four different directions is a simplistic method, but is only designed to represent the maximum error that could be expected from the Ω_{BM}^{ANLN} 1 arc-minute uncertainty. More complex simulations, e.g., Monte Carlo, are not really worthwhile as we only have very crude error estimates of the BM locations.

To test the component of εg_{BM} that could be attributable to Ω_{BM}^{ANLN} uncertainty, an estimate of εg_{BM} over Australia was computed (*Filmer et al., 2010*; *Filmer, 2010, Chapter 6*) according to

$$\varepsilon g_{BM}^{EGM} = g_{BM}^{EGM} - g_{obs}^{BM} \tag{9}$$

$$\varepsilon g_{BM}^{ANGD} = g_{BM}^{ANGD} - g_{obs}^{BM} \tag{10}$$

where εg_{BM}^{EGM} and $\varepsilon g_{BM}^{ANGD}$ are the g_{BM} errors for g_{BM}^{EGM} and g_{BM}^{ANGD} respectively. A comparison of the maximum magnitudes show that $\varepsilon\delta g_1^{EGM}$ (21.9 mGal) is ~39% of εg_{BM}^{EGM} (56.8 mGal). By comparison, $\varepsilon\Delta g_1$ (4.7 mGal) is ~20% of $\varepsilon g_{BM}^{ANGD}$ (24.2 mGal). This suggests that ~40% of the largest εg_{BM}^{EGM} identified by *Filmer et al. (2010)* could be attributable to uncertainty in Ω_{BM}^{ANLN}. Because $\varepsilon g_{BM}^{ANGD}$ are smaller in this mountainous region than εg_{BM}^{EGM} (*Filmer, 2010, Chapter 6*), and Δg_{SPB} are rougher in central and western Australia (e.g., *Zhang and Featherstone, 2004*), $\varepsilon\Delta g_1$ were computed for all Australia. The maximum magnitude for $\varepsilon\Delta g_1$ was ~18 mGal (HOC error of ~40 mm at ~2,000 m elevation), which is ~75% of the largest $\varepsilon g_{BM}^{ANGD}$, suggesting that, in these extreme cases, Ω_{BM}^{ANLN}uncertainty accounts for a larger proportion of $\varepsilon g_{BM}^{ANGD}$ than εg_{BM}^{EGM}.

3.4 Omission error in EGM2008 g_{BM}

Omission error in EGM2008 is another contributor to errors in g_{BM}^{EGM}. Terrestrial g_{obs} contain all gravity field signal frequencies, but the resolution of EGM2008 is 5 arc-minutes (~9 km at Australian latitudes) which results in the omission of high-frequency signals (e.g., *Hirt, 2010*). Moderately rugged terrain, like the Australian Alps (elevations up to 2,228 m; Fig. 2), is susceptible to omission error (e.g., *Claessens et al., 2009*).

An estimate of part of the omission error can be computed using the RTM technique (*Forsberg, 1984*). A subset of 239 g_{obs}^{BM} in the Australian Alps (cf. Fig. 2) was used to test the effect of omission error on g_{BM}^{EGM}. The RTM technique does not provide the complete omission error, due to the assumption of a linear relationship between topography and gravitational potential, uncertainty in the digital elevation model (DEM), the spatial resolution of the DEM, and the neglect of local topographic mass-density variations (a constant density of 2670 kg m^{-3} was used). Omission error estimates at BMs in the Australian Alps subset shown here were computed from the RTM, using the data and methods described in *Hirt (2010)*.

FIGURE 2 TO BE PLACED HERE

The RTM-estimated omission error (ε_{RTM}) was used as an approximate 'correction' for δg_{BM}^{EGM}

$$\delta g_{BM}^{RTM} = \delta g_{BM}^{EGM} - \varepsilon_{RTM} \tag{11}$$

δg_{BM}^{RTM} was then used to re-compute g_{BM}^{EGM} (referred to as g_{BM}^{RTM}) using Eq.(2). The ε_{RTM} effect on g_{BM}^{EGM} (εg_{BM}^{RTM}) in the moderately mountainous study region is evaluated by a comparison between εg_{BM}^{RTM}

$$\varepsilon g_{BM}^{RTM} = g_{BM}^{RTM} - g_{obs}^{BM} \tag{12}$$

and εg_{BM}^{EGM} (Eq.(9)) for the 239 g_{obs}^{BM} (results in Table 1).

TABLE 1 TO BE PLACED HERE

Table 1 indicates that as much as 50% of εg_{BM}^{EGM} (e.g., RMS ±11.2 mGal reduced to ±5.7) identified in the Australian Alps (Fig. 2) may be caused by omission error in EGM2008 (cf. *Pavlis et al., 2007; Hirt et al., 2010a, 2011*).

3.5 Aliasing of terrestrial gravity in mountains

Terrestrial g_{obs} in rugged terrain are usually taken along roads that primarily run through valleys, avoiding less accessible regions with high elevations (cf. *Janák and Vaníček, 2005*). This poor sampling of the gravity field makes it possible that the ANGD does not represent the gravity field in the Australian Alps properly (cf. *Featherstone and Kirby, 2000*). On the other hand, 5 arc minute area mean gravity anomalies used in EGM2008 were supplemented by RTM implied gravity anomalies of the same resolution (*Pavlis et al., 2012*), which may have reduced the effect of spatial aliasing of ANGD g_{obs} used by EGM2008 in the Australian Alps.

Comparison between least-squares adjusted Helmert H^O in the Australian Alps using g_{BM}^{EGM} (not g_{BM}^{RTM}) versus g_{BM}^{ANGD} to compute HOCs show Helmert H^O using g_{BM}^{EGM} to be up to 129 mm higher (mean difference of +57 mm) than Helmert H^O using g_{BM}^{ANGD} at 241 BMs (*Filmer, 2010, Chapter 7*). This indicates a systematic difference between EGM2008 and ANGD in the Australian Alps, which can only be partially explained by Ω_{BM}^{ANLN} error and EGM2008 omission error (here, ε_{RTM} is not removed from g_{BM}^{EGM}).

This can be tested, albeit indirectly, for g_{BM}^{EGM} and g_{BM}^{ANGD} by comparing any decrease in levelling loop misclosures after applying HOCs and NCs (cf. *Ramsayer, 1959*). As an example, 18 first-order levelling loops (maximum allowable misclosure of $4\sqrt{d}$ mm) in the Australian Alps were used to test any differences. Table 2 shows a decrease (as would be expected from theory) in the loop misclosures (c_L in mm, where $c_L = M_L/\sqrt{d}$; M_L is the loop misclosure; *Filmer and Featherstone, 2009*) after HOC and NC are applied to the levelling compared to when no HC is applied. The decrease for mean c_L is 0.47 mm (13%) when using g_{BM}^{EGM} compared to a decrease of 0.30 mm (8.5%) when using g_{BM}^{ANGD}.

TABLE 2 TO BE PLACED HERE

While acknowledging that the decreases in loop closures in Table 2 are small in relation to the precision of the levelling, the apparently systematic effect does suggest that, despite g_{BM}^{EGM} containing large errors in the Australian Alps (cf. *Filmer et al., 2010; Claessens et al., 2009*), g_{BM}^{EGM} appears to better represent gravity here. There are several possible reasons for this, which will be discussed next.

4. SUMMARY AND DISCUSSION

We have investigated five sources of error that can propagate into g_{BM} from two different methods (Table 3). We now summarise the effect of these on g_{BM}^{EGM}, from EGM2008 using Eq.(2) and on g_{BM}^{ANGD}, from ANGD terrestrial g_{obs} using Eqs.(3) and (4).

TABLE 3 HERE

δH_b at g_{obs} impact on both g_{BM}^{EGM} and g_{BM}^{ANGD}. δH_b can be ~2 mGal for older ANGD g_{obs} (*Barlow 1977*) and map directly into g_{BM}^{ANGD}. Densification of the ANGD with more precise GPS heighting has probably been a factor in diluting the impact of δH_b over time. $\varepsilon g_{BM}^{ANGD}$ RMS for 9,527 g_{obs}^{BM} is 2.1 mGal over all Australia (see *Filmer, 2010, Chapter 6*), so assuming that errors from Ω_{BM}^{ANLN} uncertainty/error, poor gravity sampling and interpolation are likely to comprise a large part of this error budget (see Table 4), δH_b probably contribute < 1 mGal. Their effect on g_{BM}^{EGM} is less certain, but appear to be smaller than for g_{BM}^{ANGD} based on EGM2008 error estimates from *Pavlis et al. (2012)*.

Inconsistent horizontal datums among different datasets (e.g., *Hinze et al., 2005*) impact upon both g_{BM}^{EGM} and g_{BM}^{ANGD} through δg^{EGM} and ANGD Δg_{SPB} being systematically computed in a different location to the true position of the BM (cf. *Heck 1990*; *Featherstone 1995*). In our case study, the maximum possible error for g_{BM}^{EGM} and g_{BM}^{ANGD} is ~±2 mGal which propagate into a HOC error of <10 mm at Australia's highest point. Although the magnitude of the errors are similar, it is noticeable in Fig. 1 (also cf. *Filmer, 2010, Chapter 6*) that δg (and thus g_{BM}^{EGM}) appears more susceptible to this problem in the eastern Australian mountains, while Δg_{SPB} and therefore g_{BM}^{ANGD} are more susceptible in central and western Australia, where Δg_{SPB} are rougher (*Zhang and Featherstone, 2004*).

Errors caused by large uncertainties in BM location are an extension of the inconsistent horizontal datum problem, but are random rather than systematic errors. This is a problem rarely discussed in the literature (cf. *Tscherning, 1980*), perhaps because geoid computations often use mean gravity anomalies that are not so reliant on gravity values at a specific point. Using a simple method, we found that maximum possible errors in (moderately) mountainous terrain for Ω_{BM}^{ANLN} 1 arc-minute errors can reach ~22 mGal for g_{BM}^{EGM} and propagates into gravimetric HCs causing ~55 mm errors at elevations of 2,228 m (*Filmer and Featherstone, 2011*).

This again highlights δg sensitivity in mountainous terrain, suggesting that uncertainty in Ω_{BM}^{ANLN} may be a significant impediment to using modelled gravity to predict g_{BM}. In contrast, Ω_{BM}^{ANLN} uncertainty does not appear to have such a large effect on Δg_{SPB} in the Australian Alps (<5 mGal), because Δg_{SPB} are relatively smooth in this region. Repeating this test over all Australia showed that εg_1^{ANGD} can reach 18 mGal in extreme cases in central and western Australia (see Figs. 1(a) and (b)) because Δg_{SPB} are rougher in this region. Zhang and Featherstone (2004) attributed this to observed gravity being more strongly correlated with subsurface density variations in areas where the elevations are smooth because of the heavily weathered topography, suggesting a more complicated subsurface geological structure in Central Australia. Maximum $\varepsilon g_{BM}^{ANGD}$ (magnitude) is 24 mGal (*Filmer, 2010, Chapter 6*) indicating that Ω_{BM}^{ANLN} uncertainty may account for up to 75% of $\varepsilon g_{BM}^{ANGD}$ in extreme cases, but can only account for ~40% of maximum εg_{BM}^{EGM} in the Australian Alps where εg_{BM}^{EGM} is at its largest.

We tested the effect of EGM2008 omission error on g_{BM}^{EGM} in the Australian Alps. This error was considered only for modelled gravity, finding that when ε_{RTM} was removed from g_{BM}^{EGM}, εg_{BM}^{RTM} RMS was ~50% of εg_{BM}^{EGM} RMS (11.2 mGal versus 5.7 mGal; Table 1). This can be compared to *Hirt et al. (2011)*, where an improvement of ~90% for Swiss data (European Alps) was found respectively after ε_{RTM} removal. This suggests that the ε_{RTM} contribution to εg_{BM}^{EGM} in Australia is competing against other significant errors (e.g., Ω_{BM} uncertainty) that may not be present in the Swiss data, although the larger mountains in the European Alps also affect the results.

Levelling loop closures were used to test the relative merits of g_{BM}^{EGM} and g_{BM}^{ANGD} for HCs in mountainous terrain. Because εg_{BM}^{EGM} are larger than $\varepsilon g_{BM}^{ANGD}$ in the Australian Alps test area (Eqs. (9) and (10)), we assumed that HCs using g_{BM}^{ANGD} would result in larger decreases in the loop closures. However, HCs using g_{BM}^{EGM} performed marginally better, which is somewhat enigmatic, although acknowledging the differences in the loop closures are small and that other systematic levelling errors may mask the benefit of the HCs. One plausible explanation for the smaller first-order loop closures using g_{BM}^{EGM} compared to g_{BM}^{ANGD} is unrepresentative sampling of ANGD g_{obs}. This can occur in mountainous areas, where g_{obs} are taken predominately in accessible low-lying areas, often along roads that run through valleys (e.g., *Papp and Szűcs, 2011*) although *Janák and Vaníček (2005)* describe an example in Canada where aliasing is caused by g_{obs} being taken on mountain tops.

For example, the ANGD maximum g_{obs} height is 1,975 m, while the maximum ANLN BM height is 2,228 m. This 253 m difference could cause an error in Δg_{SPB} of ~50 mGal (using 0.1967 mGal m^{-1}), which causes a HOC error of ~120 mm (*Filmer and Featherstone, 2011*), which perhaps coincidentally (assuming the presence of other large errors) is the Helmert H^O difference when using g_{BM}^{EGM} versus g_{BM}^{ANGD} (Section 3.5). This unrepresentative sampling leads to spatial aliasing in g_{BM}^{ANGD}, resulting in gravimetric HC computed using g_{BM}^{ANGD} being systematically too small.

The 5 arc-minute area mean gravity anomalies used for EGM2008 were supplemented by RTM-implied gravity anomalies computed from a 30 arc-second DEM (*Pavlis et al., 2012*), which appear to have reduced the effect of spatial aliasing of g_{obs} in the Australian Alps. This offers an explanation for the apparent superiority of g_{BM}^{EGM} over g_{BM}^{ANGD} for the loop closure comparison. Possible remedies include densification of g_{obs} at higher elevations by survey, gravity 'reconstruction' at higher elevations (e.g., *Lemoine et al., 1998; Featherstone and Kirby, 2000*), or forward modelling using a DEM to predict g_{obs} (e.g., *Papp et al., 2009*) in sparsely observed areas of high elevation.

5. CONCLUSIONS AND RECOMMENDATIONS

We have tested g_{BM} predicted from modelled and terrestrial g_{obs} for errors that may impede their use for HCs in the development of a gravimetric height system if redefining a vertical datum. The errors investigated are mostly a function of inadequacies in the metadata of the terrestrial gravity database or the levelling network. We have used 3 mGal as the maximum tolerable error for this study in Australia, below which εg_{BM} have a negligible effect on HCs (notably the HOC), but this threshold value should be lowered in more mountainous countries, depending on whether the HOC or NC is used.

g_{BM} predicted from modelled and terrestrial gravity are both affected by δH_b at terrestrial g_{obs} (g_{BM}^{ANGD} moreso) and errors caused by inconsistent horizontal datums, but these errors appear to be mostly nuisance values, rather than the cause of errors >3 mGal. Modelled gravity is a realistic option for predicting g_{BM} for computing HCs, but impediments are signal omission and approximate horizontal BM positions, which between them account for most of εg_{BM} from modelled gravity. The impact of both of these errors are larger in mountainous regions, but while RTM techniques can be used to compute 'approximate' corrections for signal omission, the solution for approximate BM position remains more problematic.

Horizontal BM position errors seem to be a little-discussed problem in the geodetic literature, but needs addressing because using state-of-the-art gravity models to predict gravity a kilometre or more away from the true position of the BM can lead to large errors in HCs and hence in gravimetric heights. By comparison, εg_{BM} from predicted terrestrial g_{obs} comprise mostly errors due to approximate BM positions, but also to unrepresentative sampling in mountainous terrain causing εg_{BM} that could lead to HOC errors of ~120 mm in the Australian Alps. Denser gravity in mountainous regions through survey or forward modelling techniques present some possible remedies.

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FIGURES

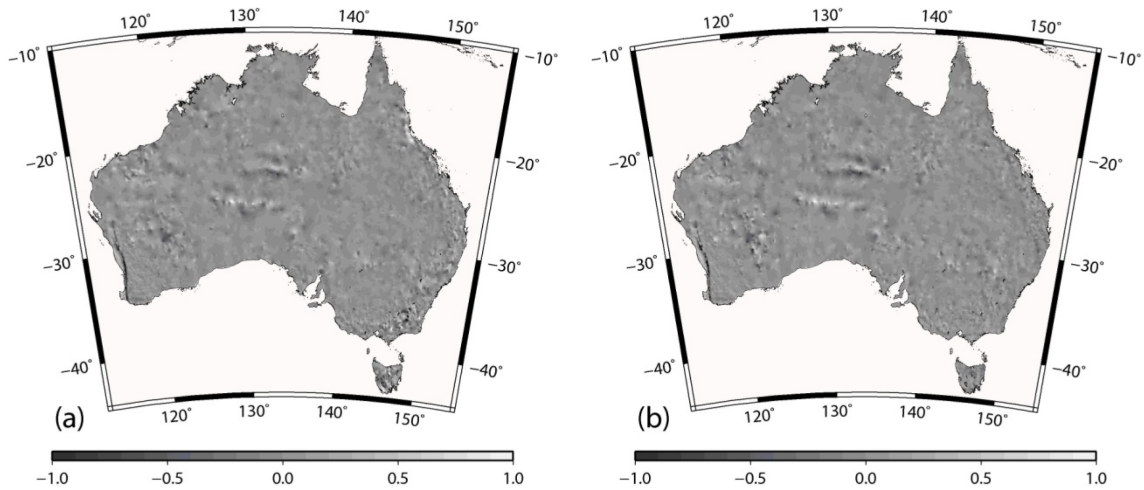

Figure 1: (a) Differences between δg^{EGM} at Ω_{BM}^{ANLN} and Ω_{BM}^{GDA} (maximum 2.1 mGal, minimum -2.33 mGal, RMS ±0.2 mGal), and (b) differences between ANGD Δg_{SPB} at Ω_{BM}^{ANLN} and Ω_{BM}^{GDA} (maximum 1.5 mGal, minimum -2.2 mGal, RMS ±0.2 mGal). Lambert projection, units in mGal.

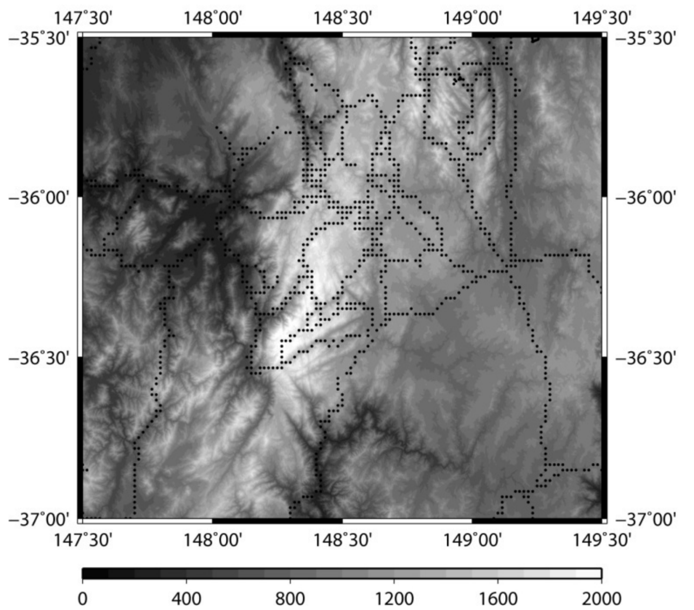

Figure 2: SRTM heights (m) in the Australian Alpine subset, and ANLN BMs (black dots), demonstrating the moderately rugged terrain and the levelling section routes.

TABLES

Statistic	εg_{BM}^{EGM}	εg_{BM}^{RTM}
Min	-16.73	-23.78
Max	46.00	28.13
Mean	5.67	2.27
STD	±9.66	±5.20
RMS	±11.20	±5.67

Table 1: Descriptive statistics for εg_{BM}^{EGM} and εg_{BM}^{RTM} at 239 BMs in the Australian Alps. Units in mGal.

	No HC	HOC		NC	
		g_{BM}^{EGM}	g_{BM}^{ANGD}	g_{BM}^{EGM}	g_{BM}^{ANGD}
Min	0.367	0.193	0.393	0.180	0.413
Max	14.464	11.630	13.518	11.629	13.544
Mean	3.540	3.068	3.237	3.072	3.241

Table 2: c_L (mm) for 18 first-order ANLN loops in the Australian Alps.

Error type	εg_{BM}^{EGM} (mGal)	δHOC (mm)	$\varepsilon g_{BM}^{ANGD}$ (mGal)	δHOC (mm)	Comment
Barometer height at g_{obs}	~0.3	<1	~2	<10	Crude estimate of STD
Horizontal datum	~2	<10	~2	<10	Maximum error
BM uncertainty	~22	~55	~18	~40	Maximum error for Australia
Omission error	~5	~15	NA	NA	ε_{RTM} RMS in the Australian Alps
Poor spatial sampling in mountains	NA	NA	50	~120	Maximum error based on difference in max. elevations ANGD/ANLN

Table 3: Summary of errors investigated and their magnitude. Note that some of these error magnitudes are STD/RMS, and some are maximum magnitude. Barometer height error is adopted from *Fraser et al. (1976)* and *Barlow (1976)*. δHOC is estimated at

2228 m in Australian Alps (maximum for Australia) and is adopted from *Filmer and Featherstone (2011)*.