Energy Coupling in Optical WDM Systems with Frequency-Dependent Attenuation Profile

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Introduction

- Capacity of linear fiber optic systems has reached a peak due to the nonlinearity of the channel.
- Multi-channel (WDM) systems also suffer from channel coupling due to spectral broadening.

Achievable rate of an WDM system with 5 channels

\[ B_{\text{channel}} = 20 \text{ GHz}, \quad B_{\text{guard}} = 5 \text{ GHz}, \quad \text{distance} = 2000 \text{ km.} \]

Nonlinear Schrödinger Equation (NLSE)

**Attenuation**
- Exponential decay in power.

**Dispersion**
- Linear term.
- Causes pulse broadening in time.

**Nonlinearity**
- Causes frequency mixing (spectral broadening, SPM, XPM, FWM).

The nonlinear term causes channel coupling in WDM systems!

Idea: Frequency-dependent attenuation profile

\[ \alpha(\omega) = \begin{cases} 0, & \omega \in W \\ \infty, & \text{otherwise.} \end{cases} \]

Such a system:
- does not allow spectral broadening.
- and turns out to still preserve the total energy of the signal:

\[ E(z) = \frac{1}{2\pi} \int_E |Q(z, \omega)|^2 d\omega = E(0) \quad \forall z. \]

Energy coupling between channels

However, the energy in each individual channel, \( E_n(z) \), is not preserved. Four-Wave Mixing (FWM) still causes coupling:

\[ \frac{d}{dz} E_n(z) = -\beta \left\{ \int_{\omega_1}^{\omega_2} |Q(z, \omega)|^2 d\omega \right\} \cdot \left\{ Q(z, \omega) \cdot \left[ Q(z, \omega) \times Q(z, \omega) \right]^{\ast} \right\}. \]

Condition for absence of energy coupling

We derived the following condition that ensures the absence of energy coupling between channels (\( dE_n(z)/dz = 0 \)):

\[ (W_n + W_m) \cap (W_n + W_m) = \emptyset, \quad \forall \{n, m\} \neq \{n, m\}. \]

where \( + \) denotes the sum of intervals:

\[ [\omega_1, \omega_2] + [\omega_1, \omega_2] = [\omega_1 + \omega_1, \omega_2 + \omega_2]. \]

Design of an energy-decoupled system

- For channels with equal width \( W \), condition (1) forces the use of a Sidon sequence [1] to place the channel centers \( \omega_n \):

\[ \omega_n = 2mW, \quad m_n \text{ is a Sidon sequence} \]

\[ m_n + m_n \neq m_m + m_m, \quad \forall \{n, m\} \neq \{n, m\}, \]

\[ \beta_2 = -21.67 \text{ psec}^2/\text{km}, \quad \gamma = 1.26 \text{ W/km}, \quad W = 2\pi \cdot 1 \text{ GHz} \]

- The maximum spectral efficiency of Sidons sequences for an \( N \)-channel system is:

\[ \eta(N) = \frac{NW}{\omega_N - \omega_1 + \gamma} \in \mathcal{O}(1/N). \]

i. e. an energy-decoupled \( N \)-channel system can asymptotically fill at most a fraction \( 1/N \) of the spectrum.

Conclusions

- The frequency-dependent attenuation profile prevents spectral broadening and conserves the total energy of the system.
- There is still energy transfer between channels, which can be avoided by using a Sidon sequence.
- The maximum spectral efficiency of an energy-decoupled \( N \)-channel system is \( \mathcal{O}(1/N) \), which is very inefficient. To design a more efficient system, energy coupling needs to be allowed.

References