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Height unification using GOCE

Research article

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Abstract:

With the gravity field and steady-state ocean circulation explorer (GOCE) (preferably combined with the gravity field and climate experiment (GRACE)) a new generation of geoid models will become available for use in height determination. These models will be globally consistent, accurate (<3 cm) and with a spatial resolution up to degree and order 200, when expressed in terms of a spherical harmonic expansion. GOCE is a mission of the European Space Agency (ESA). It is the first satellite equipped with a gravitational gradiometer, in the case of GOCE it measures the gradient components V_{xx}, V_{yy}, V_{zz} and V_{xz}. The GOCE gravitational sensor system comprises also a geodetic global positioning system (GPS)-receiver, three star sensors and ion-thrusters for drag compensation in flight direction. GOCE was launched in March 2009 and will fly till the end of 2013. Several gravity models have been derived from its data, their maximum degree is typically between 240 and 250. In summer 2012 a first re-processing of all level-1b data took place.

One of the science objectives of GOCE is the unification of height systems. The existing height offsets among the datum zones can be determined by least-squares adjustment. This requires several precise geodetic reference points available in each height datum zone, physical heights from spirit levelling (plus gravimetry), the GOCE geoid and, in addition, short wavelength geoid refinement from terrestrial gravity anomalies. GOCE allows for important simplifications of the functional and stochastic part of the adjustment model. The future trend will be the direct determination of physical heights (orthometric as well as normal) from precise global navigation satellite system (GNSS)-positioning in combination with a next generation combined satellite-terrestrial high-resolution geoid model.

Keywords:

GOCE • height • height system • height system unification

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1. Introduction

One of the science objectives of the gravity field and steady-state ocean circulation explorer (GOCE) is the worldwide unification of height systems (ESA, 1999). GOCE will provide a geoid model of unprecedented accuracy and spatial resolution. Combined with one of the latest models of the the gravity field and climate experiment (GRACE) satellite mission and with the best available terrestrial gravity information, globally consistent height determination will be possible with an accuracy of a few centimetres in regions such as North-America, Europe, Japan or Australia and of about 20 to 30 cm in areas with less advanced geodetic infrastructure. Thus, almost all practical needs concerning the use of heights in mapping, engineering, exploration and science can be met in the near future.

Heights, as discussed in this article, are physical heights such as orthometric, normal or normal-orthometric heights. Essentially, they are measured gravity potential differences converted to heights. Potential differences give us accurate information about whether points are "higher", "lower" or "on the same level". Traditionally, potential differences are obtained from spirit levelling combined with gravimetry. The theory is well established and described e.g. in (Heiskanen & Moritz, 1967, ch.4 or Torge & Müller, 2012). Ideally national height systems refer to mean sea level (MSL) at a chosen oceanic tide gauge. In other words, one connects a chosen tide gauge with a near-by geodetic benchmark, the datum point, and from the MSL at the tide gauge a reference height value at this benchmark is deduced and held fixed. The heights are therefore often referred to as "heights above mean sea level". Globally, hundreds of national and regional height systems exist. By far not all

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of them are established following the same high standards as e.g. those followed for the realization of the official height systems in North America, Europe, Australia or Japan.

MSL at the various tide gauge sites deviates from the geoid. The geoid is an equipotential surface of the Earth's gravity field. It is the one which coincides with the globally averaged elevation of MSL. "If the sea were at rest, its surface would coincide with the geopotential surface", says A. Gill, 1982, p.48. If height systems would not refer to the actual or historical MSL of a chosen tide gauge but instead to the geoid, all height systems would be consistent. They would refer to one and the same equipotential surface. See also the discussion by Sjöberg (2011). The deviation of MSL from the geoid is called mean dynamic ocean topography (MDT). Its root-mean-square (RMS) signal variation is of the order of 30 cm with maximum values of up to 1 to 2 m at the locations of strong ocean currents and along coastal boundaries. This implies, with national height systems referenced to MSL at chosen tide gauges, their height offsets with respect to the geoid are typically equal or below the above quoted numbers.

Height system unification means detection, determination, and preferably, elimination of the offsets between the height systems, so that physical heights everywhere can be related to one and the same level surface. For various scientific as well as practical reasons it is desirable, to get global height systems consistent at the level of a few centimetres or better, e.g. Plag & Pearlman (2009). This would make the accuracy of height systems comparable with that of (geometric) positioning by geodetic space techniques (GNSS, DORIS, VLBI, SLR), where one reaches even the sub-centimetre.

As said above, the classical technique of determination of physical heights is spirit levelling combined with gravimetry. The method is very precise, in principle, but easily subject to systematic distortions. In addition, levelling is time consuming and consequently expensive. Nowadays potential differences or physical height differences can also be derived by two methods completely independent from the classical one.

The first method is commonly referred to as "GPS-levelling", where we get:

$$H = h - N \quad \text{or} \quad H^n = h - \varsigma, \tag{1}$$

with h the ellipsoidal height as derived from GPS, H orthometric height, N geoid height, H^n normal height and ς height anomaly. In terms of potential differences, it is the computation of the gravity potential difference between two points with known geocentric coordinates, i.e.

$$\Delta W_{AB} = \Delta U_{AB} + \Delta T_{AB}, \tag{2}$$

where W is the gravity potential, U the normal potential and T the anomalous potential. The situation is shown in Fig. 1 for three points A, B and C, where B and C are located on one continent. The method is not related to MSL. Comparison with the classical technique offers therefore the possibility of unification of height systems, as will be discussed in section 3. Of course, this is true only if the two components of "GPS-levelling", h and N can be made consistent on a cm-level and if a geoid model is available at this level of accuracy.

The second method is not much different and is referred to as ocean method. The concept is to connect the MSL at various tide gauges by a model of mean dynamic ocean topography (MDT). The MDT could either be the result of a numerical ocean circulation model or be derived by geodetic space techniques. In the latter case, MDT corresponds to H on land and is derived from the difference of an altimetric height of the mean sea surface (MSS), corresponding to h, and a geoid model giving N, compare Eq. (1).

The theory of height unification based on the two methods "GPS-levelling" and "ocean levelling" is well established and several numerical studies have been conducted. The theoretical foundations are found e.g. in (Colombo, 1980, Rummel & Teunissen, 1988, Heck & Rummel, 1990, Rapp & Balasubramania, 1992 or more recently in Sansó & Venuti, 2002); numerical studies are (Xu & Rummel, 1990, Xu, 1992, Khafid, 1998 or Zhang et al., 2008). Ocean levelling is discussed in (Cartwright & Crease, 1963, Sturges, 1967 and 1974, Fischer, 1977, Rummel & Ilk, 1995 or Woodworth et al., 2012). Recent investigations on the improvement and unification of the height systems of Australia and New Zealand are (Featherstone & Filmer, 2012) and (Tenzer et al., 2011), respectively.

In this article, in Section 2, we will in short describe the state-of-the-art of the GOCE satellite mission, the principle of its gravitational gradiometer, the complete sensor system, the status of GOCE gravity models and plans for the remaining mission period. In Section 3 height datum connection and its realization at a cm-level will be discussed, assuming the availability of a GOCE or combined GOCE/GRACE geoid model. In the final section we will draw some conclusions.

2. GOCE and GOCE geoid model

GOCE is the acronym for "Gravity and steady-state Ocean Circulation Explorer". It is the first mission of the European Space Agency's (ESA's) Earth oriented satellite programme "Living Planet". The satellite was launched in March 2009; from November 2009 on it delivers mission data; the mission end is planned for the second half of 2013. GOCE is the first satellite carrying a gravitational gradiometer instrument. The mission objectives are the determination of the Earth's gravitational field with an accuracy of 1 ppm of "g" (corresponding to 1 mGal) and of the geoid with an accuracy of 1 to 2 cm, and to attain a spatial resolution of about 100 km, which corresponds to a spherical harmonic expansion of geoid or gravity complete to degree and order (d/o) 200. GOCE's orbit is circular and sun-synchronous, with an inclination of the orbit plane of 96.7°. This implies that a cap around the two poles with an opening angle of 6.7° is left without observations. In order to enhance the gravity signal sensitivity the orbit altitude is chosen exceptionally low, only 265 km.

The core instrument of the mission is the gravitational gradiometer. It is a three dimensional instrument and consists of three or-

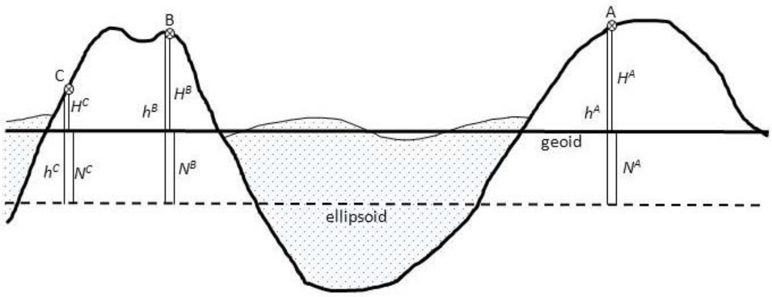

Figure 1. The principle of "GPS-leveling": Three points A, B and C are shown; B and C are located on one continent. For each of the three points the ellipsoidal heights *h* as derived from geodetic space positioning, and the geoid height *N* are given. The orthometric heights can directly be deduced using Eq. (1).

thogonally mounted one-axis gradiometers, each 50 cm long. One axis is mounted along the satellite body in flight direction, one is pointing radially towards the Earth and the third is perpendicular to these two, roughly orthogonal to the orbit plane. The gravitational gradients are delivered to the users in this gradiometer instrument reference frame (GRF). Each of the three one-axis gradiometers consists of two three-axis accelerometers mounted at the ends of each axis. The gravitational gradients are derived from taking acceleration differences in three directions along each axis of a pair of sensors. For more details we refer to (Floberghagen et al., 2011 or Frommknecht et al., 2011). Each accelerometer has two ultra precise axes, while the third axis is much more robust (Marque et al., 2008). The sensitive axes are arranged such that the gradiometer components V_{xx}, V_{yy}, V_{zz}, V_{xz} and the angular acceleration about the y-axis (approximately perpendicular to the orbit plane) are measured with very high precision. More specifically, the performance is approximately 10 mE/$\sqrt{\text{Hz}}$ for V_{xx} and V_{yy} and $2 \cdot 10$ mE/$\sqrt{\text{Hz}}$ for V_{zz} and V_{xz}, both inside the measurement band of 5 mHz to 0.1 Hz (1 mE = $10^{-12}s^{-2}$). The precision of the remaining gradiometer components V_{xy} and V_{yz} is much lower. These two components do not contribute to gravity field determination.

The gradiometer instrument is the core of the GOCE gravitational sensor system. The second main element is the European geodetic GPS receiver. From its measurements kinematic orbits are recovered with a precision of about 2 cm. This performance is verified by independent distance measurements using satellite laser ranging (Bock et al., 2011). From the kinematic orbits the long wavelength part of the gravity field is derived and combined with the short wavelength part coming from the gradiometer. Long wavelengths mean here the spherical harmonic degrees and orders below about 80 or 100, while the short wavelength signal is resolved up to d/o 240 or even 250. Three star trackers measure the orientation of the GRF relative to the celestial reference frame. These data are also used for the reconstruction of the angular rates, in combination with the angular motion as derived from the accelerometers. The air drag in flight direction is measured as "common-mode" signal by the accelerometers and proportionally compensated by a pair of ion thrusters. Angular control is performed by magnetic torques. At the end of each orbit cycle of 61 days the gradiometer is calibrated. The calibration signal is generated by randomly shaking the satellite with cold gas thrusters. Time variable gravitational signals from the satellite itself are minimized through the high stiffness of the satellite and extremely tight thermal control of the gradiometer.

All systems work well. In February and summer 2010 two interruptions occurred due to severe problems with the on-board processor units. In general, the level-1b data is of excellent quality. Unfortunately, the noise level of the components V_{zz} and V_{xz} is higher than expected by a factor 2, for still unknown reasons.

The level-2 processing is done by the High level Processing Facility (HPF), which is a scientific consortium of ten European institutes with expertise in orbit and gravity field determination. So far the HPF processed about one year of data; the resulting models are published in three consecutive releases. There exist also several combined GRACE and GOCE gravity fields. A summary of the available models is given in Table 1. During summer 2012 a first reprocessing of all level-1b mission data took place. Its main features are the combined processing of all three star trackers, an optimized attitude and angular rate determination and a linear interpolation of the calibration parameters between each of the calibrations. The expectation is that mainly the low and medium degree and order spherical harmonic coefficients will be improved, but also the higher degree and order coefficients will benefit to some extent from the reprocessing.

Currently the cumulative geoid error at d/o 200 is between 4 and 5 cm. With more and more data being included in the processing it will go down to about 2 cm to 3 cm by the end of the mission. In order to obtain a general feeling about the quality of GOCE-based gravity and geoid models, we compared geoid heights from a GOCE model with one of the best GRACE models, GRACE-ITG2010s (Mayer-Gürr et al., 2010) and with EGM2008 (Pavlis et al., 2012). Up to d/o 140 GOCE and GRACE geoid heights agree on a level of 1 to 2 cm. For a comparison up to d/o 200 with EGM2008 we selected three areas with very high geodetic standard, namely USA,

Table 1. GOCE and GOCE-GRACE combined gravity field models (taken from Gruber et al., 2012).

Model Name (Originator)	Maximum D/O	Data	Description	References
DIR1 (ESA)	240	GOCE 2m GRACE 6y CHAMP 6y Alt./Terr.	GOCE direct approach. EIGEN-5C was used as reference model.	(Bruinsma et al. 2010) (Roland Pail et al. 2011)
TIM1 (ESA)	224	GOCE 2m	GOCE time-wise approach. GOCE-only model.	(R. Pail, Goiginger, Mayrhofer, et al. 2010) (Roland Pail et al. 2011)
SPW1 (ESA)	210	GOCE 2m GRACE 5y	GOCE space-wise approach. GOCE-only plus GRACE for low degrees.	(Migliaccio et al. 2010) (Roland Pail et al. 2011)
DIR2 (ESA)	240	GOCE 6m GRACE 7y	GOCE direct approach. GRACE was used as reference model.	-
TIM2 (ESA)	250	GOCE 6m	GOCE time-wise approach. GOCE-only model.	-
SPW2 (ESA)	240	GOCE 6m	GOCE space-wise approach. GOCE-only model.	-
DIR3 (ESA)	240	GOCE 1y GRACE 6y LAGEOS 6y	GOCE direct approach. GRACE and SLR normal equations included.	-
TIM3 (ESA)	250	GOCE 1y	GOCE time-wise approach. GOCE-only model.	-
GOCO01S (GOCO Consortium)	224	GOCE 2m GRACE 7y	TIM1 model including ITG-GRACE2010S normal equations	(R. Pail, Goiginger, Schuh, et al. 2010)
GOCO02S (GOCO Consortium)	240	GOCE 6m GRACE 7y LAGEOS 5y	TIM2 model including ITG-GRACE2010S and LAGEOS SLR normal equations.	-
EIGEN-6S (GFZ/CNES)	240	GOCE 6m GRACE 7y LAGEOS 7y	GOCE direct approach including GRACE and LAGEOS normal equations.	-
EIGEN-6C (GFZ/CNES)	1420	GOCE 6m GRACE 7y LAGEOS 7y Alt./Terr.	EIGEN-6S including normal equations for EGM2008 gravity anomalies.	-
GOCO03S (GOCO Consortium)	250	GOCE 1y GRACE 7y LAGEOS 5y	TIM2 model including ITG-GRACE2010S and LAGEOS SLR normal equations.	-

Germany and Australia, three less well surveyed regions, i.e. parts of South America, Africa and Himalaya and, in addition, Antarctica. In the former areas the agreement is between 2.5 cm and 3.5 cm in terms of geoid heights. It is only between 23 cm and 36 cm in the latter. In Antarctica no terrestrial gravity data were available for EGM2008, only the incorporated GRACE model ITG2003s. The RMS-geoid difference to GOCE in Antarctica is 11 cm.

3. Height unification based on the GOCE geoid model

Remarks. In Eqs. (1) and (2) the fundamental relationship was discussed between ellipsoidal height, physical heights and the geoid heights/height anomalies or analogously between gravity potential, normal potential and anomalous potential. For simplicity, we will in the sequel only deal with the quantities H and N (Stokes case) and not in parallel with normal heights H^n and height anomalies ζ (Molodenskii case) nor with ΔW and ΔT. Furthermore, it will be assumed that the considered quantities are given in the same global terrestrial reference frame and are consistent in terms of the adopted reference ellipsoid, ellipsoidal coordinates and system of permanent tides. When dealing with the solution of the geodetic boundary value problem (GBVP) it will be assumed to be formulated in spherical and constant radius approximation. All these items need careful consideration when actually performing a global height unification; they do not affect, however, the principles of the methods discussed below.

We return to Eq. (1). Let us assume there exist a dense global set of geodetic reference points, e.g. the stations of the IGS, with precise ellipsoidal heights h given, and we have a global geoid height model available. Then orthometric heights H can be directly deduced from Eq. (1). In this case, levelling will only serve for point densification. There is a clear trend towards this approach, e.g. in the U.S. and in Canada.

However, many countries possess well surveyed height systems based solely on spirit levelling (combined with gravimetry) and these data represent a valuable asset. We assume, there exist $L+1$ non-overlapping height zones Φ_ℓ with $\ell = 0, 1, 2 \ldots L$ and the globe being sub-divided into $\Phi_E = \Phi_0 \cup \Phi_1 \cup \Phi_2 \cup \ldots \Phi_L$. Each of the given orthometric heights refers to one of these datum zones. The situation is shown in Fig. 2, which is a modified version of Fig. 1. Now the orthometric heights of points A, B and C refer to three datum zones a, b and c, respectively.

Then Eq. (1) can be turned into an adjustment model of the form:

$$\tilde{o} = \tilde{h} - \tilde{N} - \tilde{H}_\ell = N_0 + N_{\ell 0} + \tilde{\varepsilon}, \tag{3}$$

with $\tilde{o}$ the observable (observed minus computed) and the "tilde" denoting stochastic quantities, the unknown height offset between the datum zone ℓ and 0, $N_{\ell 0}$, the unknown common offset between datum zone 0 and the geoid N_0 and the residual $\tilde{\varepsilon}$. Under the assumption that the three quantities on the left hand side are of comparable accuracy and that there are several observation points per datum zone, the offsets can be estimated by least-squares adjustment.

Remark. The notation N_0 for the zero-order geoid term is generally adopted in the geodetic literature (e.g. in Heiskanen & Moritz, 1967 or Torge & Müller, 2011); the notation $N_{\ell 0}$ was introduced by Rummel & Teunissen (1988) and adopted in other work. One could regard it more logical to denote these quantities H_0 and $H_{\ell 0}$, in other words, consider them as being part of the orthometric heights and reductions from the regional height datum to the geoid.

Unfortunately the equation as formulated in 3 cannot be applied as it is. GOCE is unable to provide the "complete" geoid heights. When expanded into a series of spherical harmonics, GOCE gives N complete to a maximum d/o of about 240 or 250. The geoid error (commission error) at d/o 200 will be about 3 cm by the end of the mission. The missing part, above e.g. d/o 200 or 250 and up to infinity is denoted omission error. Unfortunately it is not negligible. Its average size can be estimated using degree variance models. For a maximum degree nmax = 250 of the GOCE field the RMS-values of the omission part are 16 cm using the model by Kaula, 34 cm using the Tscherning & Rapp model and 43 cm with the Moritz-Jekeli model.

The short wavelength part (SWL) has therefore to be derived either from a classical computation of the residual geoid signal (above nmax of GOCE) by the Stokes integral formula using terrestrial mean gravity anomalies, i.e. by the solution of the GBVP, or from a high resolution global gravity model, such as EGM2008 (Pavlis et al, 2012). In principle, the two approaches are equivalent because the best worldwide available terrestrial gravity anomaly data went into the computation of the spherical harmonic coefficients up to d/o 2160 of EGM2008. Still, a regional geoid refinement based on the best available gravity and topographic data sets may be superior in terms of accuracy and spatial resolution. Either way, along with the use of terrestrial gravity anomalies also the unknown height offsets enter into the geoid computation. Gravity anomalies require the reduction of the measured gravity at surface elevation to the geoid. The reduction is carried out using the orthometric heights H_ℓ, which are biased because of their reference to a datum ℓ instead of to the geoid, compare Eq. (4). The geoid height at point P is then composed of

$$N^P = N^P_{\mathrm{GOCE}} + N^P_{\mathrm{SWL}} + \sum_{k=1}^{L} N_{k0} f_k^P. \tag{4}$$

In (4) it is N^P_{GOCE} the geoid height as derived from a GOCE gravity field model, N^P_{SWL} the short wavelength geoid part, computed from free air gravity anomalies using Stokes integral formula (or EGM2008) and $\sum_{k=1}^{L} N_{k0} f_k^P$, the indirect bias, resulting from the height offset bias of the gravity anomalies. Thereby it is

$$f_k^P = \iint_{\Phi_k} St(\psi_{PQ}) d\Phi_Q, \tag{5}$$

the "Stokes-weight" of height offset $N_{\ell 0}$. With Eq. (4) the complete functional model of a least-squares adjustment becomes:

$$\begin{aligned} \tilde{o}^P &= \tilde{h}^P - \tilde{N}^P_{GOCE} - \tilde{N}^P_{SWL} - \tilde{H}^P = \\ &= N_0 + N_{\ell 0} + \textstyle\sum_{k=1}^{L} N_{k0} f_k^P + \tilde{\varepsilon} \end{aligned} , \tag{6}$$

with the $L+1$ unknowns N_0 and $N_{\ell 0}$.

The corresponding stochastic model is:

$$\mathrm{E}\left\{\tilde{\varepsilon}\right\} = 0, \text{ and } \mathrm{E}\left\{\tilde{\varepsilon}\tilde{\varepsilon}^T\right\} = \Sigma_h + \Sigma_{\mathrm{GOCE}} + \Sigma_{\mathrm{SWL}} + \Sigma_H. \tag{7}$$

The error variance-covariance models (VCM) Σ of h, GOCE and SWL are available. It is more of a problem to conceive a realistic VCM for the levelling part. In several studies, such as (Xu, 1992, Khafid, 1998 or Zhang et al., 2008) the above adjustment model has been tested with simulated or real data sets.

Recent studies have shown that the indirect bias term $\sum_{k=1}^{L} N_{k0} f_k^P$ in Eq. (6) is negligible, if and only if an accurate and high resolution geoid model from GOCE is available, compare (Gatti, Reguzzoni & Venuti, 2012 or Gerlach & Rummel, 2012). Gerlach & Rummel (ibid) did a series of numerical tests. They assumed a height off-set bias of 1 metre, which is rather high. Then they took height zones of various sizes and distance from the computation point P, compare Eq. (5). With the GOCE geoid model assumed to be given complete up to degree and order 200, the indirect bias stays always below 9 mm with an average value of below 5 mm. They verified this result with a numerical test based on the distribution of the national height systems of Europe. Even with the extreme case of a height off-set of 2.32 m for Belgium the indirect bias stays below 1 cm. Without this bias term the adjustment model takes a rather simple form, compare Eq. (3). A further simplification can be applied to the stochastic model. It concerns the treatment of the VCM of GOCE, which is a very large matrix. Gerlach and Fecher (2012) could show that the VCM may be reduced to its so-called m-symmetry part. With these two simplifications the adjustment is straightforward, in principle, and can be applied to the global unification of height systems.

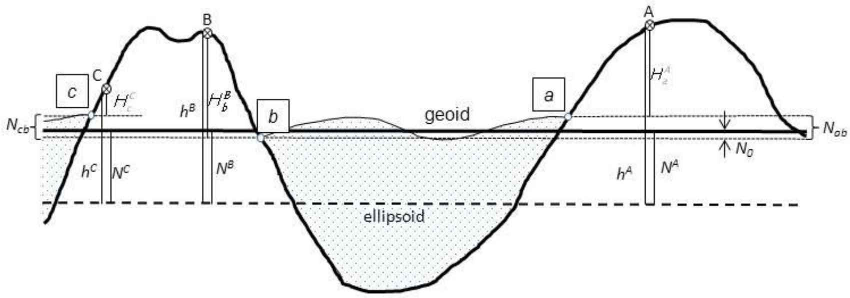

Figure 2. The basic situation is as in Figure 1. Now the orthometric heights are available as well, but each referring to a separate datum zone (a, b and c, respectively). Consequently one has to deal with the unknown height offsets between the datum zones and their common offset relative to the geoid.

4. Conclusions

A linear adjustment model is presented here for the determination of the offsets between existing geodetic height systems. It can be implemented using a high resolution geoid model from GOCE. Furthermore it requires the availability of precise ellipsoidal heights, e.g. from permanent GPS-stations, of orthometric (or normal) heights H and of terrestrial free air gravity anomalies for the computation of the short wavelength geoid part. The unknowns of this adjustment problem are the height offsets $N_{\ell 0}$ between the various datum zones and one adopted reference zone 0, and the height offset N_0 of zone 0 with respect to the geoid. In addition, one may consider solving for systematic deformations of the levelling networks. The solution requires the availability of at least one geodetic reference station with measured values of h and H per datum zone. One has also to make sure that all included data are consistent in terms of global terrestrial reference system, reference ellipsoid, coordinate type, and permanent tide system. The use of a geoid model from GOCE results in some important simplifications of the adjustment model. With GOCE and the best available terrestrial gravity anomalies global height unification at the 4 to 5 cm level seems feasible in all well surveyed parts of the world. An accuracy of below 30 cm can be realized almost everywhere. The direct realization of regional height systems based on geodetic satellite positioning and one common, internationally adopted high-resolution geoid model may be the trend of the future. In this case, the role of geodetic levelling would be more and more one of an interpolator.

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