Combination of geodetic measurements by means of a multi-resolution representation

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Introduction

Recent and in particular current satellite gravity missions provide important contributions for global Earth gravity models, and these global models can be refined by airborne and terrestrial gravity observations. The most common representation of a gravity field model in terms of spherical harmonics has the disadvantages that it is difficult to represent small spatial details and cannot handle data gaps appropriately. An adequate modeling using a multi-scale representation (MSR) is necessary in order to exploit the highest degree of information out of all these mentioned measurements.

The MSR provides a simple hierarchical framework for identifying the properties of a signal. The procedure starts from the measurements, performs the decomposition into frequency-dependent detail signals by applying a pyramidal algorithm and allows for data compression and filtering, i.e. data manipulations.

Since different geodetic measurement types (terrestrial, airborne, spaceborne) cover different parts of the frequency spectrum, it seems reasonable to calculate the detail signals of the lower levels mainly from satellite data, the detail signals of medium levels mainly from airborne and the detail signals of the higher levels mainly from terrestrial data. A concept is presented how these different measurement types can be combined within the MSR. In this presentation the basic principles on strategies and concepts for the generation of a MSR will be shown.

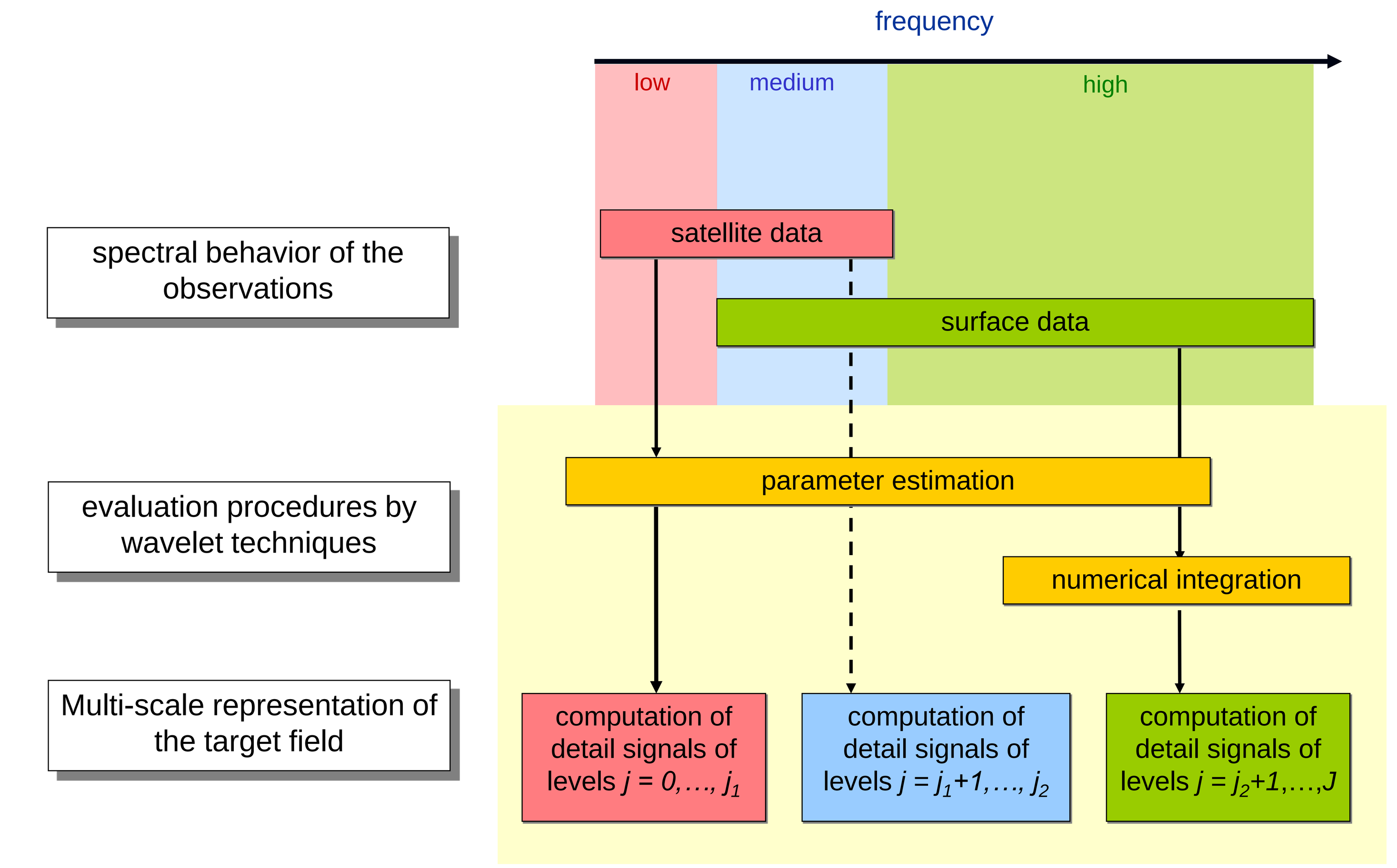

Fig. 1: The observation techniques cover different parts of the frequency spectrum. Combining all available observations in one large parameter estimation process is not always possible since main memory is limited. Fig.5 shows how to avoid large coefficient matrices processing the observation techniques separately.

References

[1] Haagmans R., Prijatna K., Dahl Omang O.: An Alternative Concept for Validation of GOCE Gradiometry Results Based on Regional Gravity, Proceedings of the 3rd meeting of the International Gravity and Geoid Commission, 2002

[2] Schmidt M., M. Fengler, T. Mayer-Gürr, A. Eicker, J. Kusche, L. Sanchez, S.-C. Han: Regional gravity modeling in terms of spherical base functions, Journal of Geodesy, 81:17-38, 2007

Motivation

In Haagmans et al. [1] a method is presented how to split a gravity signal into several frequency bands. This implies also a combination strategy for different observation types using several frequency bands to obtain a gravity field estimation. They introduced a separation in three frequency bands. Together these ranges cover the whole frequency spectrum (cf. Fig. 1) :

$$\Delta V_p = \Delta V^L + \Delta V^M + \Delta V^H \qquad (1)$$

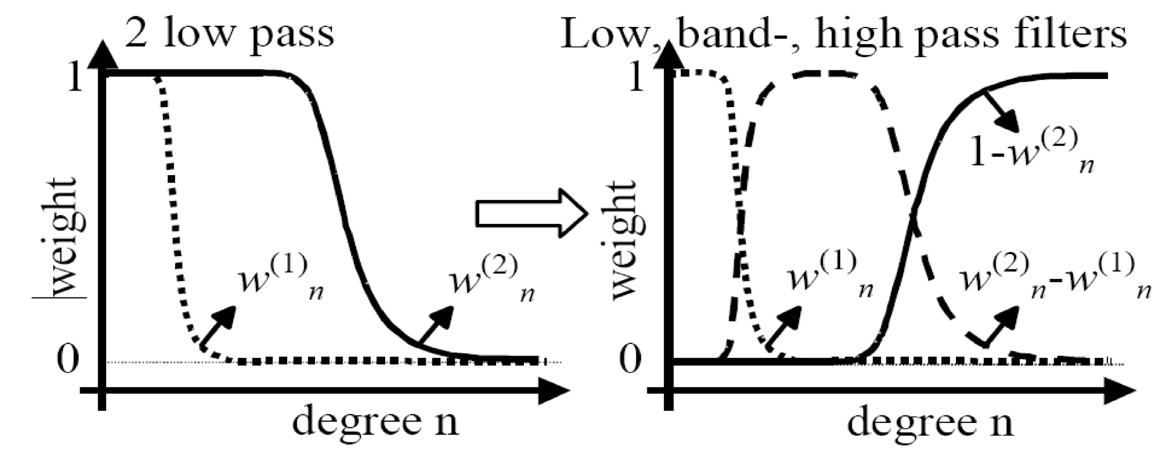

Fig. 2: Combination of two low-pass filters ($w^{(1)}_n$, $w^{(2)}_n$) yields a low-pass filter (L) and a band-pass (M) filter allowing a multi-scale representation. Figure taken from Haagmans et al.[1]

The successive application of low-pass filters is the basic idea of the MSR. Low-pass filters can be realized by scaling functions depending on a specific resolution level.

Spherical Base Functions

In order to realize a MSR with localizing spherical base functions, we use a low- and several band-pass filters. This spectral splitting allows representing the gravity field (1) under a number of different resolutions and (2) as a sum of various detail signals each related to a specific frequency bands.

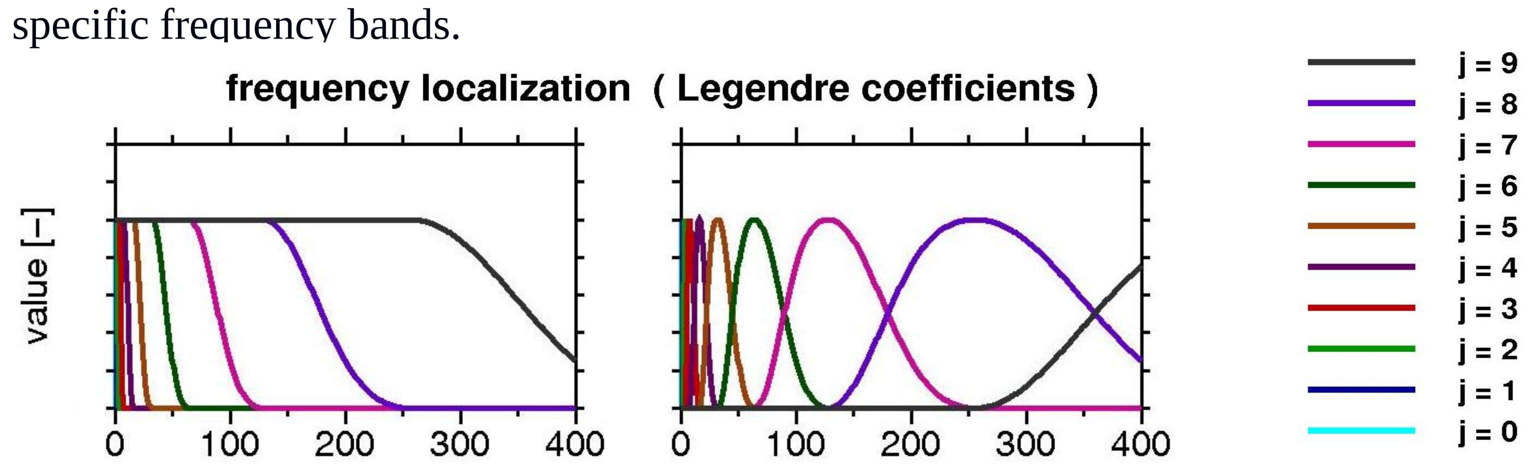

Fig. 3: Frequency (spectral) behavior of the Blackman scaling functions (left) wavelet functions (right) for different levels (scales) j. The lower the level is chosen the coarser are the structures to filtered out. The Blackman wavelet of level j = 7 (magenta), e.g. covers the frequency range between degree 64 and 255. The scaling and wavelet functions are characterized by the same behavior as the filters shown in Fig. 2.

Multi-scale Representation (MSR)

A MSR splits the input signal, e.g. the gravitational potential $\Delta V(\mathbf{x})$ into detail signals $v_j(\mathbf{x})$:

$$\Delta V(\mathbf{x}) = \Delta V_{j'}(\mathbf{x}) + \sum_{j=j'}^{J} v_j(\mathbf{x}) \quad \text{with} \quad v_j(\mathbf{x}) = \sum_{k=1}^{N_j} d_{j,k}\, \psi_j(\mathbf{x}, \mathbf{x}_{j,k}) \qquad (2)$$

The detail signals $v_j(\mathbf{x})$ are calculated by means of the the wavelet functions ψ_j of level j. They are defined, e.g. on the level j Reuter grid points $\mathbf{x}_{j,k}$.

The scaling coefficients $d_{j,k}$ for the highest level $j=J$ can be estimated using the observation equation

$$\Delta V(\mathbf{x}) + e(\mathbf{x}) = \sum_{k=1}^{N_J} d_{J,k}\, \phi_{J+1}(\mathbf{x}, \mathbf{x}_{J,k}) \qquad (3)$$

with observation error $e(\mathbf{x})$ of given potential values differences $\Delta V(\mathbf{x}) = V(\mathbf{x}) - V_0(\mathbf{x})$ and the grid points $\mathbf{x}_{J,k}$ the scaling functions φ_{J+1} are centered. $V(\mathbf{x})$ are observed potential values; $V_0(\mathbf{x})$ means a given background (initial) gravity model. The scaling coefficients of the lower levels $j < J$ can be calculated by means of the pyramid algorithm .

Combination Concept

Since the highest level covers the frequency range of all lower levels, the scaling coefficients of the lower levels can be obtained from the highest level. Based on that idea the pyramid algorithm provides the transformation equations $\mathbf{d}_j = \mathbf{H}_j\, \mathbf{d}_{j+1}$ for the scaling coefficient vectors $\mathbf{d}_j = (d_{j,k})$ with $j=j',\dots,J$ and $k=1,\dots,N_j$. With this results the detail signals are computable via equation (2). The N_j x N_{j+1} matrix $\mathbf{H}_j$ is defined on the level-j and level-($j+1$) Reuter grids. For more details see Schmidt et al. [2].

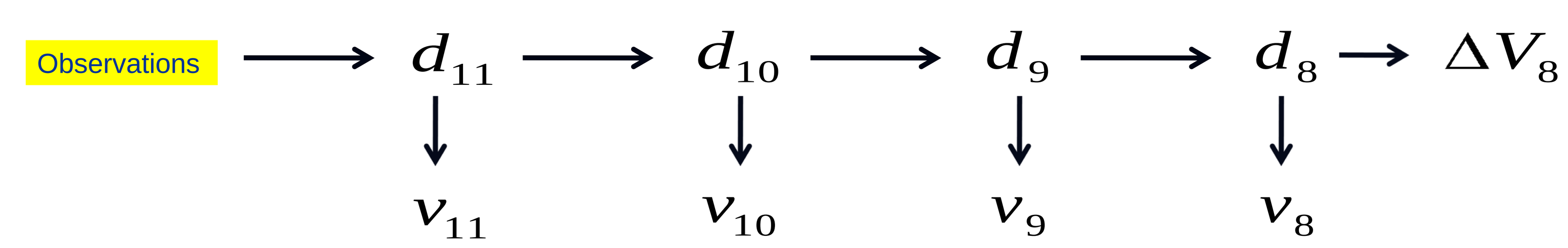

Fig. 4: Example of the pyramid algorithm with J = 11 and j' = 8. The pyramid algorithm can be symbolized as a filter bank . The coefficients d_{11} are estimated from observations.

As shown in Fig. 1 different observation techniques cover different parts of the frequency spectrum. Extending the splitting idea of Haagmans et al. [1] we now achieve a successive gravity field estimation at different levels.

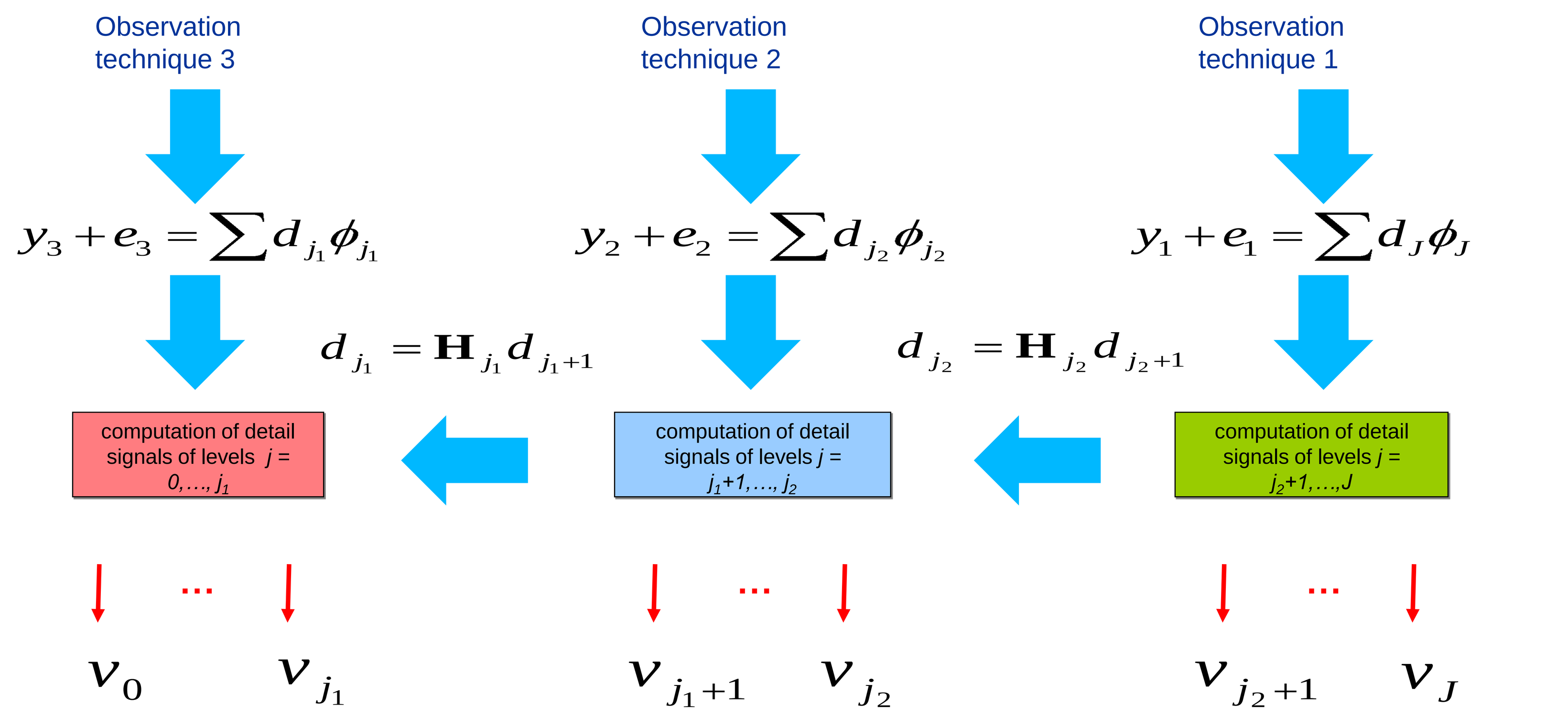

Fig. 5: Combination technique using the pyramid algorithm and the gravity potential observations y_i (functionals of V). Coefficients d_J need to be computed with the observation technique(s) that cover(s) the highest degrees in the frequency spectrum. The computation of the scaling coefficients in the lower levels d_{j_1} and d_{j_2} leads to much smaller normal equation systems, so that the computations speed up a lot.

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