Concepts for modeling VTEC as multi-scale representation

Michael Schmidt, Denise Dettmering, Wenjing Liang, Robert Heinkelmann

Deutsches Geodätisches Forschungsinstitut (DGFI), Munich, Germany, schmidt@dgfi.badw.de

1. Motivation

Modern satellite navigation and positioning systems require the use of precise and **high resolution ionosphere correction models**.

On the other side precise measurements of modern space-geodetic techniques, such as GNSS (terrestrial and space-borne) allow the **study of ionosphere** variations with an unprecedented accuracy.

Here we present **two approaches** (boxes 3 and 6) for estimating ionospheric model parameters by a **combination** of space-geodetic observations.

Due to the **distribution** of the observation sites (cf. Fig. 1) the different space-geodetic techniques allow the estimation of the ionospheric target function on **different resolution levels**.

GPS, for instance, allows the estimation of the ionospheric target function at a much higher resolution level than altimetry. Consequently, the **multi-scale representation** (MSR) comes directly into play.

As ionospheric target functions we understand the four-dimensional (4-D) **electron density** N_e or the 3-D **vertical total electron content** VTEC.

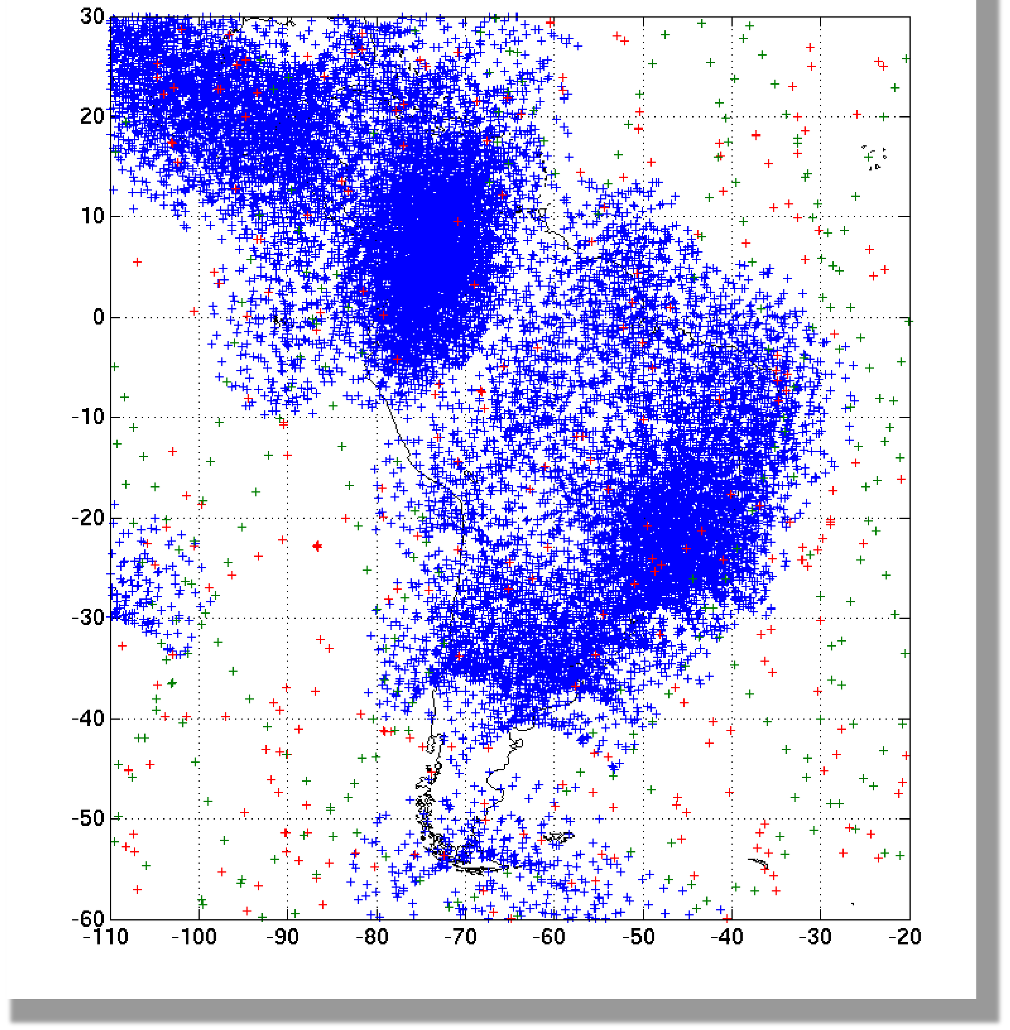

Fig. 1: Selected GPS observation sites (pierce points at a constant height of around 500 km) from SIRGAS network (blue crosses), altimetry VTEC measurement positions from Jason-1 and Envisat (green crosses), and tangent points of electron density profiles derived from the Formosat-3/COSMIC mission (red crosses) for July 21st, 2006.

2. Procedure

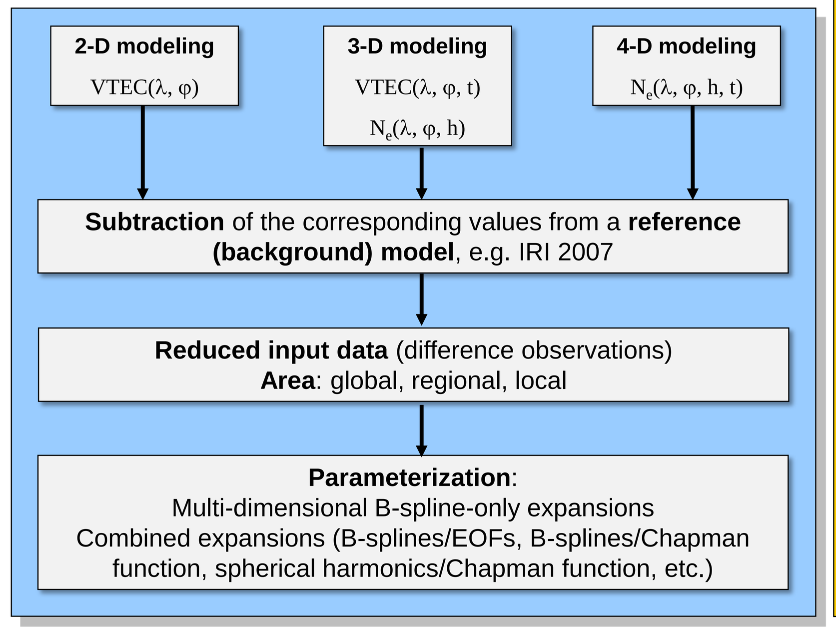

Fig. 2: Multi-dimensional models for VTEC and the electron density; λ = longitude, φ = latitude, h = height, t = time.

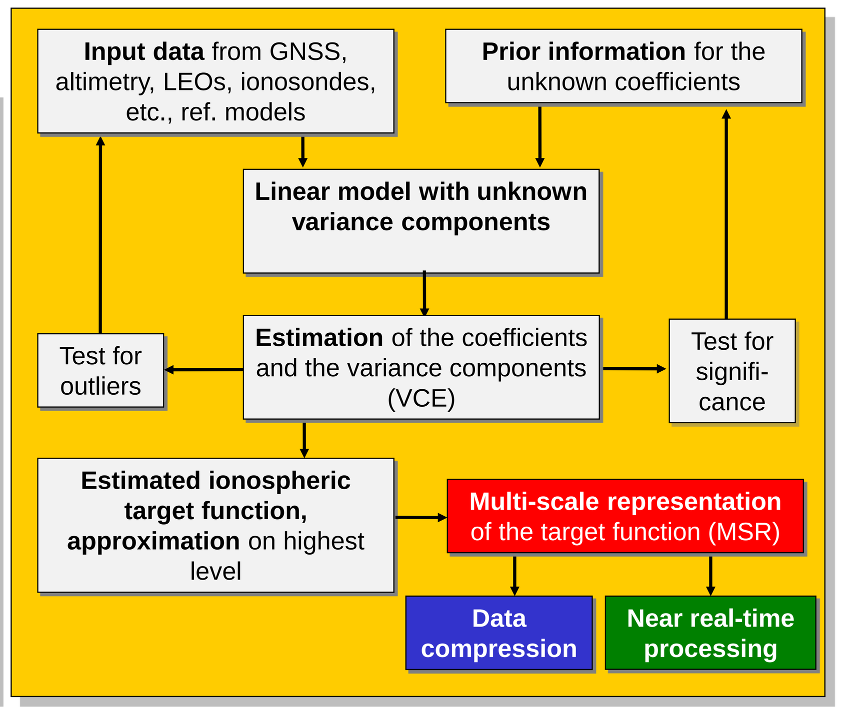

Fig. 3: Estimation procedure including prior information, variance components, MSR, compression and near real-time processing.

3. Traditional data combination

The **VTEC observation equation** at a fixed time moment $t = t_i$ reads; cf. Schmidt (2007)

$$\Delta VTEC(\lambda,\varphi,t_i) = VTEC(\lambda,\varphi,t_i) - VTEC^{back}(\lambda,\varphi,t_i)$$
$$= \sum_{k=0}^{17}\sum_{l=0}^{17} d^4_{k,l}(t_i)\,\phi^4_k(\lambda)\,\phi^4_l(\varphi) = (\phi_4(\lambda))^T\,\mathbf{D}_4(t_i)\,\phi_4(\varphi)$$

We calculate the 18 × 18 = 324 coefficients of the matrix $\mathbf{D}_4(t_i)$ for each time epoch t_i = 1:00, 3:00, 5:00, …, 23:00 by **parameter estimation** from the **combination** of the input data shown in Fig. 1.

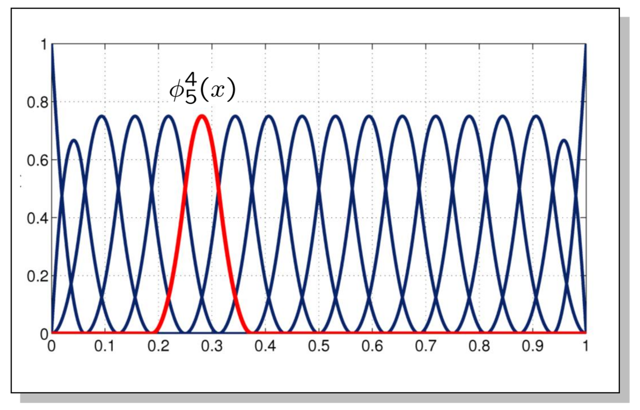

Fig. 4: 18 endpoint-interpolating polynomial B-splines of level J = 4 within the unit interval. The splines are compactly supported; the higher the level is chosen the sharper the spline

Prior information is used for handling **data gaps**.

The **relative weighting** of the observation techniques is performed by **variance component estimation**.

The **final VTEC map** is the sum of the of the background model and the estimated $\Delta VTEC$ signals, cf. Fig. 6.

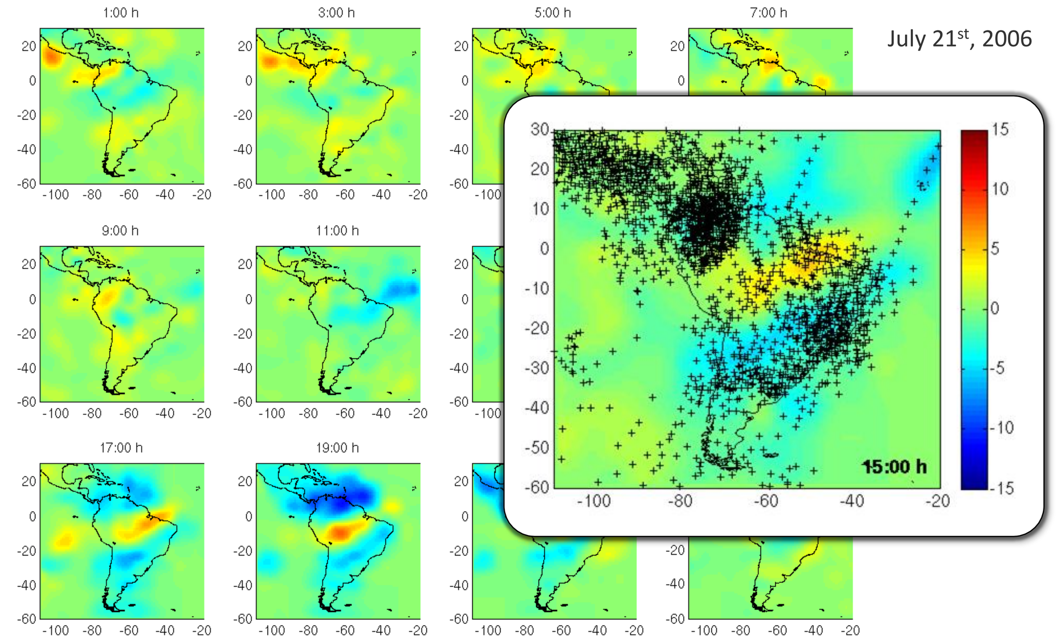

Fig. 5: Estimated $\Delta VTEC$ signals; the right panel shows the improvement of the model in areas input data are available.

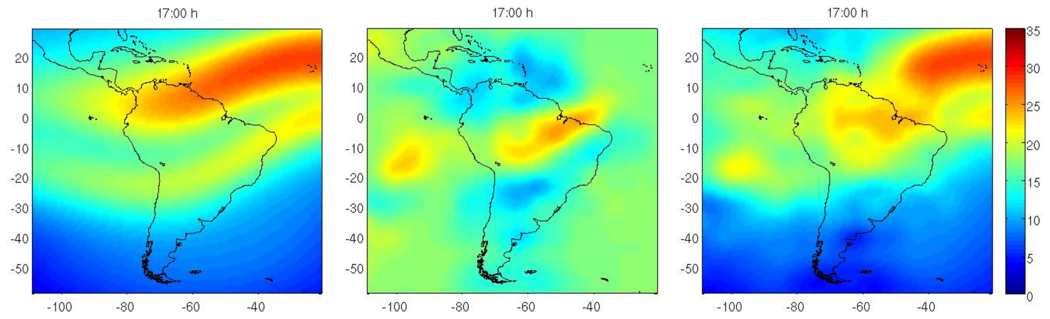

Fig. 6 (from left to right): Background model IRI 2007 for 17:00, estimated $\Delta VTEC$ signal, final estimated VTEC signal

4. Multi-scale representation (MSR)

The MSR splits a signal into a smooth approximation and a number of detail signals by **successive low-pass filtering**.

The lower the level J is chosen the smoother the **approximation**.

A **detail signal** comprises the information between two adjacent levels.

The coarser the structures the smaller the number of **wavelet coefficients**.

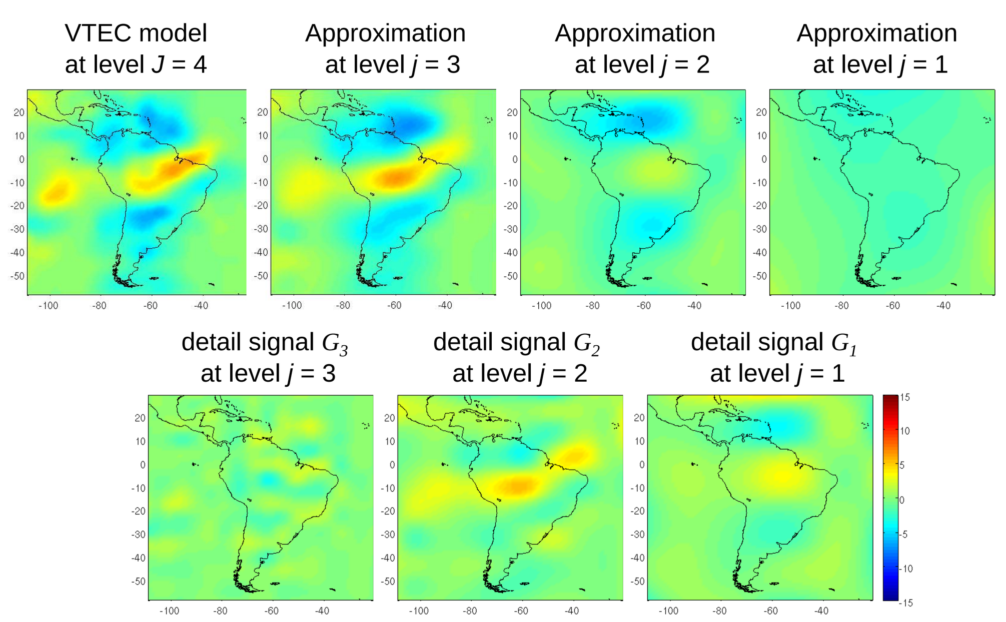

Fig. 7 (top, from left to right): Successive approximation of the estimated signal for 17:00; (bottom): detail signals signal

5. Pyramid algorithm

The **MSR** reads at a fixed time moment $t = t_i$ reads; cf. Fig. 7

$$\Delta VTEC_4(\varphi,\lambda,t_i) = \Delta VTEC_1(\varphi,\lambda,t_i) + \sum_{j=1}^{3} G_j(\varphi,\lambda,t_i)$$

The **detail signal** $G_j(\varphi,\lambda,t_i)$ of resolution level j is calculable from the three **wavelet** coefficient matrices $C^1_j(t_i), C^2_j(t_i), C^3_j(t_i)$ at a fixed time moment $t = t_i$.

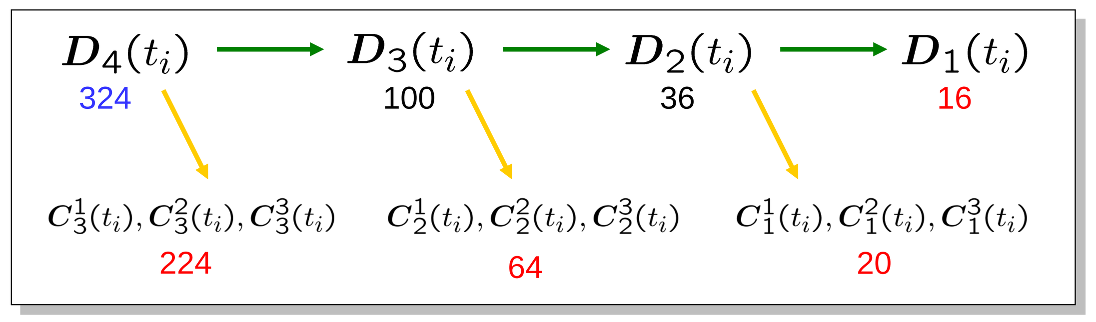

Fig. 8: Pyramid algorithm (top from left to right: scaling coefficient matrices $\mathbf{D}_j(t_i)$; (bottom) wavelet coefficient matrices $\mathbf{C}^k_j(t_i)$ for time t_i

The sum of the **red numbers** is 324, i.e., the total number of coefficients is kept.

The **green** arrows indicate a **low-pass filtering**, the orange arrows a **band-pass filtering.**

Usually, the absolute values of many wavelet coefficients are very small; **data compression** techniques can be applied effectively.

6. Alternative data combination

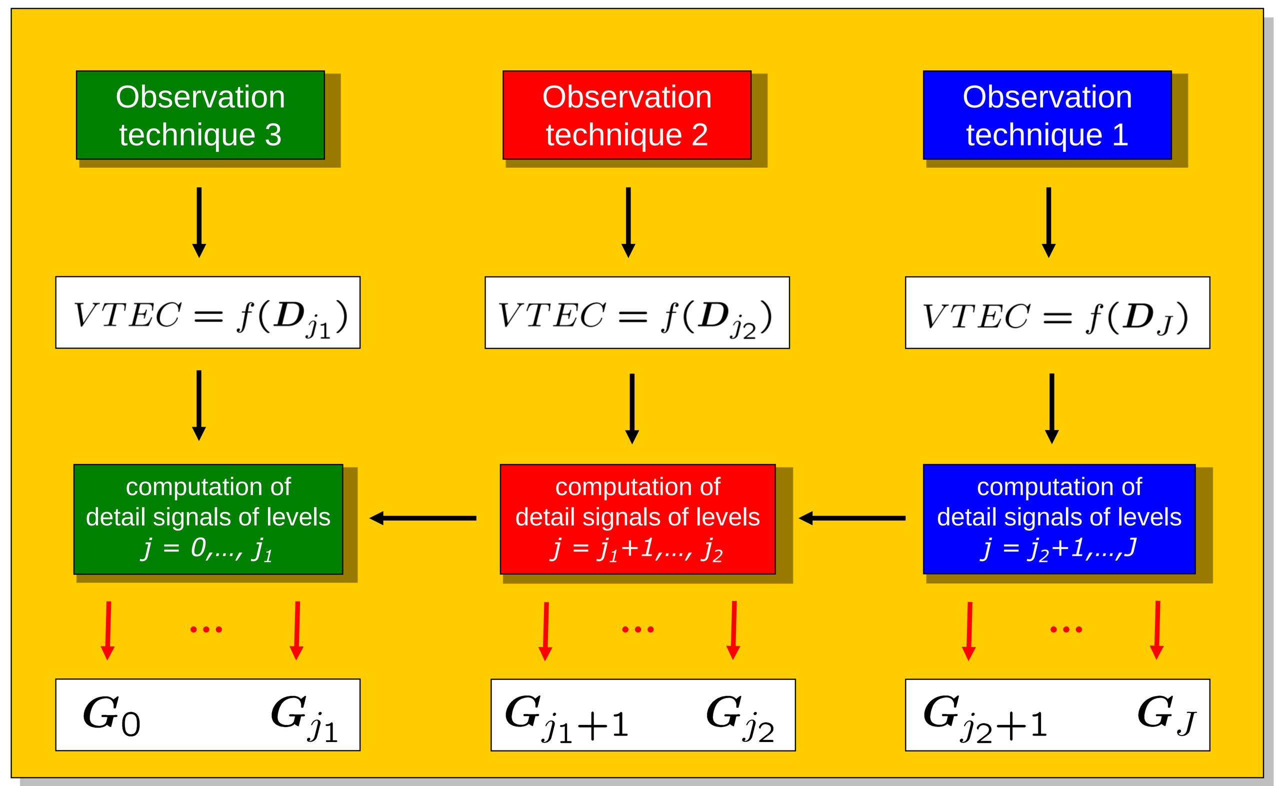

Fig. 9: Flowchart of the alternative combination procedure. The colors of the boxes are chosen according to the crosses shown in Fig. 1. For the calculation of the detail signals the pyramid algorithm and the corresponding observations are used.

In the first step we estimate the coefficient matrix $\mathbf{D}_J$ of highest resolution J level by parameter estimation from the **high resolution observation technique 1**, e.g. **GPS over continents**.

In the next step we apply the pyramid algorithm until level j_2+1 and calculate the scaling coefficient matrix $\mathbf{D}_{j2}$.

In the following step we improve the matrix $\mathbf{D}_{j2}$ by introducing the **observation technique 2 of mid resolution**, e.g. **space-borne GPS measurements** on LEOs.

Continuation of the procedure until the last step from **low resolution data** , e.g. for level $j = 0$ from **altimetry**.

References

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