Flexible combination of GOCE gravity gradients

with various observation techniques in regional gravity field modelling

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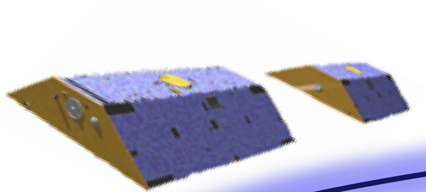
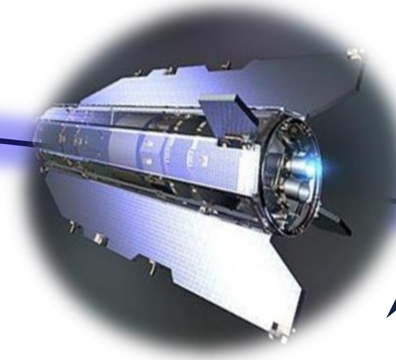
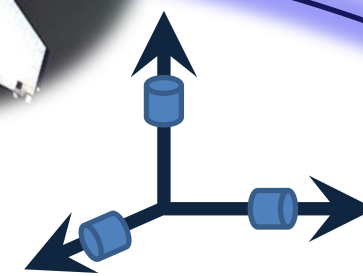
Introduction





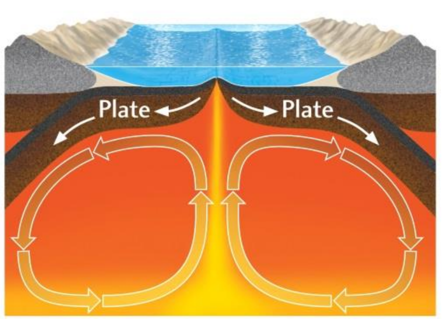


Why REGIONAL gravity field modelling?

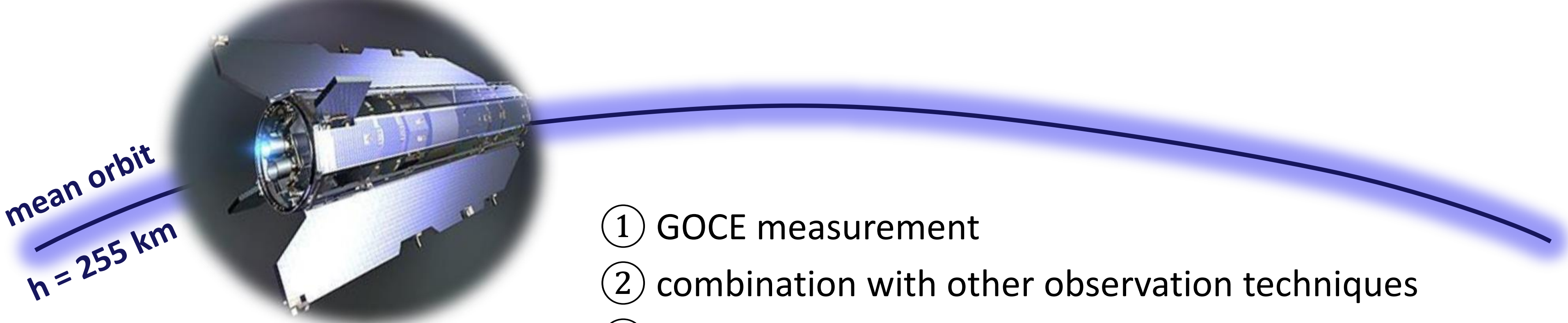
- different measurement systems, resolutions and accuracies
- regional mass variations (ice melting, groundwater storage, earthquakes, …)

Why GOCE?

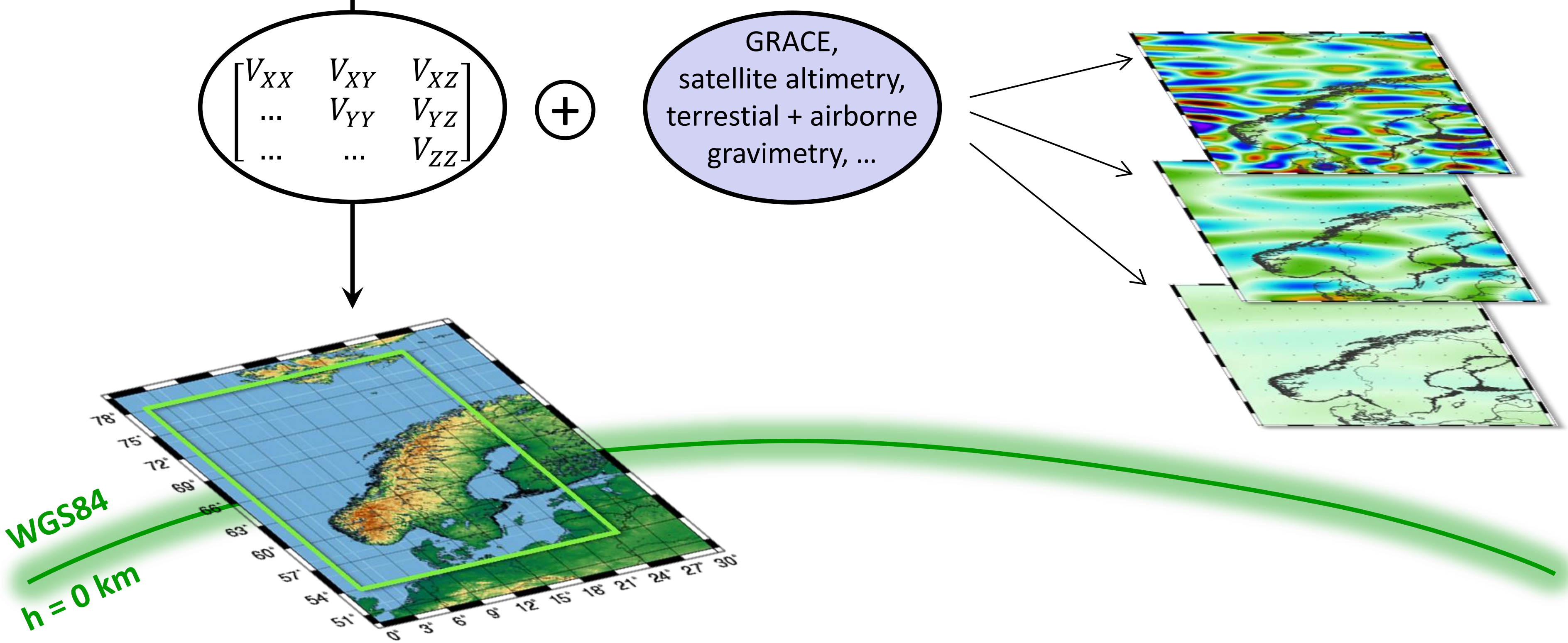
- multi-dimensional measurement system
- research on the Earth's interior and for geophysical exploration



Introduction



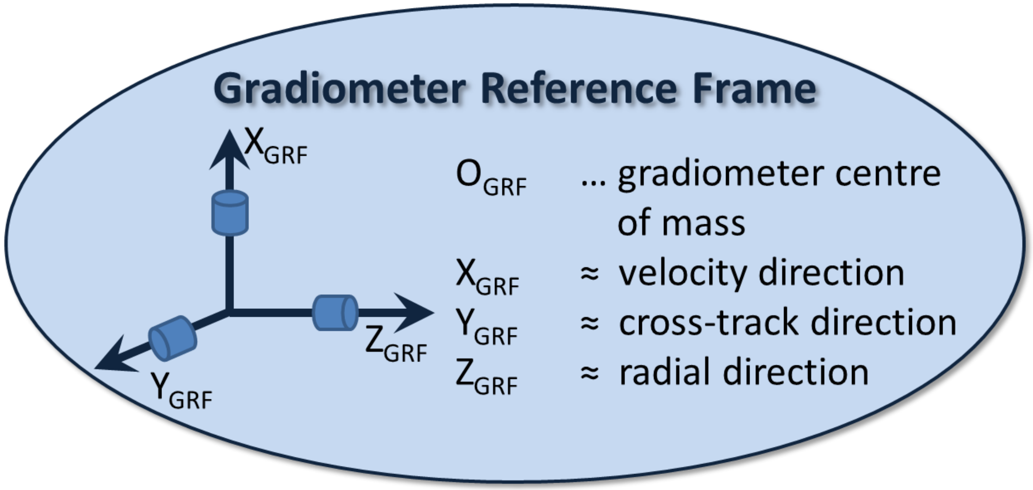
1. GOCE measurement
2. combination with other observation techniques
3. regional gravity field representations at different resolution levels



Measurement

Gravity gradients

$$\boldsymbol{V}_{ab} = \left(\frac{\partial^2 V}{\partial a \partial b}\right) = \begin{bmatrix} V_{xx} & V_{xy} & V_{xz} \\ \dots & V_{yy} & V_{yz} \\ \dots & \dots & V_{zz} \end{bmatrix}$$



Spectral sensitivity

- measurement band width MBW: $5 \dots 100\ mHz$
- **resolution levels j** split spectrum into frequency bands
- upper boundary corresponds to the **maximum degree l‘** in a series expansion
- relation to the **spatial resolution r** on the Earth‘s surface

$$l_j' = 2^j - 1$$

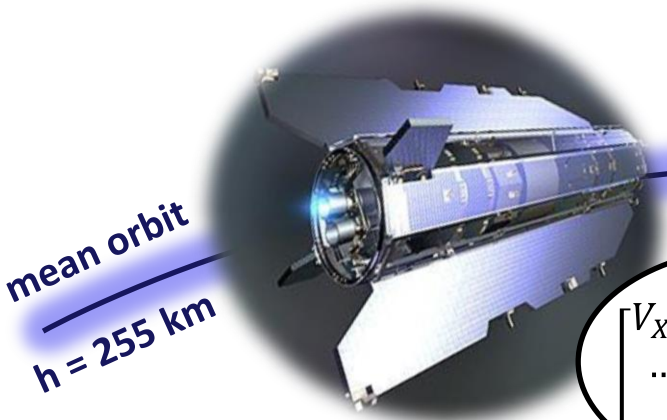
$$r = \frac{20000[km]}{l_j'}$$

GOCE MBW

j [level]	1	2	3	4	5	6	7	8	9	10	11	12
l [deg] *r* [km]	1 20000	3 6667	7 2857	15 1333	31 645	63 317	127 157	255 78	511 39	1023 20	2047 10	4095 5

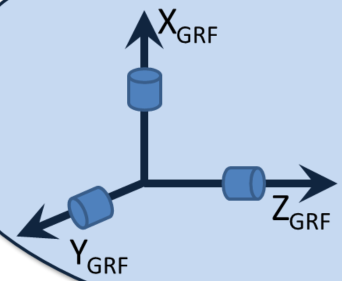
frequency [deg] →

Modelling Approach



$$\begin{bmatrix} V_{XX} & V_{XY} & V_{XZ} \\ \dots & V_{YY} & V_{YZ} \\ \dots & \dots & V_{ZZ} \end{bmatrix}$$

Gradiometer Reference Frame



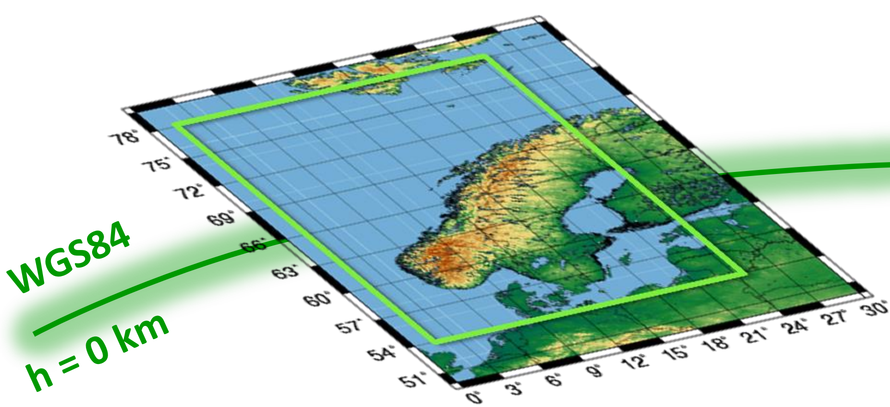
O_{GRF} … gradiometer centre of mass
X_{GRF} ≈ velocity direction
Y_{GRF} ≈ cross-track direction
Z_{GRF} ≈ radial direction

aim: using original non-rotated gradients

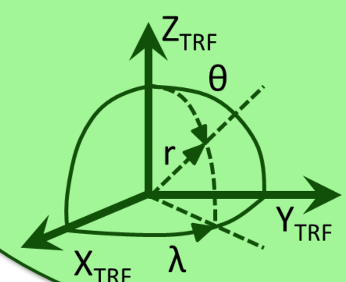
Rotation

gravitational potential $V(\boldsymbol{x})$ at observation point $P(\boldsymbol{x})$

$$V(\boldsymbol{x}) = \sum_{q=1}^{N} \sum_{l=0}^{l_J'} \frac{2l+1}{4\pi} d_{J,q} \phi_{J+1} \left(\frac{R}{r}\right)^{l+1} P_l(\cos\psi)$$



Terrestrial Reference Frame



O_{TRF} … Earth's centre of mass
λ … longitude
θ … co-latitude
r … radial distance

Modelling Approach

$$V(\boldsymbol{x}) = \sum_{q=1}^{N} \sum_{l=0}^{l_J'} \frac{2l+1}{4\pi} d_{J,q} \phi_{J+1} \left(\frac{R}{r}\right)^{l+1} P_l(\cos\psi)$$

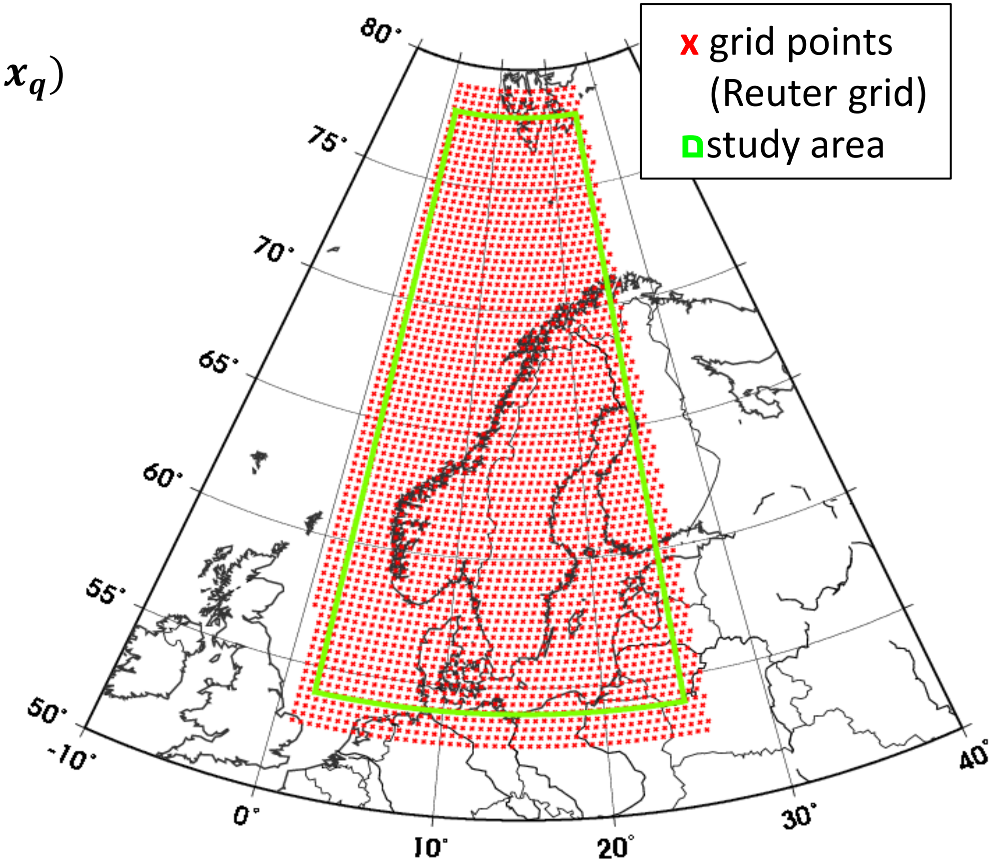
- J — max. resolution level (max. degree: l_J')
- $\boldsymbol{\phi_{J+1}}$ — **scaling functions (located on a Reuter grid)**
- $\boldsymbol{d_J}$ — **scaling coefficients**
- N — total number of grid points $\boldsymbol{x_q}$
- $P_l(\cos\psi)$ — Legendre polynomials
- ψ — spherical distance angle $\sphericalangle(\boldsymbol{x}, \boldsymbol{x_q})$

Rotation:

Spherical TRF – Cartesian GRF

$$\begin{bmatrix} V_r \\ V_\lambda \\ V_\theta \end{bmatrix}, \begin{bmatrix} V_{rr} & V_{r\lambda} & V_{r\theta} \\ \dots & V_{\lambda\lambda} & V_{\lambda\theta} \\ \dots & \dots & V_{\theta\theta} \end{bmatrix}$$

$$\begin{bmatrix} P_r \\ P_\lambda \\ P_\theta \end{bmatrix}, \begin{bmatrix} P_{rr} & P_{r\lambda} & P_{r\theta} \\ \dots & P_{\lambda\lambda} & P_{\lambda\theta} \\ \dots & \dots & P_{\theta\theta} \end{bmatrix}$$



Modelling Approach - Analysis

Estimation of unknown scaling coefficients $\boldsymbol{d}_{J,q}$

For each tensor element V_{ab} an observation equation is formulated (**deterministic part**)

... subtracting a background model V_{GOCO} : GOCO03S (d/o 250),
... using as prior information $\boldsymbol{\mu}_d$ due to rank deficiencies (singularity),
... using reproducing kernels $\phi_{J+1,ab}$.

$$\Delta \boldsymbol{V}_{ab} = \boldsymbol{V}_{ab} - \boldsymbol{V}_{GOCO,ab}$$

The unknown scaling coefficients $\boldsymbol{d}_J$ are estimated by relative weighting of all observations

... using variance components (**stochastic part**).

Deterministic part

$$\Delta V_{ab}(\boldsymbol{x}) + e_{ab}(\boldsymbol{x}) = \boldsymbol{\phi}_{J+1,ab}{}^T(\boldsymbol{x}, \boldsymbol{x}_q)\, \boldsymbol{d}_J$$

IN:	ΔV_{ab}	observation
	$\boldsymbol{e}_{ab}$	measurement error
	$\boldsymbol{\phi}_{J+1,ab}$	(Nx1) vector of scaling functions
OUT:	$\hat{\boldsymbol{d}}_J$	(Nx1) vector of scaling coefficients

Stochastic part

$$D(\Delta \boldsymbol{V}_{ab}) = \sigma_{ab}^2\, \boldsymbol{P}_{ab}{}^{-1}$$

IN:	$\Delta \boldsymbol{V}_{ab}$	vector of observations
	$\boldsymbol{P}_{ab}$	weighting matrix of observations
OUT:	$\hat{\sigma}_{ab}^2$	variance components (VCs)

DGFI CGE

Modelling Approach - Analysis

$$D\begin{pmatrix}\begin{bmatrix}\Delta\boldsymbol{V}_{xx}\\ \Delta\boldsymbol{V}_{xy}\\ \Delta\boldsymbol{V}_{xz}\\ \Delta\boldsymbol{V}_{yy}\\ \Delta\boldsymbol{V}_{yz}\\ \Delta\boldsymbol{V}_{zz}\\ \boldsymbol{\mu}_d\end{bmatrix}\end{pmatrix} = \hat{\sigma}_{xx}{}^2\begin{bmatrix}\boldsymbol{P}_{xx}{}^{-1} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & & \boldsymbol{0}\\ \vdots & & \ddots & \vdots\\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0}\end{bmatrix} + \ldots + \hat{\sigma}_d{}^2\begin{bmatrix}\boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & & \boldsymbol{0}\\ \vdots & & \ddots & \vdots\\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{\Sigma}_d{}^{-1}\end{bmatrix}$$

prior information GOCO03S

$$\hat{\boldsymbol{d}}_J = \left(\sum_{k=1}^{6}\frac{1}{\hat{\sigma}_k{}^2}\boldsymbol{\Phi}_{J+1,\boldsymbol{k}}{}^T P_k \boldsymbol{\Phi}_{J+1,\boldsymbol{k}} + \frac{1}{\hat{\sigma}_d{}^2}\boldsymbol{\Sigma}_d{}^{-1}\right)^{-1} \left(\sum_{k=1}^{6}\frac{1}{\hat{\sigma}_k{}^2}\boldsymbol{\Phi}_{J+1,\boldsymbol{k}}{}^T P_k \Delta\boldsymbol{V}_k + \frac{1}{\hat{\sigma}_d{}^2}\boldsymbol{\mu}_d\right)$$

Stochastic part

$$D(\Delta\boldsymbol{V}_{\boldsymbol{ab}}) = \sigma_{ab}^2 \boldsymbol{P}_{\boldsymbol{ab}}{}^{-1}$$

IN: $\Delta\boldsymbol{V}_{\boldsymbol{ab}}$ vector of observations
$\boldsymbol{P}_{\boldsymbol{ab}}$ weighting matrix of observations

OUT: $\hat{\sigma}_{ab}$ variance components (VCs)

Deterministic part

$$\Delta V_{ab}(\boldsymbol{x}) + e_{ab}(\boldsymbol{x}) = \boldsymbol{\Phi}_{J+1,ab}{}^T(\boldsymbol{x},\boldsymbol{x}_q)\,\boldsymbol{d}_J$$

IN: ΔV_{ab} observation
$\boldsymbol{e}_{ab}$ measurement error
$\boldsymbol{\Phi}_{J+1,ab}$ (Nx1) vector of scaling functions

OUT: $\hat{\boldsymbol{d}}_J$ (Nx1) vector of scaling coefficients

Modelling Approach – Observation Equations

$$\Delta \boldsymbol{V}_{ab} = \begin{bmatrix} \Delta V_{xx} & \Delta V_{xy} & \Delta V_{xz} \\ \dots & \Delta V_{yy} & \Delta V_{yz} \\ \dots & \dots & \Delta V_{zz} \end{bmatrix} = \left(\sum_{q=1}^{N_J} \hat{d}_{J,q} \; \boldsymbol{\Phi}_{J+1,ab}(\boldsymbol{x}, \boldsymbol{x}_q) \right)$$

reduced observations $\Delta \boldsymbol{V}_{ab}$

- described by series expansion
- using estimated scaling coefficients $\hat{\boldsymbol{d}}_J$ and
- modified scaling functions $\boldsymbol{\Phi}_{J+1,ab}$

$j = J + 1$

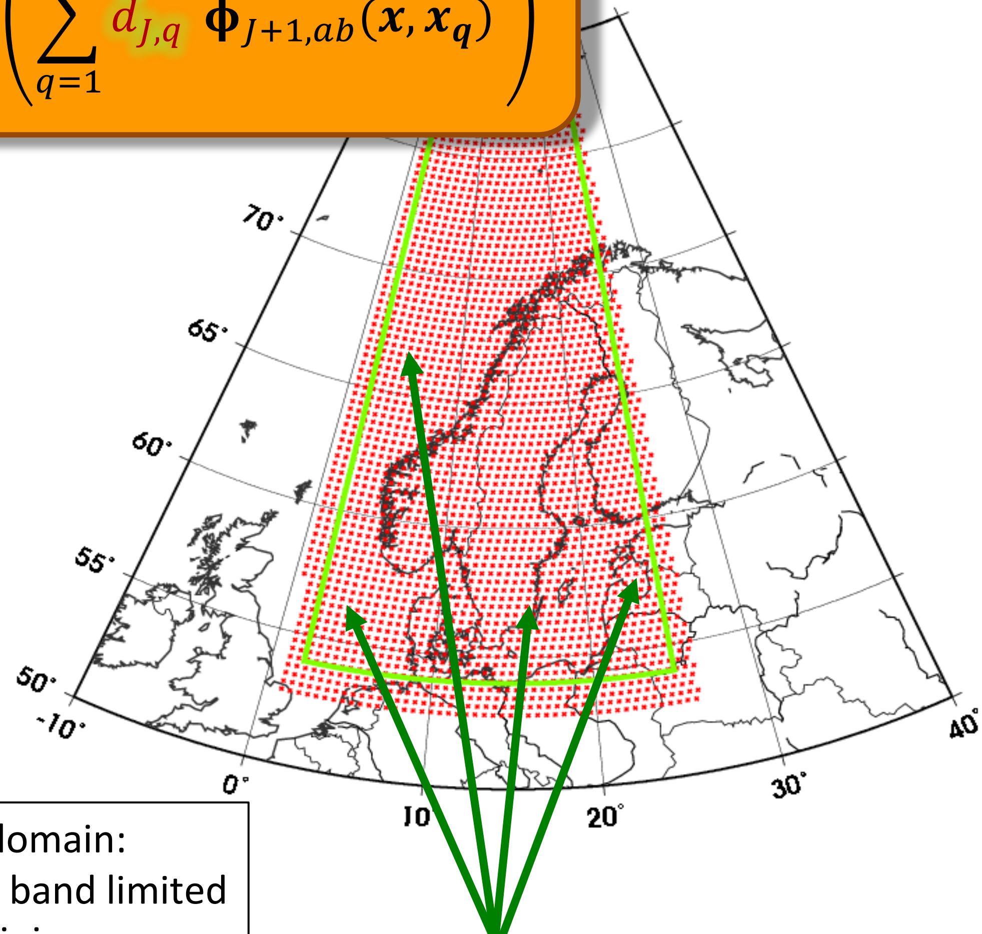
$$\phi_{j,ab} = \sum_{l=0}^{l_j'} \frac{2l+1}{4\pi} \Phi_{j,l} \left(\frac{R}{r}\right)^{l+1} \begin{cases} \frac{1}{r} \cdot P_l(\cos\psi)\left(-\frac{l+1}{r}\right) + \frac{1}{r^2} \cdot \frac{\partial P_l(\cos\psi)}{\partial \theta^2} & \cdots \Delta V_{xx} \\ \frac{1}{r^2 \sin\theta} \cdot \frac{\partial P_l(\cos\psi)}{\partial\lambda\,\partial\theta} - \frac{1}{r^2 \sin^2\theta} \cdot \frac{\partial P_l(\cos\psi)}{\partial\lambda} & \cdots \Delta V_{xy} \\ \frac{1}{r^2} \cdot \frac{\partial P_l(\cos\psi)}{\partial\theta} - \frac{1}{r} \cdot \left(-\frac{l+1}{r}\right) \cdot \frac{\partial P_l(\cos\psi)}{\partial\theta} & \cdots \Delta V_{xz} \\ \frac{1}{r} \cdot P_l(\cos\psi)\left(-\frac{l+1}{r}\right) + \frac{1}{r^2 \tan\theta} \cdot \frac{\partial P_l(\cos\psi)}{\partial\theta} + \frac{1}{r^2 \sin^2\theta} \cdot \frac{\partial P_l(\cos\psi)}{\partial\lambda^2} & \cdots \Delta V_{yy} \\ \frac{1}{r^2 \sin\theta} \cdot \frac{\partial P_l(\cos\psi)}{\partial\lambda} - \frac{1}{r \sin\theta} \cdot \left(-\frac{l+1}{r}\right) \cdot \frac{\partial P_l(\cos\psi)}{\partial\lambda} & \cdots \Delta V_{yz} \\ P_l(\cos\psi) \cdot \left(-\frac{l+1}{r}\right) & \cdots \Delta V_{zz} \end{cases}$$

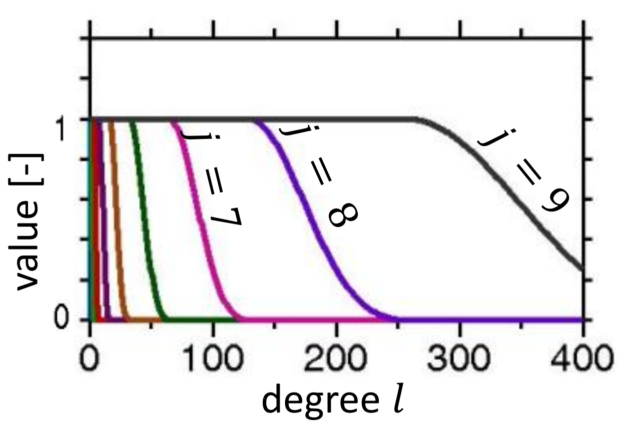
Modelling Approach - Synthesis

$$\Delta \boldsymbol{V}_{ab} = \begin{bmatrix} \Delta V_{xx} & \Delta V_{xy} & \Delta V_{xz} \\ \dots & \Delta V_{yy} & \Delta V_{yz} \\ \dots & \dots & \Delta V_{zz} \end{bmatrix} = \left(\sum_{q=1}^{N_J} \hat{d}_{J,q} \; \boldsymbol{\Phi}_{J+1,ab}(\boldsymbol{x}, \boldsymbol{x}_q) \right)$$

IN: $\hat{\boldsymbol{d}}_J$ estimated coefficients
OUT: $\Delta \boldsymbol{V}_{ab}$ gradients of the reduced gravitational potential

- setting up tensor $\Delta \boldsymbol{V}_{ab}$ of observation equations
- using Blackman scaling functions $\boldsymbol{\Phi}_{J+1,ab}$





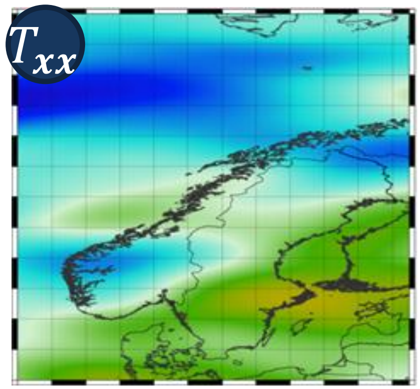
frequency domain:
- strongly band limited
- but declining

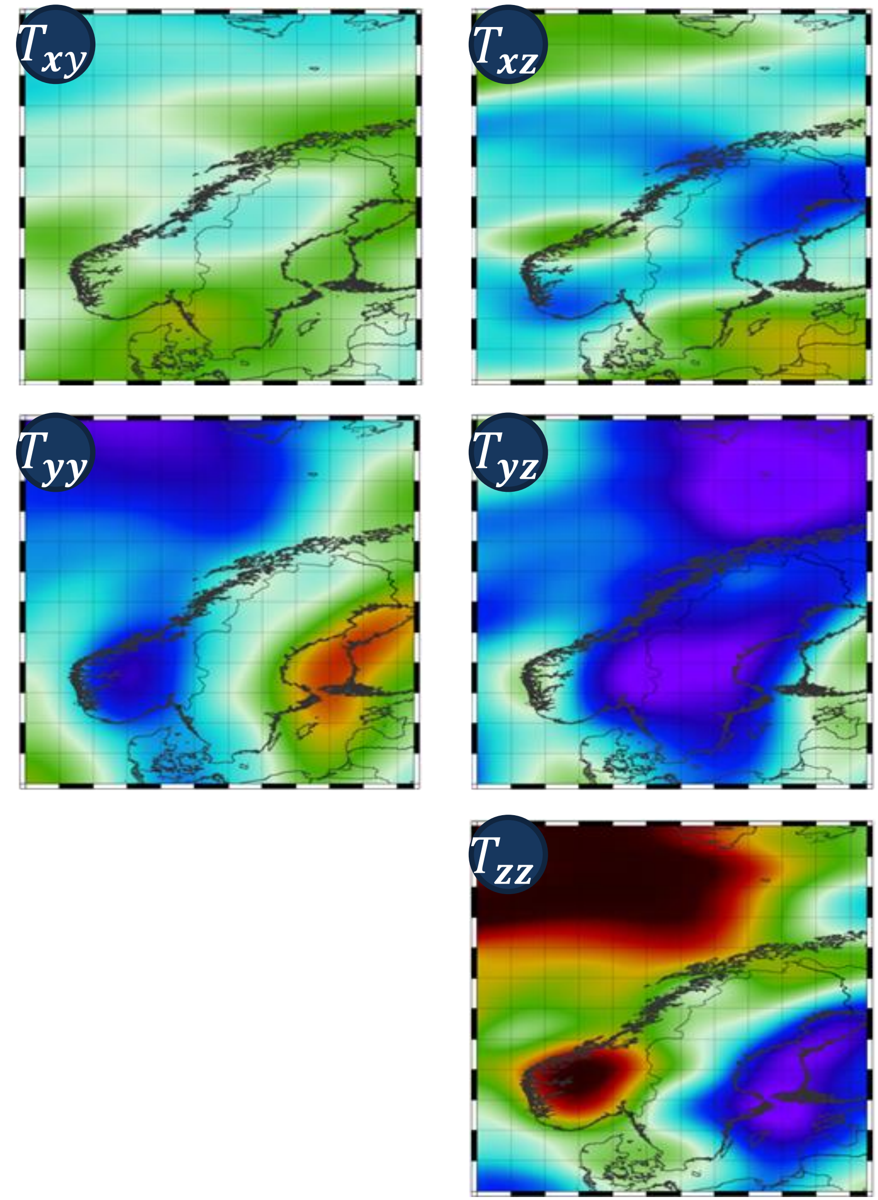
spatial domain:
- oscillations

level j = 3

Results

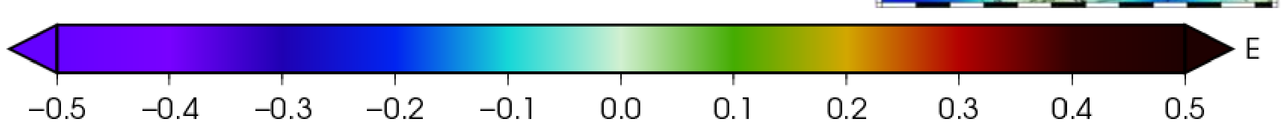
GOCE j = 8





IN: V_{ab} @ $270\ km$
OUT: T_{ab} @ $270\ km$
… gradients of the disturbing potential $T = V - U_{WGS84}$

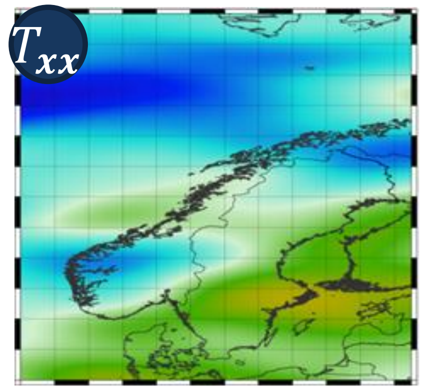
- different structures depending on spatial directions
- radial zz component: largest magnitude



Results – Laplace Condition

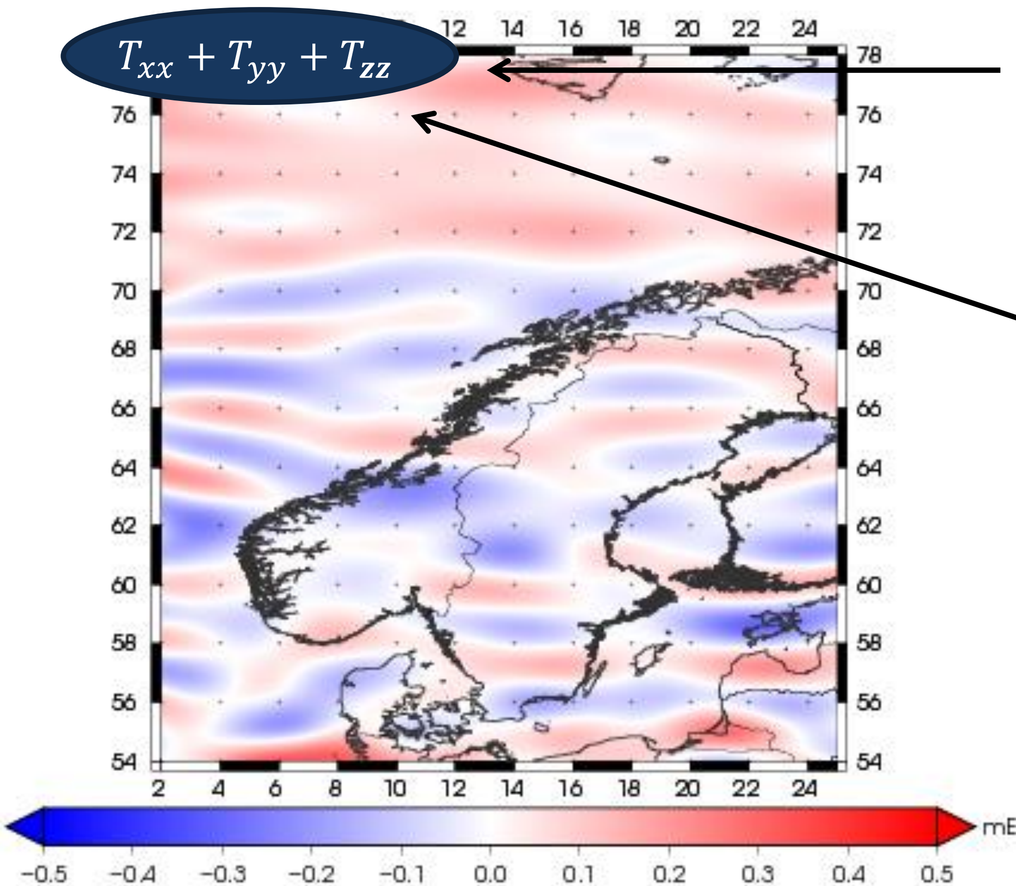
$$T_{xx} + T_{yy} + T_{zz} = 0$$

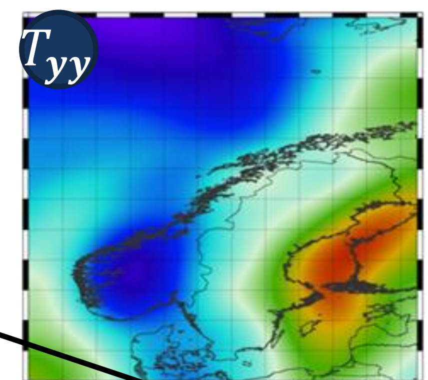
mean [µE]	std [µE]
9.6	85.1



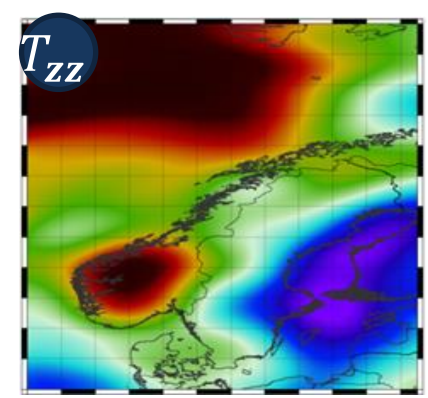
… fulfilled in the frame of the modelling accuracy, depending on

- oscillations
- edge effects
- smoothing
- omission errors
- interpolation effects





sum vs. single components: 3 orders of magnitude smaller



Results – Relative Weighting

Variance Component Estimation (VCE)

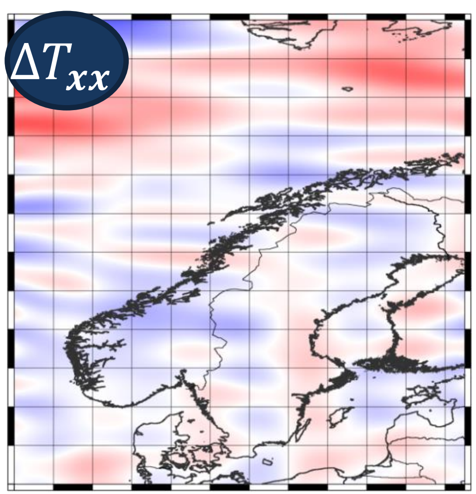
- high VC $\hat{\sigma}_{ab}^2$ → low weight $\frac{1}{\hat{\sigma}_{ab}^2}$
- V_{GOCO} (reference)

Relative weighting of observations

→ **down-weighting** of less accurate components V_{xy}, V_{yy}, V_{yz}
→ reduction of influence of systematic errors
→ smaller differences compared with GOCO03S

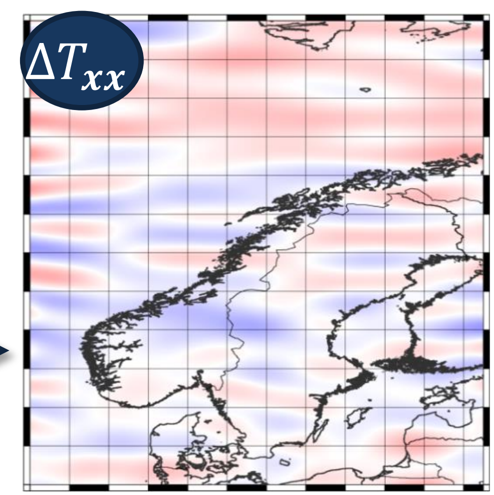
IN	VC [order of magnitude]	
	① est	② fix
V_{back}	E+00	E+00
V_{xx}	E-02	E-02
V_{xy}	**E+00**	**E+11**
V_{xz}	E+01	E+00
V_{yy}	**E-02**	**E+03**
V_{yz}	**E+03**	**E+11**
V_{zz}	E-02	E-02

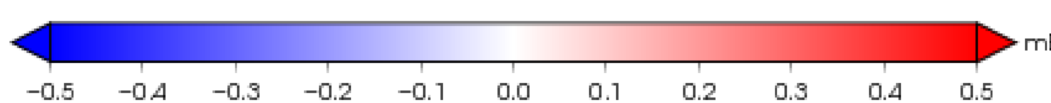
Comparison with GOCO03S (d/o 250)



differences
ΔT_{xx} @ $270\ km$
std. $\sigma \approx 0.3mE$

← ① VC est
② VC fix →



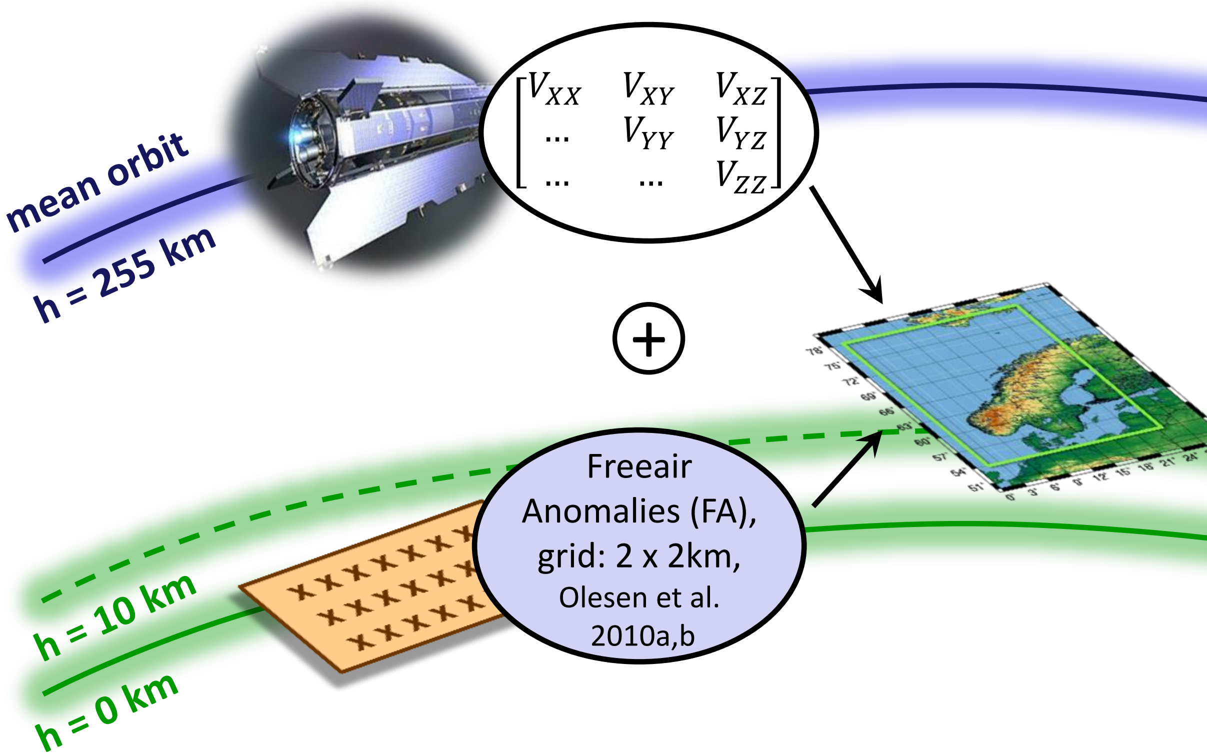


DGFI
CGE

Combination

frequency [deg] →

j [level]	1	2	3	4	5	6	7	8	9	10	11	12
l [deg] r [km]	1 20000	3 6667	7 2857	15 1333	31 645	63 317	127 157	255 78	511 39	1023 20	2047 10	4095 5
	satellite gravimetry (levels 1–8)											
		altimetry (levels 3–10; transition at 2 and 11)										
							terrestrial gravimetry (levels 8–12; transition at 7)					



combination @ 10 km, j = 9:

- low frequency domain: background model GOCO03S
- mid frequency domain: GOCE
- high frequency domain (topography): FA

- fine structures of FA not needed in our geophysical applications

➢ compromise: modelling @ 10 km height

Combination – Observation Equations

Combination of different measurement techniques

IN: observations | **MODEL: expression in terms of scaling functions $\tilde{\phi}_{J+1}$**

measure-ment technique	functional	series expansion
GRACE	ΔV	$\sum_{q=1}^{N_J} d_{J,q}\, \tilde{\phi}_{J+1}(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_q)$
GOCE	V_{ab}	$\sum_{q=1}^{N_J} d_{J,q} \begin{bmatrix} \phi_{J+1,xx}(\boldsymbol{x},\boldsymbol{x}_q) & \phi_{J+1,xy}(\boldsymbol{x},\boldsymbol{x}_q) & \phi_{J+1,xz}(\boldsymbol{x},\boldsymbol{x}_q) \\ \phi_{J+1,yx}(x,x_q) & \phi_{J+1,yy}(\boldsymbol{x},\boldsymbol{x}_q) & \phi_{J+1,yz}(\boldsymbol{x},\boldsymbol{x}_q) \\ \phi_{J+1,zx}(x,x_q) & \phi_{J+1,zy}(x,x_q) & \phi_{J+1,zz}(\boldsymbol{x},\boldsymbol{x}_q) \end{bmatrix}$
altimetry	SSH	$\sum_{q=1}^{N_J} d_{J,q}\, \tilde{\phi}_{J+1}(\boldsymbol{x}, \boldsymbol{x}_q) + DOT(\boldsymbol{x})$
terrestr. gravimetry	Δg	$\sum_{q=1}^{N_J} d_{J,q}\, \tilde{\phi}_{J+1}(\boldsymbol{x}, \boldsymbol{x}_q)$

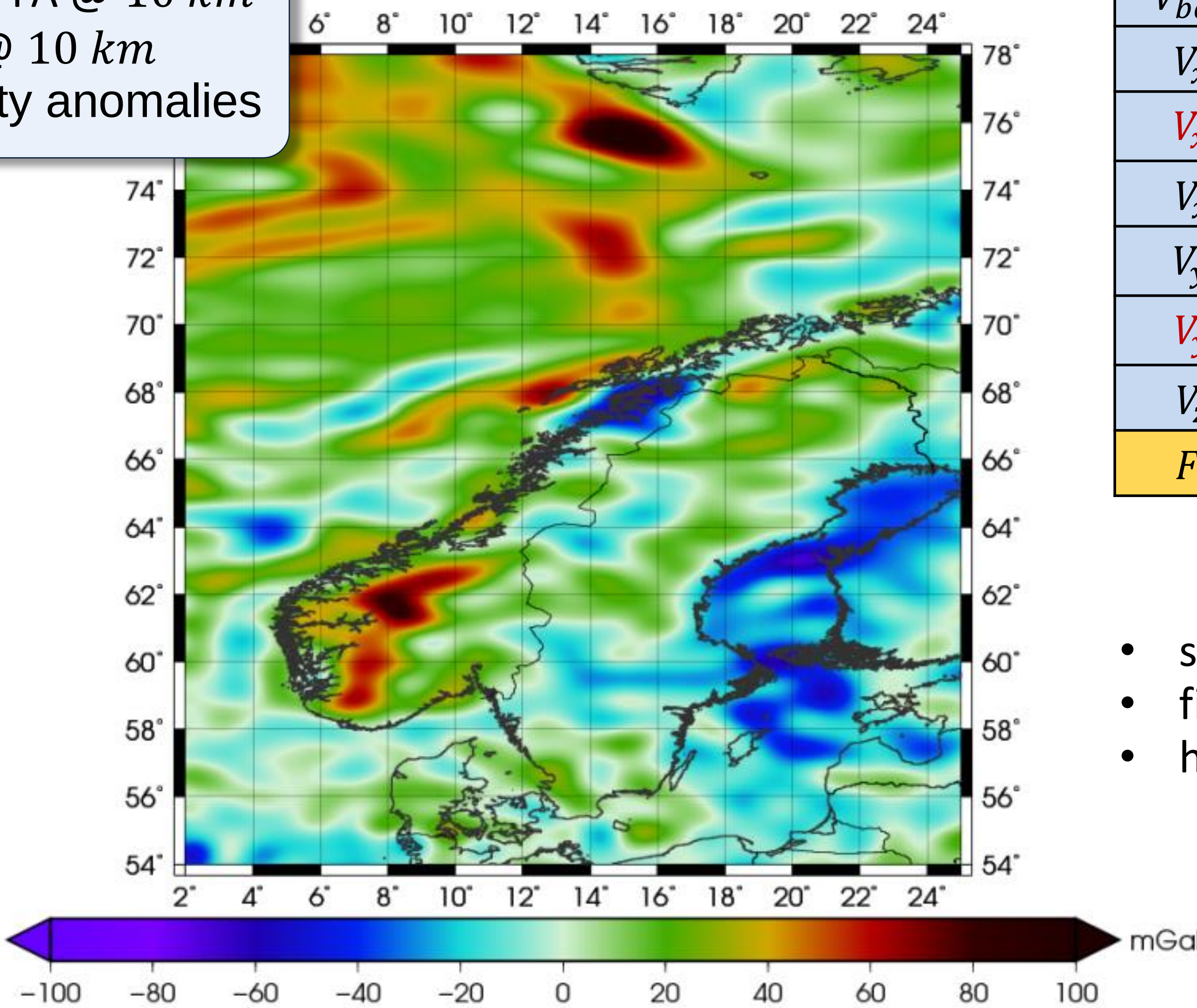
unknown parameters:
scaling coefficients d_J

DGFI CGE

Combination

IN: V_{ab} , FA @ $10\ km$
OUT: Dg @ $10\ km$
gravity anomalies

GOCE + FA
$j = 9, l' = 511$



IN	VC [order of magnitude]
V_{back}	E-00
V_{xx}	E-02
V_{xy}	E+06
V_{xz}	E-00
V_{yy}	E+03
V_{yz}	E+06
V_{zz}	E-02
FA	E-04

- signal $\pm 100\ mGal$
- fine structures
- highest weight: FA

Combination - MRR

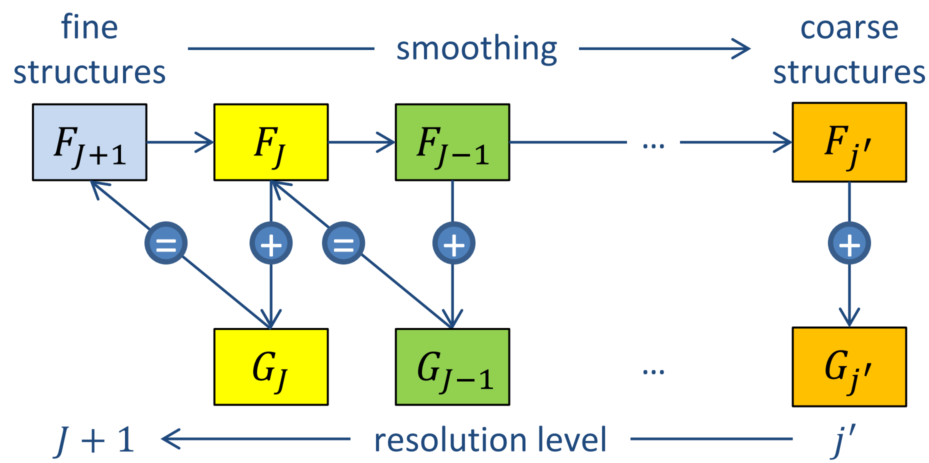
Multi-resolution representation (MRR)

… splitting a signal F_{J+1} into

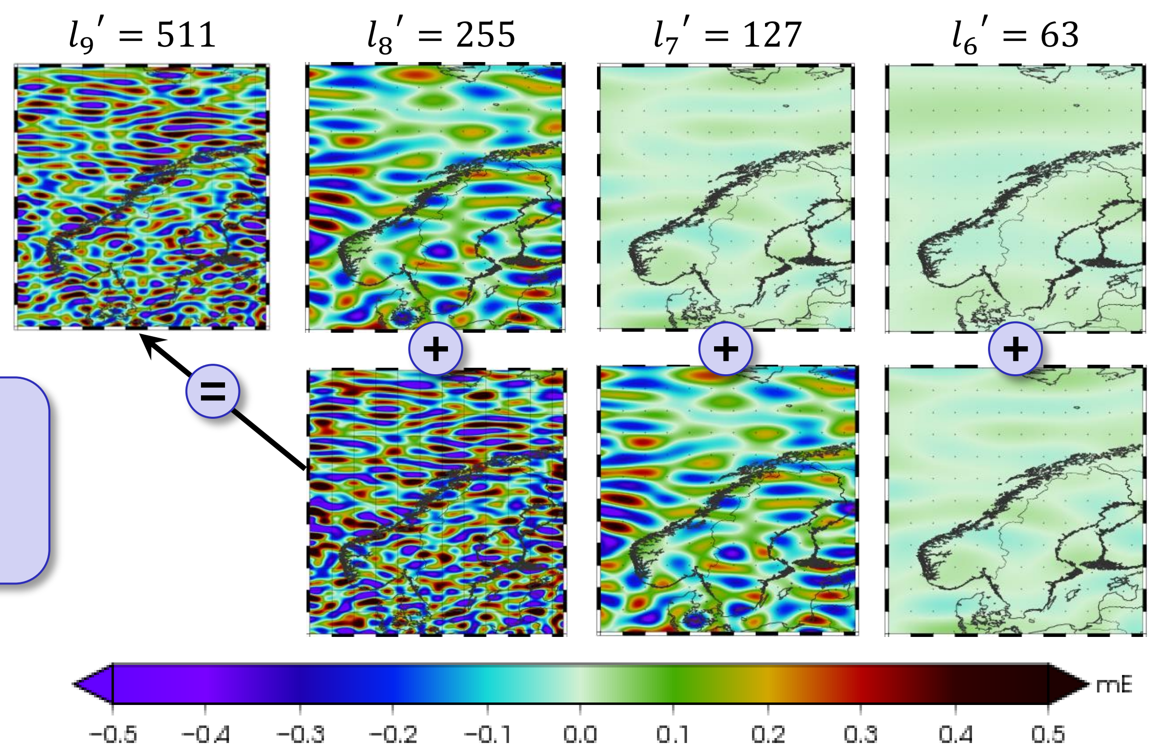
- a smoothed version $F_{j'}$

  and

- a number of detail signals G_j (related to specific frequency bands of level j)

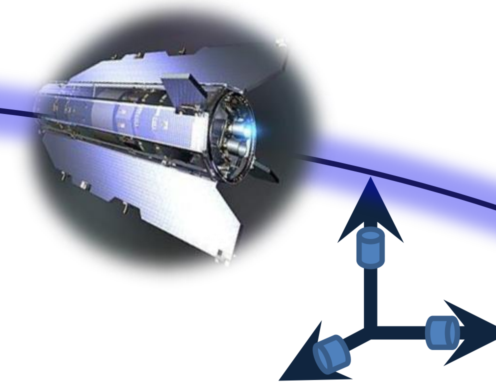


$$F_{J+1} = T_{zz,9}$$



$$F_{J+1} = F_{j'} + \sum_{j=j'}^{J} G_j$$

Summary & Outlook



GOCE gravity gradient grids

… show different structures

… give information on the gravitational field depending on different spatial directions

- ➢ comparison to a consistent filtered EGM2008 model
- ➢ time-variations

Regional gravity field modelling approach

… using variance components enables a flexible relative weighting of the data sets

… allows consistent (spectral) combination with other gravity field observations exploiting the highest degree of information out of each observation technique

… might add details to global models

- ➢ full error propagation
- ➢ Pyramidal algorithm