Closed-loop tests of a regional gravity field modelling approach using radial basis functions

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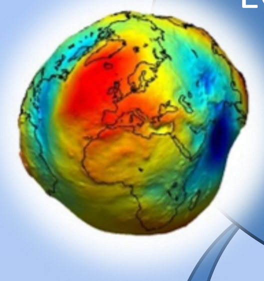
INPUT



EGM2008, zero tide global spherical harmonic model (Pavlis et al., 2008)

Simulated observations

ΔT	T_r	T_{rr}
potential differences GRACE	disturbing potential terrestrial & aiborne	gravity gradients GOCE
spectral res.: d/o 250 + noise	spectral res.: d/o 2190 + noise	spectral res.: d/o 250 + noise

Regional gravity field modelling

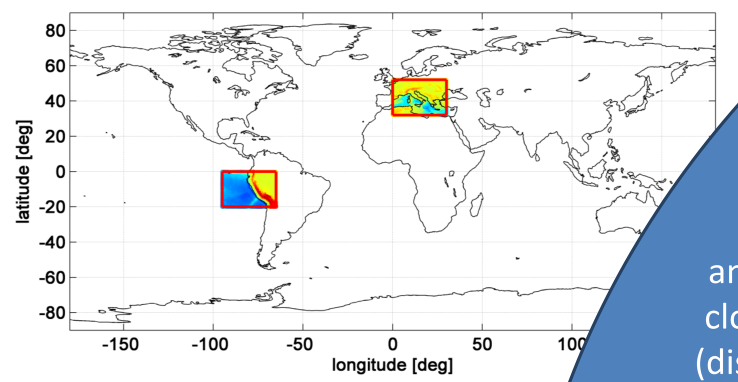
using radial basis functions

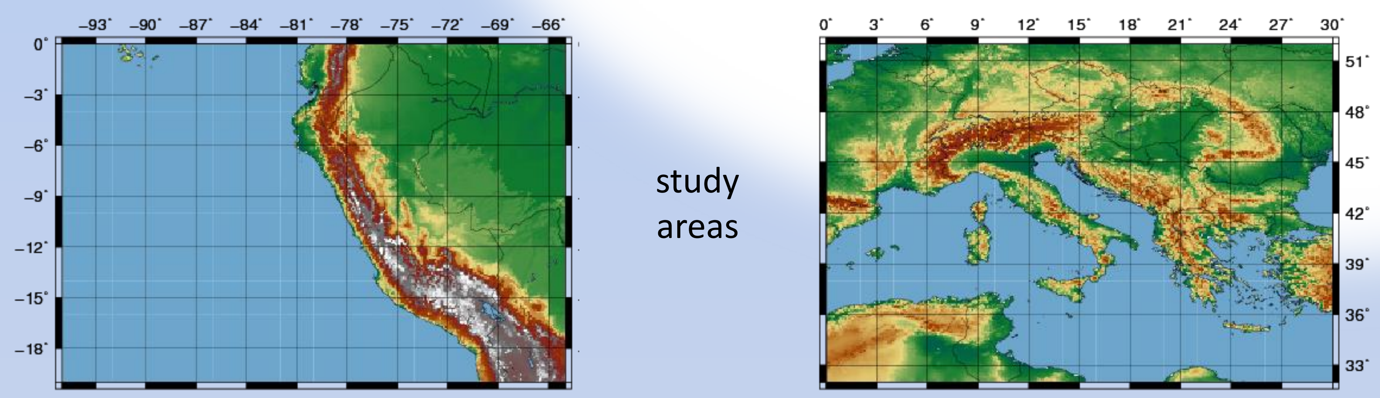
Validation data sets

disturbing potential T
geographic grid: ellipsoidal lon, lat, height (DTM2006.0)

grid res.: 30'x30'
spectral res.: d/o 250

grid res.: 5'x5'
spectral res.: d/o 2190





study areas

Motivation

Since many high-resolution observations, e. g. from terrestrial gravimetry are only available in regional areas, regional gravity field modelling becomes more and more important as an extension of the traditional global modelling of the Earth's gravitational potential. However, a lot of open questions have to be answered and problems concerning a consistent model combination and application have to be solved.

In contrast to global approaches using spherical harmonic functions, the choice of the set of basis functions for regional analysis is a central question. In the context of inter-comparing different approaches within the IAG ICCT JSG0.3* we set up a closed-loop simulation using an approach based on radial scaling functions. All results are compared with validation data sets (disturbing potential T) on geographic grids on the surface of topography for study areas in Europe (E) and South America (SA).

Summary

- We test 3 different scaling functions in the synthesis: Shannon, Blackman and Cubic Polynomials (CuP). The Blackman approach delivers the smallest standard deviations w. r. t. the validation data set (see bottom plots) and thus is used for all computations.
- We use the simulated gravity field observations for terrestrial, airborne and satellite measurement techniques with and without white noise. Noisy data sets deliver slightly larger standard deviations (see table "Differences", keys o1 vs. o3; c1 vs. c5), except for the combination of all observations (c4 vs. c7).
- We consider different spectral and spatial resolutions. GOCE data contribute the most in the mid frequency domain (j = 8, up to degree l' = 255, spatial resolution 30' ≈ 80 km) while terrestrial and airborne data contribute the most in the high frequency domain (j = 11, up to degree l' = 2047, spatial resolution 5' ≈ 10 km).
- We compute "only" as well as "combined" solutions. At the higher resolution level j = 11 the combined solutions show better results than the "only" solutions (c7 vs. o4), but at level j = 8 the GOCE-only solution (with noise) fits better to the validation data than the combination with GRACE (o3 vs. c5). For noise-free data there is no significant difference (o1 vs. c1).
- We test different background models with different resolutions and weighting procedures, as T_{back} is introduced as prior information as well. The chosen background model and resolution do not influence the solution (c3 vs. c4) while an observation dependent weighting of T_{back} slightly improves the solutions (o1 vs. o2; c5 vs. c6).

In general the closed-loop tests give information on the external accuracy of our regional gravity field modelling strategy. This helps to understand the interactions and relationships between different parametrisations and implementations (see Schmidt et al., EGU2014-12952/poster B770).

Differences

„only" solutions (key „o")

key	area	obs	spectral res.	spatial res.	noise	T_{back}	weight T_{back}	ΔT [m²/s²]	std . dev. [m²/s²]
o1	E	GOCE	j = 8	30'	-	GOCO03s d/o 60	const.	-26.89 ... 10.58	4.35
o2	E	GOCE	j = 8	30'	-	GOCO03s d/o 60	obs. d.	-26.62 ... 10.63	4.34
o3	E	GOCE	j = 8	30'	white	GOCO03s d/o 60	const.	-26.15 ... 10.88	4.43
o4	E	ter.	j = 11	5'	white	EGM2008 d/o 120	const.	-17.71 ... 4.34	1.59

„combined" solutions (key „c")

key	area	obs	spectral res.	spatial res.	noise	T_{back}	weight T_{back}	ΔT [m²/s²]	std. dev. [m²/s²]
c1	E	GR+GO	j = 8	30'	-	GOCO03s d/o 60	const.	-26.76 ... 10.66	4.34
	SA	GR+GO	j = 8	30'	-	GOCO03s d/o 60	const.	-33.01 ... 14.71	4.28
c2	E	all	j = 11	30'	-	GOCO03s d/o 60	const.	-49.60 ... 79.48	8.54
	SA	all	j = 11	30'	-	GOCO03s d/o 60	const.	-53.37 ... 41.02	8.09
c3	E	all	j = 11	5'	-	GOCO03s d/o 60	const.	-15.30 ... -5.51	1.03
	SA	all	j = 11	5'	-	GOCO03s d/o 60	const.	-12.45 ... -6.01	0.47
c4	E	all	j = 11	5'	-	EGM2008 d/o 120	const.	-15.11 ... -0.08	1.06
c5	E	GR+GO	j = 8	30'	white	GOCO03s d/o 60	const.	-26.97 ... 14.34	4.87
c6	E	GR+GO	j = 8	30'	white	GOCO03s d/o 60	obs. d.	-26.55 ... 11.85	4.53
c7	E	all	j = 11	5'	white	EGM2008 d/o 120	const.	-16.35 ... -3.22	0.95

Note: we introduce T_{back} as prior information with a constant weight (const.) or an observation depending weight (obs. d.) which means that it gets a higher weight in case of data gaps to avoid rank deficiencies and singularity problems. We interpret the smallest standard deviations of the differences ΔT as "best" solutions (highlighted in green).

Modelling approach

We use series expansions in terms of radial basis functions (scaling functions) based on Legendre polynomials acting as low-pass filters in the frequency domain.
The maximum degree (l) of the series expansion is related to a specific resolution level (j). It defines the cut-off frequency of the low-pass filter and thus the spectral (j) and the spatial resolution (r).

j [level]	1	2	3	4	5	6	7	8	9	10	11	...
l' [deg]	1	3	7	15	31	63	127	255	511	1023	2047	...
r [km]	20000	6667	2857	1333	645	317	157	78	39	20	10	...

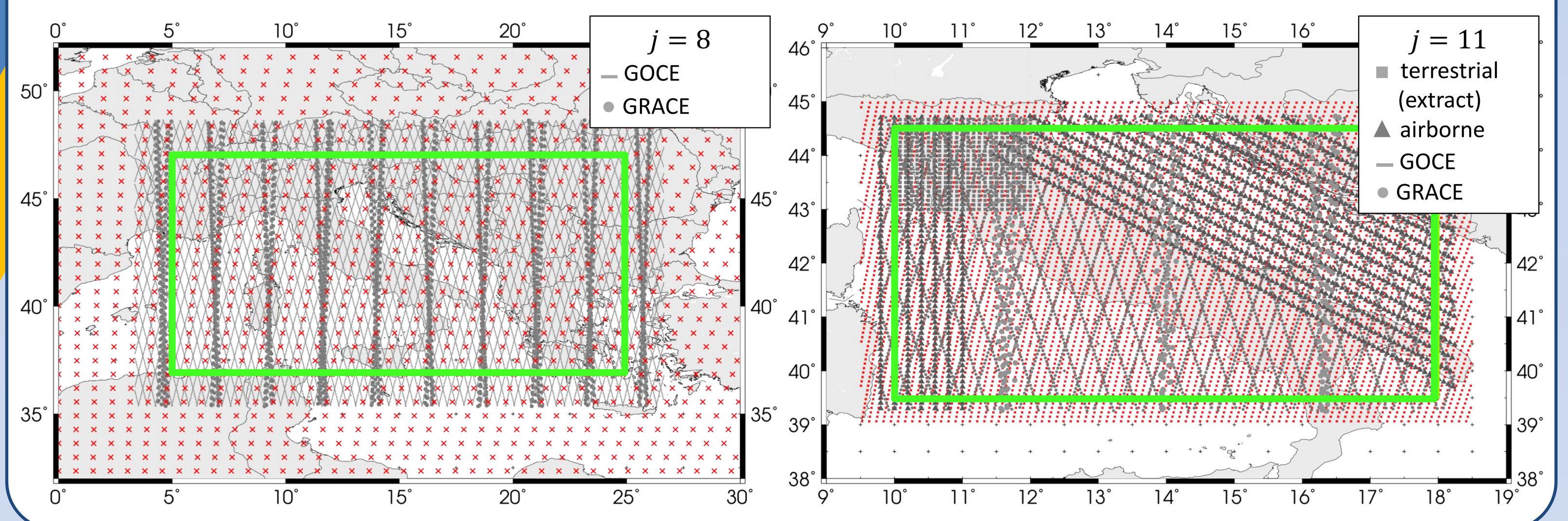
Observation equations

We set up observation equations for each observation type (ΔT, T_r, T_{rr}), subtracted by a background model T_{back}. We define an extended Gauß-Markov model and calculate the unknown **scaling coefficients** d_j by variance component estimation. In this analysis step we use **Shannon scaling functions** ϕ_{j+1} with appropriate **Legendre coefficients** $\Phi_{j+1,l}$.

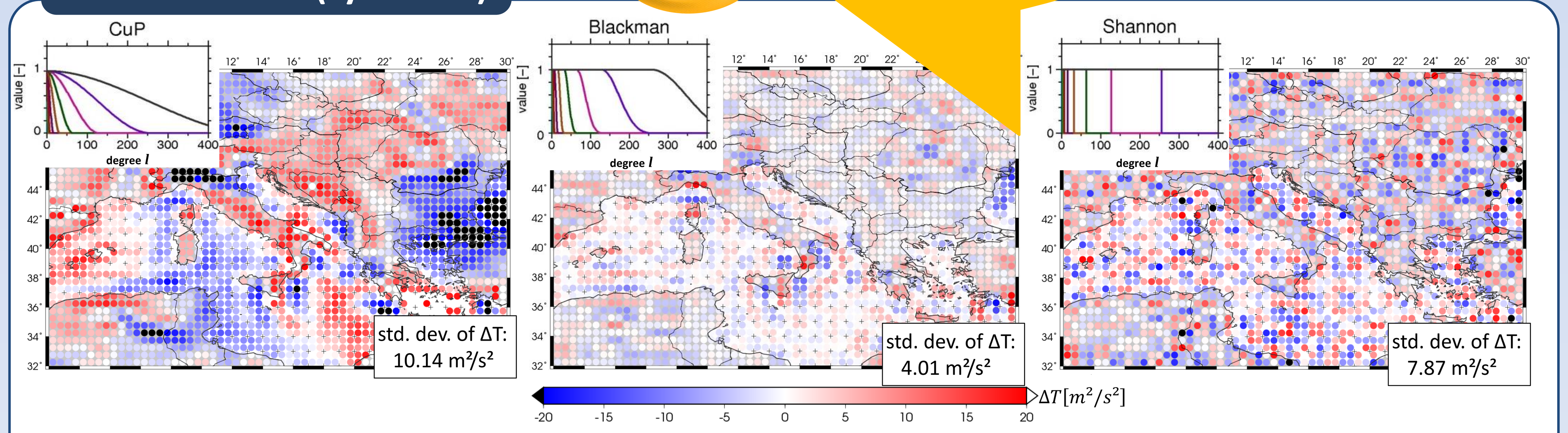
$$\left.\begin{array}{r}\Delta T(\boldsymbol{x})\\ T_r(\boldsymbol{x})=\dfrac{\partial T(\boldsymbol{x})}{\partial r}\\ T_{rr}(\boldsymbol{x})=\dfrac{\partial^2 T(\boldsymbol{x})}{\partial r^2}\end{array}\right\} = \sum_{q=1}^{N} d_{j,q}\,\phi_{j+1}(\boldsymbol{x},\boldsymbol{x}_q) = \sum_{q=1}^{N} d_{j,q} \sum_{l=0}^{l_j'} \frac{2l+1}{4\pi}\Phi_{j+1,l} \left\{\begin{array}{l}\left(\dfrac{R}{r_1}\right)^{l+1} P_l(\boldsymbol{x}_1,\boldsymbol{x}_q)-\left(\dfrac{R}{r_2}\right)^{l+1} P_l(\boldsymbol{x}_2,\boldsymbol{x}_q)\\ \left(\dfrac{R}{r}\right)^{l+1}\dfrac{l-1}{r}P_l(\boldsymbol{x},\boldsymbol{x}_q)\\ \left(\dfrac{R}{r}\right)^{l+1}\dfrac{(l+1)(l+2)}{r^2}P_l(\boldsymbol{x},\boldsymbol{x}_q)\end{array}\right.$$

- l_j' max. degree of resolution level j
- N total number of grid points $\boldsymbol{x}_q$
- $P_l(\boldsymbol{x},\boldsymbol{x}_q)$ Legendre polynomials

The functions are located on a Reuter grid (**area of computation**). The **area of investigation** is the smallest area. The area of observations should be larger to avoid edge effects. The higher the resolution level, the sharper the decline of the functions, the denser the grid points (see Figures below: Reuter grids at $j = 8$ vs. $j = 11$ and data distribution for two study areas in Europe).



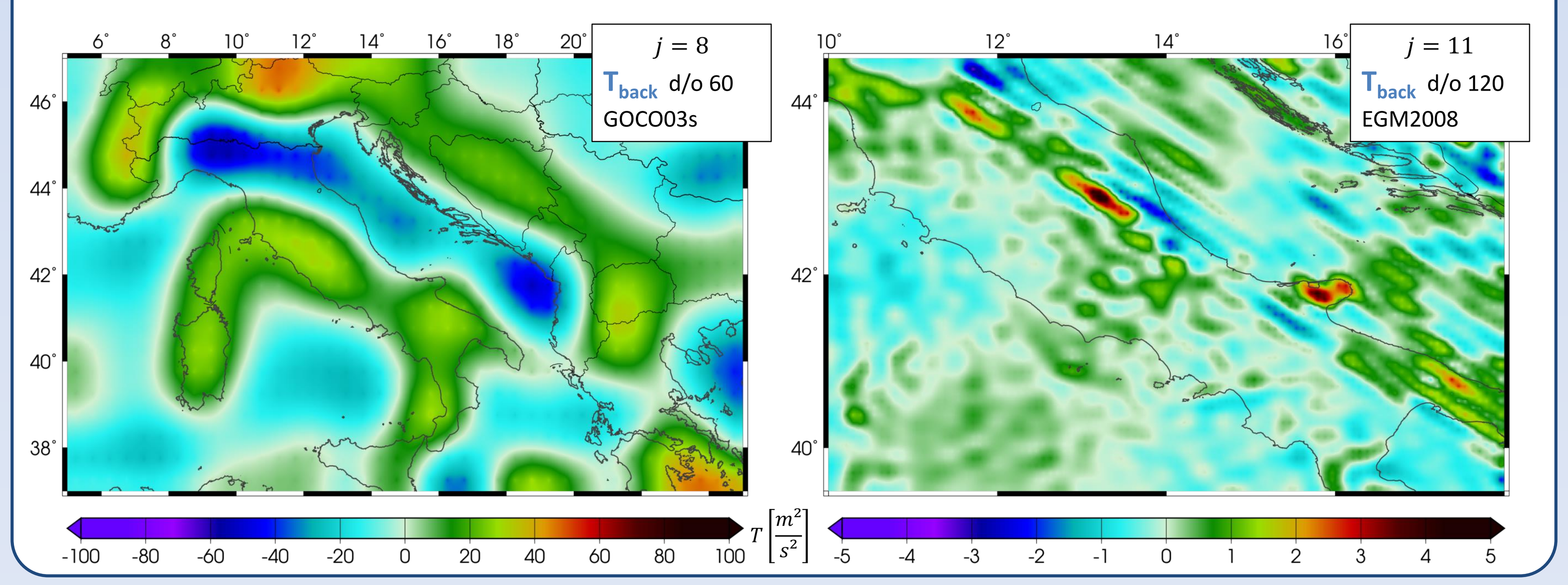
Comparison of different basis functions (synthesis)



Blackman scaling functions deliver the smallest standard deviations (std. dev.) compared with Shannon and CuP, w. r. t. the validation data set. They are a compromise between declining functions in the frequency domain and oscillating functions in the spatial domain: Thus they are used for the synthesis step.

Results

In the synthesis step we use series expansions in terms of **Blackman scaling functions** (multiplied with the estimated coefficients d_j). We model functionals of the gravitational potential (e.g. the disturbing potential T) on the locations of the validation data in the **area of investigation**. The figures below show the resulting signals w. r. t. two different background models.



* IAG (International Association of Geodesy) ICCT (Inter-Commission Committee on Theory)
JSG (Joint Study Group) 0.3: Comparison of Current Methodologies in Regional Gravity Field Modeling.
Further information on JSG0.3 can be found on the website http://jsg03.dgfi.badw.de.

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References

Lieb et al.: Combination of GOCE gravity gradients in regional gravity field modelling using radial basis functions. IAG Proceedings 142, 2014
Schmidt et al.: Regional Gravity Modelling in Terms of Spherical Base Functions. J Geod, 81, 17-38, doi: 10.1007/s00190-006-0101-5, 2007