A new best estimate for the conventional value W_0

- Final Report of the WG on Vertical Datum Standardization -

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Introduction

- W_0 is understood as the potential value of the geoid;
- Since there are an infinite number of equipotential surfaces, the geoid is to be defined arbitrarily by convention;
- Usual convention: the geoid is the equipotential surface of the Earth's gravity field that best fits (in a least square sense) the undisturbed mean sea level;
- Since to satisfy this condition is not possible and since the sea level changes, a convention about mean sea level (time span and area) is also needed:
  - mean value at a local tide gauge $W_0 = W_0^{(i)}$
  - mean value a several tide gauges $W_0 = \frac{1}{n}\sum_{i=1}^{n} W_0^{(i)}$
  - potential value of a best fitting ellipsoid in ocean areas $W_0 = U_0$
  - mean value over ocean areas sampled globally $\int_S (W - W_0)^2 \, dS = \min$

W_0 and the IERS Conventions

- In 1991, the International Astronomical Union introduced timescales for the relativistic definition of the celestial space-time reference frame;
- The relationship between Geocentric Coordinate Time (TCG), and Terrestrial Time (TT) depends on the constant $L_G = W_0 / c^2$
- For this reason, the IERS Conventions included a W_0 value and updated this value regularly according to new best-estimates:

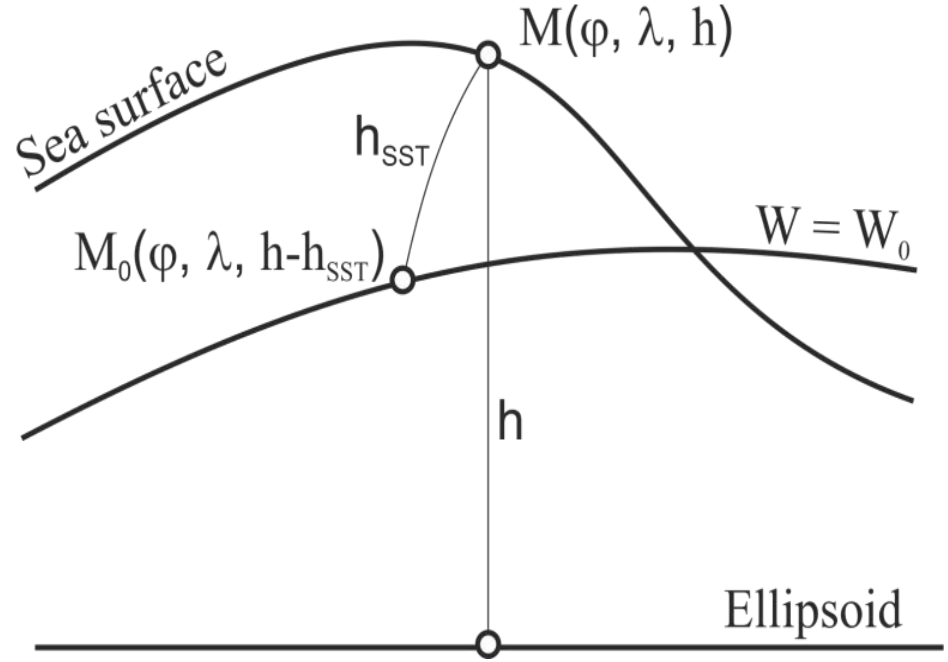
Year	W_0	L_G
1991	62 636 860 ± 30 m^2s^{-2} (Chovitz 1988)	6.969 291 × 10^{-10} ± 3 × 10^{-16} (IAU 1991, Recommendation IV, note 6)
1992	62 636 856.5 ± 3 m^2s^{-2} (Burša et al. 1992)	6.969 290 19 × 10^{-10} ± 3 × 10^{-17} (Fukushima 1995)
1995	62 636 856.85 ± 1 m^2s^{-2} (Burša 1995a)	6.969 2903 × 10^{-10} ± 1 × 10^{-17} (McCarthy 1996, Tab. 4.1)
1999	62 636 856.0 ± 0.5 m^2s^{-2} (Burša et al. 1998, Groten 1999)	6.969 290 134 × 10^{-10} (as defining constant) (IAU2000, Resolution B1.9)

- In 2000, L_G is declared as “defining constant”, i.e. it should not change with new estimations of W_0. The corresponding W_0 value is the best-estimate available in 1998.

Concept underlying the 1998 W_0 value ($62\,636\,856.0 \pm 0.5\ m^2s^{-2}$)

- Satellite altimetry provides the coordinates φ, λ, h of points M describing the sea surface.
- If the sea surface topography h_{SST} is reduced, points (M_0) on the geoid are obtained;
- Using the coordinates ($\varphi, \lambda, h\text{-}h_{SST}$) and a global gravity model (GGM), the potential value at any point M_0 on the geoid (i.e. W_0) can be computed;
- Since points on the geoid cannot be materialised in practice, W_0 is estimated by satisfying the condition (cf. Burša et al. 1998, Eq. [5]):

$$\int_S h_{SST}^2 \, dS = \min$$

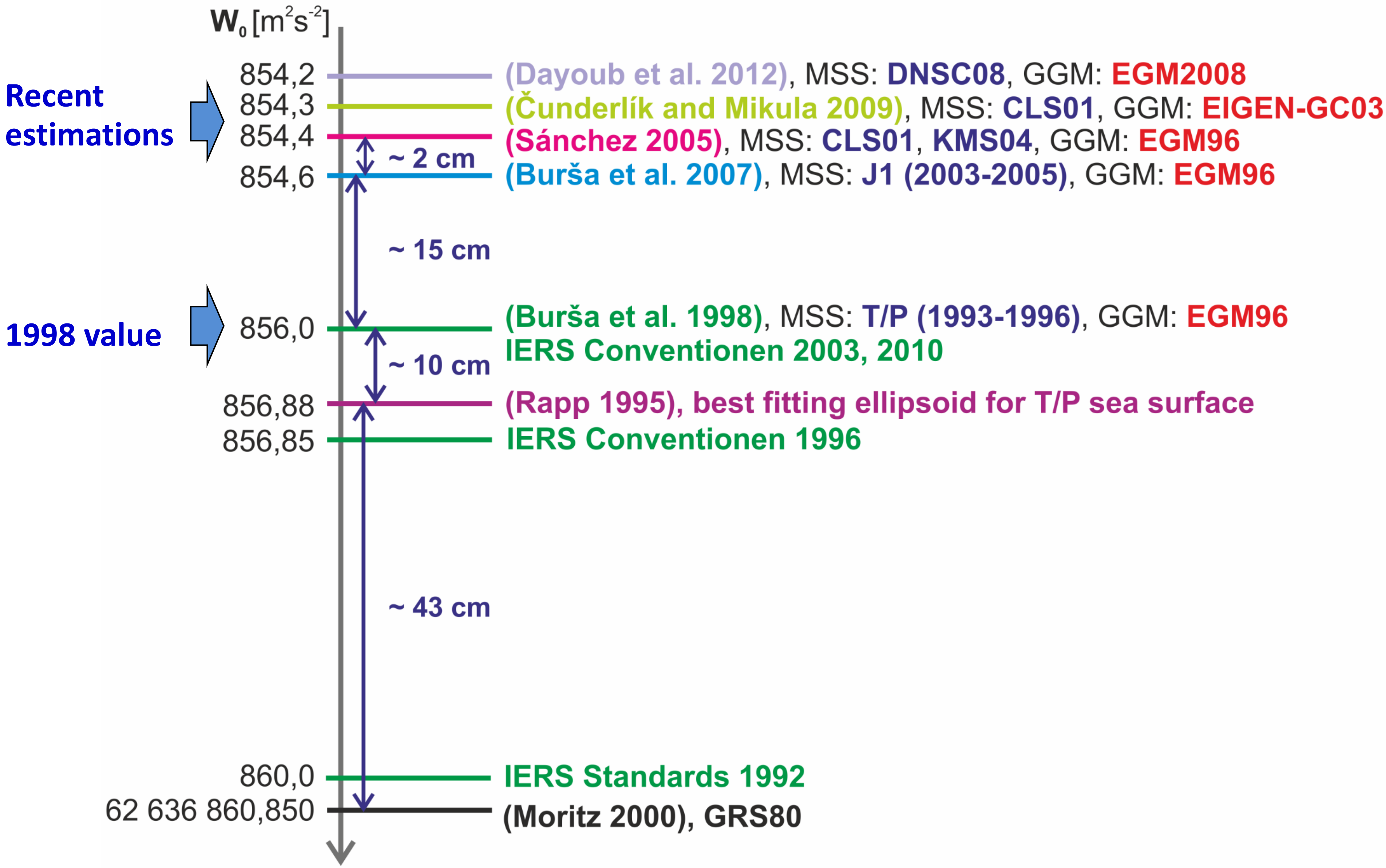


Approximation applied for the estimation of the 1998 W_0 value (adapted from Burša et al. 1997, Fig. 4, and Burša et al. 1998, Fig. 1).

Models applied by Burša et al. 1998:

- $S \rightarrow$ Burša's own mean sea surface model from T/P (1993 to 1996)
- $h_{SST} \rightarrow$ POCM4b (Stammer et al. 1996)
- GGM $\rightarrow$ EGM96 (Lemoine et al. 1998)

Recent W_0 computations (since 2005) based on newer models of the sea surface and the Earth's gravity field



Working Group on "Vertical Datum Standardisation"

- **Common initiative of**

GGOS Theme 1: Global Height System	International Gravity Field Service (IGFS)	IAG Commission 2: Gravity Field	IAG Commission 1: Reference Frames

- **Main objective**: to provide a recommendation of the W_0 **value** to be appointed as the **reference level** of the international height reference system;
- **Term:** 2011 – 2015;
- **Members:** J. Ågren (Sweden), R. Čunderlík (Slovakia), N. Dayoub (Syria), J. Huang (Canada), R. Klees (The Netherlands), J. Mäkinen (Finland), K. Mikula (Slovakia), Z. Minarechová (Slovakia), P. Moore (United Kingdom), D. Roman (USA), Z. Šima (Czech Republic), C. Tocho (Argentina), V. Vatrt (Czech Republic), M. Vojtiskova (Czech Republic), Y. Wang (USA).

Working Group on "Vertical Datum Standardisation"

Aspects analysed within the WG-VDS:

- Different estimation methodologies but the same input models (for redundancy);
- Sensitivity of the W_0 estimation on the Earth's gravity field model
- Dependence of W_0 on the omission error of the global gravity model
- Influence of the time-dependent Earth's gravity field changes on W_0
- Sensitivity of the W_0 estimation on the mean sea surface model
- Influence of time-dependent sea surface changes on W_0
- Effects of the sea surface topography on the estimation of W_0
- Dependence of the W_0 empirical estimation on the tide system
- Rigorous error propagation analysis to estimate the influence of the input data uncertainties on the W_0 estimation.

Basic approach for the empirical estimation of W_0

As proposed by Sacerdote and Sansò (2001):

Sea surface topography:

$$\Xi^k \cong \frac{W_S - W_0}{\bar{\gamma}^k}$$

Condition:

$$\int_{S_O} \left(W_0^k - W_0\right)^2 dS_O = \min$$

Solution:

$$W_0 = \frac{\int_{S_O} \frac{W_S}{\bar{\gamma}_S^2} d\sigma}{\int_{S_O} \frac{1}{\bar{\gamma}_S^2} d\sigma}$$

for error propagation analysis

$$W_0 = U_0 + \frac{\sum_1^k \left[\frac{T^k - h^k \bar{\gamma}^k}{\bar{\gamma}^{k^2}}\right] \delta S_O^k}{\sum_1^k \left[\frac{1}{\bar{\gamma}^{k^2}}\right] \delta S_O^k}$$

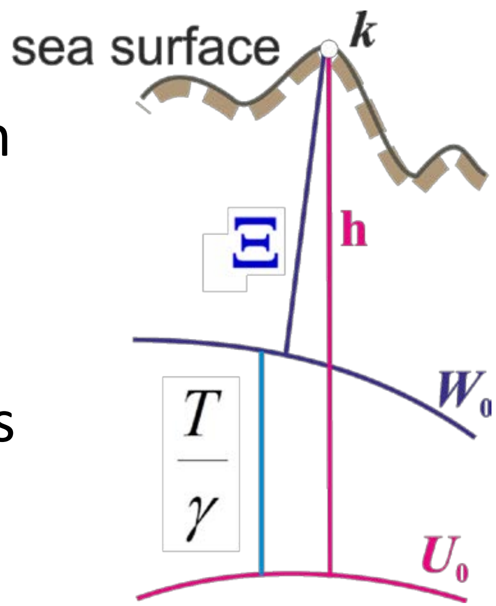
$$\delta S_O^k = \cos\varphi \ \delta\varphi \ \delta\lambda$$

If the sea surface topography Ξ is reduced from the see surface heights h (as Burša et al. 1998 proposed):

$$W_0 = U_0 + \frac{\sum_1^k \left[T^k - \left(h^k - \Xi^k\right)\bar{\gamma}^k\right] \delta S_O^k}{\sum_1^k \delta S_O^k}$$

Input data for the empirical estimation of W_0

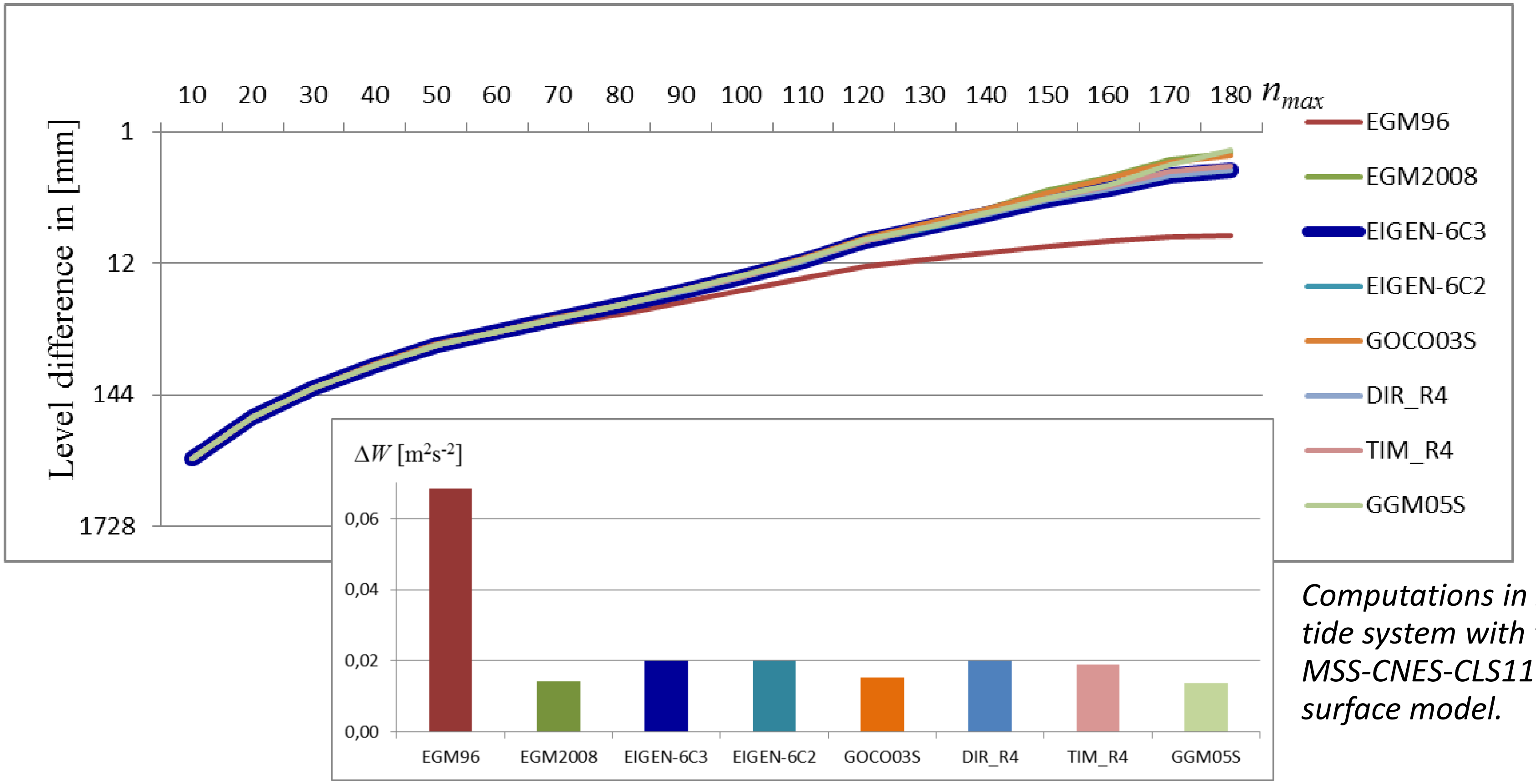
- h^k from a mean sea surface model:
  - CLS11 (Schaeffer et al. 2012), DTU10 (Andersen 2010)
  - own computed yearly models (1992-2013)
  - cross calibrated data from the DGFI-OpenADB (Schwatke et al. 2010) with covariance matrixes (Bosch et al. 2014), 9 missions.



- T^k from a global gravity model:
  - EGM96 (Lemoine et al. 1998), EGM2008 (Pavlis et al. 2012), EIGEN-6C and 6EIGEN-6C3stat (Förste et al. 2012), GOCO03S (Mayer-Gürr et al. 2012), DIR-R4 (Bruinsma et al. 2013), TIM-R4 (Pail et al. 2011), GGM05S (Tapley et al. 2013), monthly models from GRACE GFZ Release 05.
- Ξ^k from the (oceanographic) Model ECCO-2 (Menemenlis et al. 2008)
- U_0, γ from GRS80

OpenADB: Open Altimeter Database,
http://openadb.dgfi.badw.de

Dependence of the W_0 estimate on the choice of the gravity model

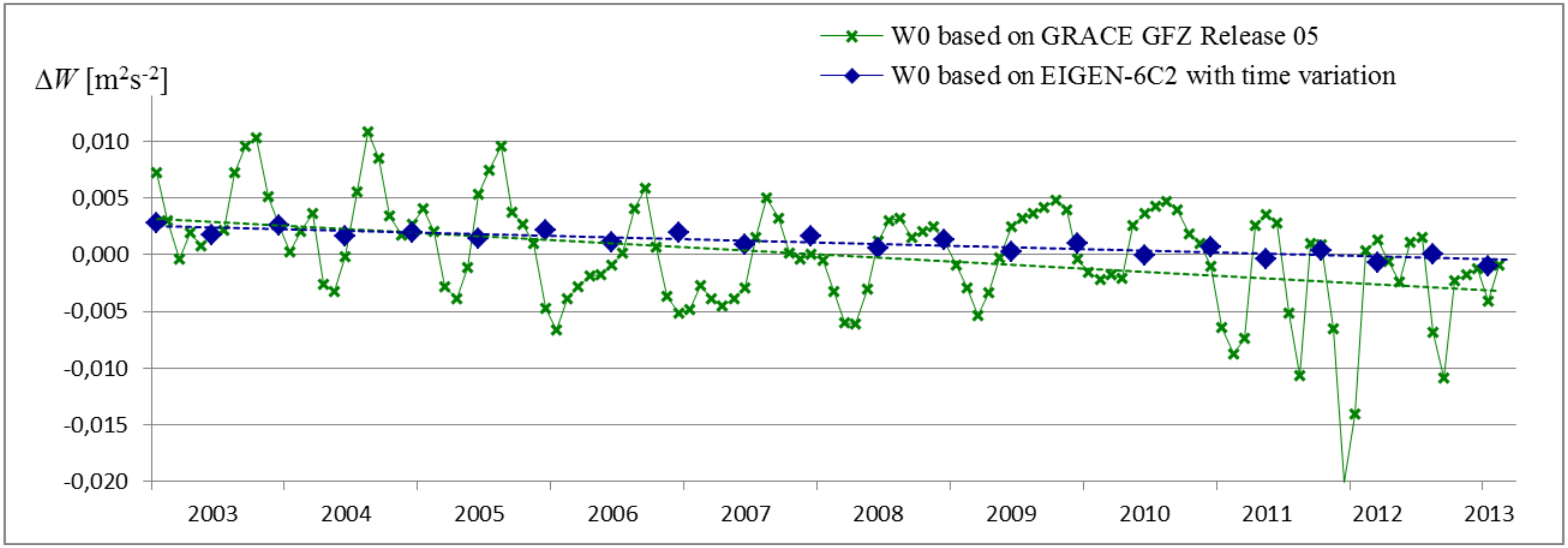


Computations in zero tide system with the MSS-CNES-CLS11 sea surface model.

1) W_0 estimations based on models including GRACE, GOCE and Satellite Laser Ranging (Lageos) data are practically identical. Max. differences 0.01 m^2s^{-2}.

2) The use of a satellite-only gravity model is suitable. After n = 200 the largest differences are 0.001 m^2s^{-2}.

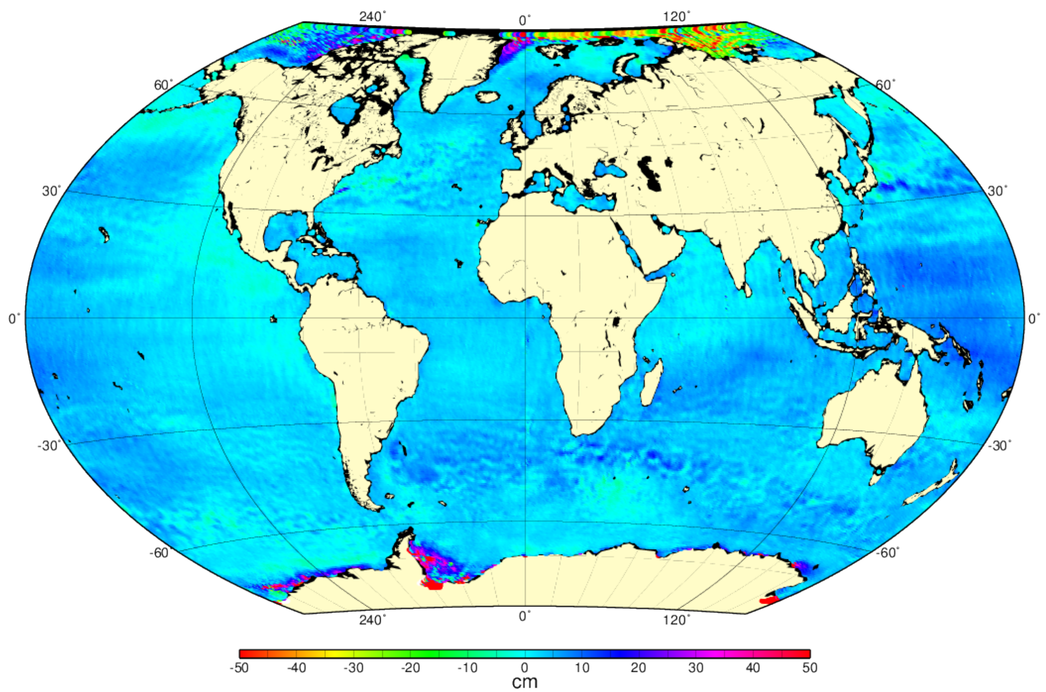
Dependence of the W_0 estimate on the choice of the gravity model



Changes in the W_0 estimates after applying the monthly GRACE-based models GFZ Release 05 and the time-dependent harmonics of the model EIGEN-6C2. The linear trend of W_0 using the GFZ Release 05 is -6.617x10^{-4} m^{2}s^{-2}a^{-1}*, while the linear trend using EIGEN-6C2 is* -2.647x10^{-4}m^{2}s^{-2}a^{-1}.

3) Seasonal variations of the Earth's gravity model can be neglected (max. variation 0.03 m^{2}s^{-2}).

Dependence of the W_0 estimate on the mean sea surface model



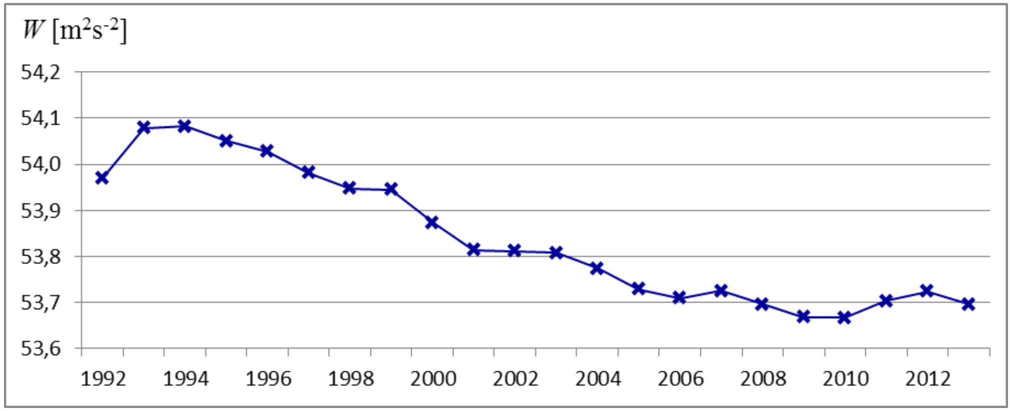
Potential differences (divided by the normal gravity) between the estimations derived from the models MSS-CNES-CLS11 and DTU10 (computations in zero tide system with the GGM EIGEN-6C3).

By using the models CLS11 and DTU10 there is a difference of 0.31 m^2s^{-2}, which reflects the mean discrepancy of ~ 3 cm between both models. Possible causes:

- Different strategies to process the altimetry data;
- Different reductions taken into account in each model;
- Different periods (inter-annual ocean variability).

Dependence of the W_0 estimate on the mean sea surface model

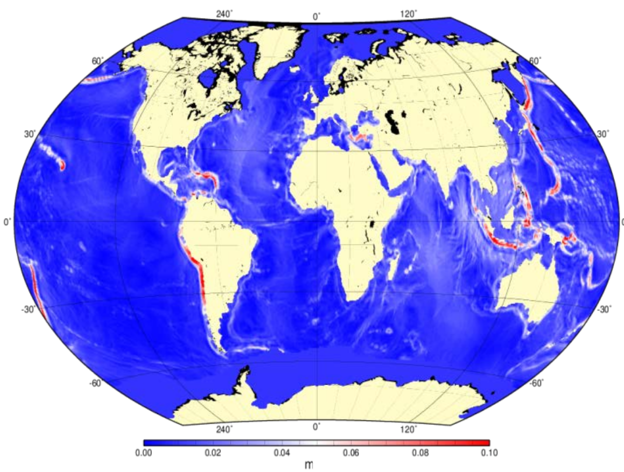
Alternative: use of yearly mean sea surface models



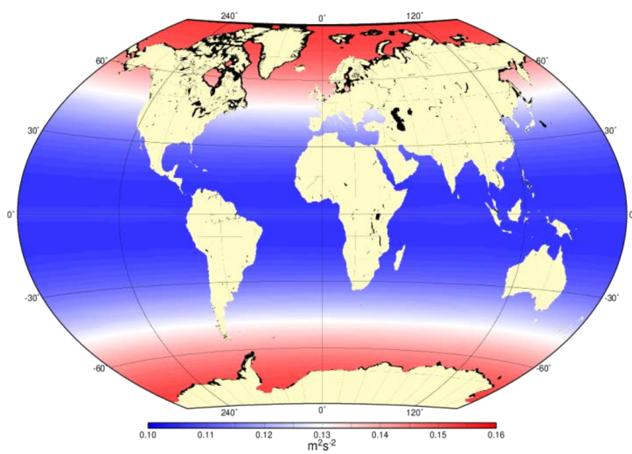
- The W_0 estimates reflect (with opposite sign) the sea level rise measured by satellite altimetry;
- Max. difference 0.46 m^2s^{-2};
- These variations shall not be understood as a change in W_0, but in the sea level; i.e. the geoid is not growing/decreasing with the mean sea level!
- This only means that the mean sea level coincides with a different equipotential surface depending on the period utilized for the average of the sea surface heights.

Reliability of the W_0 estimate

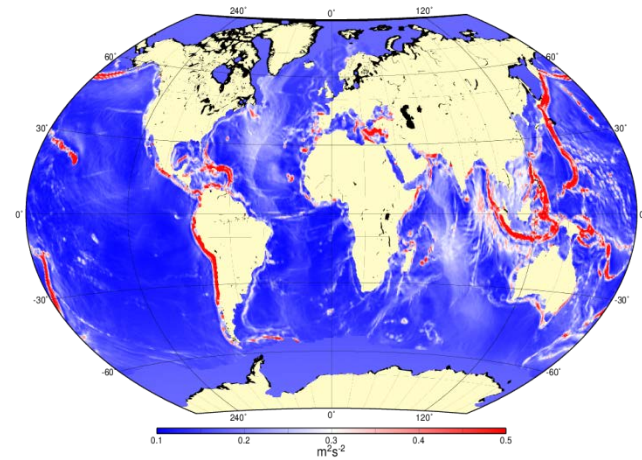
Until now, all the computations assumed error free input data (MSS and GGM). By rigorous error propagation analysis (as Sjöberg 2011 proposed), the W_0 value estimate decreases by about 0.3 m^2s^{-2}.



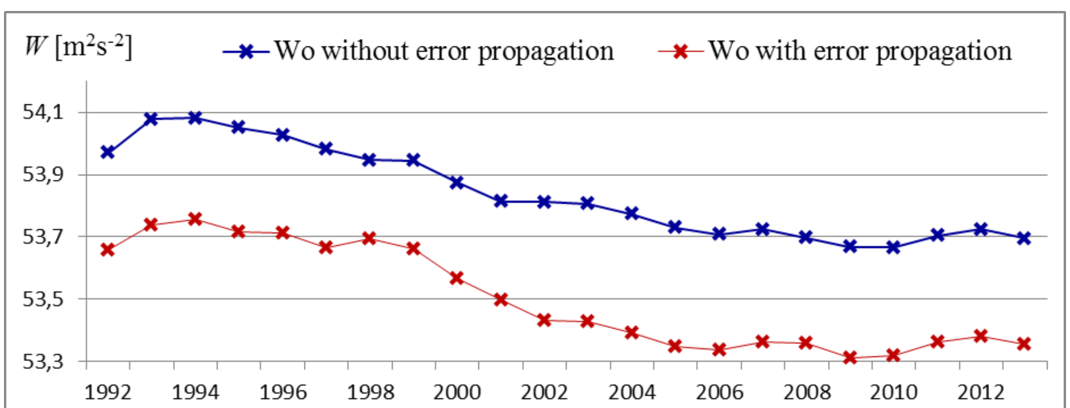
Standard deviation (σ_h) of the mean sea surface heights for the year 2005.



Standard deviation (σ_T) of the anomalous potential derived from the model EIGEN-6C3 (n = 200).



Standard deviation ($\sigma_W = \sqrt{\sigma_T^2 + \gamma^2 \sigma_h^2}$) of the gravity potential values computed at the sea heights (h) for the year 2005 with the model EIGEN-6C3 (n = 200).



W_0 estimates assuming error free input data (blue series) and applying a proper error propagation computation (red series).

Dependence of the W_0 estimate on the tide system

In theory:

1) the value W_0 depends only on
    - the volume enclosed by the surface $W = W_0$
    - the geocentric gravity constant GM
    - angular velocity of the Earth's rotation ω

2) Direct and indirect effects of the tidal potential on W_0 are compensated by the deformation of the corresponding level surface, but the volume enclosed by this surface does not change; i.e. the potential value W_0 does not change.

Empirically: W_0 determination in the three tide systems:

$$\left[W_0^{TF} - W_0^{ZT}\right] = 0.0278\,\mathrm{m^2s^{-2}}$$

$$\left[W_0^{MT} - W_0^{TF}\right] = 0.0665\,\mathrm{m^2s^{-2}}$$

$$\left[W_0^{MT} - W_0^{ZT}\right] = 0.0943\,\mathrm{m^2s^{-2}}$$

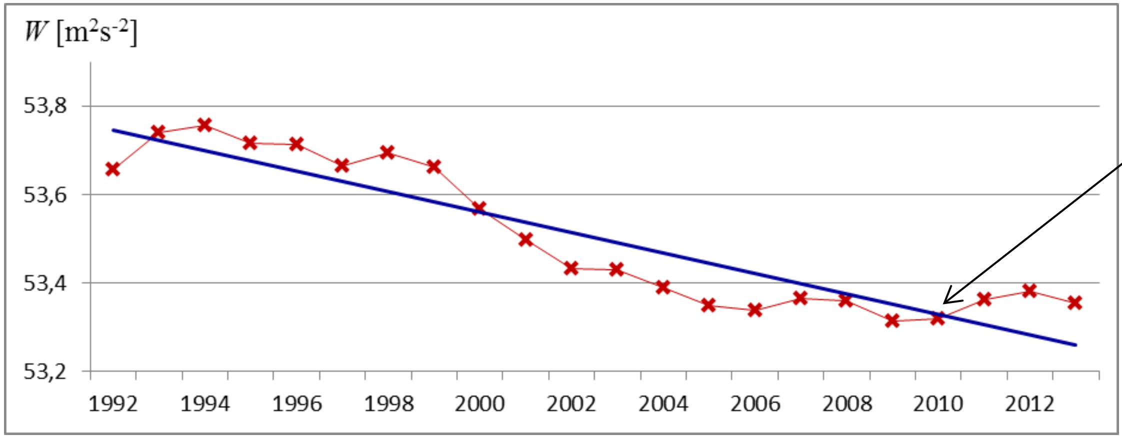
IAG GGOS

Conclusions

1) Computations carried out within the WG-VDS demonstrate that the 1998 W_0 value (62 636 856.0 ± 0.5 m^2s^{-2}) is not in agreement (and consequently it is not reproducible) with the newest geodetic models describing geometry and physics of the Earth.
2) The 1998 W_0 value is not suitable as a conventional reference value and a *better estimate* for W_0 has to be adopted by the IAG for the definition and realization of the IHRS.
3) As reference level, the conventional value W_0 has to be fixed (without time variations); but it has to have a clear relationship with the sea surface (as convention for the realization of the geoid).

Conclusions

4) We propose to adopt the potential value obtained for the year 2010 after fitting the yearly W_0 estimations by means of a lineal regression:



W_0 = 62 636 853.353 m^2s^{-2}
rounded to
W_0 = 62 636 853.4 m^2s^{-2}

5) The formal error of this value is **± 0.02 m^2s^{-2}**. However, as convention the adopted W_0 is understood free of error.

6) The introduction of a reference W_0 value is not accepted by the whole geodetic community. There are a variety of approaches to avoid a W_0 value.

7) Results provided by the WG-VDS are for those approaches requesting a reliable W_0 value.