A new best estimate for the conventional value $W_0$
- Final Report of the WG on Vertical Datum Standardization -

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Introduction

- $W_0$ is understood as the potential value of the geoid;
- Since there are an infinite number of equipotential surfaces, the geoid is to be defined arbitrarily by convention;
- Usual convention: the geoid is the equipotential surface of the Earth’s gravity field that best fits (in a least square sense) the undisturbed mean sea level;
- Since to satisfy this condition is not possible and since the sea level changes, a convention about mean sea level (time span and area) is also needed:
  - mean value at a local tide gauge $W_0 = W_0^{(i)}$
  - mean value a several tide gauges $W_0 = \frac{1}{n} \sum_{i=1}^{n} W_0^{(i)}$
  - potential value of a best fitting ellipsoid in ocean areas $W_0 = U_0$
  - mean value over ocean areas sampled globally $\int (W - W_0)^2 \, dS = \min_S$
$W_0$ and the IERS Conventions

- In 1991, the International Astronomical Union introduced timescales for the relativistic definition of the celestial space-time reference frame;
- The relationship between Geocentric Coordinate Time (TCG), and Terrestrial Time (TT) depends on the constant $L_G = W_0/c^2$
- For this reason, the IERS Conventions included a $W_0$ value and updated this value regularly according to new best-estimates:

<table>
<thead>
<tr>
<th>Year</th>
<th>$W_0$</th>
<th>$L_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>62 636 856.5 ± 3 m²s⁻² (Burša et al. 1992)</td>
<td>6.969 290 19 × 10⁻¹⁰ ± 3 × 10⁻¹⁷ (Fukushima 1995)</td>
</tr>
<tr>
<td>1995</td>
<td>62 636 856.85 ± 1 m²s⁻² (Burša 1995a)</td>
<td>6.969 2903 × 10⁻¹⁰ ± 1 × 10⁻¹⁷ (McCarthy 1996, Tab. 4.1)</td>
</tr>
<tr>
<td>1999</td>
<td>62 636 856.0 ± 0.5 m²s⁻² (Burša et al. 1998, Groten 1999)</td>
<td>6.969 290 134 × 10⁻¹⁰ (as defining constant) (IAU2000, Resolution B1.9)</td>
</tr>
</tbody>
</table>

- In 2000, $L_G$ is declared as “defining constant”, i.e. it should not change with new estimations of $W_0$. The corresponding $W_0$ value is the best-estimate available in 1998.
A new best estimate for the conventional value $W_0$

Concept underlying the 1998 $W_0$ value

(62 636 856.0 ± 0.5 m$^2$s$^{-2}$)

- Satellite altimetry provides the coordinates $\varphi$, $\lambda$, $h$ of points $M$ describing the sea surface.
- If the sea surface topography $h_{SST}$ is reduced, points ($M_0$) on the geoid are obtained;
- Using the coordinates ($\varphi$, $\lambda$, $h-h_{SST}$) and a global gravity model (GGM), the potential value at any point $M_0$ on the geoid (i.e. $W_0$) can be computed;
- Since points on the geoid cannot be materialised in practice, $W_0$ is estimated by satisfying the condition (cf. Burša et al. 1998, Eq. [5]):

$$\int h_{SST}^2 \, dS = \min_S$$

Models applied by Burša et al. 1998:
- $S \rightarrow$ Burša’s own mean sea surface model from T/P (1993 to 1996)
- $h_{SST} \rightarrow$ POCM4b (Stammer et al. 1996)
- GGM $\rightarrow$ EGM96 (Lemoine et al. 1998)
Recent $W_0$ computations (since 2005) based on newer models of the sea surface and the Earth’s gravity field.

Recent estimations:
- 854.3, (Čunderlík and Mikula 2009), MSS: CLS01, GGM: EIGEN-GC03
- 854.4, (Sánchez 2005), MSS: CLS01, KMS04, GGM: EGM96
- 854.6, (Burša et al. 2007), MSS: J1 (2003-2005), GGM: EGM96

1998 value:
- 856.88, (Rapp 1995), best fitting ellipsoid for T/P sea surface
- 856.85, IERS Conventionen 2003, 2010

1992 standards:
- 860.0, IERS Standards 1992
- 62 636 860.850, (Moritz 2000), GRS80

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Working Group on “Vertical Datum Standardisation”

- **Common initiative of**
  - GGOS Theme 1: Global Height System
  - International Gravity Field Service (IGFS)
  - IAG Commission 2: Gravity Field
  - IAG Commission 1: Reference Frames

- **Main objective:** to provide a recommendation of the $W_0$ value to be appointed as the **reference level** of the international height reference system;

- **Term:** 2011 – 2015;

- **Members:** J. Ågren (Sweden), R. Čunderlík (Slovakia), N. Dayoub (Syria), J. Huang (Canada), R. Klees (The Netherlands), J. Mäkinen (Finland), K. Mikula (Slovakia), Z. Minarechová (Slovakia), P. Moore (United Kingdom), D. Roman (USA), Z. Šima (Czech Republic), C. Tocho (Argentina), V. Vatrt (Czech Republic), M. Vojtiskova (Czech Republic), Y. Wang (USA).
Aspects analysed within the WG-VDS:

- Different estimation methodologies but the same input models (for redundancy);
- Sensitivity of the $W_0$ estimation on the Earth's gravity field model
- Dependence of $W_0$ on the omission error of the global gravity model
- Influence of the time-dependent Earth's gravity field changes on $W_0$
- Sensitivity of the $W_0$ estimation on the mean sea surface model
- Influence of time-dependent sea surface changes on $W_0$
- Effects of the sea surface topography on the estimation of $W_0$
- Dependence of the $W_0$ empirical estimation on the tide system
- Rigorous error propagation analysis to estimate the influence of the input data uncertainties on the $W_0$ estimation.
Basic approach for the empirical estimation of $W_0$

As proposed by Sacerdote and Sansò (2001):

**Sea surface topography:**

$$
\Xi^k \cong \frac{W_S - W_0}{\bar{\gamma}^k}
$$

**Condition:**

$$
\int_{S_o} \left( W_0^k - W_0 \right)^2 dS_O = \text{min}
$$

**Solution:**

$$
W_0 = \frac{\int_{S_o} \frac{W_S}{\bar{\gamma}^2} d\sigma}{\int_{S_o} \frac{1}{\bar{\gamma}^2} d\sigma}
$$

If the sea surface topography $\Xi$ is reduced from the sea surface heights $h$ (as Burša et al. 1998 proposed):

$$
W_0 = U_0 + \frac{1}{\sum_{1}^{k} \delta S_O^k} \sum_{1}^{k} \left[ T^k - \left( h^k - \Xi^k \right) \bar{\gamma}^k \right] \delta S_O^k
$$

$$
\delta S_O^k = \cos \varphi \ \delta \varphi \ \delta \lambda
$$

For error propagation analysis

$$
W_0 = U_0 + \frac{1}{\sum_{1}^{k} \left[ \frac{1}{\bar{\gamma}^k} \right] \delta S_O^k} \sum_{1}^{k} \left[ \frac{T^k - h^k \bar{\gamma}^k}{\bar{\gamma}^k} \right] \delta S_O^k
$$
Input data for the empirical estimation of $W_0$:

- $h^k$ from a mean sea surface model:
  - CLS11 (Schaeffer et al. 2012), DTU10 (Andersen 2010)
  - own computed yearly models (1992-2013)
  - cross calibrated data from the DGFI-OpenADB (Schwatke et al. 2010) with covariance matrixes (Bosch et al. 2014), 9 missions.

- $T^k$ from a global gravity model:

- $\Xi^k$ from the (oceanographic) Model ECCO-2 (Menemenlis et al. 2008)

- $U_0, \gamma$ from GRS80

OpenADB: Open Altimeter Database, http://openadb.dgfi.badw.de
Dependence of the $W_0$ estimate on the choice of the gravity model

1) $W_0$ estimations based on models including GRACE, GOCE and Satellite Laser Ranging (Lageos) data are practically identical. Max. differences 0.01 m$^2$s$^{-2}$.

2) The use of a satellite-only gravity model is suitable. After $n = 200$ the largest differences are 0.001 m$^2$s$^{-2}$. 
Dependence of the $W_0$ estimate on the choice of the gravity model

3) Seasonal variations of the Earth’s gravity model can be neglected (max. variation 0.03 m$^2$s$^{-2}$).
Dependence of the $W_0$ estimate on the mean sea surface model

By using the models CLS11 and DTU10 there is a difference of 0.31 m$^2$s$^{-2}$, which reflects the mean discrepancy of ~ 3 cm between both models. Possible causes:

- Different strategies to process the altimetry data;
- Different reductions taken into account in each model;
- Different periods (inter-annual ocean variability).

Potential differences (divided by the normal gravity) between the estimations derived from the models MSS-CNES-CLS11 and DTU10 (computations in zero tide system with the GGM EIGEN-6C3).
Dependence of the $W_0$ estimate on the mean sea surface model

Alternative: use of yearly mean sea surface models

- The $W_0$ estimates reflect (with opposite sign) the sea level rise measured by satellite altimetry;
- Max. difference 0.46 m²s⁻²;
- These variations shall not be understood as a change in $W_0$, but in the sea level; i.e. the geoid is not growing/decreasing with the mean sea level!
- This only means that the mean sea level coincides with a different equipotential surface depending on the period utilized for the average of the sea surface heights.
Reliability of the $W_0$ estimate

Until now, all the computations assumed error free input data (MSS and GGM). By rigorous error propagation analysis (as Sjöberg 2011 proposed), the $W_0$ value estimate decreases by about 0.3 m$^2$s$^{-2}$.

Standard deviation ($\sigma_h$) of the mean sea surface heights for the year 2005.

Standard deviation ($\sigma_T$) of the anomalous potential derived from the model EIGEN-6C3 (n = 200).

Standard deviation ($\sigma_w = \sqrt{\sigma_T^2 + \gamma^2 \sigma_h^2}$) of the gravity potential values computed at the sea heights (h) for the year 2005 with the model EIGEN-6C3 (n = 200).

$W_0$ estimates assuming error free input data (blue series) and applying a proper error propagation computation (red series).
Dependence of the $W_0$ estimate on the tide system

In theory:
1) the value $W_0$ depends only on
   - the volume enclosed by the surface $W = W_0$
   - the geocentric gravity constant $GM$
   - angular velocity of the Earth’s rotation $\omega$

2) Direct and indirect effects of the tidal potential on $W_0$ are compensated by the deformation of the corresponding level surface, but the volume enclosed by this surface does not change; i.e. the potential value $W_0$ does not change.

Empirically: $W_0$ determination in the three tide systems:

\[
\begin{align*}
[W_0^{TF} - W_0^{ZT}] &= 0.0278 \text{ m}^2\text{s}^{-2} \\
[W_0^{MT} - W_0^{TF}] &= 0.0665 \text{ m}^2\text{s}^{-2} \\
[W_0^{MT} - W_0^{ZT}] &= 0.0943 \text{ m}^2\text{s}^{-2}
\end{align*}
\]
Conclusions

1) Computations carried out within the WG-VDS demonstrate that the 1998 $W_0$ value (62 636 856.0 ± 0.5 m²s⁻²) is not in agreement (and consequently it is not reproducible) with the newest geodetic models describing geometry and physics of the Earth.

2) The 1998 $W_0$ value is not suitable as a conventional reference value and a better estimate for $W_0$ has to be adopted by the IAG for the definition and realization of the IHRS.

3) As reference level, the conventional value $W_0$ has to be fixed (without time variations); but it has to have a clear relationship with the sea surface (as convention for the realization of the geoid).
4) We propose to adopt the potential value obtained for the year 2010 after fitting the yearly $W_0$ estimations by means of a lineal regression:

\[ W_0 = 62,636,853.353 \text{ m}^2\text{s}^{-2} \]

rounded to

\[ W_0 = 62,636,853.4 \text{ m}^2\text{s}^{-2} \]

5) The formal error of this value is $\pm 0.02 \text{ m}^2\text{s}^{-2}$. However, as convention the adopted $W_0$ is understood free of error.

6) The introduction of a reference $W_0$ value is not accepted by the whole geodetic community. There are a variety of approaches to avoid a $W_0$ value.

7) Results provided by the WG-VDS are for those approaches requesting a reliable $W_0$ value.