Realisation of a vertical reference system for South America as a densification of an International Height Reference System

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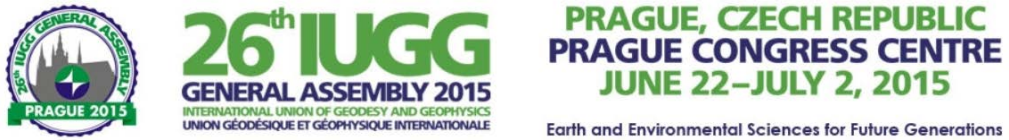
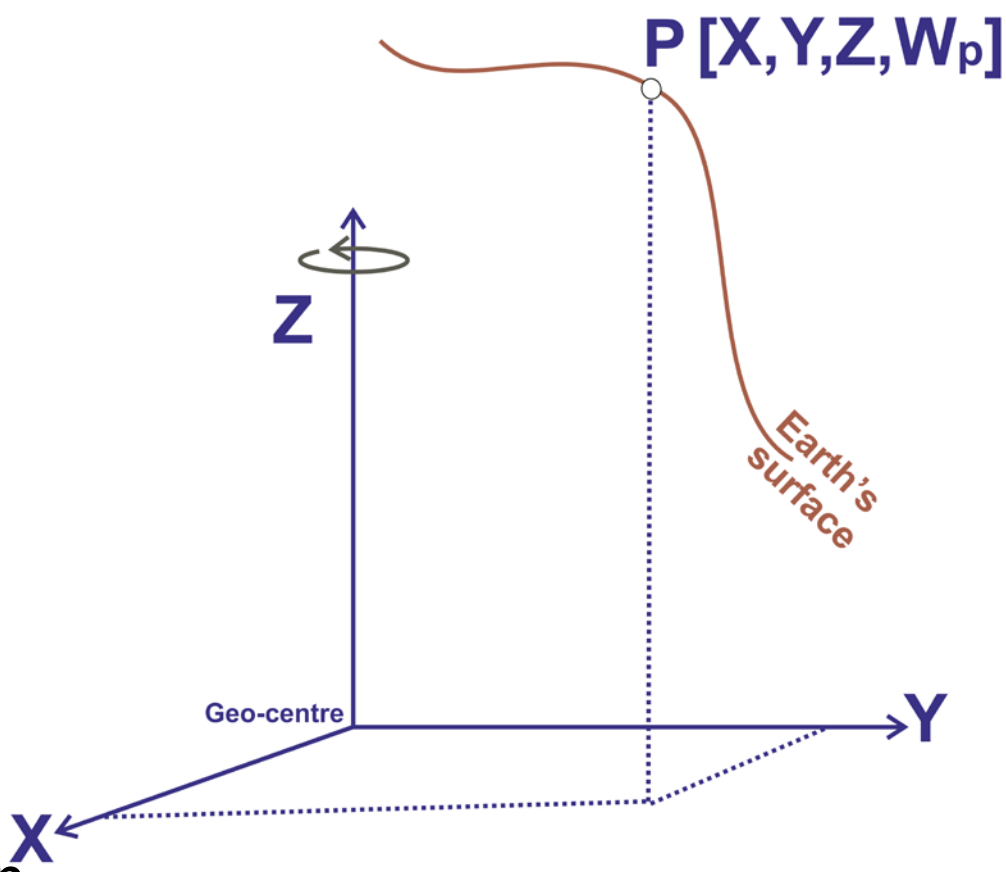
SIRGAS: Geocentric Reference System for the Americas



International Height Reference System - IHRS

1) IHRS: Geopotential reference system co-rotating with the Earth.
2) Coordinates of points attached to the solid surface of the Earth are given by
   - **geopotential values $W(\mathbf{X})$** (and their changes with time $dW(\mathbf{X})/dt$), and
   - **geocentric Cartesian coordinates $\mathbf{X}$** (and their changes with time $d\mathbf{X}/dt$) in the ITRS.



See talk tomorrow:
Ihde et al.: Definition and realisation of an International Height Reference System

International Height Reference System - IHRS

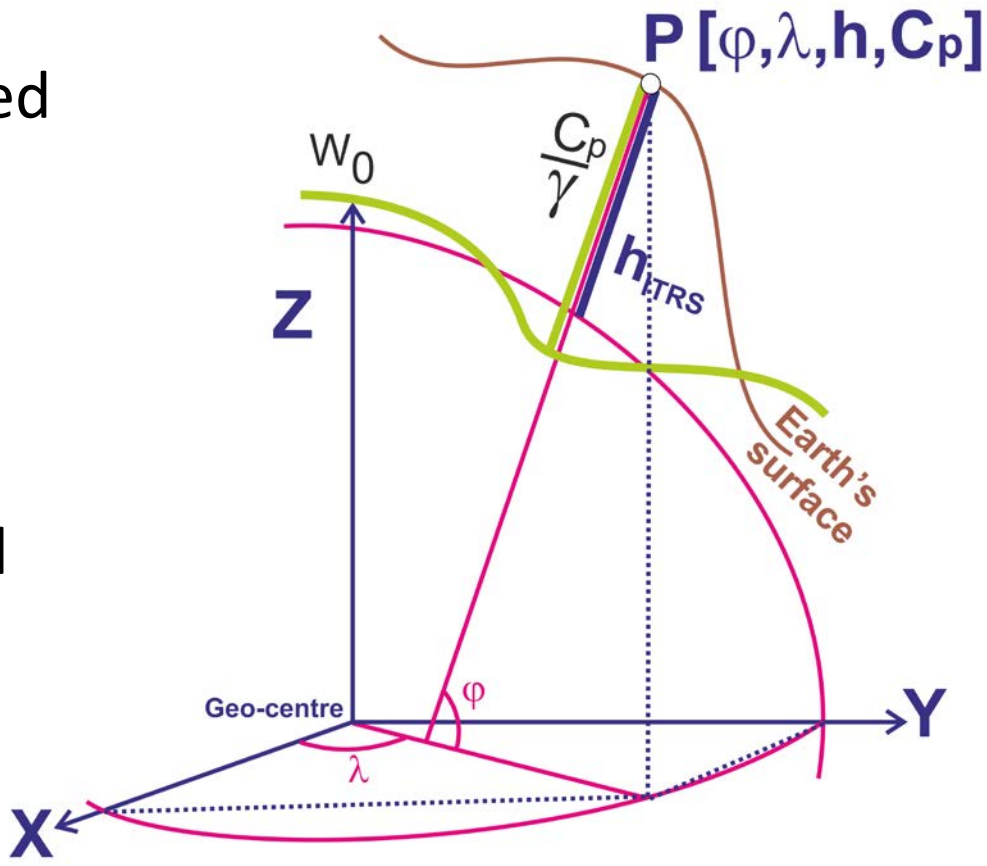
For practical purposes, potential values $W(\mathbf{X})$ and geocentric positions $\mathbf{X}$ are to be transformed into vertical coordinates with respect to a reference level:

1) geometrical component

- $h(t_0,\mathbf{X})$; $dh(\mathbf{X})/dt$
- conventional level ellipsoid
  $U_0 = const.$

2) physical component

- $C_p(t_0,\mathbf{X})$; $dC_p(\mathbf{X})/dt$
- conventional fixed value
  $W_0 = const.$



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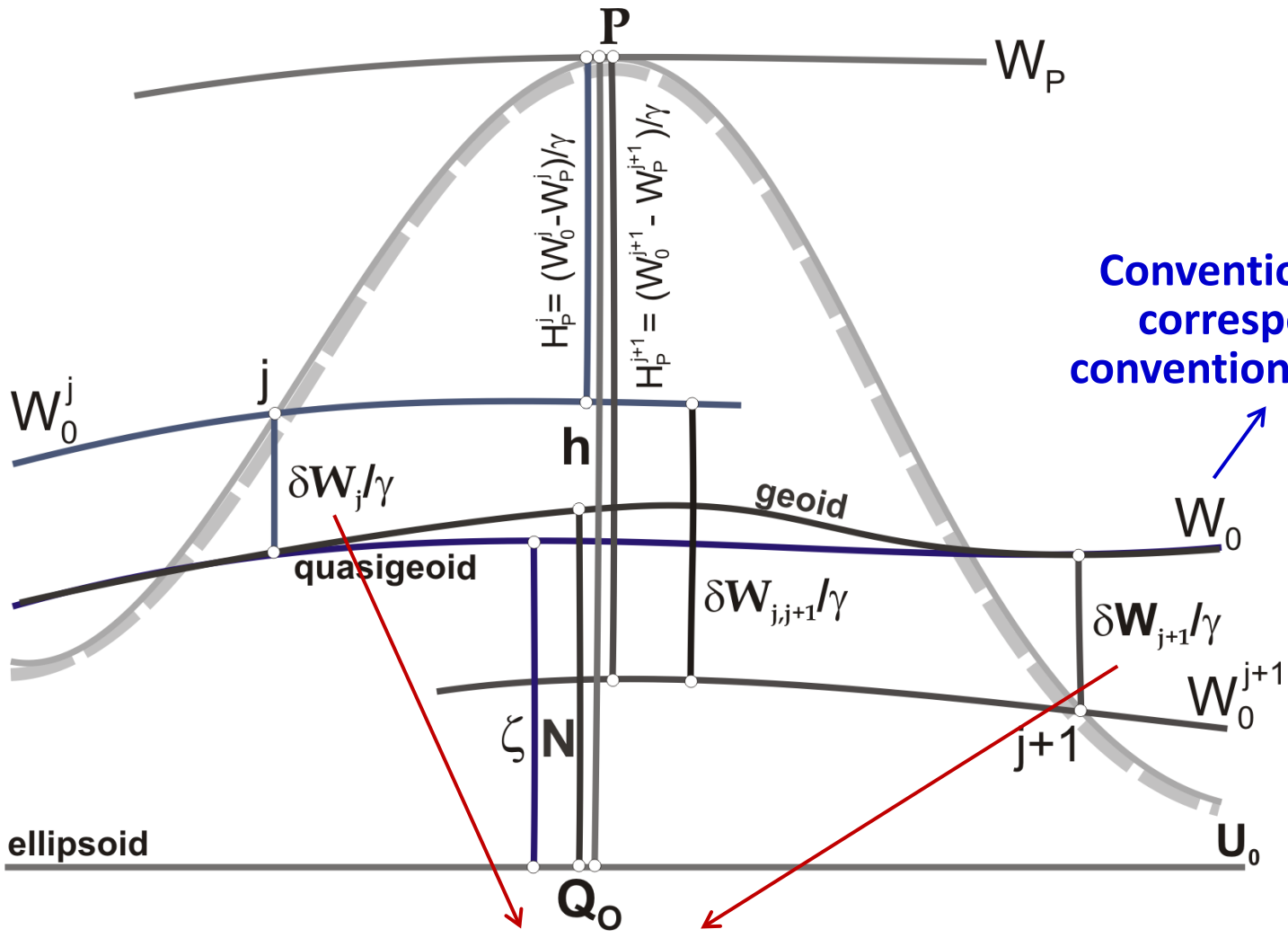
Remarks on the vertical reference level W_0

- W_0 = potential value of the geoid;
- Since there is an infinite number of equipotential surfaces, the **geoid** is to be **defined arbitrarily by convention**;
- Geoid = equipotential surface of the Earth's gravity field that best fits (in a least square sense) the mean sea level:
- Since the mean sea level changes, a **convention** about mean sea level (i.e. epoch, area) is also needed;
- Reference level ellipsoid = ellipsoid that best fits the geoid;
- Therefore, it is expected that $U_0 = W_0$
- Proposal for the IHRS:
  - the **current (2015) best-estimated value for W_0** shall be defined (and fixed) as the potential value of the geoid:

$$U_0 = W_0 = 62\,636\,853.4\ \mathrm{m^2 s^{-2}}$$

See talk tomorrow: Sánchez et al.: A new best estimate for the conventional value W_0

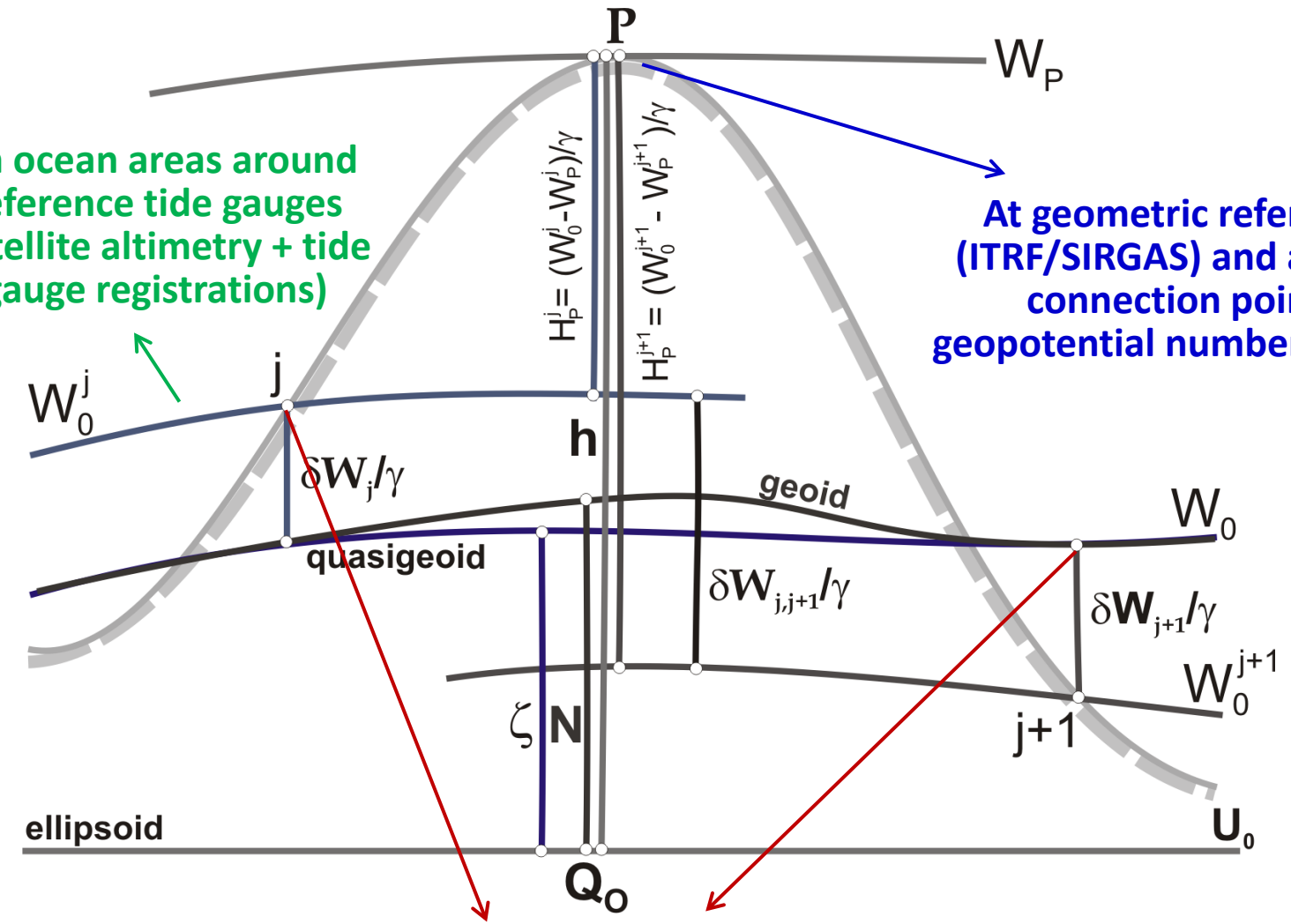
Integration of local (regional) physical height systems into the IHRS



Conventional W_0 value corresponding to a conventional global geoid

Transformation parameters between local levels W_{0i} and the global one W_0

Integration of local (regional) physical height systems into the IHRS



On ocean areas around reference tide gauges (satellite altimetry + tide gauge registrations)

At geometric reference stations (ITRF/SIRGAS) and at international connection points (GNSS + geopotential numbers from levelling)

Solution by combined least squares adjustment.

At reference tide gauges (GNSS + tide gauge registrations)

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Observation equations for the vertical datum unification

(after Rummel und Teunissen 1988, Heck and Rummel 1990)

- at border points connecting neighbouring vertical datum zones:

$$H^{N,i+1}(P) - H^{N,i}(P) = q\left(\delta W_0^{i+1} - \delta W_0^{i}\right)$$

- at tide gauges, levelling nodes, geometric reference stations

$$h(P) - H^{N,i}(P) - q\Delta W_0 - E(\zeta(P)) = e^i(P)\,\delta W_0^i + \sum_{\substack{j=1 \\ i\neq j}}^{J} f_0^j(P)\,\delta W_0^j(P)$$

$$\Delta W_0 = W_0 - U_0$$

GNSS positioning on land and satellite altimetry on sea areas around tide gauges

heights from geop. numbers on land and sea surface topography around tide gauges

height anomalies from GBVP [GGM (n=200) + terrestrial gravity + terrain models]

vertical datum discrepancies (to be determined)

indirect effects (negligible)

$$q := -\frac{1}{\gamma}, \quad e^i(P) := -q + f_0^i(P), \quad f_0^i(P) := \frac{1}{2\pi\gamma}\iint_{\sigma_i} S\left(\psi_{P,P_k}\right) d\sigma$$

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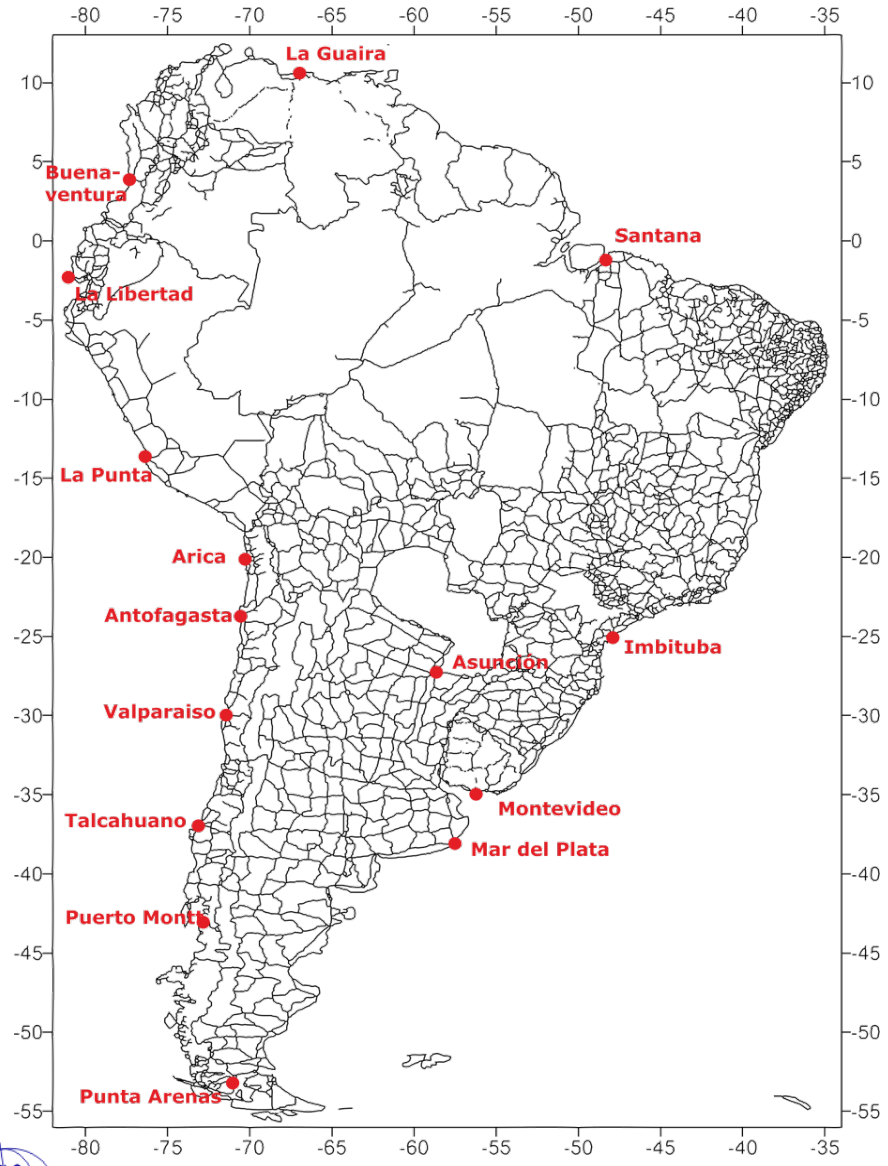
Existing height systems in South America



- *15 reference tide gauges*;
- mean sea surface level referred to *different epochs* (some unknown);
- Levelling *since ~1940* with *dH/dt = 0*;
- in general *no gravity reductions applied*;
- *no common adjustment*;
- First and second order levelling networks comprise more than *360 000 km* and *200 000 bench marks*.

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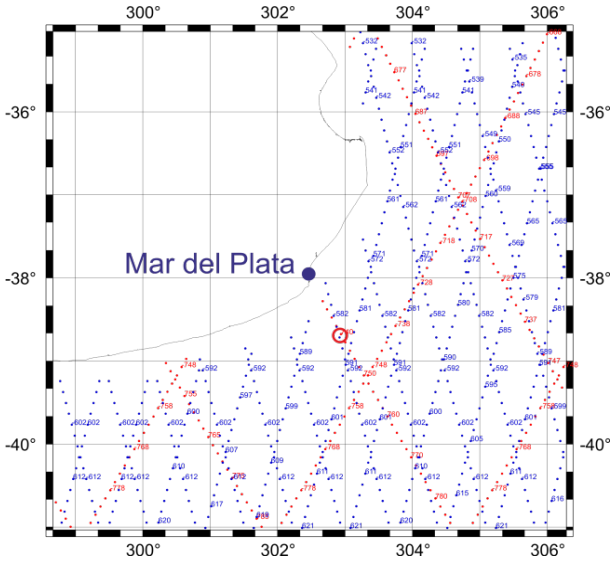
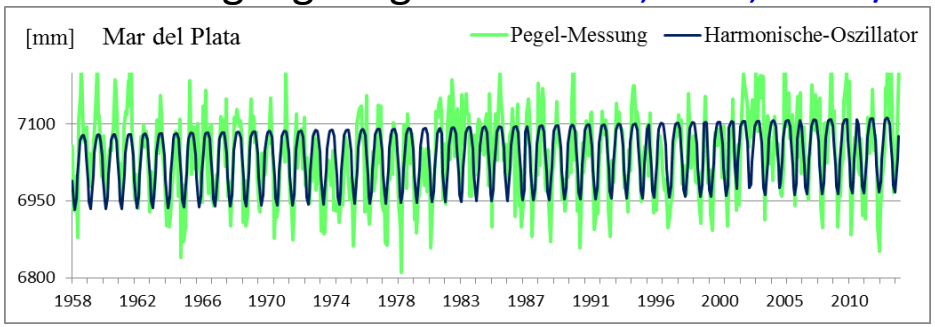
Observables available in South America for the regional realisation of the global IHRS

Trend from satellite altimetry: 2.4 ± 0.8 mm/a



Trend from gauge registrations: 0,6 ± 0,2 mm/a



Trend from GPS: 2.2 ± 2.2 mm/a

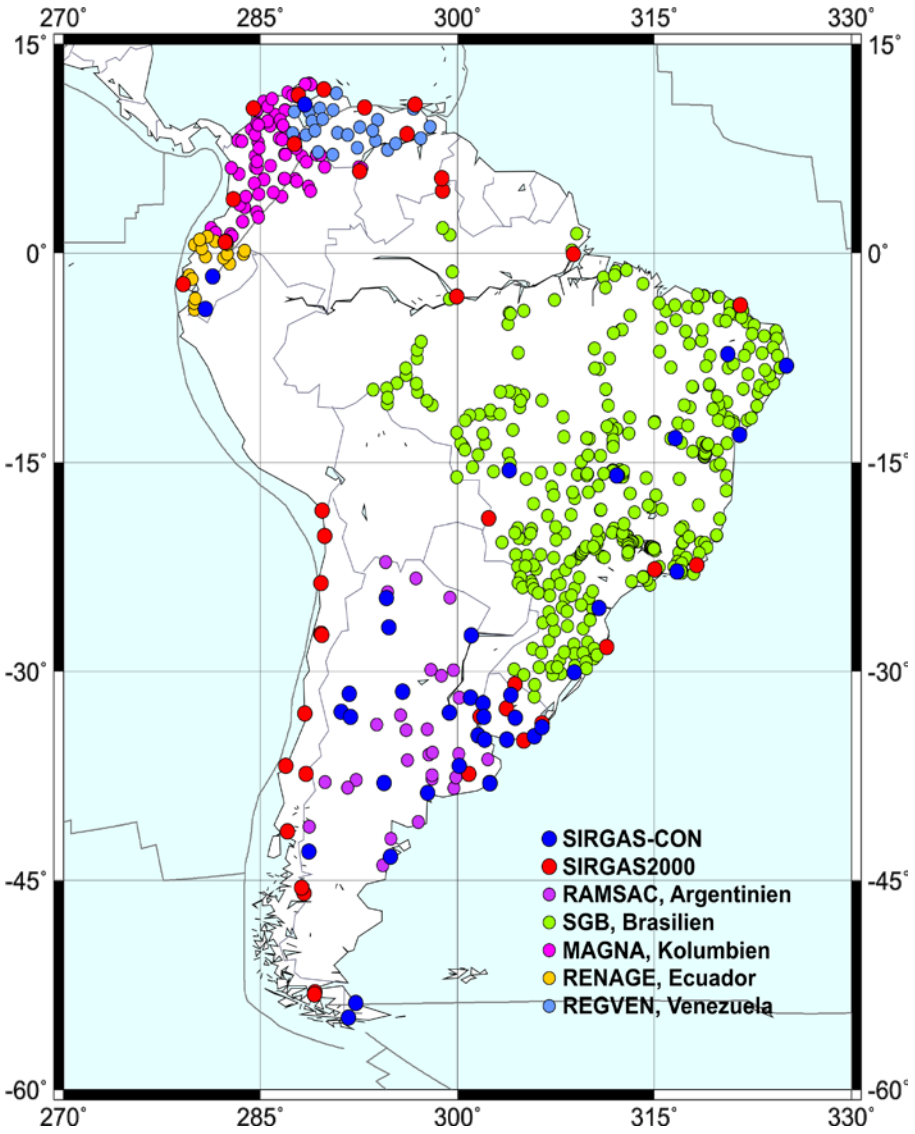
→ Discrepancy: 2.4 – (2.2 + 0.6) = 0.8 mm/a

Geometric heights in sea areas around tide gauges

- mean sea surface heights from satellite altimetry (OpenADB);
- Tide gauge registrations from PSMSL;
- GNSS positioning at tide gauges.
- **Data standardisation** (TIGA objectives):
  - Determination of *vertical trends* from satellite altimetry, tide gauge registrations, and GPS;
  - It is assumed that the trends $(dh/dt)_{Altimetry} = (dh/dt)_{(Gauge + GPS)}$
  - Reduction of the reference sea levels to a common *epoch (2005.0)*.

OpenADB: Open altimetry database @ DGFI-TUM
PSMSL: Permanent Service for Mean Sea Level

Observables available in South America for the regional realisation of the global IHRS



Geometrical heights on land areas

- Reference stations (663):
  ITRF stations (10) +
  SIRGAS stations (74) +
  national densifications (579);
- **Data standardisation**:
  - Transformation of previous ITRF solutions to the IGb08;
  - Stations positions given at a *common epoch (2005.0)* (with station velocities or a kinematics model - VEMOS);
  - Transformation *from conventional tide-free to zero-tide*.

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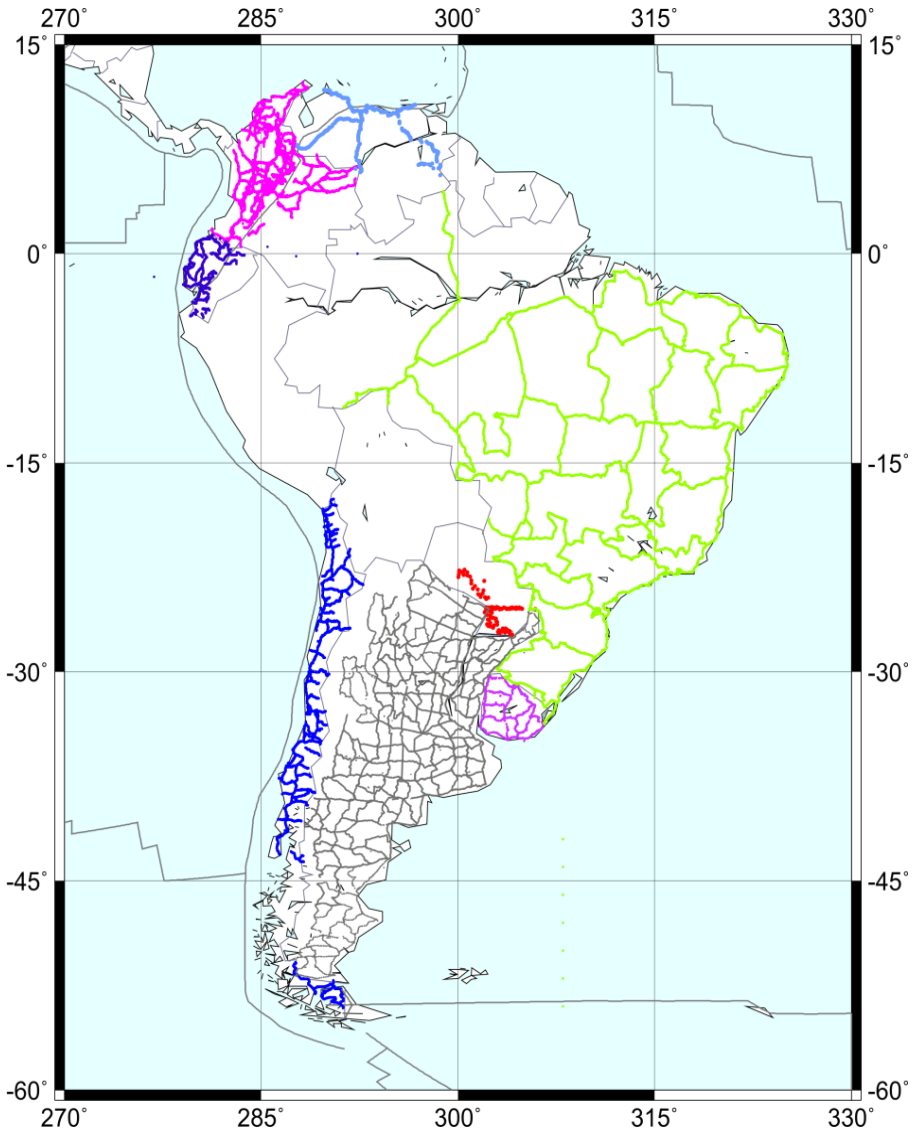
Observables available in South America for the regional realisation of the global IHRS



Observed height differences

- Levelling lines provided by the countries
- **Data standardisation**:
  - least squares adjustment country by country to build *free normal equations for each vertical datum zone*;
  - astronomical correction + indirect effect (levelling in *zero-tide system*);
  - *kinematic adjustment* assuming $dH/dt \approx dh/dt$;
  - *combination of free normal equations* for countries with international levelling connections

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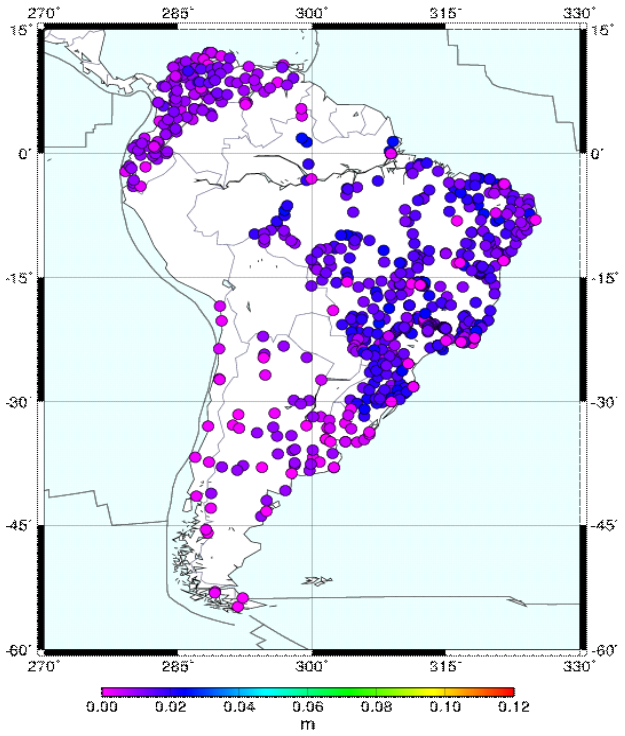
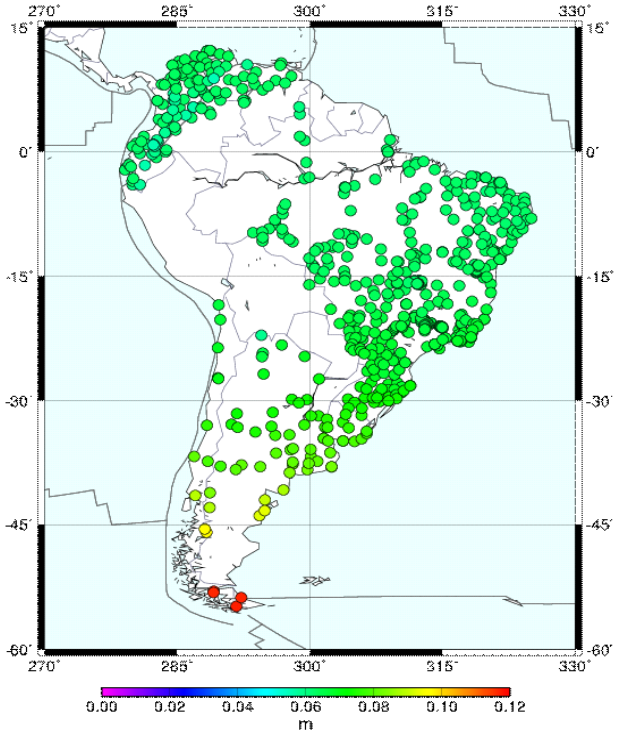
Observables available in South America for the regional realisation of the global IHRS

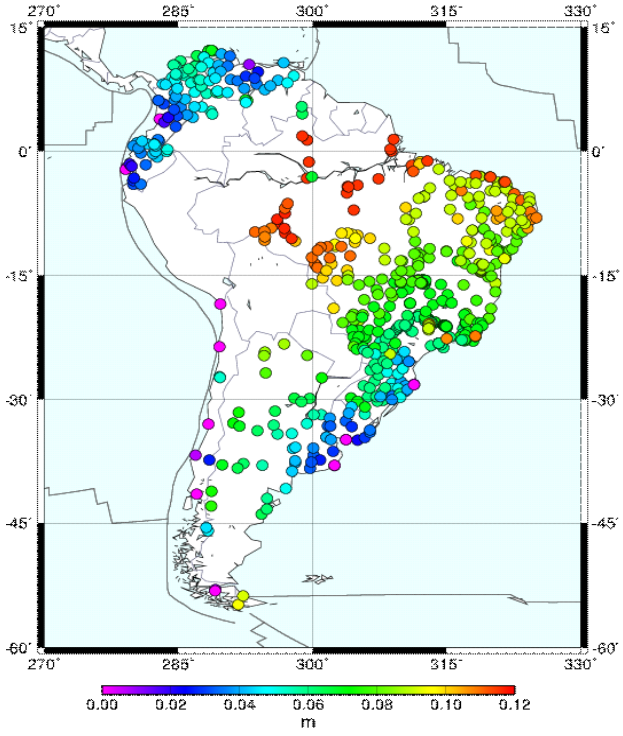
Uncertainty of the input data



Ellipsoidal heights



Height anomalies



Normal heights

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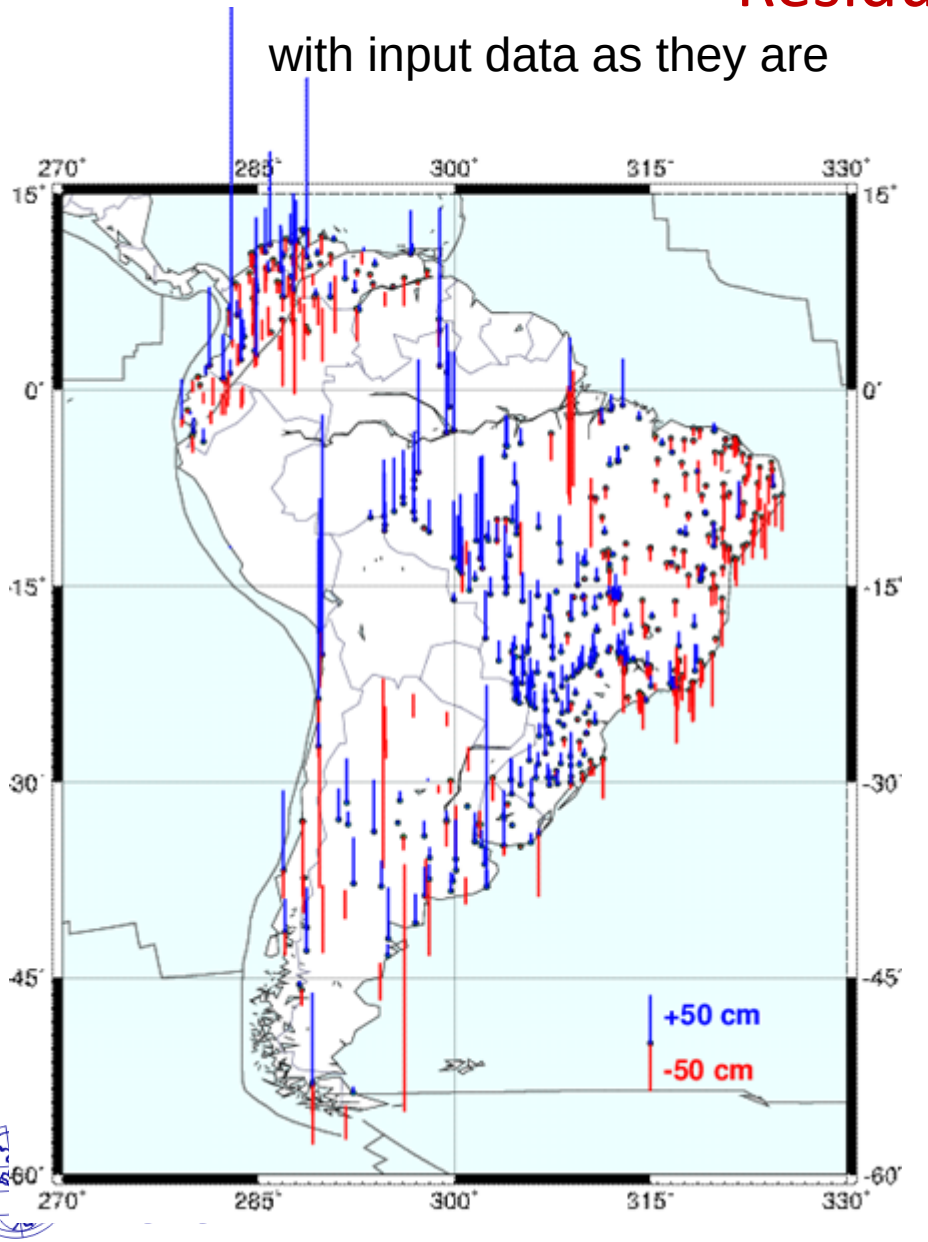
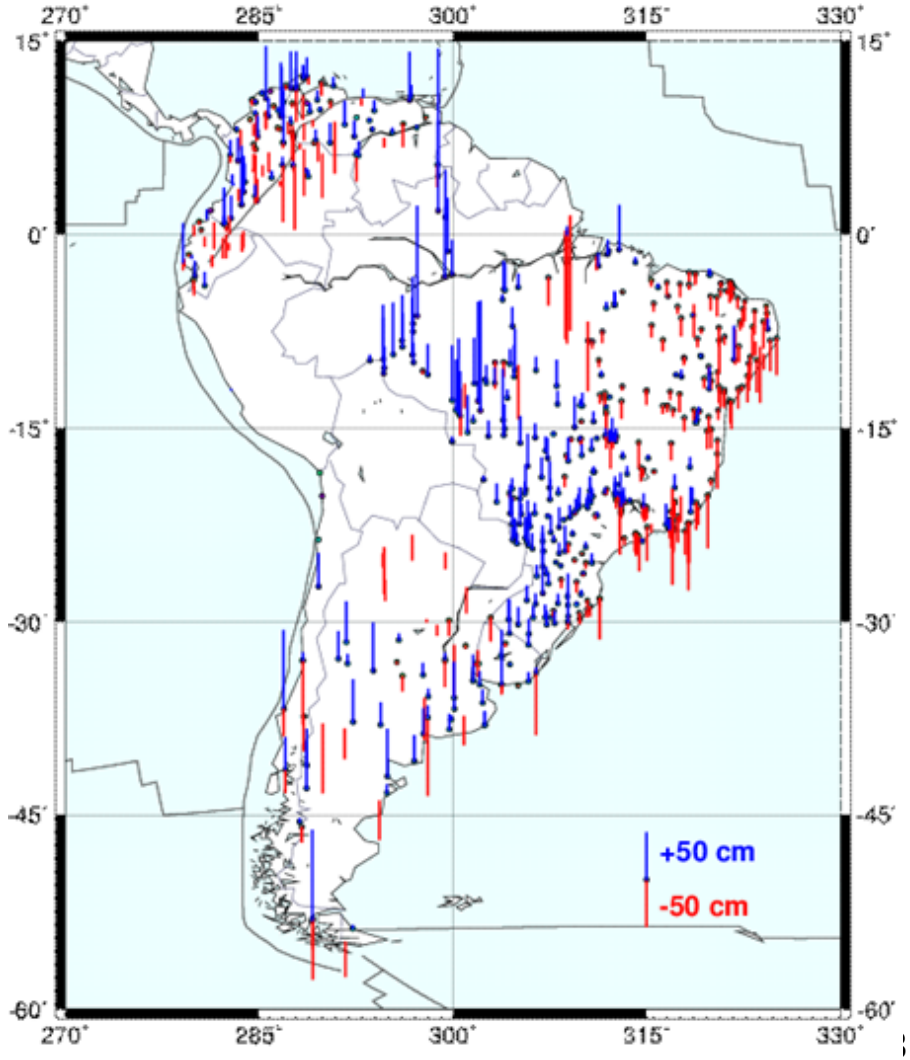
Residuals

with input data as they are

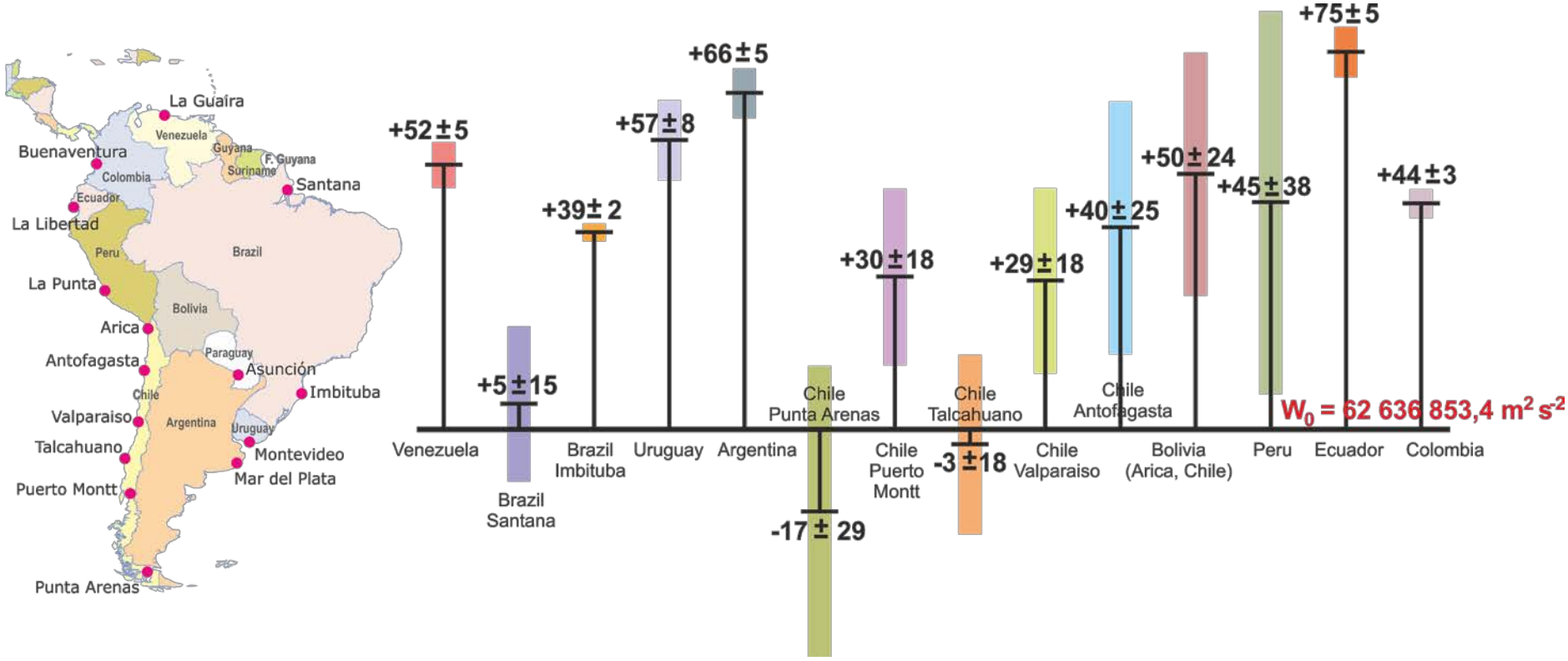


with input data standardized



Realisation of a vertical reference system for South America as a densification of the IHRS

Vertical datum parameters with respect to W_0= 62 636 853.4 m^2s^{-2}



- Uncertainty of about **±5 cm** in those countries with **good data coverage**;
- Uncertainty of about **±20 ... 40 cm** in those countries with **poor data coverage** (similar uncertainties have been found by other authors in other regions, e.g. Gruber et al. 2012, Rülke et al. 2014, Gerlach and Rummel 2013).

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Final comments

- Observation equations at **tide gauges** are required for the integration of **isolated zones** (where levelling connection is not possible, e.g.: Southern part of Chile, between Colombia and Panama, islands);
- More SIRGAS reference stations and more connections between countries must be levelled in order to get more observation equations, i.e. to **increase redundancy**;
- The vertical datum unification requires essentially **levelling-based geopotential numbers** and **(quasi)-geoid models of high-resolution**;
- The precise combination of physical heights, ellipsoidal heights and (quasi-)geoid models requires a **standardisation of conventions, constants, and procedures** (e.g. tide system, reference epoch for vertical positions, etc.).