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Consistent adjustment of combined terrestrial and celestial reference frames

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Abstract. Today, the realization of the International Terrestrial Reference System (ITRS) and the International Celestial Reference System (ICRS) is performed separately. Consequently, the two realizations are not fully consistent and show differences in the network geometry and the realized scale of the terrestrial reference frame (TRF) and in the simultaneously estimated Earth Orientation Parameter (EOP) series. The paper deals with the common adjustment of the Terrestrial Reference Frame (TRF), the Celestial Reference Frame (CRF) and the linking EOP. It presents a computation strategy, which is based on the combination of normal equations. The main focus of the paper is on the impact of a combination of different space geodetic techniques on the CRF, which was not in detail studied so far. The influence of the local tie handling as well as the impact of the EOP combination are studied. The results show, that the combination leads only to small but partly systematic changes of the CRF. The maximum systematic effect is about 0.5 mas for source positions derived in regional networks only.

Keywords. International Terrestrial Reference Frame · International Celestial Reference Frame · EOP · VLBI · SLR · GNSS · local ties · combination of normal equations

1 Introduction

The International Terrestrial Reference System (ITRS) and the International Celestial Reference System (ICRS) are realized separately and by different processing centres today (*Altamimi et al.* (2011) and *IERS* (2009), respectively). Consistency between the two realizations - the International Terrestrial Reference Frame (ITRF) and the International Celestial Reference Frame (ICRF) - is achieved to a certain extent by an alignment of the VLBI station network to the ITRF within ICRF computation and by fixing the source coordinates on ICRF2 in the IVS normal equation systems contributing to ITRF. However, consistency of the frames can be ensured by a simultaneous estimation of both frames and the linking Earth Orientation Parameter (EOP) series. The paper deals with the consistent computation of the Terrestrial Reference Frame (TRF) and the Celestial Reference Frame (CRF) based on the combination of the space geodetic techniques Very Long Baseline Interferometry (VLBI), Satellite Laser Ranging (SLR) and Global Satellite Navigation Systems (GNSS). While the effect of combining the four techniques on the station coordinates and EOP was well studied for the different ITRF computations and also in other combination projects (*Altamimi et al.* (2011), *Rothacher et al.* (2011), *Seitz et al.* (2012)), the special interest of this paper is on the effect of the combination on the CRF. In *Seitz et al.* (2011) first results of the simultaneous adjustment of TRF and CRF are published. The present paper shows results of a more detailed investigation. Open questions regarding systematic effects in the results can now be answered.

2 Realization of ITRS and ICRS

The realization of the ITRS is based on the combination of the four space geodetic techniques VLBI, SLR, GNSS and DORIS (*Altamimi et al.* (2011), *Seitz et al.* (2012)). Parameters common to all techniques are station coordinates and EOP, i.e. the coordinates of the terrestrial pole and Length of Day (LOD), which is related to ΔUT1=UT1-UTC by LOD= $-\delta\Delta\text{UT1}/\delta\text{t}$. The station networks and the EOP series are estimated consistently. The positions of the radio sources are fixed to the ICRF2 (*IERS*, 2009), the latest realization of the ICRS (Fig. (1)). The four techniques provide individual potentials regarding the determination of geodetic parameters. SLR observations are very sensitive to the centre of mass of the Earth. Thus, the origin of the ITRF is defined to be realized from SLR observations only. VLBI and SLR provide the scale with a high accuracy. Therefore, the ITRF scale is realized as a weighted mean of VLBI and SLR scale. The strengths of the GNSS are the high density of the station network and the high precision of the GNSS-derived parameters. Hence, GNSS is very important for a stable linking of all techniques.

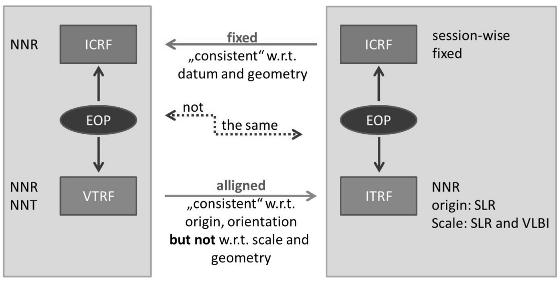

Fig. 1: Current situation for ITRF and ICRF computation. Abbreviations: NNR = no-net-rotation, VTRF = VLBI Terrestrial Reference Frame.

The ICRS is realized from VLBI observations only (Fig. (1)). VLBI observes extragalactic radio sources and is, therefore, the only technique that allows for the determination of the TRF and the CRF. In the current realization (ICRF2) the TRF – consisting of the VLBI network only, VTRF – is aligned (via VTRF2008 (*Nothnagel*, 2009)) to ITRF2005 (reference epoch 2000 Jan 01, 0 h) with respect to origin and orientation by no-net-translation and no-net-rotation conditions, respectively. The scale is realized from the considered VLBI observations. The ICRF2, the VTRF and the related EOP series are estimated consistently in one adjustment (*IERS*, 2009). The solution used for generating ICRF2 is the "gsf008a" solution, run by the VLBI group of the Goddard Space Flight Center (GSFC).

Due to the separate computation of TRF and CRF, deficiencies in consistency of the two frames exist: (i) the VLBI station networks of both realizations differ with respect to the scale: While the scale of the VTRF is realized from VLBI only, the ITRF scale is a weighted mean of the SLR and VLBI scales; (ii) the two station networks differ with respect to the network geometry, which is slightly changed by the combination due to discrepancies between local ties and coordinate differences derived from the space geodetic techniques; (iii) the variations of the EOP series differ significantly, because the one resulting from the ICRF computation is based on VLBI observations only, while the series related to the ITRF is derived from a combination. In order to achieve full consistency of all parameters, station coordinates, source coordinates and the EOP must be adjusted simultaneously as shown in Fig. (2). The realization of the geodetic datum of a TRF-CRF solution is performed as it is conventional: The origin is realized by SLR observations and the scale as a weighted mean of VLBI and SLR scales. The orientation of the station network is realized by applying no-net-rotation conditions (NNR) with respect to the ITRS realization DTRF2008 (*Seitz et al.*, 2012). The orientation of the CRF is realized by a no-net-rotation condition with respect to the ICRF2.

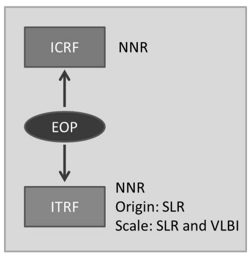

Fig. 2: Simultaneous determination of ITRF and ICRF.

3 Combination strategy

The computation strategy used is based on the combination of constraint-free normal equations

(NEQ). Constraint-free in the context of this paper means, the normal equations are free from pseudo-observations related to the realization of the geodetic datum. Constraints necessary for the stabilization of weak parameters (e.g., tropospheric or clock parameters) are included. The parameters, which are combined are station coordinates, the terrestrial pole coordinates, LOD and nutation parameters. The terrestrial pole, UT1-UTC and the nutation paramters (the latter only for VLBI and GNSS) are set up using a piece-wise-linear representation with estimates at 0 h for all techniques, even if the satellite techniques do not allow for determining UT1-UTC and nutation in an absolute sense but only for the determination of LOD and nutation rates. LOD and the nutation rates are represented indirectly in this parametrization by the linear relation between two subsequent UT1-UTC or nutation parameters, respectively.

The combination software used is the sublibrary for combination applications (CS) of the DGFI Orbit and Geodetic Parameter Estimation Software DOGS (*Gerstl et al.*, 2001). The input data are homogeneously processed time series of session-wise VLBI, daily GNSS, and weekly SLR normal equations (*Rothacher et al.* (2011), *Seitz et al.* (2011). Table (1) gives an overview of the input data.

Table 1: Input data.

Technique	Time span	Resolution	Institution
VLBI	1984-2007	session (24h)	combined: DGFI,IGG
GNSS	1994-2007	daily	GFZ
SLR	1993-2007	weekly	DGFI

The combination strategy is shown in Fig. (3). First, the time series of NEQ are solved and the resulting time series of the station positions, EOP and datum parameters are analyzed in order to identify discontinuities and outliers, which have to be considered in the combination. After that, the NEQ series are accumulated to one NEQ per technique. Thereby, station velocities are introduced as new parameters.

In a second computation step, the different technique-specific NEQ are combined. As observations of different space geodetic techniques are usually not related to common reference points, terrestrial difference vectors (local ties) between closely located instruments at so called co-location sites must be introduced to combine the networks. Figure (4) shows the global distribution of the co-location sites.

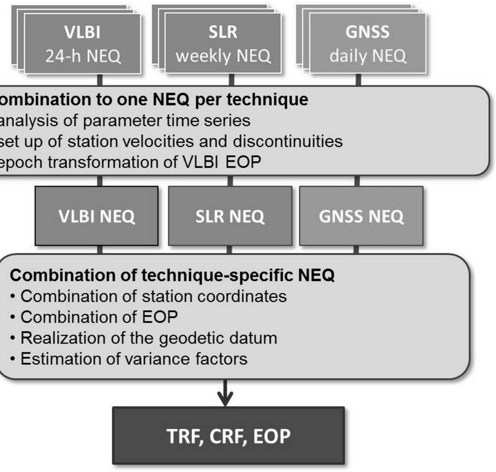

Fig. 3: Computation strategy for a simultaneous TRF-CRF determination.

Differences between the local ties and the results of the space geodetic techniques, which can reach up to several centimetres, are an accuracy limiting factor. Thus, local ties must be introduced applying adequate standard deviations, which allow for minimizing the deformation of the combined networks and guarantee at the same time a stable combination of the networks (*Seitz et al.*, 2011). In order to investigate the impact of local ties on the estimates, four solutions were set up with different levels of standard deviations for the local ties. The standard deviations are computed from the discrepancies between local ties and space geodetic techniques by dividing them by quotients of 1, 2, 3 and 6. E.g., for a discrepancy of 3 mm, standard deviations of 3.0, 1.5, 1.0 and 0.5 mm are obtained. Station velocities of co-locating instruments are combined by introducing pseudo-observations in form of equality equations applying a standard deviation of 0.1 mm/yr. In contrast to the station coordinates, EOP are "real" common parameters. Thus, the corresponding elements of the different NEQ are accumulated in the combination. The datum of the combined solution is realized as described in Sec. (2). Stable sources according to *Feissel-Vernier et al.* (2006)

are used to set up the no-net-rotation conditions for realizing the orientation of the CRF.

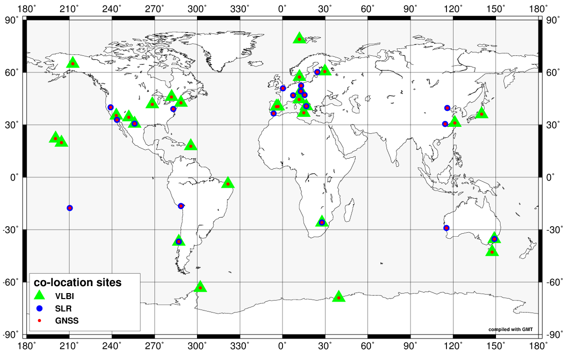

Fig. 4: Global distribution of co-location sites used.

In addition to the combined TRF-CRF solutions a VLBI-only solution was computed for comparisons. The datum was realized applying no-net-translation and -rotation conditions with respect to DTRF2008. Furthermore, a special combined solution was performed, in which CRF, TRF and EOP are estimated but only the EOP series are combined and no local ties are introduced. The datum of the station network was realized by setting up individual conditions per technique, which are all related to DTRF2008 in order to ensure consistency. Table (2) gives an overview about the different solution types.

Table 2: Solution types performed. The last column gives the quotient used for the computation of the standard deviations of the local ties.

Solution	Parameters combined	Quotient
VLBI-only	-	-
EOP-combined	EOP	-
TRF-CRF 1	TRF, EOP	1
TRF-CRF 2	TRF, EOP	2
TRF-CRF 3	TRF, EOP	3
TRF-CRF 6	TRF, EOP	6

4 Combination results

The effect of the combination on the CRF is expected to be twofold. On the one hand, the combination of the station networks by introducing local ties might have an impact in the CRF. On the other hand, the combination of the EOP series is assumed to have an impact on the source coordinates, in particular on such sources, which are observed only in one or two sessions. Figure (5) shows the declination (DE) and right ascension multiplied by cos(DE), RA·cos(DE), of the combined solutions with respect to the VLBI-only solution. Figure (6) shows the differences for the datum sources only.

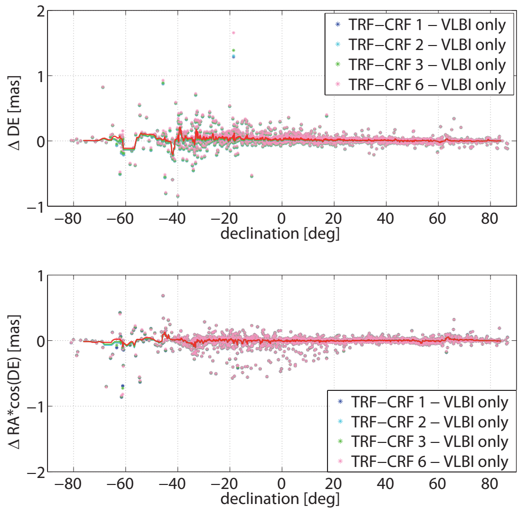

Fig. 5: DE and RA·cos(DE) of the four combined solutions with respect to the VLBI-only solution. All sources are considered. The solid lines represent the sliding medians.

The results obtained from the different solution types are very similar. Considering the datum sources only, only a very small systematic difference in DE can be identified (Fig. (6)): The smaller the standard deviations for the local ties is, the larger is the DE-shift of sources. The values of the sliding median of the DE-shift itself increases from North to South for all solution types and has a maximum of 0.04 mas at about -40° of DE for the solution TRF-CRF 6. With respect to RA·cos(DE) the solution types do not show any systematic difference for the datum sources. The systematic effect found in RA·cos(DE) when considering all sources (Fig.(5)) will be discussed later in the paper.

In order to study the effect of local ties in more detail, the station positions of VLBI co-location sites are analyzed. Figure (7) compares the horizontal components of station position dif-

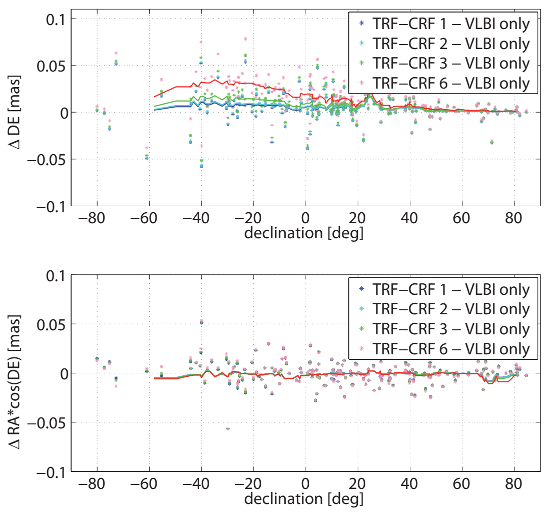

Fig. 6: DE and RA·cos(DE) of the four combined solutions with respect to the VLBI-only solution. Datum sources are considered only. The solid lines represent the sliding medians.

ferences for the four combined solutions. Most of the co-location sites show small differences for all TRF-CRF solutions. But, in particular for TIGO (Concepciòn, Chile) and O'Higgins (Antarctica) an increase of the differences can be found if smaller standard deviations for the local ties are applied. Due to the low density of the VLBI station network on the southern hemisphere, the systematics in the source positions (Fig. (6)) might be related to the two co-location sites.

The influence of the local ties on the CRF is only one aspect that has to be considered. The implementation of local ties is also crucial for the homogeneity of the combined network. In order to measure the homogeneity, the origin realized in the VLBI part of the combined network is analyzed. The origin of the combined solutions is realized from SLR observation only and is transferred to the VLBI part of the combined network via local ties. Figure (8) shows the translation parameters of the VLBI part of the network with respect to DTRF2008. The smallest transformation parameters of less than 2 mm are obtained for the solution TRF-CRF 2 and TRF-CRF 3. In Fig. (7) the deformation of the network due to the combination was shown for the horizontal components. The lower plot of Fig. (8) gives the mean deformation values considering all three components of station position. The results of Fig. (6), Fig. (7) and Fig. (8) show, that when introducing local ties, the two contradicting requirements – a minimal network deformation and a maximal consistency of the combined frame – must be balanced. A quotient of 2 or 3 for the computation of the local tie standard deviations from the local tie discrepancies provides a homogeneous frame and keeps the mean network deformation of TRF and CRF limited to 1 mm and 0.02 mas, respectively.

Besides the local ties, also the combination of the EOP has an effect on the CRF. The RA·cos(DE) differences plotted in Fig. (5) show a systematic effect for some sources with a DE between about -40° and +30°. In maximum the effect reaches about 0.5 mas. The datum sources (Fig. (6)), however, are not affected. In order to analyse this effect, a combined solution was performed in which only the EOP but not the station networks are combined. Thus, the CRF can only be influenced by the EOP combination but not by the local ties. Fig. (9) shows the RA·cos(DE) values of the combined solution TRF-CRF 2 with respect to the EOP-combined solution.

Comparing Fig. (9) with the results in Fig. (5) it can be seen that the systematic effect visible in Fig. (5) vanishes comparing two solutions with combined EOP. That means, the effect can be related to the combination of EOP only, while the combination of the station networks does not contribute to this systematic effect. It becomes also visible very well, that especially sources closely located to the celestial South pole are shifted due to the combination of the station networks. This confirms again that co-location sites on the southern hemisphere have the largest impact.

Investigating the sources affected by the systematics in RA·cos(DE) it is found, that 108 sources observed in only 21 sessions show a difference to the VLBI-only solution of larger than 0.1 mas. 18 of the sessions are VLBA (Very Long Baseline Array, http://www.vlba.nrao.edu/) sessions in which 105 of the 108 sources are observed, most of them by one session only and some under low elevation angles (the mean latitude of the regional VLBA network is about 34°). Going further into detail, it can be found that 96 of the sources are observed by only three VLBA sessions (Fig. (10)).

It can be concluded, that the combination of

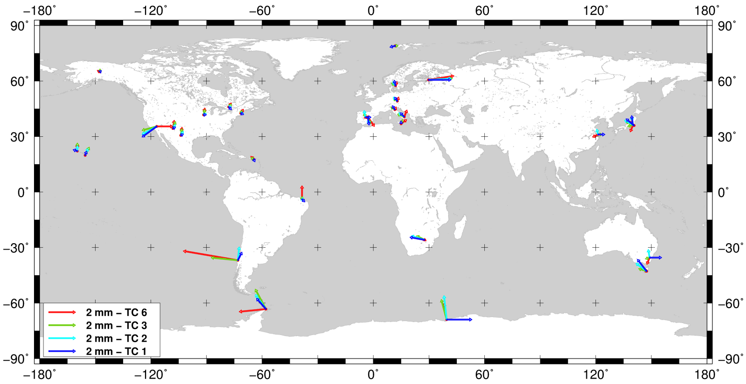

Fig. 7: Station position differences of VLBI co-location sites for the four combined TRF-CRF solutions (TC) with respect to the VLBI-only solution.

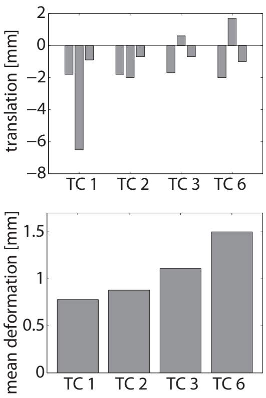

Fig. 8: Translation parameters [x,y,z] of VLBI part of combined solutions TRF-CRF 1, 2, 3, 6 (TC1, TC2, TC3, TC6) with respect to DTRF2008 (top). Mean change of network geometry (mean network deformation) (bottom).

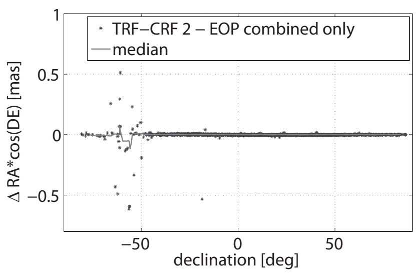

Fig. 9: RA·cos(DE) of combined solution TRF-CRF 2 with respect to a solution in which only the EOP but not the station coordinates are combined.

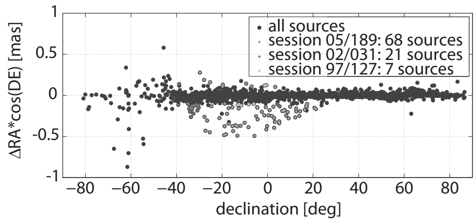

Fig. 10: RA·cos(DE) of TRF-CRF 2 with respect to VLBI-only. Most of the sources affected by the systematics are observed by three VLBA sessions only (grey coloured).

EOP can have a systematic effect on such sources of the CRF, which are observed rarely and by regional sessions only. The reason can be the differences between the EOP derived from the global GNSS and SLR networks and the VLBI estimates derived from regional sessions. With respect to the standard deviations of the RA angles of the corresponding sources of 0.4 mas or larger, the systematic effect is not significant. However, a discussion on sources in the ICRF observed in a few regional sessions only might be worthwhile.

5 Summary and Conclusions

The results presented in the paper show, that a simultaneous estimation of TRF, CRF and the EOP series is possible and provides promising results. The origin is realized from SLR observations, the scale as a weighted mean of VLBI and SLR scales, and only the orientation of the TRF and the CRF have to be realized by conditions:

non-deforming no-net-rotation conditions are applied. The effect of the combination on the CRF is small: the maximum systematic effect is about 0.5 mas, but with respect to the standard deviation of the position of a single source, it is not significant. However, it has to be investigated in the future, if the impact on the CRF as a whole is significant. Two different kinds of impact on the CRF are assumed and confirmed. The effect of combining station networks, and here in particular the impact of local tie discrepancies, becomes visible for regions with a low density of VLBI stations (and, thus, also a low density of co-location sites). However, a systematic effect can be seen only in the DE of the datum sources. The maximum median value is about 0.04 mas. Considering all sources this small systematics cannot be detected because of the high noise of the source differences. It could be shown, that the combination of the EOP series has also an effect on the CRF. A systematic effect in RA was found for some of the sources between -40° and +30° of DE, which are observed once by VLBA sessions only. In maximum a shift in RA·cos(DE) of about 0.5 mas for some sources is reached. The largest position changes occur for sources observed by regional sessions only. This effect can be related to the combination of the combination of the terrestrial pole and LOD. The assumption, that the coordinates of the terrestrial pole and LOD derived from regional VLBI sessions show systematic differences with respect to the parameters derived from a global GNSS network will be investigated in detail in a future publication.

It should be mentioned, that IUGG recently made a resolution on the improvement of the consistency of ITRF and ICRF (resolution 3 of *IUGG* (2011)). It reads: "The International Union of Geodesy and Geophysics urges that highest consistency between the ICRF, the International Terrestrial Reference Frame (ITRF), and the Earth Orientation Parameters (EOP) as observed and realized by the IAG and its components such as the IERS should be a primary goal in all future realizations of the ICRS."

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