The role of VLBI in the weekly inter-technique combination

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Abstract

The geodetic space-techniques VLBI, GPS and SLR show a different grade of sensitivity on geodetic and geophysical parameters like station coordinates and Earth orientation parameters (EOP). While the satellite techniques GPS and SLR are, in principle, able to determine the Earth's center of mass, VLBI is, as an unique technique, able to determine all EOP like the motion of the celestial and the terrestrial pole and the rotation angle of the Earth. The accuracy of all obtained parameters depends strongly on the network geometry of the technique-specific observing stations. In order to benefit from the strength of each geodetic space-technique and in order to get a most stable solution of station coordinates and all EOP within one adjustment, the techniques could be combined in an inter-technique combination. Within the common approach of combining multi-year technique-specific normal equations, the weights of the techniques are assumed to be constant over time.
In this paper, weekly combined solutions are presented. The weighting is realized by a variance component estimation (VCE). The obtained relative weights are not constant over time. The VCs of the 'IVS-R1' sessions show a clear seasonal variation which is not yet fully understood. Nevertheless, an improvement of the EOP due to a session-wise VCE-based weighting could be achieved in some cases compared to a constant weighted solution.

1. Introduction

The geodetic space techniques GPS, SLR and VLBI show a different sensitivity on geodetic and geophysical parameters like the station coordinates, the Earth orientation parameters (EOP) or the Earth's gravity field coefficients. The satellite techniques GPS and SLR are both able to determine the Earth's center of mass but they are only sensitive to the rates of change of the Earth rotation angle UT1-UTC and the celestial pole coordinates X and Y [2]. The outstanding ability of VLBI to determine all EOP (celestial and the terrestrial pole, UT1-UTC) in an absolute sense makes a combination of the different techniques very reasonable. Up to now, the combination of the techniques for computing terrestrial reference frames was based on technique-specific normal equations (NEQs) containing all observations of one technique (multi-year reference frame (MRF)). Therein, the station motion is parameterized with a position at a reference epoch t_0 and a constant velocity. Hence, the relative weighting of the NEQs is assumed to be constant over time. In this paper, weekly combined solutions are computed. The weighting can be done individually for each week (session) or constant over time. A variable weighting has the advantage, that differences in the quality inherent in the input data can be taken into account. In this paper, the variable weighting of the techniques is done using a variance component estimation (VCE) algorithm [1].

2. Processing algorithm

The epoch-wise combination of the different techniques is done at DGFI at the normal equation level. The normal equation matrices of the geodetic space techniques GPS, SLR and VLBI are added to a weekly combined normal equation matrix. A simplified flow chart of the combination process is given in Figure 1. An important step in the combination is the relative weighting of the NEQs. Since the weights are not assumed to be constant over time, for every weekly NEQ an iterative estimation of the relative weights using a VCE is performed.

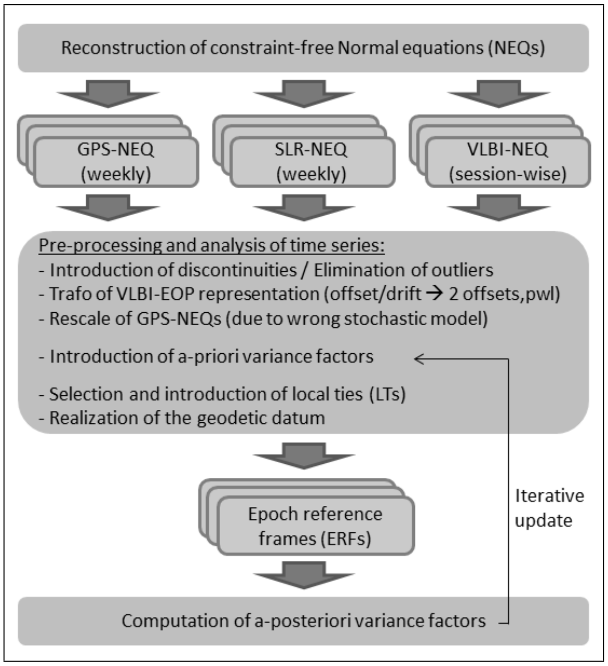

Figure 1. Simplified flow chart of the epoch combination approach.

The aposteriori variance component (VC) of the i-th individual normal equation matrix of the $(k+1)$-th iteration step is computed according to

$$\sigma_i^{2(k+1)} = \frac{\Omega_{c,i}^{(k)}}{r_{c,i}^{(k)}} \tag{1}$$

with the weighted sum of the residuals squared

$$\Omega_{c,i}^{(k)} = \boldsymbol{\hat{x}_c}^{(k)^T} \boldsymbol{N_i} \boldsymbol{\hat{x}_c}^{(k)} - 2\boldsymbol{y_i}^T \boldsymbol{\hat{x}_c}^{(k)} + \boldsymbol{l_i}^T \boldsymbol{P_i} \boldsymbol{l_i} \tag{2}$$

and the partial redundancy (degree of freedom)

$$r_{c,i}^{(k)} = m_i - \frac{1}{\sigma_i^{2(k)}} tr(\boldsymbol{N_i} \boldsymbol{N_c}^{(k)^{-1}}). \tag{3}$$

$\boldsymbol{\hat{x}_c}^{(k)}$ is the vector of the estimated parameters in the (k)-th iteration step.

$\boldsymbol{N_i}$ and $\boldsymbol{y_i}$ are the normal equation matrix and the corresponding right hand side of the i-th individual equation system. m_i is the number of observations (including the number of pre-reduced parameters). The matrix $\boldsymbol{N_c}$ is the weighted sum of the individual matrices $\boldsymbol{N_i}$ according to

$$\boldsymbol{N_c}^{(k)} = \sum_i \frac{1}{\sigma_i^{2(k)}} \boldsymbol{N_i}. \tag{4}$$

The matrix $\boldsymbol{l_i}^T \boldsymbol{P_i} \boldsymbol{l_i}$ is the weighted sum of the observations squared. In the combination, one VC for each weekly GPS- and SLR-NEQ is estimated. For VLBI, one VC for each session NEQ is estimated. At the end, time series of combined station coordinates, EOP and VCs for each technique are obtained.

3. Variance components

Figure 2 shows the time series of VCs for GPS, SLR and VLBI between 1994.0 and 2007.0. As weights for the NEQs, the reciprocal values of the VCs are used (equation (4)). Therefore, a low VC means a high weight in the combination process. For GPS, all VCs are nearly equal to 1.0 except the VCs between 1994.0 and 1996.0. This might be due to the fact that within this time period, less globally well-distributed stations were available. Hence, the accuracy of these

GPS NEQs w.r.t. the NEQs after 1998.0 is decreased and their impact on the combined NEQs is varying (by about 5%). After 1998.0, a small seasonal variation of the GPS VCs is visible. The mean VC for SLR before 2000.0 is 6.12. After this epoch, it decreases to a mean value of 2.34 (see also table 1). This decrease by about 62% is explained with the improvement of the SLR observation network since 1994.0. The VLBI VCs can be allocated to the particular session types. Between 1994.0 and 1998.0, a bulge of the VCs occurs. This bulge seems to be in coincidence with the signature of the GPS VCs at this time. Although the VLBI sessions 'NEOS', 'CORE' and 'IRIS' are scheduled in order to determine accurate EOP measurements (therefore, a good station distribution is used and the VCs of these session types are expected to be near one), the VCs of these sessions are much larger than one (table 1). The large VCs are dominated by the bulge in the VCs before 1998.0 which is caused by the combination.

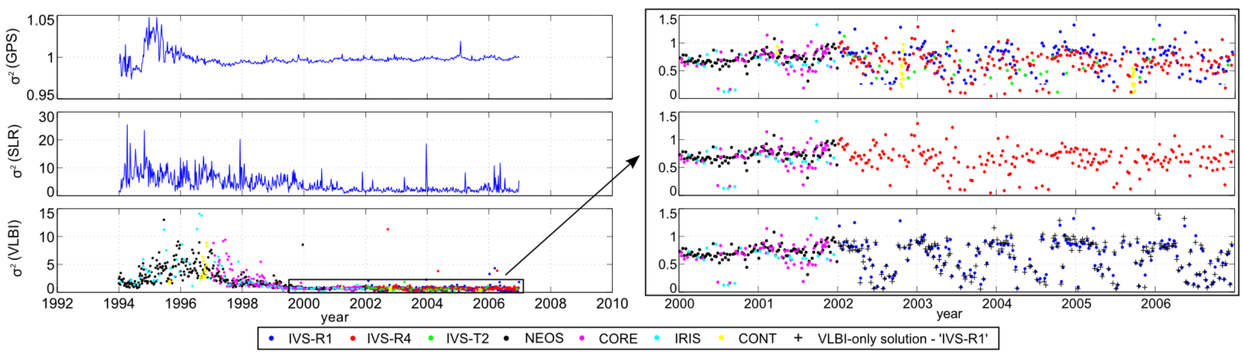

Figure 2. Left plots: a posteriori VCs of the techniques GPS (upper), SLR (middle), and VLBI (lower). In the case of VLBI, one VC per session is estimated. Right plots: zoom of VLBI plot for 2000.0-2007.0. All VLBI sessions are shown for 2000 and 2001. Data shown for 2002.0-2007.0 is all VLBI sessions (upper), IVS-R4 sessions only (middle), and IVS-R1 sessions only (lower). In addition, the lower right plot adds the a posteriori VCs of the VLBI-only solutions for the IVS-R1 sessions for comparison.

The right part of Figure 2 shows the VLBI variance components between 2000.0 and 2007.0. Since 2002.0, a periodic variation of the VCs is visible. If the components of the session types 'IVS-R1' and 'IVS-R4' are separated (these sessions are scheduled by the IVS since 2002.0), it is clearly visible, that only the 'IVS-R1' sessions show an annual variation with a minimum in the summer. This means that the impact of the VLBI NEQ on the combined NEQ during the summer is higher than during the winter time (equation (4)). As described above, also the GPS VCs show a small annual variation after the epoch 2002.0 which is in phase with the VLBI variation. In order to find the technique responsible for this variation, the aposteriori variance factors of the VLBI-only 'IVS-R1' solutions are shown additionally to the VLBI VCs of the VCE in the lower right plot of Figure 2. The annual variation occurs not only in the VCs of the VCE but also in the variance factors of the single technique solutions. This proofs, that the annual variation is caused by VLBI. The reason for the periodic behaviour of the VLBI VCs is not finally explained until now.

Table 1. Mean VCs for GPS, SLR and each VLBI session type before and after the epoch 2000.0.

solution	$t < 2000.0$	$t \geq 2000.0$	solution	$t < 2000.0$	$t \geq 2000.0$
GPS	1.0	1.0	IVS-T2	—	0.64
SLR	6.12	2.34	NEOS	2.199	—
VLBI (all sessions)	2.63	0.91	CORE	1.633	—
IVS-R1	—	0.70	IRIS	3.547	—
IVS-R4	—	0.73	CONT	2.123	0.926

4. Earth orientation parameter

One type of the estimated parameters in the combined solutions are the EOP. All EOP are parameterized as a piecewise linear polygon with estimated offsets at the midnight epochs. Per weekly NEQ, eight EOP offsets are included. Since the satellite techniques are only sensitive to the rates of change of UT1-UTC and the celestial pole coordinates (X,Y), at least one offset of the estimated polygon has to be fixed to its apriori value in order to repair the rank deficiency of the NEQ. The other midnight offsets are extrapolated using the rates. Since the rates are highly correlated with the orbit parameters and consequentially are affected by orbit systematics, the extrapolated offsets show a systematic deflection w.r.t. the reference time series IERS 08 C04 (see right plot of Figure 3). If the NEQs of the satellite techniques are combined with a VLBI

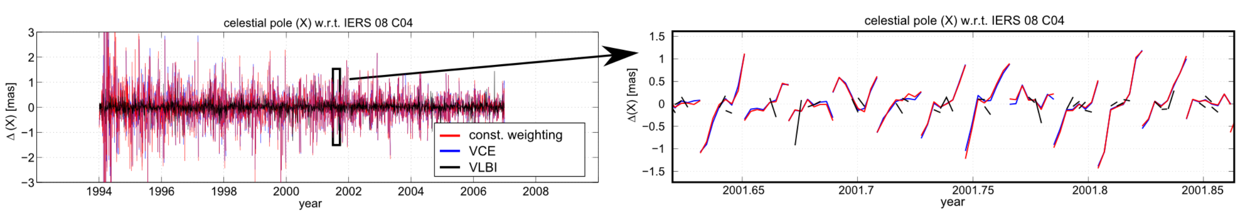

Figure 3. Estimated celestial pole coordinates in x-direction w.r.t. the IERS 08 C04 time series for the VLBI-only solution and the combined solution with and without variable weighting of the techniques.

NEQ, which contains absolute information about the offsets, the constraints are not necessary any longer. Usually, at least two VLBI sessions are scheduled during one week ('IVS-R1' on Monday and 'IVS-R4' on Thursday). The offsets in the combined solution in between the VLBI epochs are extrapolated with the rates delivered by the satellite techniques. As follows, the polygon in between the VLBI epochs shows systematic differences w.r.t. the reference time series (Figure 3). In table 2, the weighted mean RMS values of the EOP of the single technique solutions and of the two combined solutions (constant weighted and VCE-based weighted) are given. In the case of the celestial pole coordinates and UT1-UTC, the constant weighted combination shows the largest scatter. If a VCE is used, the scattering decreases slightly. The large scatter is explained by the deflecting polygon parts in between two VLBI epochs. If only the epochs with three techniques contributing are considered (VCE-based solution at VLBI epochs), the WRMS values decrease significantly but still are larger than the WRMS values for the VLBI-only solution (celestial pole:

Table 2. Weighted mean RMS values of the terrestrial and celestial pole coordinates and UT1-UTC w.r.t. the IERS 08 C04 time series for the GPS-only, the VLBI-only and for different combined solutions.

WRMS	GPS	VLBI	const. weighting	VCE	VCE (VLBI epochs)
cel. pole (X) [μas]	—	88.7	240.9	239.8	94.8
cel. pole (Y) [μas]	—	95.3	112.5	112.4	100.0
UT1-UTC [μs]	—	12.1	39.9	39.5	17.5
terr. pole (x) [μas]	123.0	213.7	142.3	122.7	109.8
terr. pole (y) [μas]	114.2	248.2	136.6	117.9	107.5

5 to 7%, UT1-UTC: 45%). In the case of the terrestrial pole coordinates, the constant weighted combination shows larger scatter than the GPS only solution. In contrast to this, if a VCE is used, the scattering is at the level of the GPS-only solution (slight improvement of the x-coordinate and slight degradation for the y-coordinate).

5. Conclusions

Within the inter-technique combination, VLBI plays a central role. It is the unique technique to determine the absolute offsets of UT1-UTC and the celestial pole coordinates. Therefore, no constraints for these parameters for the satellite techniques are necessary in the combination. The results have shown, that the combination using a VCE-based weighting allows to consider quality differences inherent in the input data. If a variable weighting of the techniques using a VCE is realized, the weights of the VLBI NEQs w.r.t. the other NEQs in the combination show some systematics, which have to be further investigated. Since 2002.0, the VCs of the 'IVS-R1' sessions show an annual variation between zero and one with a minimum in the summer. This variation also occurs in the aposteriori VCs of the VLBI-only solutions. It verifies that the reason for this variation is caused by the VLBI technique.
The estimates of the UT1-UTC and the celestial pole coordinates of the combined solutions, where a VCE is used for the weighting, are comparable to those of the VLBI-only solutions if only VLBI epochs are considered. In the case of the terrestrial pole coordinates, GPS is providing the most stable parameter time series w.r.t. the IERS 08 C04 time series. The weighted scattering of the estimates of the combined solutions using a VCE are here at the same level as the GPS-only solutions (differences below 3%).

6. Acknowledgements

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