Conferencing in Wyner's Asymmetric Interference Network: Effect of Number of Rounds

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***Abstract*—In this paper, we study how the number of conferencing rounds effects the capacity of large interference networks. We take Wyner's asymmetric linear (soft-handoff) model, include conferencing links between closely located transmitters and receivers, and we consider the per-user asymptotic multiplexing gain. Our results show, for example, that when the capacities of the conferencing links scale at most $(1/4)\log P$ with the power P and when one can choose which transmitters and receivers cooperate, then there is no loss in terms of asymptotic multiplexing gain in having only one round of conferencing. In contrast, when the capacities of the conferencing links grow faster than $(1/4)\log P$, then the asymptotic multiplexing gain with one round of conferencing is strictly smaller than that achieved with multiple rounds.**

I. Introduction

Conferencing in networks started with Willems' work [1] on the two-user discrete memoryless multiple-access channel (MAC). Willems' idea was to include a *conferencing phase*, prior to communicating over the MAC, during which the transmitters exchanged messages over noiseless rate-limited channels. Interestingly, Willems showed that a simple conferencing scheme is capacity achieving: Each transmitter shares part of its message with the other during conferencing, and both transmitters treat these shared parts as a common message when communicating over the MAC.

Further capacity results for networks with transmitter conferencing were obtained in [2]–[5]. A common feature of the proposed capacity-achieving strategies is that the transmitters share parts of their messages during the conferencing phase, and these shared parts are treated as common messages during the communications phase. Moreover, the capacity-achieving strategies in [1]–[4] employ *one-shot* conferencing (the information shared by a transmitter during conferencing depends only on that transmitter's message). This is notable because the conferencing models in [1]–[4] actually permit *multiple-round* conferencing, where the transmitters may interactively exchange many messages over multiple rounds. One-shot conferencing, however, is not always optimal; for example, the scheme in [5] requires two conferencing rounds.

Dabora and Servetto studied the two-user broadcast channel (BC) with conferencing receivers in [6, 7], and they obtained capacity results for some special cases of the general BC. In all such cases, the capacity-achieving strategies only required one-shot conferencing between the receivers: The receivers either quantize their observed channel outputs and shared the quantizer index with the other receivers, or they decode parts of their messages solely based on their observed channel outputs and shared parts of the decoded messages. For more general BCs, where the capacity region has not yet been established, Dabora and Servetto presented achievable interactive two-step conferencing strategies.

In an interesting recent paper [8], Ntranos, Maddah-Ali and Caire studied large Gaussian interference networks with conferencing links between transmitters and between receivers, assuming one-shot interference alignment (IA) techniques [9] are used for communications. In this scenario, they showed that it is suboptimal for the transmitters to share parts of their messages and for the receivers to share quantized versions of the channel output signals. In particular, strictly better performance can be achieved if the transmitters exchange quantized versions of their transmit signals, and the receivers share parts of their decoded messages.

The setup in [8] can be modelled as a Gaussian interference network with conferencing links between transmitters and between receivers. The achievability scheme in [8] requires multiple conferencing rounds. Multiple conferencing rounds between the transmitters is necessary because the transmitters wish to exchange quantized versions of their transmit signals, and the transmit signals, in turn, depend on conferencing messages from previous rounds. Similarly, multiple conferencing rounds between the receivers is necessary because the receivers wish to exchange parts of their decoded messages, and each decoded message depends on conferencing messages from previous rounds.

In practice, it is not always feasible to employ complicated multiple-round conferencing strategies. In mobile cellular networks, for example, conferencing can be implemented over a cloud radio access network (C-RAN) with limited storage and computational capabilities. In such cases, the C-RAN is typically oblivious [9] — not aware of the employed codebooks — and limited to quantizing the receive signals before relaying them or reroute functions of incoming messages.

In this work, we consider the information-theoretic limits of a simple mobile cellular network model with two conferencing modes: In Mode 1, the transmitters and the receivers can hold their conferences over an arbitrary number of rounds, and in Mode 2 only over a single round. We will present optimal strategies for Mode 1 and Mode 2, where the latter does not require any knowledge of the codebooks during the conferencing phase.

We consider Wyner's *asymmetric linear* model [10]–[12], and we use the per-user *asymptotic multiplexing gain* (also called *degrees of freedom* or *prelog*) as a performance metric. In our model, each transmitter obtains conferencing messages from the t_{L} transmitters to its left and the t_{R} transmitters to its right. Similarly, each receiver obtains conferencing messages from the r_{L} receivers to its left and the r_{R} receivers to its right. Prelog constraints μ_{Tx} and μ_{Rx} are imposed on the capacities of the transmitter and receiver conferencing links respectively. The asymptotic multiplexing gain per user for Mode 2, without receiver cooperation $(r_{\mathrm{L}} = r_{\mathrm{R}} = 0)$ and equal transmitter cooperation $(t_{\mathrm{L}} = t_{\mathrm{R}})$ was previously derived in [13]. For unbounded $\mu_{\mathrm{Tx}} = \mu_{\mathrm{Rx}} = \infty$, the asymptotic multiplexing gain per user is the same for Mode 1 and 2 and was found in [14]. See [12] for an overview over various forms of cooperation in the Wyner network.

We present general upper and lower bounds on the per-user asymptotic multiplexing gain for Mode 1, and the bounds match when conferencing is limited to adjacent transmitters and receivers $(t_{\mathrm{L}}, t_{\mathrm{R}}, r_{\mathrm{L}}, r_{\mathrm{R}} \in \{0, 1\})$. We show that this setup also achieves the *optimized per-user asymptotic multiplexing gain*, i.e., the largest asymptotic multiplexing gain per user subject to *total* conferencing prelog constraints $\mu_{\mathrm{Tx}}(t_{\mathrm{L}}+t_{\mathrm{R}}) \leq \eta_{\mathrm{Tx}}$ and $\mu_{\mathrm{Rx}}(r_{\mathrm{L}} + r_{\mathrm{R}}) \leq \eta_{\mathrm{Rx}}$.

For Mode 2, we present a general lower bound on the asymptotic multiplexing gain per user and show that it is tight when only the transmitters or only the receivers conference, i.e., when $\mu_{\mathrm{Rx}} = 0$ or $\mu_{\mathrm{Tx}} = 0$. We present the optimized per-user asymptotic multiplexing for these two special cases.

Our results demonstrate a duality between transmitter conferencing (parameters $t_{\mathrm{L}}, t_{\mathrm{R}}, \mu_{\mathrm{Tx}}$) and receiver conferencing (parameters $r_{\mathrm{L}}, r_{\mathrm{R}}, \mu_{\mathrm{Rx}}$). They also show that while for some setups Mode 2 has a strictly smaller asymptotic multiplexing gain per user than Mode 1, for others this is not the case. The same is true also for the optimized asymptotic multiplexing gain per user.

II. Problem Setup

Consider a communications system with K pairs of transmitters and receivers, labeled with the index $k = 1, 2, \ldots, K$. Assume that all transmitters and receivers are equipped with a single antenna, and that all channel inputs and outputs are real valued. We envision a network with short-range interference, à la [10, 11, 13, 14], so that the signal sent by transmitter k is only observed by receivers k and $(k + 1)$, see Fig. 1. Specifically, the time-t channel output at receiver k is

$$Y_{k,t} = X_{k,t} + \alpha_k X_{k-1,t} + Z_{k,t}, \tag{1}$$

where $X_{k,t}$ and $X_{k-1,t}$ are the symbols sent by transmitters k and $(k-1)$ at time t respectively; $\{Z_{k,t}\}$ are independent and identically distributed (i.i.d.) standard Gaussians for all k and t; $\alpha_k \neq 0$ is a given real number; and $X_{0,t} = 0$ for all t.

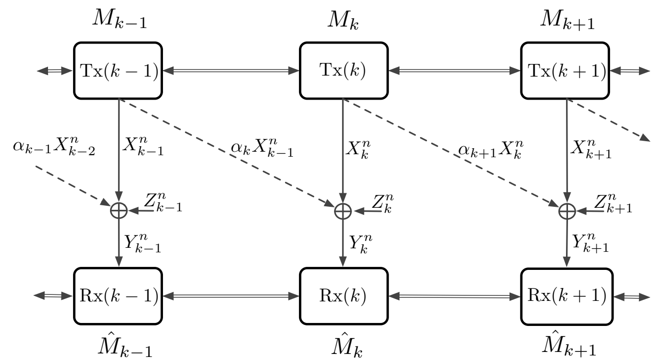

Fig. 1. Wyner's asymmetric network with rate-limited conferencing links between neighboring transmitters and receivers. The figure depicts our setup with parameters $t_{\mathrm{L}} = t_{\mathrm{R}} = r_{\mathrm{L}} = r_{\mathrm{R}} = 1$.

Transmitter k (for all k) is required to reliably communicate a message M_k to receiver k, where M_k is uniform on

$$\mathcal{M}_k \triangleq \{1, 2, \ldots, \lfloor e^{nR_k} \rfloor\}.$$

We assume that all messages $(M_1, M_2, \ldots, M_K)$ are independent of one another and the noise $\{Z_{k,t}\}$, and we impose a symmetric average block-power constraint $P > 0$ on the input sequences

$$\frac{1}{n}\sum_{t=1}^{n} X_{k,t}^2 \leq P, \quad \text{a.s.} \quad \forall\ k \in \{1, \ldots, K\}. \tag{2}$$

A key feature of this work is that we allow *rate-limited local cooperation* between neighboring transmitters and neighboring receivers; for example, see Fig. 1. Specifically, let us suppose that the communications process consists of four phases.

1) *Tx-conferencing phase:* Each transmitter may send individual messages to the nearest t_{L} and t_{R} transmitters on its left and right respectively. We assume that this conferencing takes place over *noiseless* channels, and that each channel has a maximum rate budget of R_{Tx} bits. We parametrise R_{Tx} as
$$R_{\mathrm{Tx}} \triangleq \mu_{\mathrm{Tx}}\ \frac{1}{2}\log(1 + P), \tag{3}$$
where μ_{Tx} is the *prelog* constant. Our main interest in this parametrisation is μ_{Tx}, which is the dominant parameter in the power regime of interest $P \gg 1$. Figure 1 shows the case where $t_{\mathrm{L}} = t_{\mathrm{R}} = 1$.
2) *Cooperative-communication phase:* The transmitters communicate over the K-user interference channel (1). Transmitter k's channel input is a function of the message M_k and the conferencing messages it received during the transmitter-conferencing phase.
3) *Rx-conferencing phase:* Each receiver may send individual messages to the nearest r_{L} and r_{R} receivers on its left and right respectively. The conferencing takes place over noiseless channels, each with a maximum rate budget of
$$R_{\mathrm{Rx}} \triangleq \mu_{\mathrm{Rx}}\ \frac{1}{2}(1 + P). \tag{4}$$
Figure 1 shows that case where $r_{\mathrm{L}} = r_{\mathrm{R}} = 1$.

4) *Side-information-aided decoding phase* in which the receivers decode their desired messages from the channel outputs and conferencing messages received during the Rx-conferencing phase.

We consider two modes for the conferencing phases.

Mode 1: The Tx and Rx conferencing phases consist of κ_{Tx} and κ_{Rx} conferencing rounds respectively, where κ_{Tx} and κ_{Rx} are parameters of the coding scheme.

Mode 2: The Tx and Rx conferencing phases both consist of a single "one shot" round. This mode is motivated by systems with limited terminal computational power or stringent delay constraints on the conferencing phases.

We now describe the communications phases for Mode 1. Mode 2 is obtained by setting $\kappa_{\text{Tx}} = \kappa_{\text{Rx}} = 1$.

A. Tx-Conferencing Phase

Let us denote the indices of transmitter k's neighbors by

$$\text{Tx-Nbhood}(k) \triangleq \big\{k' \in \{1,\dots,K\}\backslash\{k\} \\ : k - t_{\text{L}} \leq k' \leq k + t_{\text{R}}\big\}.$$

There are κ_{Tx} conferencing rounds in Mode 1. In the j-th round ($j = 1, 2, \dots, \kappa_{\text{Tx}}$), transmitter k sends

$$U^{(j)}_{k\to k'} = \phi^{(j)}_{k,k'}\left(M_k, \mathbf{U}^{(1)}_{\text{all}\to k}, \mathbf{U}^{(2)}_{\text{all}\to k}, \dots, \mathbf{U}^{(j-1)}_{\text{all}\to k}\right) \tag{5}$$

to its neighbor $k' \in \text{Tx-Nbhood}(k)$. Here

$$\mathbf{U}^{(j')}_{\text{all}\to k} \triangleq \left(U^{(j')}_{k'\to k}\ ;\ k' \in \text{Tx-Nbhood}(k)\right)$$

denotes the tuple of all conferencing messages sent to transmitter k in the earlier conferencing round $j' \in \{1,\dots,j-1\}$,

$$\phi^{(j)}_{k,k'} : \mathcal{M}_k \times \prod_{j'=1}^{j-1} \prod_{\tilde{k}:\ k\in\text{Tx-Nbhood}(\tilde{k})} \big\{1,2,\dots,\lfloor 2^{nR^{(j')}_{\text{Tx},\tilde{k}\to k}}\rfloor\big\} \\ \longrightarrow \big\{1,2,\dots,\lfloor 2^{nR^{(j)}_{\text{Tx},k\to k'}}\rfloor\big\},$$

and $R^{(j')}_{\text{Tx},k\to k'}$ is the rate of the j'-th conferencing message from transmitter k to transmitter k'. We require that the total conferencing rate from any transmitter k to any receiver $k' \in \text{Tx-Nbhood}(k)$ does not exceed the budget R_{Tx} in (3),

$$\sum_{j=1}^{\kappa_{\text{Tx}}} R^{(j)}_{\text{Tx},k\to k'} \leq R_{\text{Tx}}. \tag{6}$$

B. Cooperative-Communication Phase

The channel input signals of transmitter k are a function of its message M_k and the conferencing messages it received during Tx-conferencing. Specifically, transmitter k sends

$$X^n_k = f_k\left(M_k, \mathbf{U}^{(1)}_{\text{all}\to k}, \mathbf{U}^{(2)}_{\text{all}\to k}, \dots, \mathbf{U}^{(\kappa_{\text{Tx}})}_{\text{all}\to k}\right), \tag{7}$$

over the interference channel, where

$$X^n_k = (X_{k,1}, X_{k,2}, \dots, X_{k,n})$$

and

$$f_k : \mathcal{M}_k \times \prod_{j=1}^{\kappa_{\text{Tx}}} \prod_{k'\in\text{Tx-Nbhood}(k)} \big\{1,2,\dots,\lfloor 2^{nR^{(j)}_{\text{Tx},k\to k'}}\rfloor\big\} \to \mathbb{R}^n.$$

C. Rx-Conferencing Phase

The Rx-conferencing phase takes place after all the channel outputs have been been observed by the receivers. Denote receiver k' channel outputs by

$$Y^n_k = (Y_{k,1}, \dots, Y_{k,n}).$$

Let

$$\text{Rx-Nbhood}(k) \triangleq \big\{k' \in \{1,\dots,K\}\backslash\{k\}) \\ : k - r_{\text{L}} \leq k' \leq k + r_{\text{R}}\big\}.$$

In round j, receiver k sends

$$V^{(j)}_{k\to k'} = \psi^{(j)}_{k,k'}\left(Y^n_k, \boldsymbol{V}^{(1)}_{\text{all}\to k}, \boldsymbol{V}^{(2)}_{\text{all}\to k}, \dots, \boldsymbol{V}^{(j-1)}_{\text{all}\to k}\right) \tag{8}$$

to its neighbor $k' \in \text{Rx-Nbhood}(k)$. Here,

$$\psi^{(j)}_{k,k'} : \mathbb{R}^n \times \prod_{j'=1}^{j-1} \prod_{\tilde{k}:\ k\in\text{Rx-Nbhood}(\tilde{k})} \big\{1,2,\dots,\lfloor 2^{nR^{(j')}_{\text{Rx},\tilde{k}\to k}}\rfloor\big\} \\ \longrightarrow \big\{1,2,\dots,\lfloor 2^{nR^{(j)}_{\text{Rx},k\to k'}}\rfloor\big\},$$

where $R^{(j')}_{\text{Tx},k\to k'}$ is the rate of the round j' conferencing message from receiver k to receiver k', and

$$\boldsymbol{V}^{(j')}_{\text{all}\to k} \triangleq \big(V^{(j')}_{k'\to k}\ ;\ k' \in \text{Rx-Nbhood}(k)\big). \tag{9}$$

is the tuple of all conferencing messages received at transmitter k in round j'. The total conferencing rate from any receiver k to any receiver $k' \in \text{Tx-Nbhood}(k)$ has to satisfy the total rate-budget in (4),

$$\sum_{j=1}^{\kappa_{\text{Rx}}} R^{(j)}_{\text{Rx},k\to k'} \leq R_{\text{Rx}}. \tag{10}$$

D. Side-Information-Aided Decoding Phase

After observing the channel output Y^n_k and conferencing messages $\mathbf{V}^{(1)}_{\text{all}\to k}$, $\mathbf{V}^{(2)}_{\text{all}\to k}$ to $\mathbf{V}^{(\kappa_{\text{Rx}})}_{\text{all}\to k}$, receiver k produces

$$\hat{M}_k \triangleq g_k\left(Y^n_k, \mathbf{V}^{(1)}_{\text{all}\to k}, \mathbf{V}^{(2)}_{\text{all}\to k}, \dots, \mathbf{V}^{(\kappa_{\text{Rx}})}_{\text{all}\to k}\right) \tag{11}$$

as its guess of Message $\hat{M}_k$, where

$$g_k : \mathbb{R}^n \times \prod_{j'=1}^{\kappa_{\text{Rx}}} \prod_{k'\in\text{Rx-Nbhood}(k)} \{1,2,\dots,\lfloor 2^{nR^{(j')}_{\text{Rx},k\to k'}}\rfloor\} \to \mathcal{M}_k.$$

We call the collection of all transmitter and receiver mappings in (5)–(9) a $(R_1, R_2, \dots, R_K, \mu_{\text{Tx}}, \mu_{\text{Rx}}, P)$-*code*.

E. Performance Measures

We say that a rate tuple $(R_1, R_2, \dots, R_K)$ is $(\mu_{\text{Tx}}, \mu_{\text{Rx}}, P)$-*achievable* if for any $\epsilon > 0$ there exists a $(R_1, \dots, R_K, \mu_{\text{Tx}}, \mu_{\text{Rx}}, P)$-code, with sufficiently large blocklength n, such that

$$\Pr\left[(\hat{M}_1, \dots, \hat{M}_K) \neq (M_1, \dots, M_K)\right] \leq \epsilon.$$

The *capacity region* $\mathcal{C}(\mu_{\text{Tx}}, \mu_{\text{Rx}}, P)$ is the closure of the set of all achievable rate tuples. The *sum capacity* is

$$C_\Sigma(\mu_{\text{Tx}}, \mu_{\text{Rx}}, P) \triangleq \max_{(R_1,\dots,R_K)\in\mathcal{C}} \sum_{k=1}^{K} R_k$$

Our main focus in this work is on the high-SNR asymptote of the sum-capacity, the per-user *asymptotic multiplexing gain*

$$\mathcal{S}(\mu_{\mathrm{Tx}},\mu_{\mathrm{Rx}}) \triangleq \varlimsup_{K\to\infty} \varlimsup_{P\to\infty} \frac{C_{\Sigma}(\mu_{\mathrm{Tx}},\mu_{\mathrm{Rx}},P)}{\frac{K}{2}\log P},$$

where $\varlimsup$ denotes the limit supremum. To make its dependency on the mode explicit, we write $\mathcal{S}_{\mathrm{Mode1}}(\mu_{\mathrm{Tx}},\mu_{\mathrm{Rx}})$ and $\mathcal{S}_{\mathrm{Mode2}}(\mu_{\mathrm{Tx}},\mu_{\mathrm{Rx}})$.

III. Results and Discussion

A. Main Results for Mode 1

Theorem 1: Let υ_{Tx} denote the number of Tx-conferencing parameters $t_{\mathrm{L}},t_{\mathrm{R}}$ that are positive, and let υ_{Rx} the number of Rx-conferencing parameters $r_{\mathrm{L}},r_{\mathrm{R}}$ that are positive. For Mode 1, we have

$$\begin{aligned}\frac{1}{2}\min\{2,1+\mu_{\mathrm{Tx}}\upsilon_{\mathrm{Tx}}+\mu_{\mathrm{Rx}}\upsilon_{\mathrm{Rx}}\} &\leq \mathcal{S}_{\mathrm{Mode1}}(\mu_{\mathrm{Tx}},\mu_{\mathrm{Rx}})\\ &\leq \frac{1}{2}\min\{2,1+\mu_{\mathrm{Tx}}(t_{\mathrm{L}}+t_{\mathrm{R}})+\mu_{\mathrm{Rx}}(r_{\mathrm{L}}+r_{\mathrm{R}})\}.\end{aligned}$$

Proof: Omitted. ■

If all the conferencing parameters are either 0 or 1, then the bounds in Theorem 1 match.

Corollary 1: If $t_{\mathrm{L}},t_{\mathrm{R}},r_{\mathrm{L}},r_{\mathrm{R}},\in\{0,1\}$, then

$$\begin{aligned}&\mathcal{S}_{\mathrm{Mode1}}(\mu_{\mathrm{Tx}},\mu_{\mathrm{Rx}})\\ &\quad=\frac{1}{2}\min\{2,1+\mu_{\mathrm{Tx}}(t_{\mathrm{L}}+t_{\mathrm{R}})+\mu_{\mathrm{Rx}}(r_{\mathrm{L}}+r_{\mathrm{R}})\}.\end{aligned}$$

Let $\mathcal{S}^{\star}_{\mathrm{Mode1}}(\mu_{\mathrm{Tx}},\mu_{\mathrm{Rx}})$ denote Mode 1's *optimised* asymptotic multiplexing gain per user:

$$\mathcal{S}^{\star}_{\mathrm{Mode1}}(\eta_{\mathrm{Tx}},\eta_{\mathrm{Rx}}) \triangleq \max_{\substack{t_{\mathrm{L}}',t_{\mathrm{R}}',r_{\mathrm{L}}',r_{\mathrm{R}}',\in\mathbb{N},\ \mu'_{\mathrm{Tx}},\mu'_{\mathrm{Rx}}\geq 0:\\ \mu'_{\mathrm{Rx}}(r_{\mathrm{L}}'+r_{\mathrm{R}}')\leq\eta_{\mathrm{Rx}}\\ \mu'_{\mathrm{Tx}}(t_{\mathrm{L}}'+t_{\mathrm{R}}')\leq\eta_{\mathrm{Tx}}}} \mathcal{S}_{\mathrm{Mode1}}(\mu'_{\mathrm{Tx}},\mu'_{\mathrm{Rx}})$$

Corollary 1 and Theorem 1 yield the following result.

Corollary 2:

$$\mathcal{S}^{\star}_{\mathrm{Mode1}}(\eta_{\mathrm{Tx}},\eta_{\mathrm{Rx}}) = \frac{1}{2}\min\{2,1+\eta_{\mathrm{Tx}}+\eta_{\mathrm{Rx}}\}.$$

B. Results for Mode 2

Our results for this Mode 2 depend on the ordering of

$$\pi_{t_{\mathrm{L}}}\triangleq\frac{t_{\mathrm{L}}}{\mu_{\mathrm{Tx}}},\quad \pi_{t_{\mathrm{R}}}\triangleq\frac{t_{\mathrm{R}}}{\mu_{\mathrm{Tx}}},\quad \pi_{r_{\mathrm{L}}}\triangleq\frac{r_{\mathrm{L}}}{\mu_{\mathrm{Rx}}}\quad\text{and}\quad \pi_{r_{\mathrm{R}}}\triangleq\frac{r_{\mathrm{R}}}{\mu_{\mathrm{Rx}}}.$$

Let $\pi_{\ell_1},\pi_{\ell_2},\pi_{\ell_3},\pi_{\ell_4}$ denote the set $\{\pi_{t_{\mathrm{L}}},\pi_{t_{\mathrm{R}}},\pi_{r_{\mathrm{L}}},\pi_{r_{\mathrm{R}}}\}$ arranged in descending order; that is,

$$\begin{gathered}\ell_1,\ell_2,\ell_3,\ell_4 \text{ are distinct elements of } \{t_{\mathrm{L}},t_{\mathrm{R}},r_{\mathrm{L}},r_{\mathrm{R}}\}\\ \text{such that } \pi_{\ell_1}\geq\pi_{\ell_2}\geq\pi_{\ell_3}\geq\pi_{\ell_4}.\end{gathered}$$

Let

$$\mu_{t_{\mathrm{L}}}\triangleq\mu_{t_{\mathrm{R}}}\triangleq\mu_{\mathrm{Tx}} \text{ and } \mu_{r_{\mathrm{L}}}\triangleq\mu_{r_{\mathrm{R}}}\triangleq\mu_{\mathrm{Rx}}.$$

Theorem 2: In Mode 2, the asymptotic multiplexing gain per user is as shown in (12) on the next page.

Proof: Omitted. ■

The achievability bound in Theorem 2 is optimal when we are limited to either transmitter conferencing or receiver conferencing.

Theorem 3: In the absence of receiver conferencing,

$$\begin{aligned}&\mathcal{S}_{\mathrm{Mode2}}(\mu_{\mathrm{Tx}},\mu_{\mathrm{Rx}}=0)\\ &=\begin{cases}\frac{1+2\mu_{\mathrm{Tx}}}{2} & \text{if } \mu_{\mathrm{Tx}}\leq\frac{\min\{t_{\mathrm{L}},t_{\mathrm{R}}\}}{2\min\{t_{\mathrm{L}},t_{\mathrm{R}}\}+2}\\ \frac{\min\{t_{\mathrm{L}},t_{\mathrm{R}}\}+1+\mu_{\mathrm{Tx}}}{\min\{t_{\mathrm{L}},t_{\mathrm{R}}\}+2} & \text{if } \frac{\min\{t_{\mathrm{L}},t_{\mathrm{R}}\}}{2\min\{t_{\mathrm{L}},t_{\mathrm{R}}\}+2}<\mu_{\mathrm{Tx}}\leq\frac{\max\{t_{\mathrm{L}},t_{\mathrm{R}}\}}{t_{\mathrm{L}}+t_{\mathrm{R}}+2}\\ \frac{t_{\mathrm{L}}+t_{\mathrm{R}}+1}{t_{\mathrm{L}}+t_{\mathrm{R}}+2} & \text{if } \mu_{\mathrm{Tx}}>\frac{\max\{t_{\mathrm{L}},t_{\mathrm{R}}\}}{t_{\mathrm{L}}+t_{\mathrm{R}}+2}.\end{cases}\end{aligned}$$

If in the above we replace parameters $t_{\mathrm{L}},t_{\mathrm{R}},\mu_{\mathrm{Tx}}$ by $r_{\mathrm{L}},r_{\mathrm{R}},\mu_{\mathrm{Rx}}$, we obtain $\mathcal{S}_{\mathrm{Mode2}}(\mu_{\mathrm{Tx}}=0,\mu_{\mathrm{Rx}})$.

Proof: Achievability by Theorem 2. Converse omitted. ■

There is a duality between Tx conferencing and Rx conferencing in the sense that the parameters $(r_{\mathrm{L}},r_{\mathrm{R}},\mu_{\mathrm{Rx}})$ influence the per-user asymptotic multiplexing gain in the same way as the parameters $(t_{\mathrm{L}},t_{\mathrm{R}},\mu_{\mathrm{Tx}})$. Thus, Tx conferencing and Rx conferencing are equally useful.

Let $\mathcal{S}^{\star}_{\mathrm{Mode2}}(\eta_{\mathrm{Tx}},\eta_{\mathrm{Rx}})$ denote Mode 2's optimised asymptotic multiplexing gain per user

$$\mathcal{S}^{\star}_{\mathrm{Mode2}}(\eta_{\mathrm{Tx}},\eta_{\mathrm{Rx}}) \triangleq \max_{\substack{t_{\mathrm{L}}',t_{\mathrm{R}}',r_{\mathrm{L}}',r_{\mathrm{R}}'\in\mathbb{N},\ \mu'_{\mathrm{Tx}},\mu'_{\mathrm{Rx}}\geq 0:\\ \mu'_{\mathrm{Rx}}(r_{\mathrm{L}}'+r_{\mathrm{R}}')\leq\eta_{\mathrm{Rx}}\\ \mu'_{\mathrm{Tx}}(t_{\mathrm{L}}'+t_{\mathrm{R}}')\leq\eta_{\mathrm{Tx}}}} \mathcal{S}_{\mathrm{Mode2}}(\mu'_{\mathrm{Tx}},\mu'_{\mathrm{Rx}}).$$

From Theorem 3 we obtain the following theorem. An analogous result holds in the absence of transmitter conferencing.

Corollary 3: In the absence of receiver conferencing,

$$\mathcal{S}^{\star}_{\mathrm{Mode2}}(\eta_{\mathrm{Tx}},\eta_{\mathrm{Rx}}=0)=\begin{cases}\frac{1}{2}+\frac{\eta_{\mathrm{Tx}}}{2} & \text{if } \eta_{\mathrm{Tx}}\in\left[0,\frac{1}{2}\right]\\ \frac{3}{4} & \text{if } \eta_{\mathrm{Tx}}\in\left(\frac{1}{2},\frac{3}{4}\right]\\ \frac{2}{3}+\frac{\eta_{\mathrm{Tx}}}{9} & \text{if } \mu_{\mathrm{Tx}}\in\left(\frac{3}{4},1\right].\end{cases}$$

A choice of conferencing parameters $(r_{\mathrm{L}},r_{\mathrm{R}})$ that achieves $\mathcal{S}^{\star}_{\mathrm{Mode2}}(\eta_{\mathrm{Tx}},\eta_{\mathrm{Rx}}=0)$ is:

$$\begin{cases}(r_{\mathrm{L}},r_{\mathrm{R}})=(1,1) & \text{if } \eta_{\mathrm{Rx}}\in[0,3/4]\\ (r_{\mathrm{L}},r_{\mathrm{R}})=(2,1) & \text{if } \eta_{\mathrm{Rx}}\in(3/4,1].\end{cases}$$

C. Comparison of Modes 1 and 2

Example 1: Let $t_{\mathrm{L}}=t_{\mathrm{R}}=r_{\mathrm{L}}=r_{\mathrm{R}}=1$ and $\mu_{\mathrm{Tx}}\geq\mu_{\mathrm{Rx}}\geq 0$. We have

$$\mathcal{S}_{\mathrm{Mode1}}(\mu_{\mathrm{Tx}},\mu_{\mathrm{Rx}})=\frac{1}{2}+\mu_{\mathrm{Tx}}+\mu_{\mathrm{Rx}},$$

and

$$\mathcal{S}_{\mathrm{Mode2}}(\mu_{\mathrm{Tx}},\mu_{\mathrm{Rx}})\geq\begin{cases}\frac{1}{2}+\mu_{\mathrm{Tx}}+\mu_{\mathrm{Rx}}, & \text{if } 2\mu_{\mathrm{Tx}}+4\mu_{\mathrm{Rx}}\leq 1\\ \frac{5}{6}, & \text{if } 6\mu_{\mathrm{Rx}}\geq 1\\ \frac{3}{4}+\frac{1}{2}\mu_{\mathrm{Rx}}, & \text{otherwise.}\end{cases}$$

Thus, sometimes the per-user asymptotic multiplexing gains for Modes 1 and 2 coincide and sometimes the former is larger.

$$
\mathcal{S}_{\text{Mode2}}(\mu_{\text{Tx}},\mu_{\text{Rx}}) \geq \begin{cases} \frac{1+\mu_{\ell_1}+\mu_{\ell_2}+\mu_{\ell_3}+\mu_{\ell_4}}{2} & \text{if } \frac{\ell_1}{\pi_{\ell_1}}+\frac{\ell_2}{\pi_{\ell_2}}+\frac{\ell_3}{\pi_{\ell_3}}+\frac{\ell_4+2}{\pi_{\ell_4}} < 1 \\ \frac{\ell_4+1+\mu_{\ell_1}+\mu_{\ell_2}+\mu_{\ell_3}}{\ell_4+2} & \text{if } \frac{\ell_1}{\pi_{\ell_1}}+\frac{\ell_2}{\pi_{\ell_2}}+\frac{\ell_3}{\pi_{\ell_3}}+\frac{\ell_4+2}{\pi_{\ell_4}} \geq 1 > \frac{\ell_1}{\pi_{\ell_1}}+\frac{\ell_2}{\pi_{\ell_2}}+\frac{\ell_3+\ell_4+2}{\pi_{\ell_3}} \\ \frac{\ell_4+\ell_3+1+\mu_{\ell_1}+\mu_{\ell_2}}{\ell_4+\ell_3+2} & \text{if } \frac{\ell_1}{\pi_{\ell_1}}+\frac{\ell_2}{\pi_{\ell_2}}+\frac{\ell_3+\ell_4+2}{\pi_{\ell_3}} \geq 1 > \frac{\ell_1}{\pi_{\ell_1}}+\frac{\ell_2+\ell_3+\ell_4+2}{\pi_{\ell_2}} \\ \frac{\ell_4+\ell_3+\ell_2+1+\mu_{\ell_1}}{\ell_4+\ell_3+\ell_2+2} & \text{if } \frac{\ell_1}{\pi_{\ell_1}}+\frac{\ell_2+\ell_3+\ell_4+2}{\pi_{\ell_2}} \geq 1 > \frac{\ell_1+\ell_2+\ell_3+\ell_4+2}{\pi_{\ell_1}} \\ \frac{\ell_4+\ell_3+\ell_2+\ell_1+1}{\ell_4+\ell_3+\ell_2+\ell_1+2} & \text{if } \frac{\ell_1+\ell_2+\ell_3+\ell_4+2}{\pi_{\ell_1}} \geq 1. \end{cases} \tag{12}
$$

In particular, Mode 2 seems to perform as well as Mode 1 when the prelog constraints $\mu_{\text{Tx}}, \mu_{\text{Rx}}$ are small.

Our observation applies also to the optimized asymptotic multiplexing gain per user, see Corollaries 2 and 3 and the following example.

Example 2: Let $\eta_{\text{Tx}} = 0$ and $\eta_{\text{Rx}} = 1$. Then,

$$
\mathcal{S}^{\star}_{\text{Mode1}}(\eta_{\text{Tx}}, \eta_{\text{Rx}}) = 1 \quad \text{and} \quad \mathcal{S}^{\star}_{\text{Mode2}}(\eta_{\text{Tx}}, \eta_{\text{Rx}}) = \frac{7}{9}.
$$

IV. Rough Sketch of Coding Schemes

We briefly sketch our coding schemes for Modes 1 and 2 leading to the lower bounds in Theorems 1 and 2.

A. Scheme for Mode 1

For Mode 1, we silence some of the transmitters so as to partition the network into one big subnet that exploits the conferencing links and several isolated point-to-point channels that ignore the conferencing links completely. If $t_{\text{L}} > 0$ ($t_{\text{R}} > 0$), then $K\mu_{\text{Tx}}$ transmitters of the big subnet send a "prelog-1" quantisation of their input signal to their closest right-neighbor (left-neighbor). If $r_{\text{L}} > 0$ ($r_{\text{R}} > 0$), then $K\mu_{\text{Rx}}$ receivers in the big subnet send their decoded messages—which are also of prelog 1—to their immediate right-neighbor (left-neighbor). The conferenced quantized signals permit to cancel interference at the transmitters by means of dirty-paper coding, and the conferenced decoded messages permit to reconstruct and subtract interfering signals at the receivers. Time-sharing K instances of the described scheme where we vary the transmitters and receivers that conference, allows to satisfy the conferencing prelog constraints μ_{Tx} and μ_{Rx}.

In this scheme each transmitter and receiver conferences only with its immediate left- and right-neighbor. Nevertheless, our conferencing protocol makes that information from some of the transmitters (receivers) propagates to the $K\mu_{\text{Tx}}$ transmitters (the $K\mu_{\text{Rx}}$ receivers) to its left or right.

B. Scheme for Mode 2

Our scheme for Mode 2 is inspired by [13, 14]. We silence every $t_{\text{L}} + t_{\text{R}} + r_{\text{L}} + r_{\text{R}} + 2$-th transmitter so as to split the network into many subnets. In each of these subnets a subset of transmitters share their "prelog-1" messages to the t_{L} (t_{R}) transmitters to their left (right), and a subset of receivers send a "prelog-1" quantisation of their receive signals to the r_{L} (r_{R}) receivers to their left (right). The conferenced messages permit to cancel interference at the transmitters by means of dirty-paper coding, and the conferenced quantised signals permit to cancel interference through successive interference cancellation at the receivers. Carefully time-sharing several instances of the described scheme for different parameters allows to achieve the desired multiplexing gain per user and to satisfy the prelog constraints μ_{Tx} and μ_{Rx}.

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