Linear and nonlinear combustion dynamics analysis and stability prediction

T. Schuller, D. Durox, F. boudy, P. Palies, S. Candel

Laboratoire EM2C, CNRS and Ecole Centrale Paris
92295 Châtenay-Malabry, France

thierry.schuller@em2c.ecp.fr

Summer school, Non-normal and Nonlinear Effects in Aero- and Thermoacoustics
Technische Universität Münichen, Germany, May 17-20, 2010

Objectives

1. Identify the main mechanisms governing the flame response to flow perturbations.
2. Give some theoretical and modelling tools to analyze combustion dynamics
3. Provide some elements for the prediction of linear and nonlinear stability of combustors

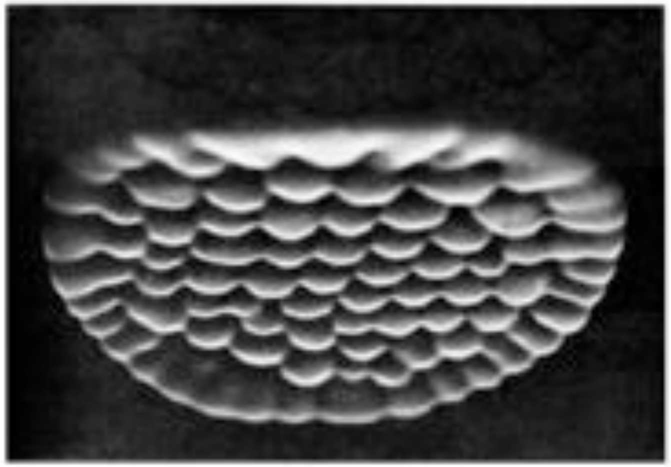
Material :

Literature review
EM2C production

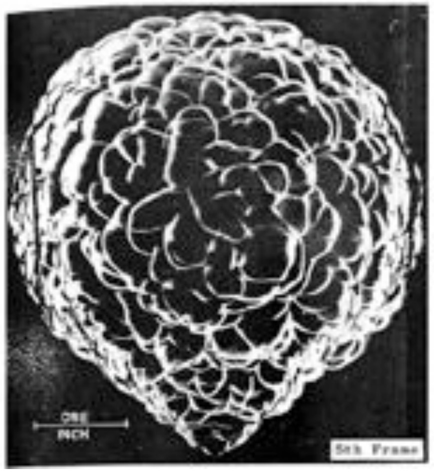
What is not considered?

Intrinsic instabilities == unstable combustion

Darrieus-Landau



Thermo-diffusive ?

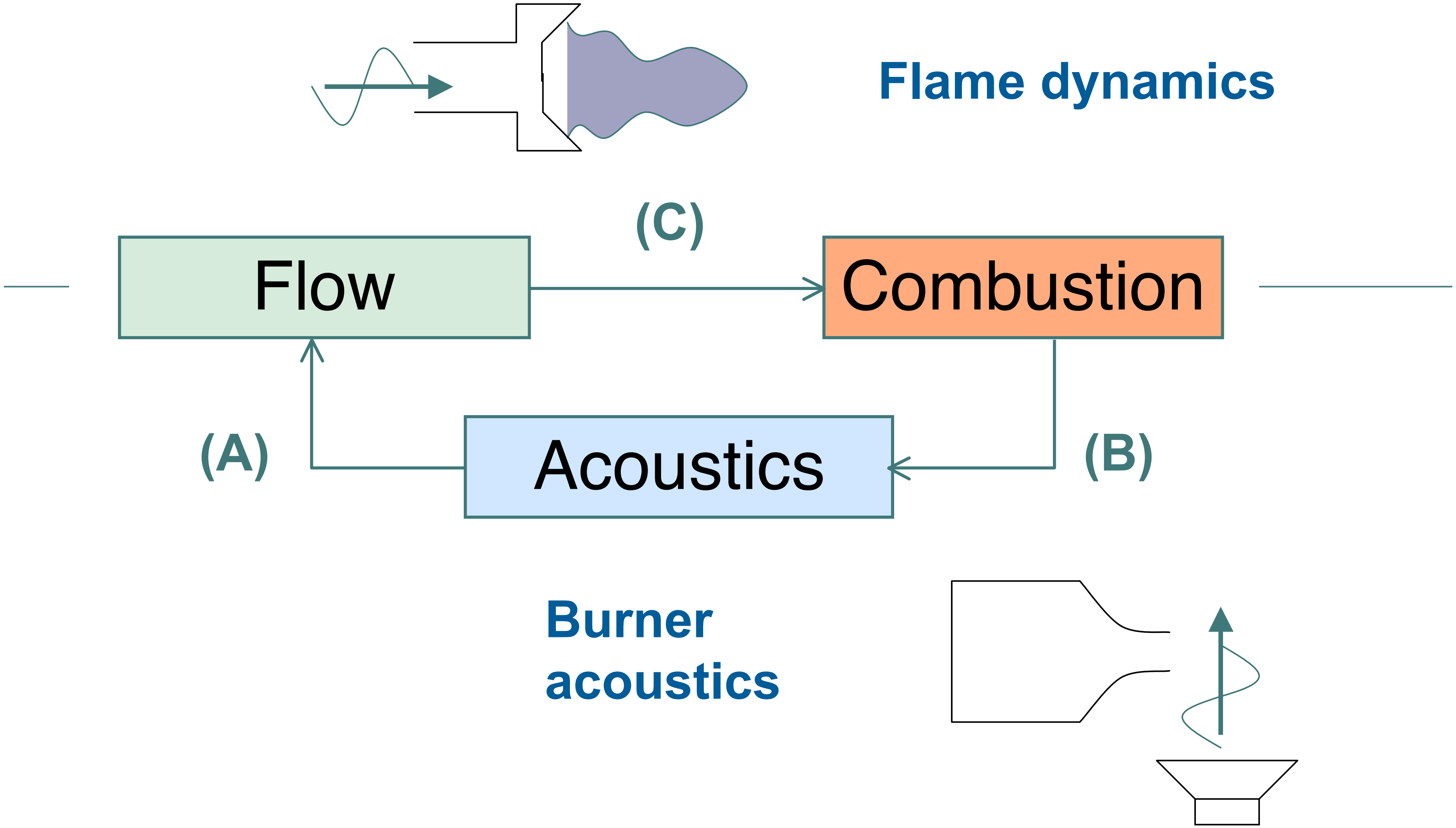


Palm-Leis&Strehlow (1969)
Markstein (1964)
Clavin et al. (1990)
Quinard (1990)
Clanet & Searby (1998)
Searby et al. (2001)

Growth rates are relatively weak : in many practical systems other mechanisms dominate

Exception : Oxy-fuel welding torch D-L instability

Acoustic induced combustion instabilities



Flame dynamics

(C)

Flow

Combustion

(A)

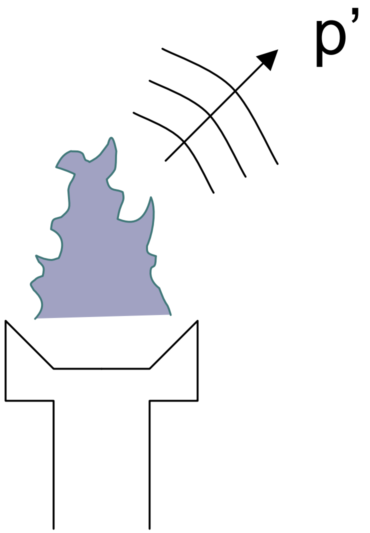
Acoustics

(B)

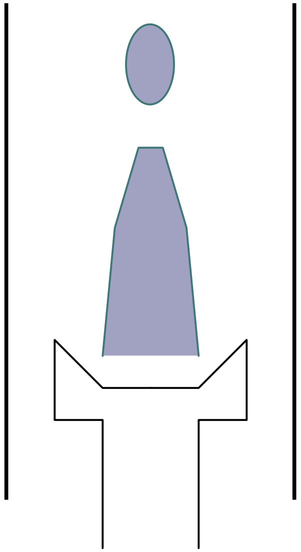
Burner acoustics

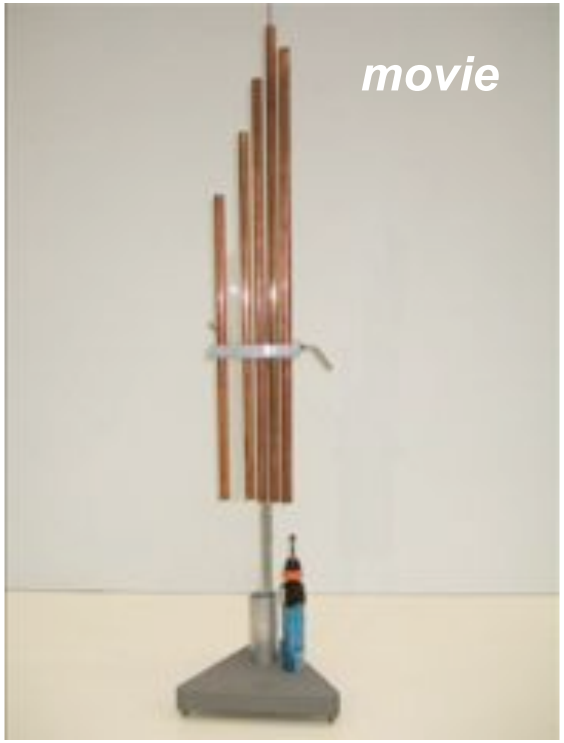
Organ pipe

Stable regime



Unstable regime





Turbulent fluctuations
Small amplitudes
Broad band noise

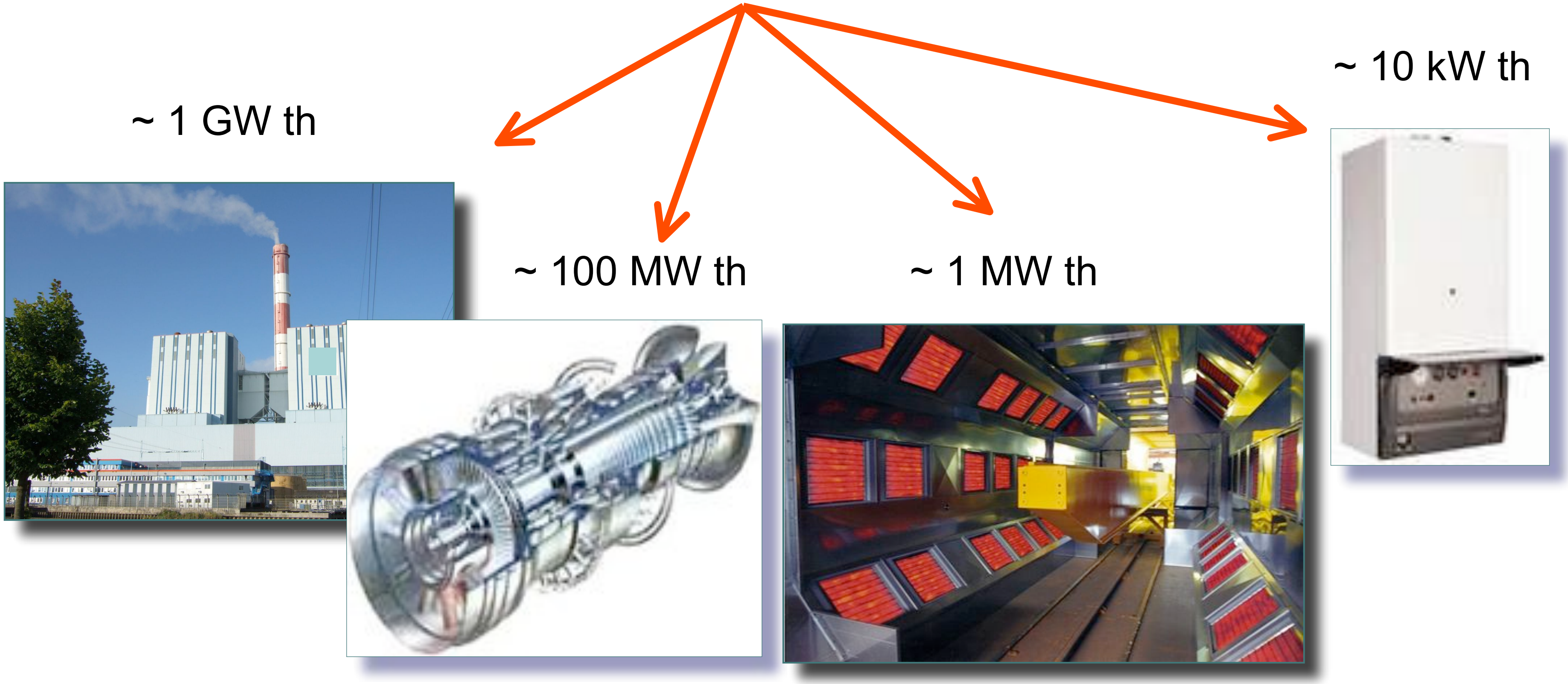
Combustion noise

High amplitude self-sustained cyclic oscillation.
The frequency depends on:
- the flame position within the tube
- the tube length
- the boundary conditions

Industrial configurations

New technologies favor acoustic coupled problems :

o High pressure	Increase efficiency	High energy densities
o Compact design	Weight gain	Highly reflective
o Lean combustion	Low NOx	Stabilization problems



~ 1 GW th

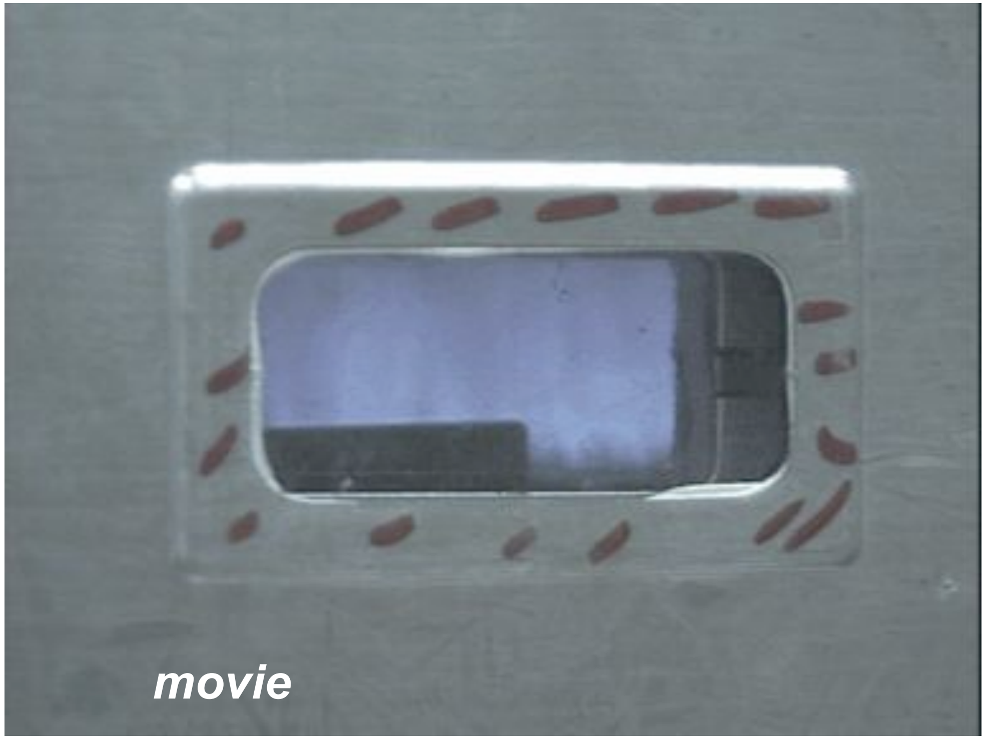
~ 100 MW th

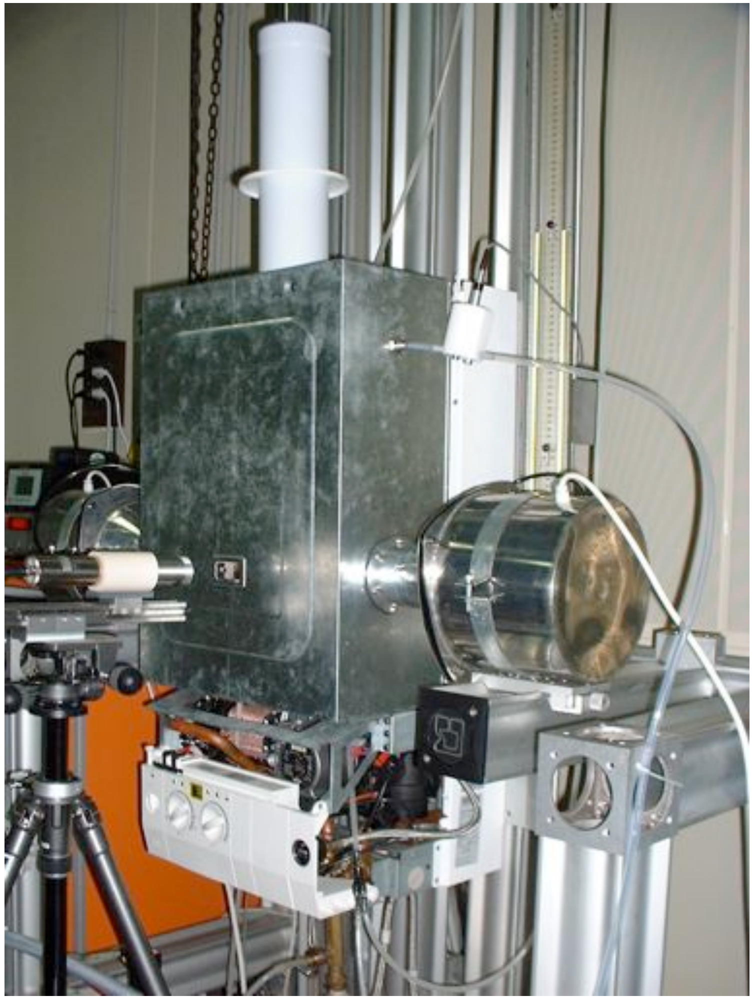
~ 1 MW th

~ 10 kW th

First example

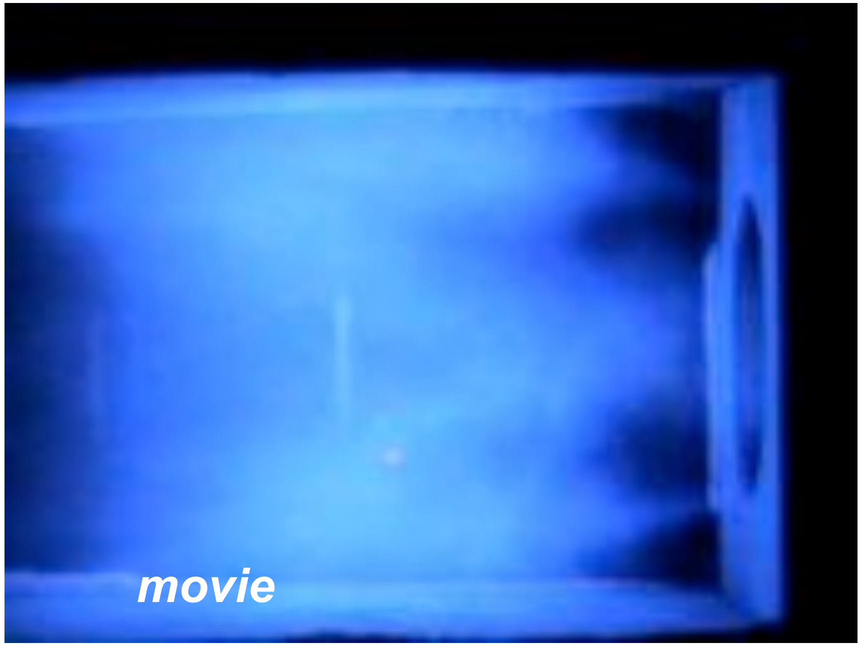
Domestic boiler





During unstable regime, walls are "breathing": large pressure fluctuations within the burner

Second example



Multipoint injection swirled burner

S. Barbosa, S. Ducruix,
P. Scouflaire (2007)

Stable regime: combustion zone (luminous zone) features small stochastic fluctuations around its mean location. Radiated noise remains weak and broad band : “combustion roar”.

Unstable regime : Large synchronized motions with a peak noise emission. Intensification of luminosity near the wall : higher heat fluxes transfered. Induce flame flashback.

Roadmap

1. **Elementary mechanisms**
2. *Flame dynamics*
3. *Linear and nonlinear stability analysis*

Heat release rate fluctuations

Premixed and partially premixed flames

$$\dot{Q} = \int_A \underline{\dot{\omega}(\Phi, \epsilon)}\,\underline{dA(\Phi, \mathbf{v})}$$

$\underline{\dot{\omega}(\Phi, \epsilon)}$ Reaction rate per unit flame surface area

- equivalence ratio
- stretch effects

$\underline{dA(\Phi, \mathbf{v})}$ Flame surface area

- equivalence ratio
- velocity

$$\frac{\dot{Q}_1}{\dot{Q}_0} = \frac{\int \dot{\omega}_1(\Phi, \epsilon)\, dA_0}{\dot{\omega}_0 \int dA_0} + \frac{\int dA_1(\Phi, \mathbf{v})}{\int dA_0}$$

local reaction rate fluctuations

local reaction rate fluctuations

flame surface area fluctuations

Cho & Lieuwen CB (2005)

Interactions leading to heat release rate fluctuations

1. ## Flame interactions with flow perturbations
   A. Unsteady strain rates
   B. Organized flow structures
   C. Velocity perturbations
   D. Mixture composition oscillations

Reviews :
Candel (2002)
Ducruix et al. (2003)
Lieuwen & Culick (2005)

2. ## Flame interactions with solid boundaries
   E. Combustion chamber walls confining the flame
   F. Anchoring devices used to stabilize the flame

Boyer & Quinard (1990), Dowling (1999),
Durox et al. (2002), Birbaud et al. (2007),
Kornilov et al. (2007)

Strain rate effects

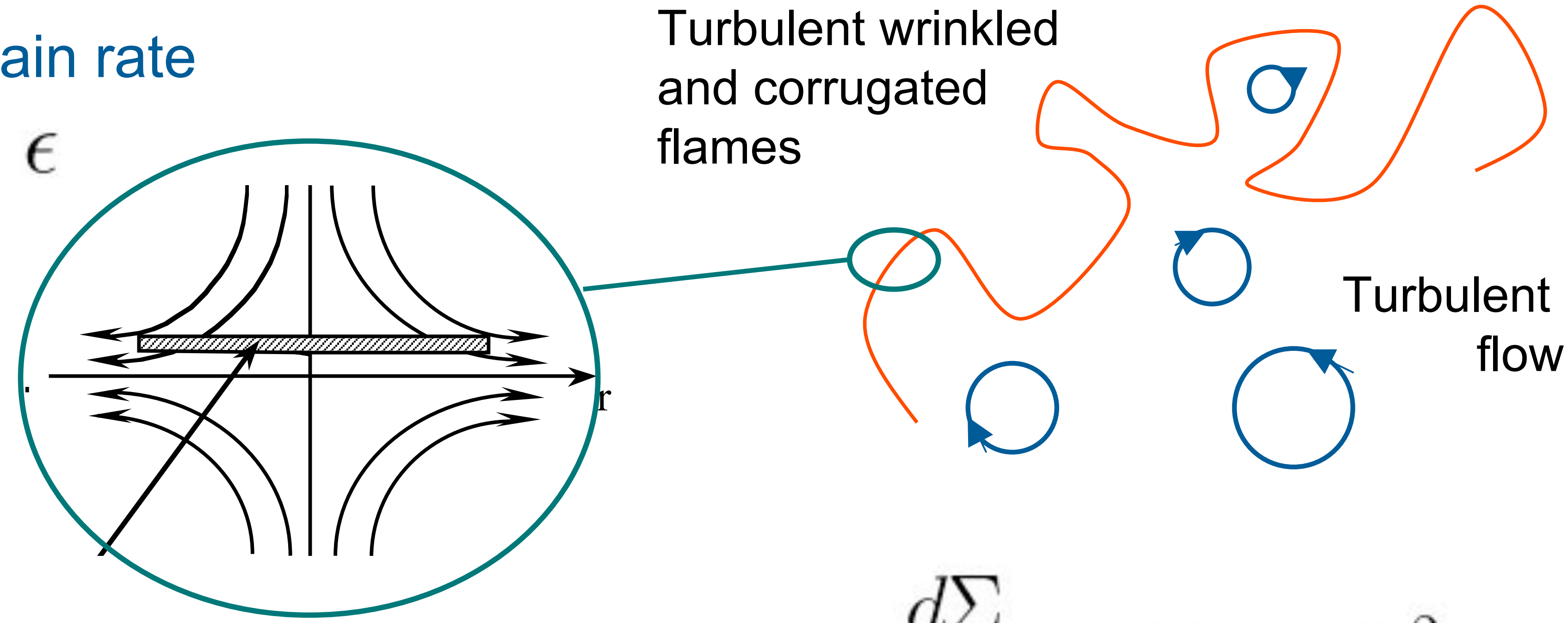
Flame surface density $\Sigma = \frac{A}{V}$

Flame surface area

Fluid volume

Strain rate

ϵ



Turbulent wrinkled and corrugated flames

Turbulent flow

Planar strained laminar flame

$$\frac{d\Sigma}{dt} = \epsilon\Sigma - \beta\Sigma^2$$

Strain rate oscillations

A. Fluctuations of flame surface density

Equilibrium $$\frac{d\Sigma_0}{dt} = 0 \qquad 0 = \epsilon_0 \Sigma_0 - \beta_0 \Sigma_0^2$$

Perturbations $$\epsilon = \epsilon_0 + \epsilon_1 \cos(\omega t) \qquad \Sigma = \Sigma_0 + \Sigma_1$$

Perturbed balance equation $$\frac{d\Sigma_1}{dt} + \epsilon_0 \Sigma_1 = [\epsilon_1 \cos(\omega t)] \Sigma_0$$

Low frequency limit $$\frac{\Sigma_1}{\Sigma_0} = \frac{\epsilon_1}{\epsilon_0} \cos(\omega t)$$

Flame surface density fluctuations are frequency dependent.

High frequency limit $$\frac{\Sigma_1}{\Sigma_0} = \frac{\epsilon_1}{\omega} \sin(\omega t)$$

High frequencies are low pass filtered

Candel (2002)

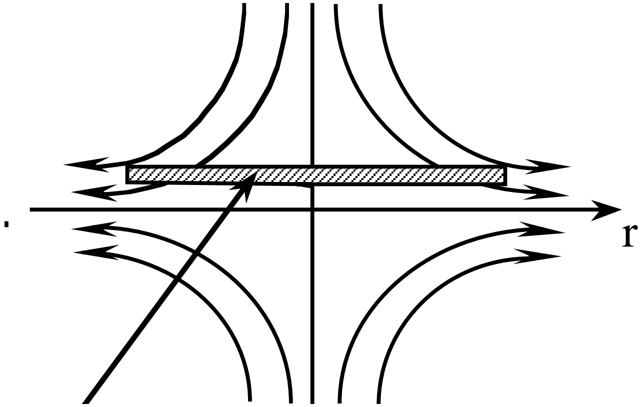
Strain rate oscillations

B. Fluctuations of the local reaction rate

$\dot{\omega} \simeq \sqrt{\epsilon}$ Non premixed flames Law (1988)

Response of the local mass rate consumption to strain rate oscillations

$$F(\omega) = \left(\frac{\dot{m}(\omega) - \dot{m}_0}{\dot{m}_0}\right) / \left(\frac{\epsilon - \epsilon_0}{\epsilon_0}\right)$$



Limit of infinitely fast chemistry

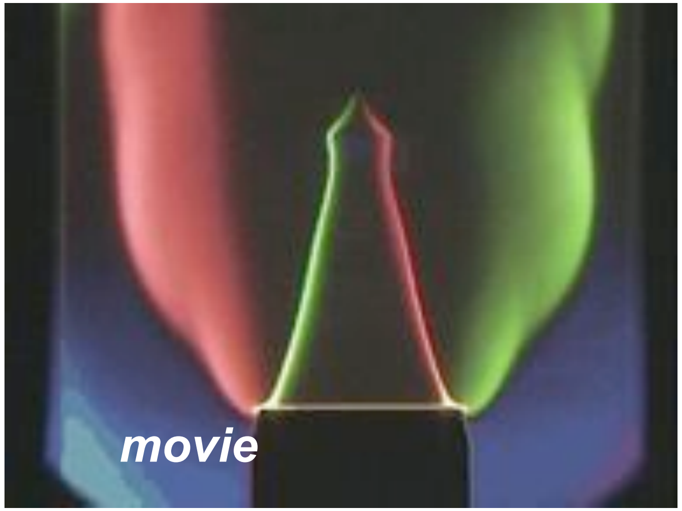
$$F(\omega) = \frac{1}{2}\frac{1}{1 + i(\omega/2\epsilon_0)}$$

Local reaction rate fluctuations are frequency dependent :
High frequencies are low pass filtered

Cut-off frequency
$\omega_0 = 2\epsilon_0$

Candel (2002)

Velocity perturbations



Bulk flow oscillations

f = 10 Hz

Acoustic wave $v_1 = \tilde{v}_1 \cos(\omega t - ky)$

$k = \omega / c_0$ $kL \ll 1$

Compact treatment

Fleifil et al. (1996), Ducruix et al. (2000)

Flow perturbations

f =150 Hz

Convective wave $v_1 = \tilde{v}_1 \cos(\omega t - ky)$

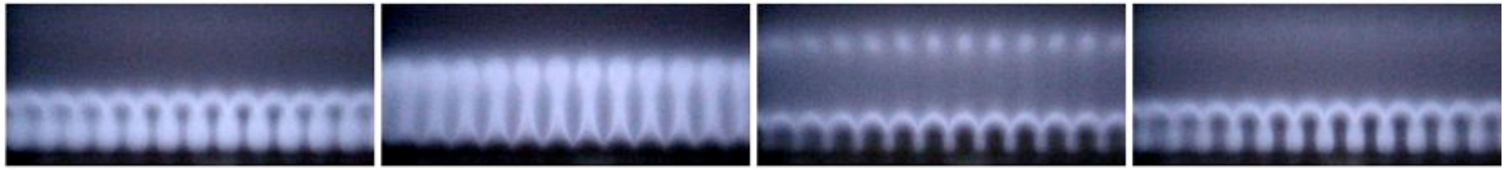
$k = \omega / u_0$ $kL \sim 1$

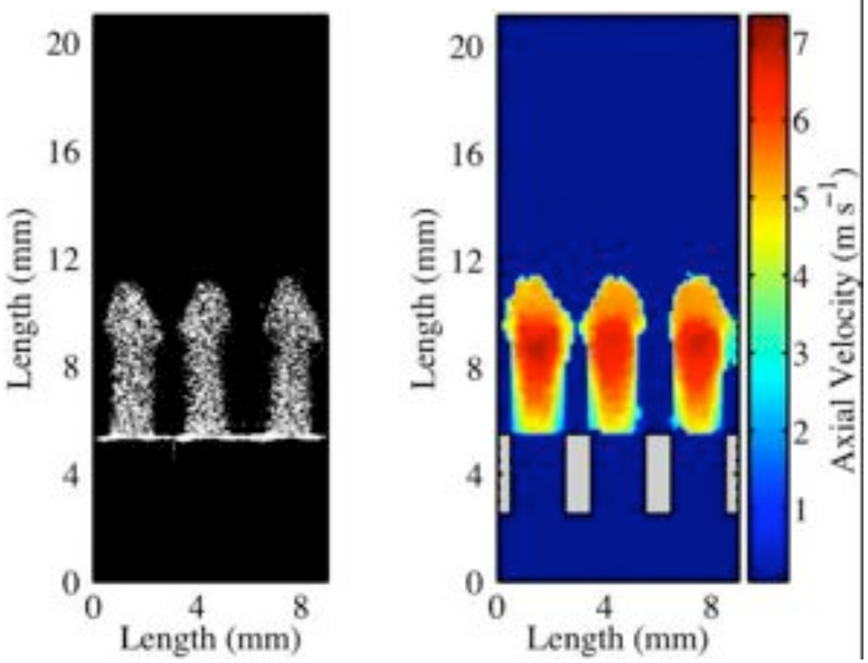
Non compact treatment $\nabla \cdot \mathbf{v}_1 = 0$

Baillot et al. (1992), Schuller et al. (2002)

Velocity perturbations

Collection of flames anchored on a perforated plate





Convective waves are often present near solid boundaries used to stabilize the combustion zone as soon as the flame is not perfectly flat.

They constitute an important component which determines the time lag of the flame response

Noiray et al. (2006, 2007, 2008)
Kornilov et al. (2009)
Altay et al. (2009)

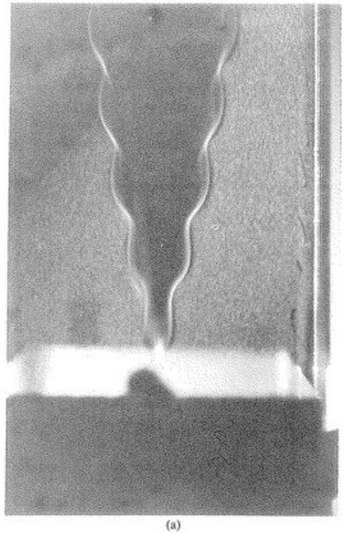
$\Phi = 0.85, V_d = 5.2$ m/s, $v' = 1.35$ m/s, $f = 500$ Hz

Anchoring point dynamics

Vibrating rod

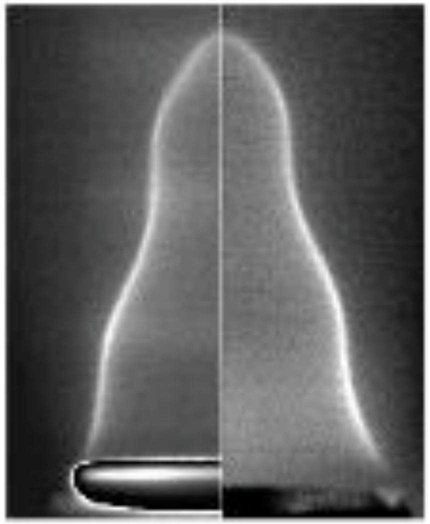
Petersen & Emmons (1961)

Acoustic modulation



Boyer & Quinard. (1996)

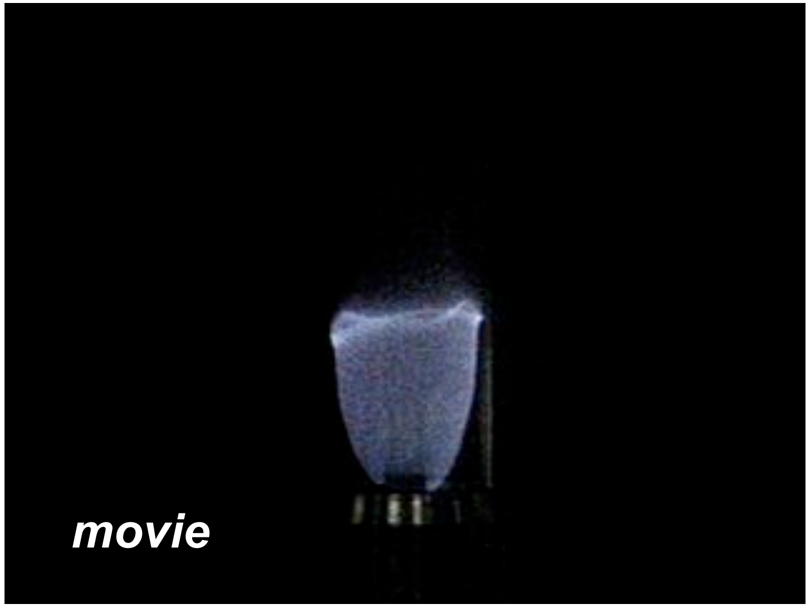
Ring modulation | Acoustic modulation



Kornilov et al. (2007)

Ring modulation and acoustic waves produce the same type of wrinkles along the flame front

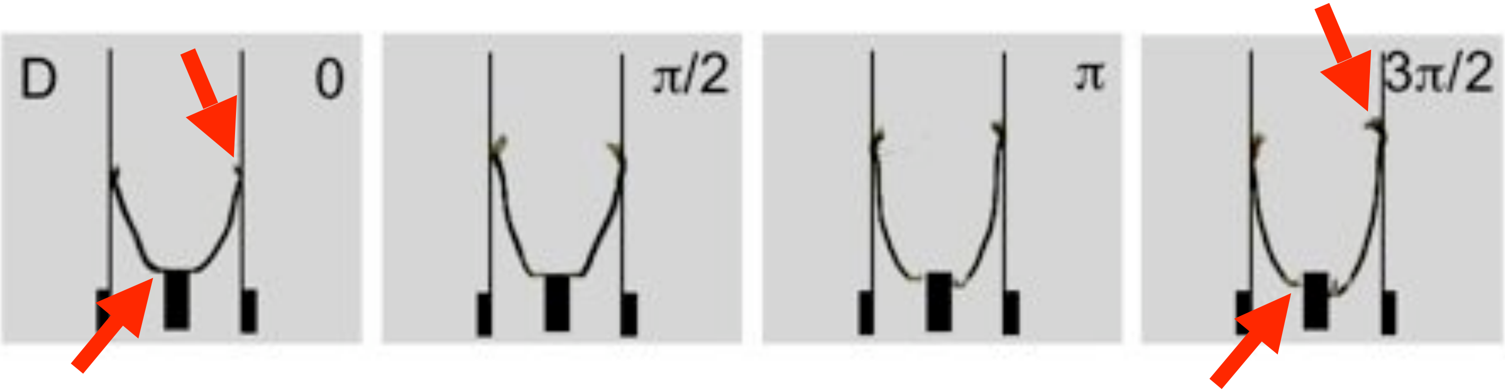
Interactions with solid boundaries



Modulated flame oscillates in a breathing mode : anchor point and flame tip feature a bulk oscillation.

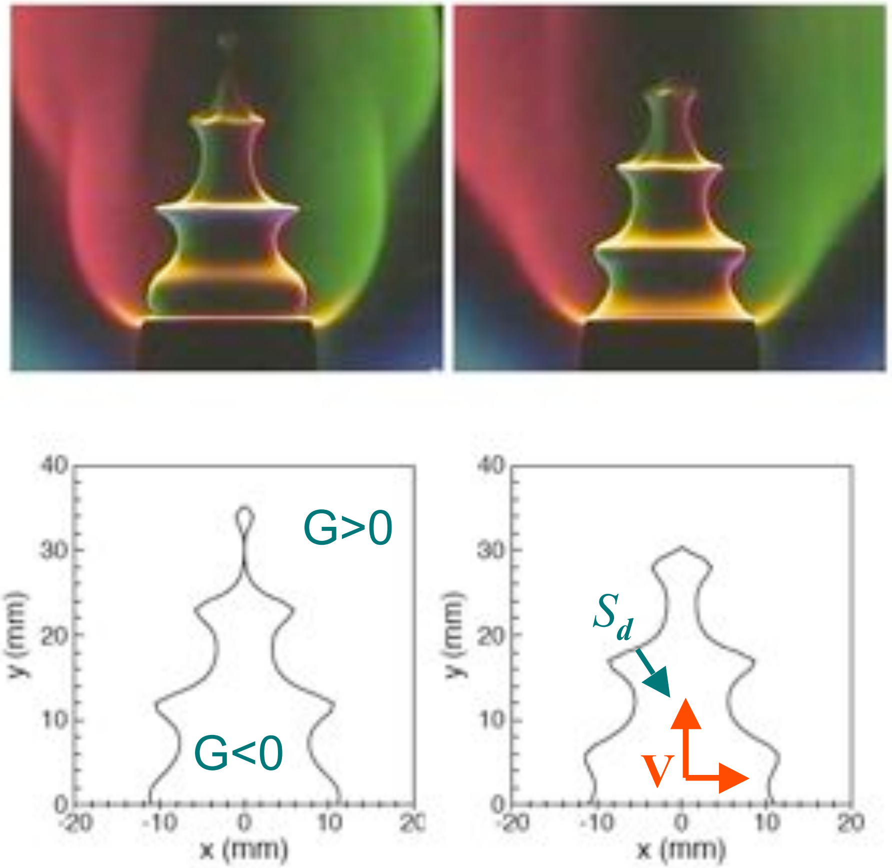
Flame anchoring point and flame tip dynamics are responsible of strong nonlinearities

Dowling (1999), Birbaud et al. (2009)



Kinematic description

Numerical modelling tools



Combine a kinematic description of the flame front with a prescribed perturbation field

$$\frac{\partial G}{\partial t} + \mathbf{v} \cdot \nabla G = S_d |\nabla G|$$

Schuller et al. (2002)

Nonlinear response

Lieuwen (2005), Preetham et al. (2008)

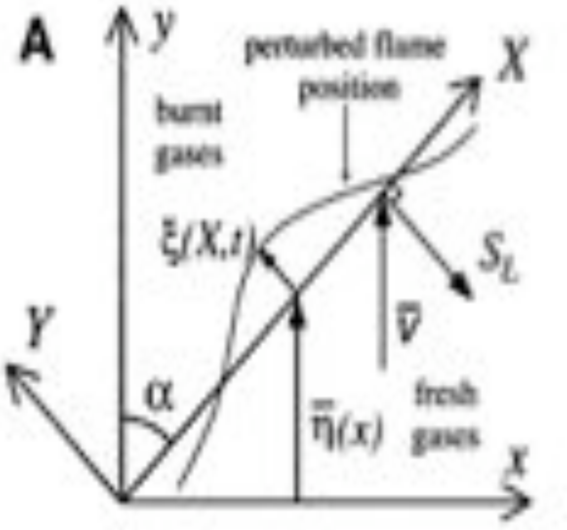
Turbulent flames

Preetham & Lieuwen (2007),
Hemchandra & Lieuwen (2010)

Kinematic description

Modelling elements

Flame response is easier to analyze in a reference frame attached to the flame front



Interference integral for flame front perturbations

$$\xi(X,t)=\underbrace{\frac{1}{\overline{U}}\int_0^X V\left(X',t-\frac{X-X'}{\overline{U}}\right)dX'}_{\text{Perturbed velocity field contribution}}+\underbrace{\xi_0\left(t-\frac{X}{\overline{U}}\right)}_{\text{Anchoring point dynamics}}$$

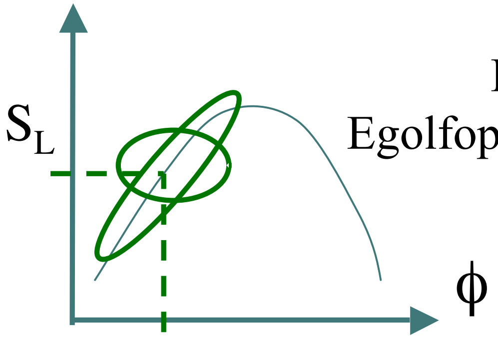
Boyer & Quinard (1990), Schuller et al. (2003), Lee & Liewen (2009)

Mixtures composition oscillations

Affect both the local reaction rate and flame surface area

$$\dot{Q} = \int_A \rho S_L \Delta h_R dA$$

$$\rho \simeq Cte$$



Lauvergne & Egolfopoulos (2000)

Flame speed describes cycles around steady conditions for increasing modulation frequencies

$$\frac{\dot{Q}'}{\dot{Q}} = \frac{\dot{Q}_{S_L}}{\dot{Q}} + \frac{\dot{Q}_{h_R}}{\dot{Q}} + \frac{\dot{Q}_A}{\dot{Q}}$$

Three contributions :

- Fluctuations in the flame speed
  Quasi-steady approach

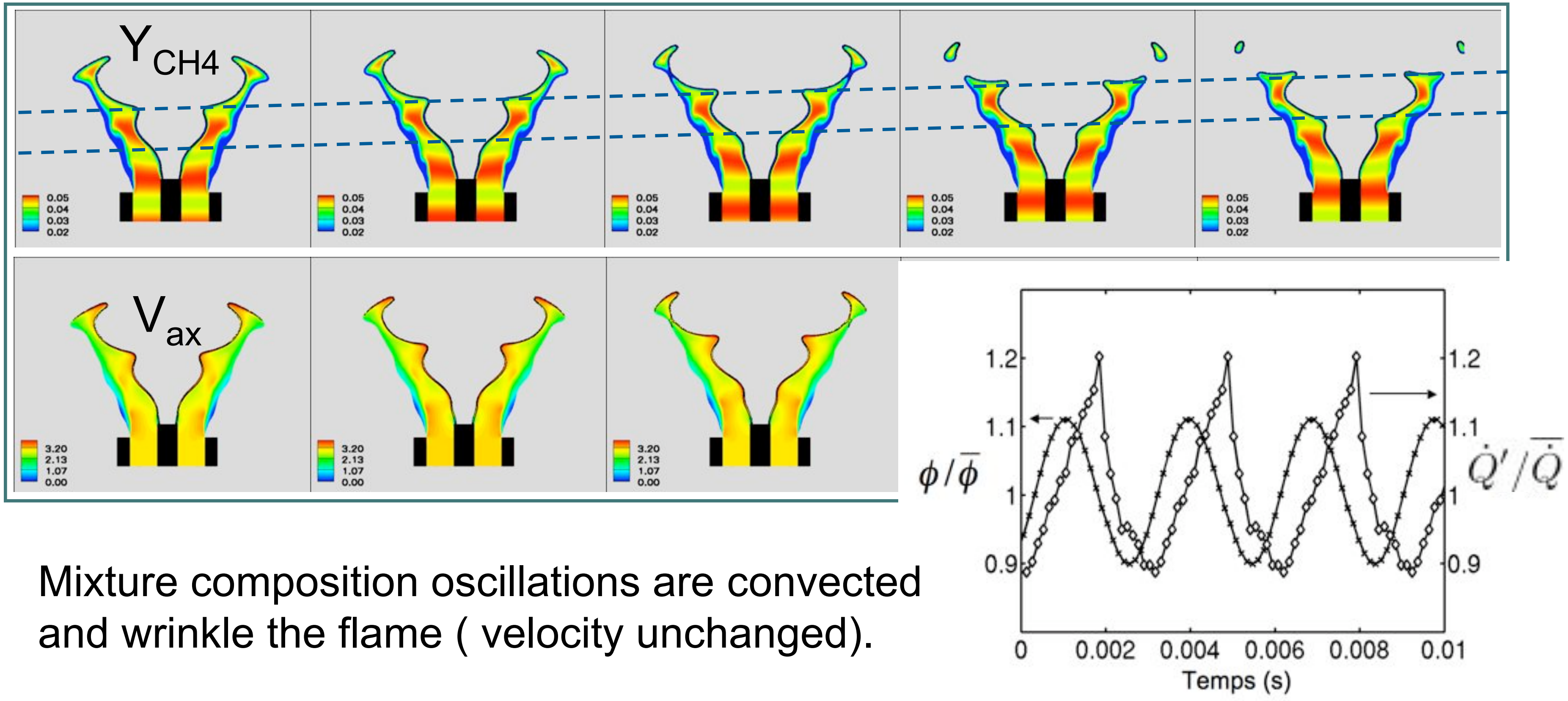
2. Fluctuation in the heat of reaction
   Use of correlations

3. Fluctuation in flame surface
   Kinematic approach

Cho & Lieuwen (2005)

Mixtures composition oscillations

Unsteady Navier-Stokes simulations : Modulated V flame



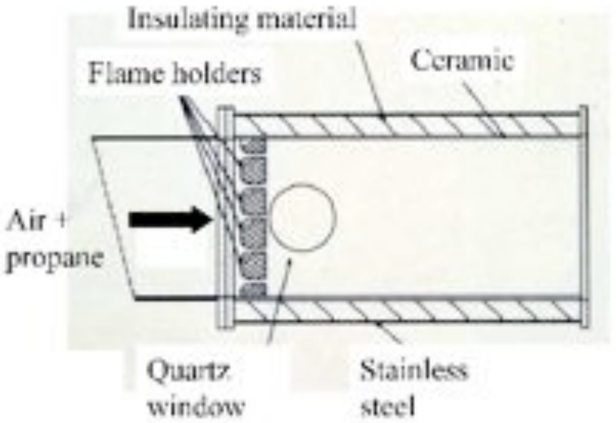
Mixture composition oscillations are convected and wrinkle the flame (velocity unchanged).

Fluctuations in the burning speed, local heat reaction and flame wrinkling result in large heat release rate fluctuations (nonlinear)

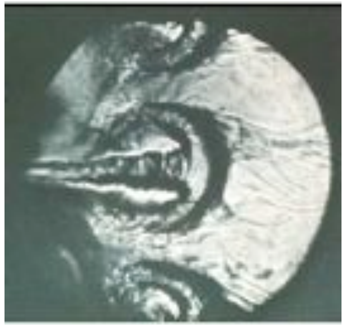
Birbaud et al. (2008)

Coherent structures

Vortex generation at the flame holder unit

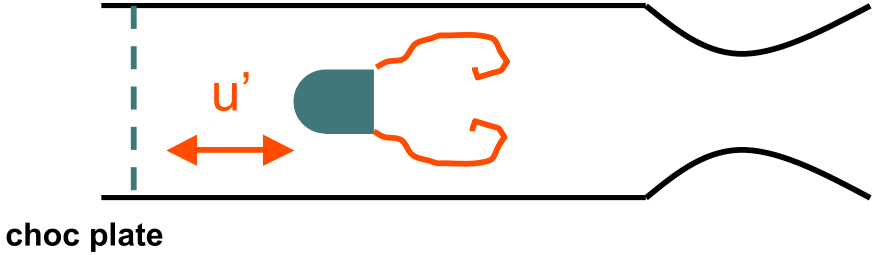


f = 535 Hz



Poinsot et al. (1987)

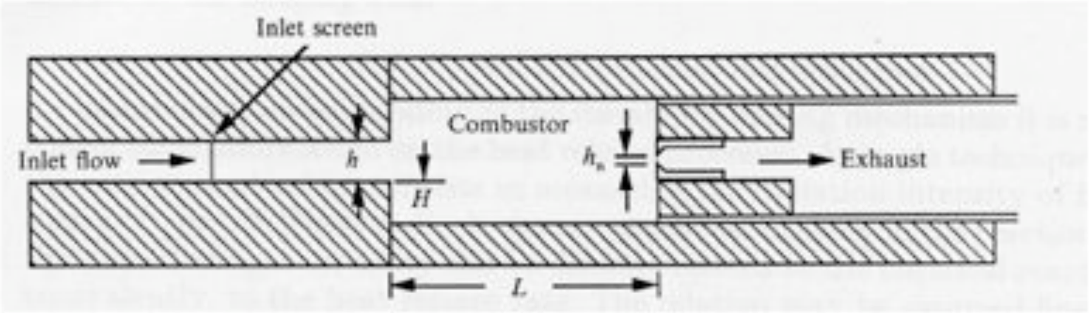
Vortices entrain hot combustion products, collisions with adjacent vortices induce large flame surface annihilations and a large pressure pulse.



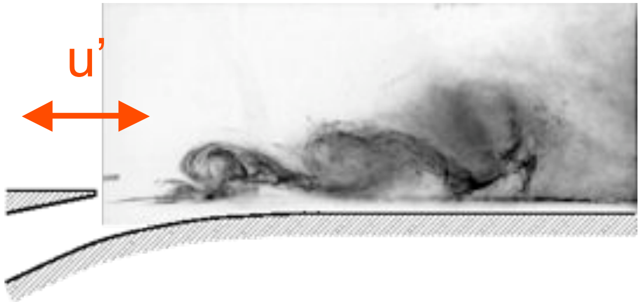
Langhorne (1988)
Bloxidsge et al. (1988)

Coherent structures

Acoustic - convective coupling



Variable geometry combustor Yu et al. (1991)



Shear layers are very sensitive to acoustic perturbations over a broad range of frequencies

Crow&Champagne (1971)

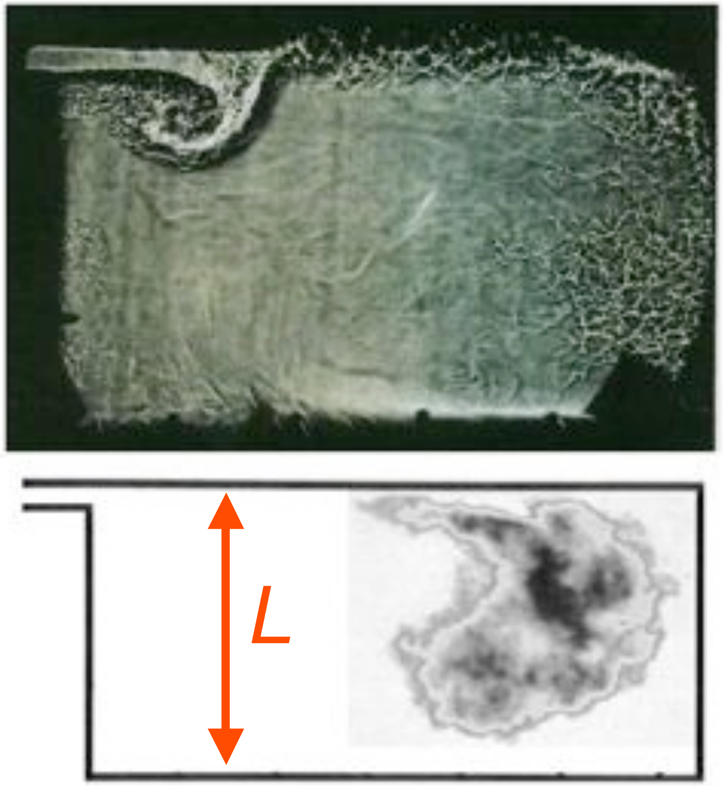
Resonant frequencies selection rule

$$\frac{1}{4N-1} \leq \frac{\tau_v}{\tau_f} \leq \frac{3}{4N-3}$$

N is the mode of oscillation and τ_v is the time for vortices to be convected from inlet to exhaust with τ_f being the feedback time taken for a pressure disturbance to travel up the inlet system and back.

Coherent structures

Flame vortex interactions with solid boundaries



Zsak et al. (1991), Sterling et al. (1991)

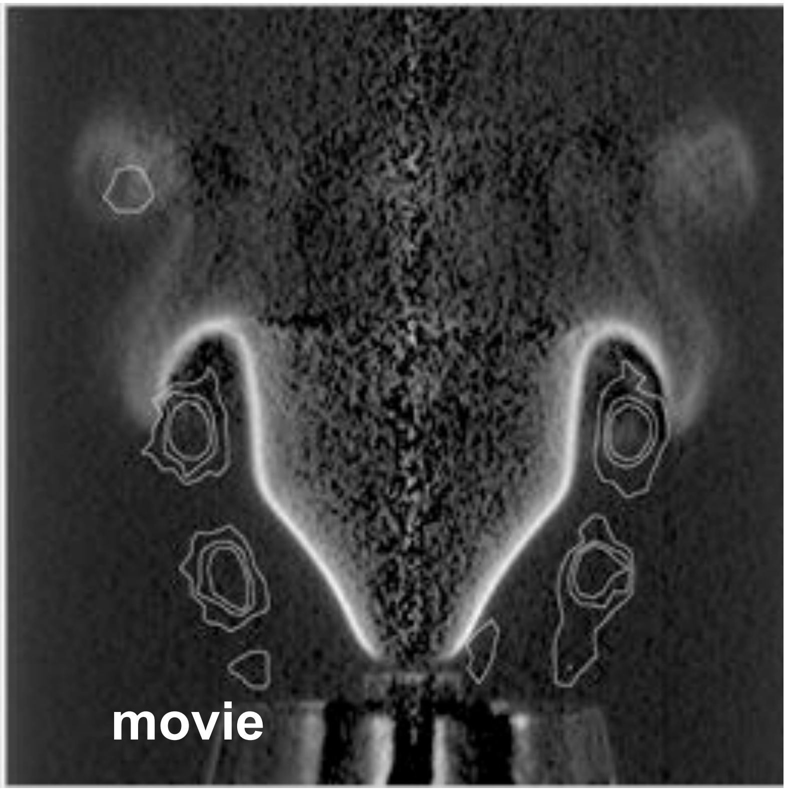
Vortex synchronized by a longitudinal mode

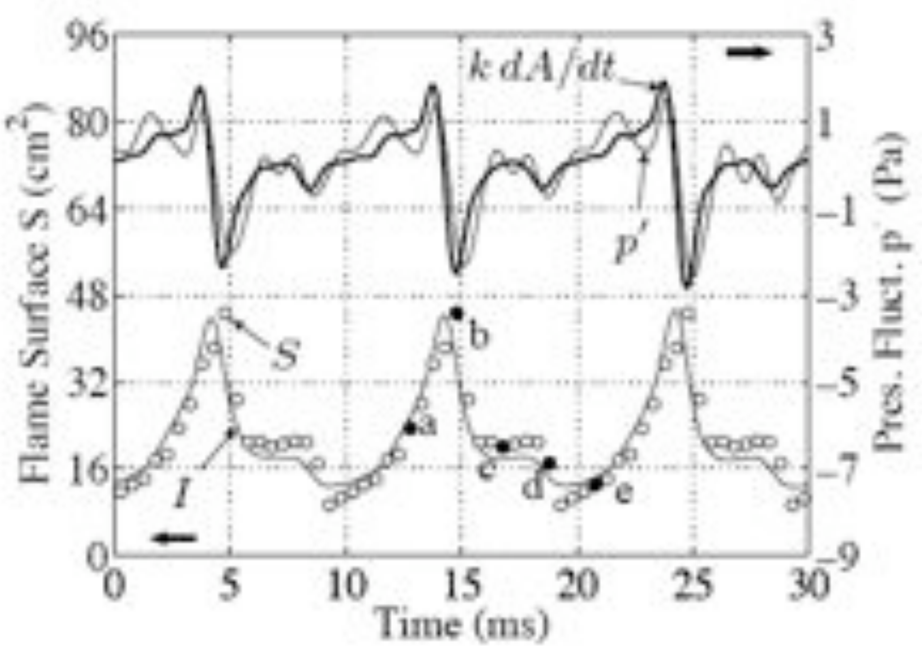
Collision with walls induces rapid burning of the fresh reactants entrained

Large L: delayed reactants combustion within large structure

Small L: flame wall interaction

Flame vortex interactions





(c) **VF** : $f_e = 100\,Hz$, $\overline{v} = 2.3\,m\,s^{-1}$, $v_{rms} = 0.6\,m\,s^{-1}$, $\Phi = 1.11$

Vortices generated in the shear layer are responsible of rapid flame surface destruction when impacting the flame periphery (strong nonlinearity)

Durox et al. (2005)

Swirled Flames

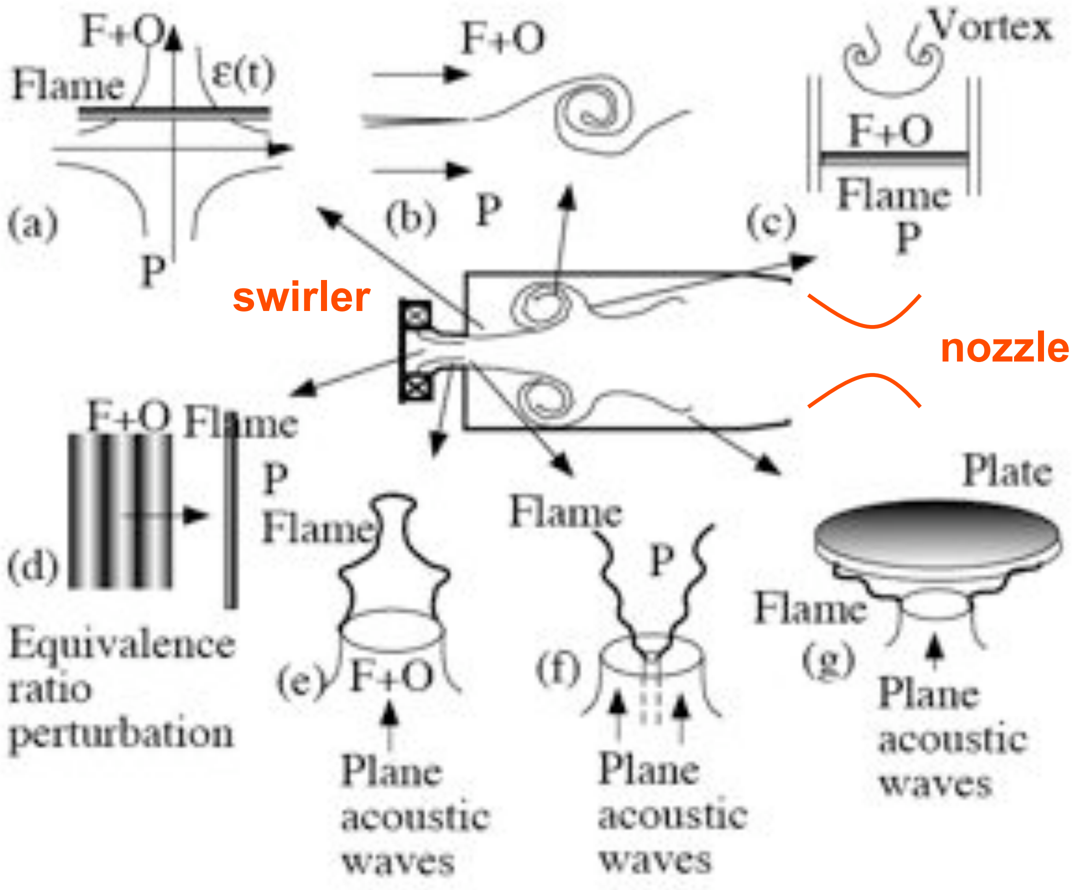
Presence of the swirler must be taken into account

In progress

- Swirl number and swirl number fluctuations
- Mode conversion
- Coherent structures

Practical burners

Additional difficulties : many competing mechanisms



Complex geometry
Turbulent flow
Complex boundaries

It is important to identify the dominant one(s) in each configuration

Roadmap

1. *Elementary mechanisms*
2. **Flame dynamics**
3. *Linear and nonlinear stability analysis*

Flame Transfer Function

Non premixed systems

Candel (2002), Tyagi (2007), Balasubramanian (2008)

Premixed/partially premixed systems

Flame frequency response relating heat release rate fluctuations to incoming flow perturbations represented :

$$\frac{\dot{Q}_1}{\dot{Q}_0} = \frac{\int \dot{\omega}_1 dA_0}{\dot{\omega}_0 A_0} + \frac{\int dA_1}{A_0}$$

- Mixture composition oscillations
- Velocity fluctuations

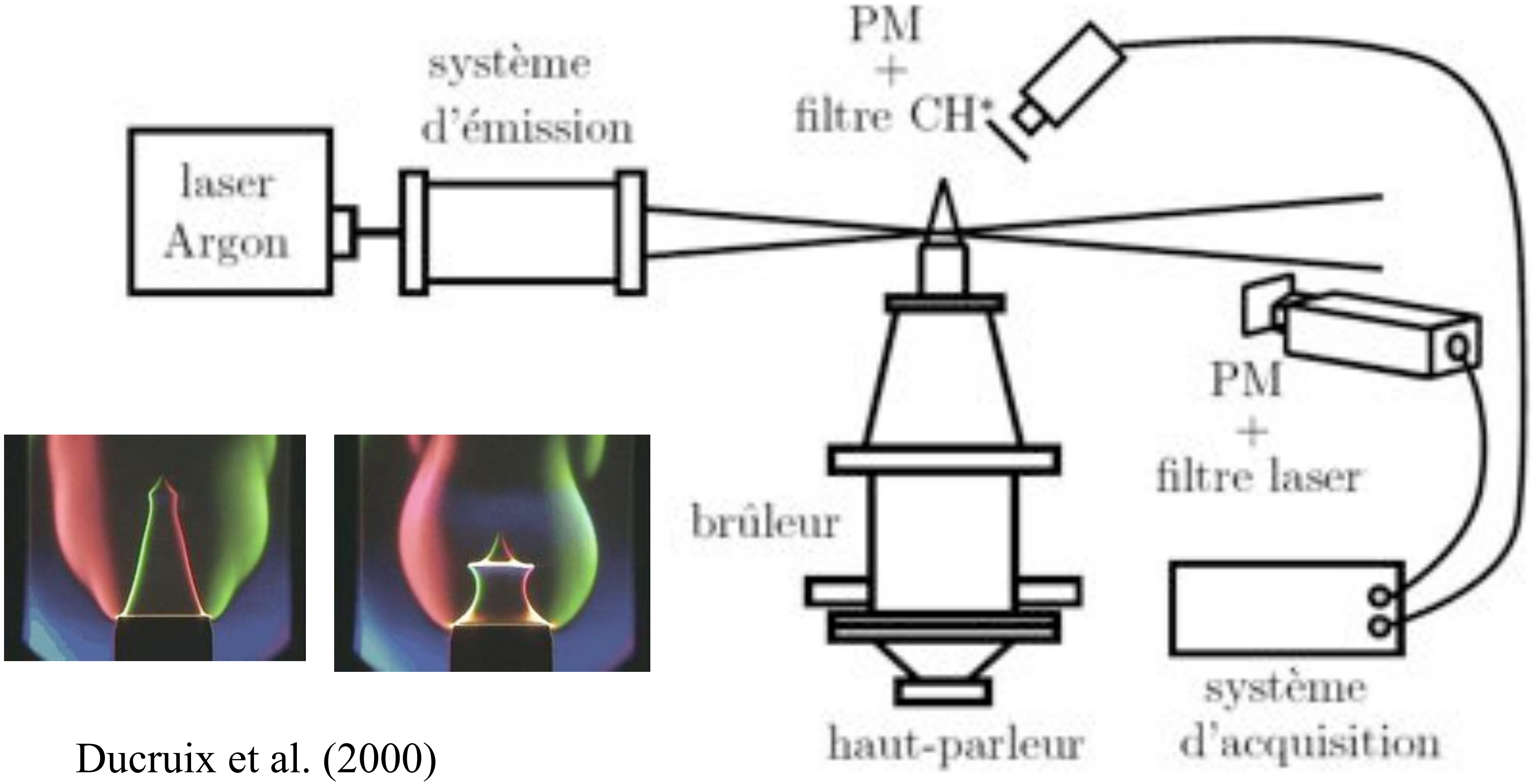
$$\begin{matrix} \dot{\omega}_1(\phi_1) \\ dA_1(\phi_1, \mathbf{v}_1) \end{matrix} \longrightarrow \dot{Q}_1$$

$$\frac{\dot{Q}_1}{\dot{Q}_0}(\omega) = F_v(\omega, v_0^u, \phi_0, v_1^u)\frac{v_1^u}{v_0^u} + F_\phi(\omega, v_0^u, \phi_0, \phi_1)\frac{\phi_1}{\phi_0}$$

velocity input mixture input

Flame Transfer Function

Example 1 : Premixed conical flame FTF submitted to incoming velocity perturbations in a CH4/air mixture

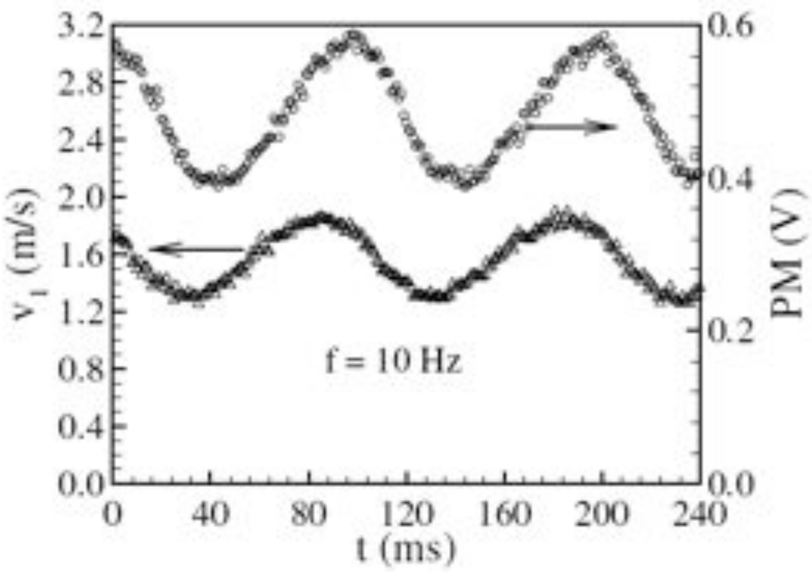


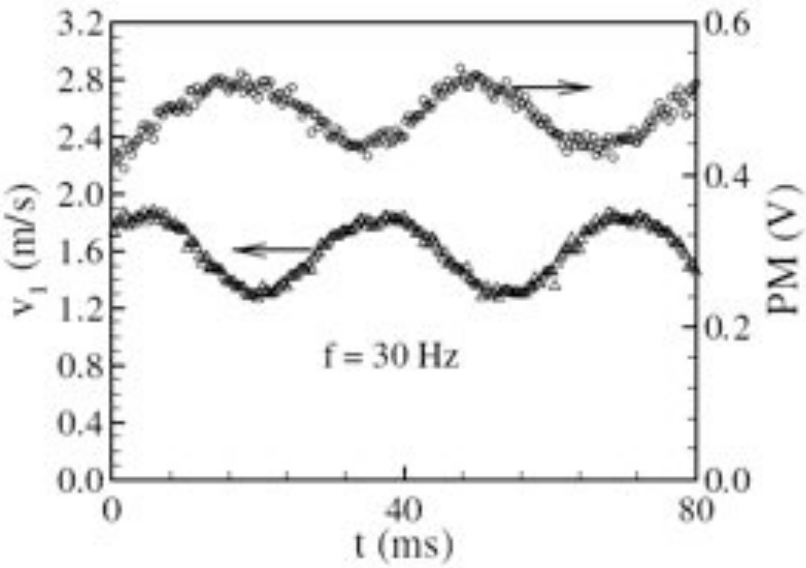
Ducruix et al. (2000)

Flame Transfer Function

Mixture kept at constant equivalence ratio

Modulation level kept constant Φ=0.95, v_0=1.20 m/s, v_{1rms}=0.19 m/s





The velocity input is harmonic and the flame response (heat release rate fluctuation) remains also harmonic

Linear response independent of the modulation level (in these two cases)

Flame Transfer Function

$$\frac{\dot{Q}_1}{\dot{Q}_0}(\omega) = F_v(\omega, v_0^u, \phi_0, v_1^u)\frac{v_1^u}{v_0^u}$$

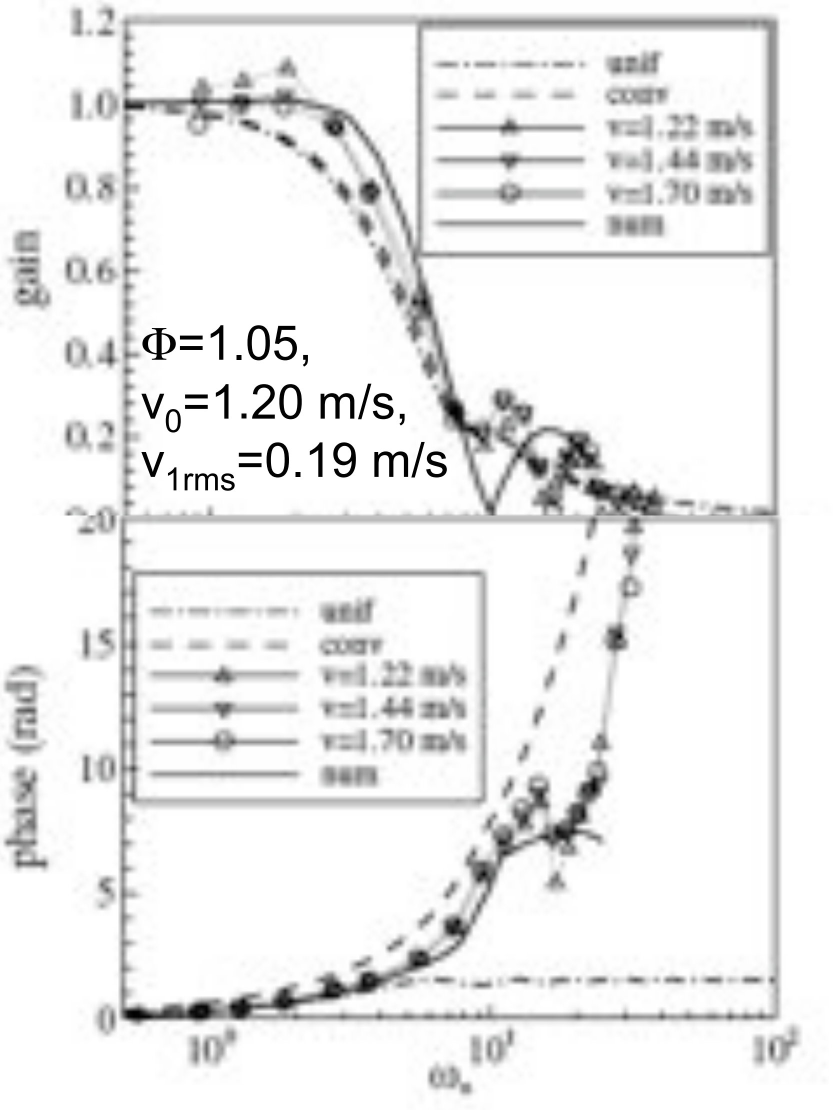
$$F_v = G(\omega)\exp(i\varphi)$$

Gain :

- relative fluctuation amplitude
- G>1 amplification
- G<1 attenuation
- Low pass filter

Phase :

- time lag $\varphi = \omega\tau$
- convective
- saturation

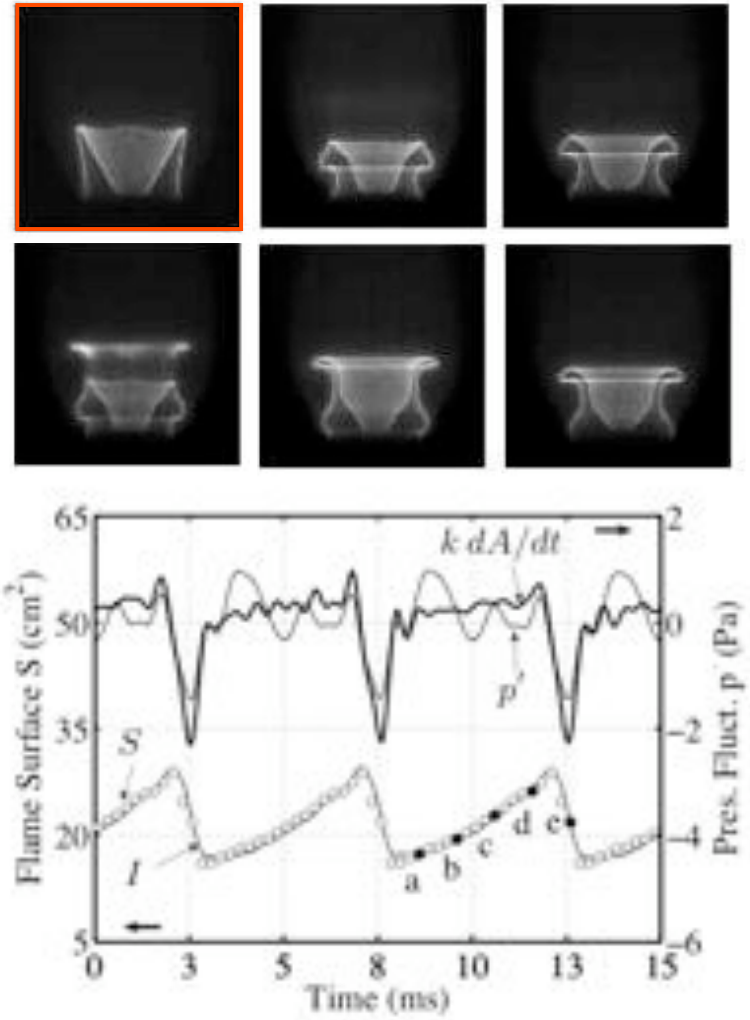


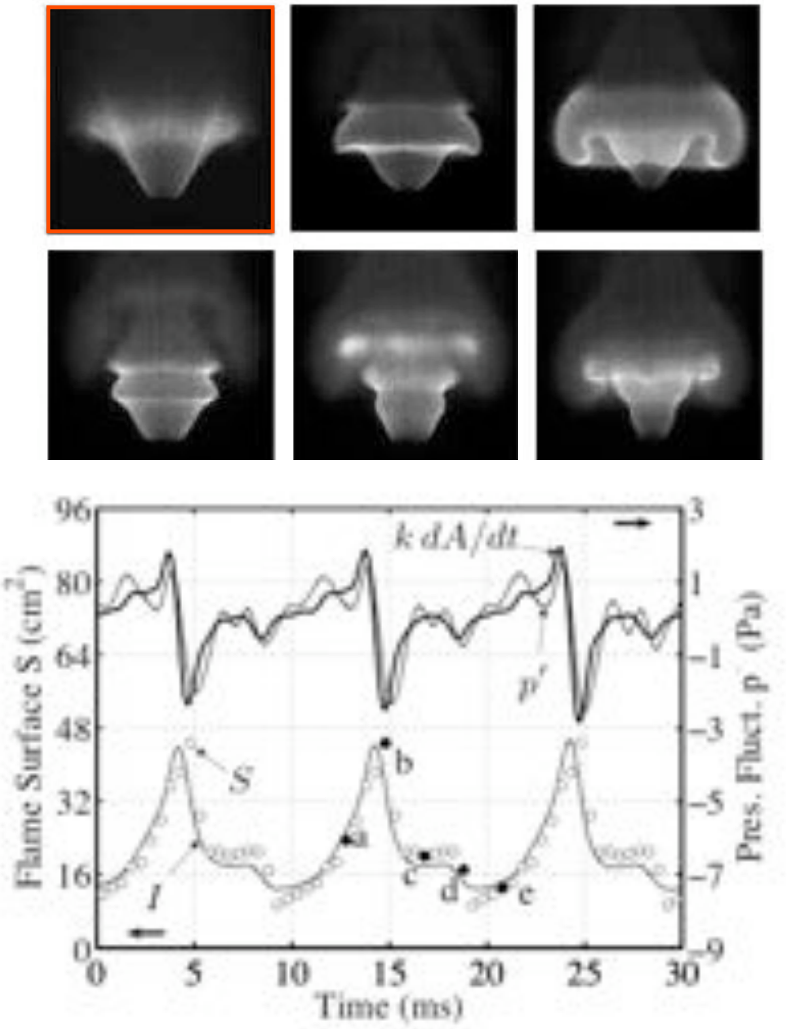
Φ=1.05,
v_0=1.20 m/s,
v_{1rms}=0.19 m/s

Schuller et al. (2002), Schuller et al. (2003), Kornilov et al. (2007)

Flame Transfer Function

The flame response depends on the type of perturbations and initial flame geometry





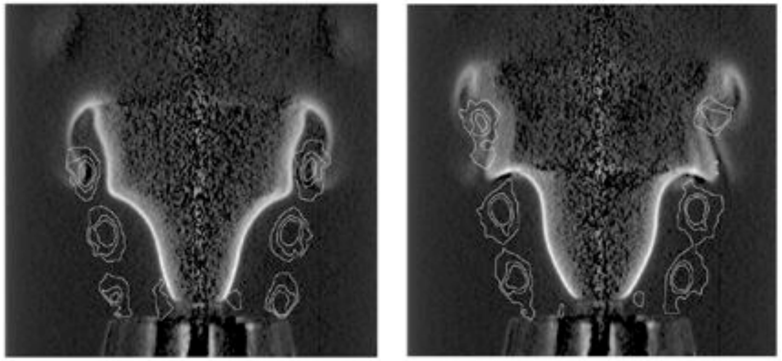
Flame Transfer Function

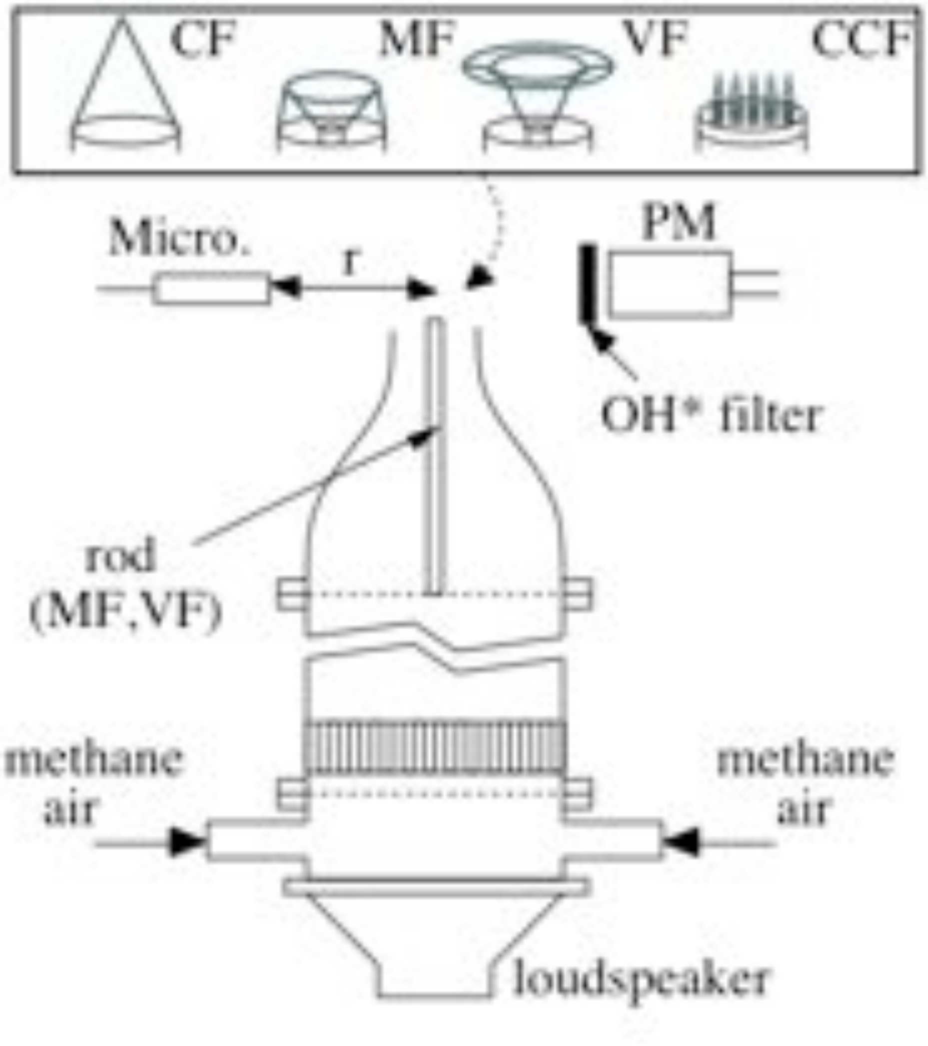
Example 2 :

Premixed “V” flame submitted to incoming velocity perturbations in a CH4/air mixture

Mixture kept at constant equivalence ratio

Modulation level kept constant

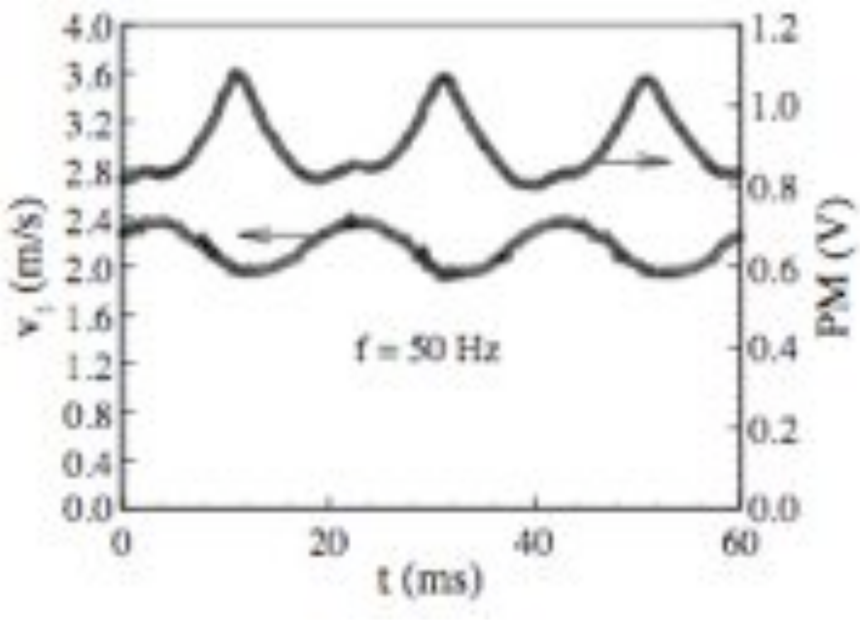


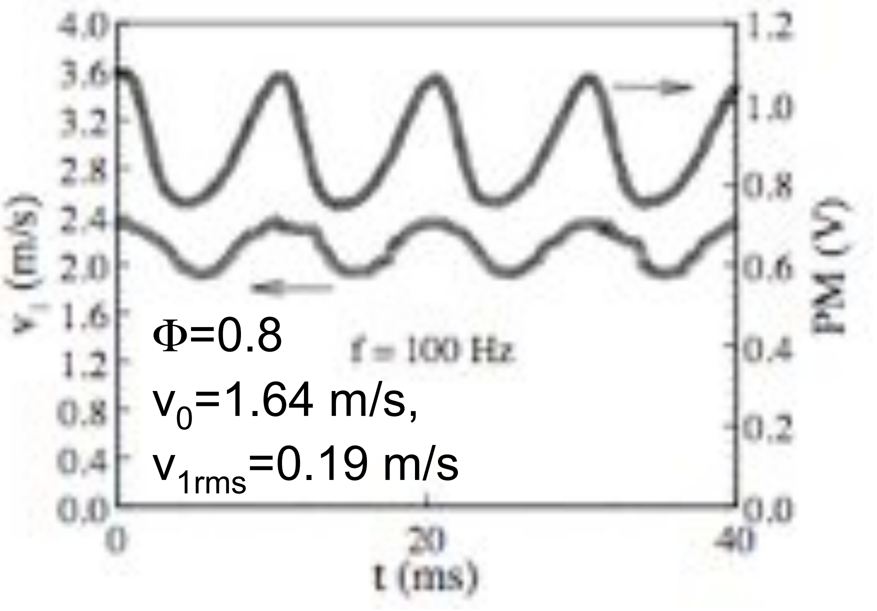


Durox et al. (2005)

Flame Transfer Function

The velocity input is harmonic, but the flame response is nonlinear and depends on the modulation level





Φ=0.8

v_0=1.64 m/s,

v_{1rms}=0.19 m/s

The FTF should be defined using spectral analysis tools examined at the forcing frequency : Principal harmonic analysis

$$FTF = \frac{S_{xy}}{S_{xx}}$$

Cross power spectral density

Power spectral density

Flame Transfer Function

$$\frac{\dot{Q}_1}{\dot{Q}_0}(\omega) = F_v(\omega, v_0^u, \phi_0, v_1^u)\frac{v_1^u}{v_0^u}$$

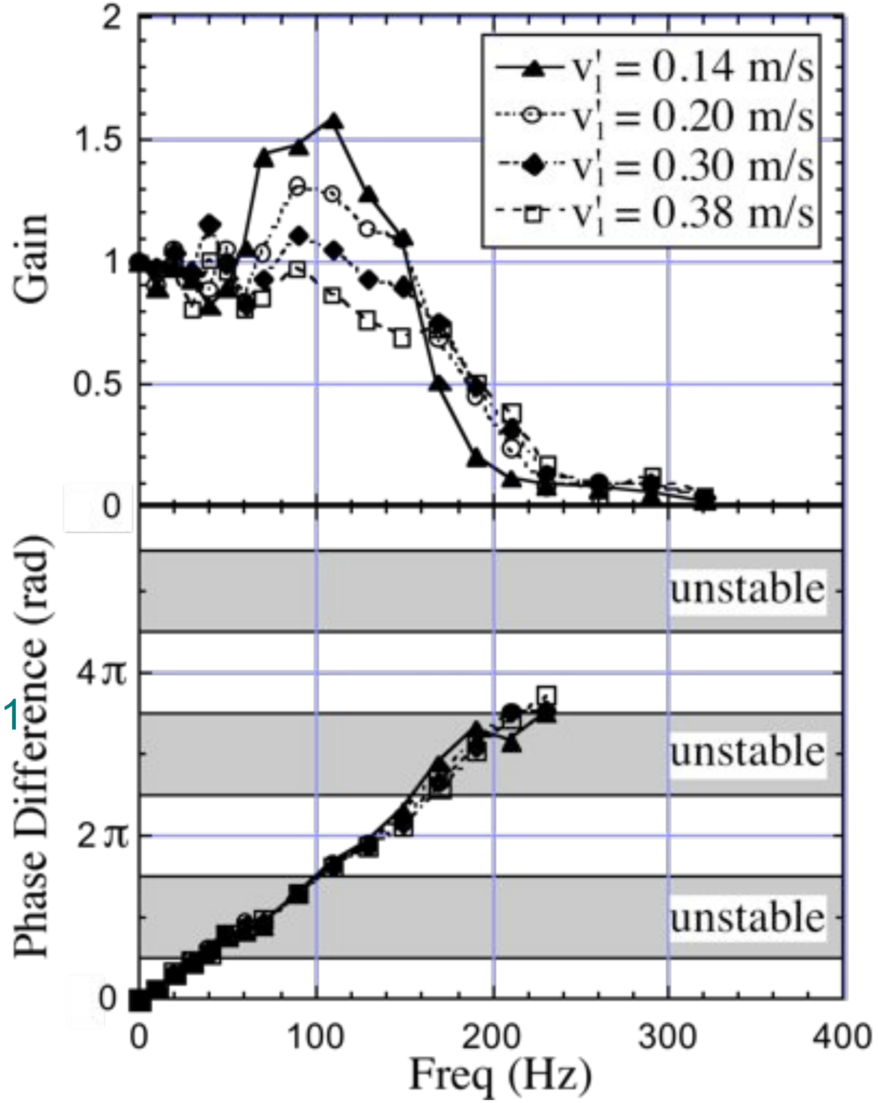
$$F_v = G(\omega)\exp(i\varphi)$$

Gain :

- relative fluctuation amplitude
- Large overshoot G>1
- Gain reduces with increasing v_1
- Low pass filter

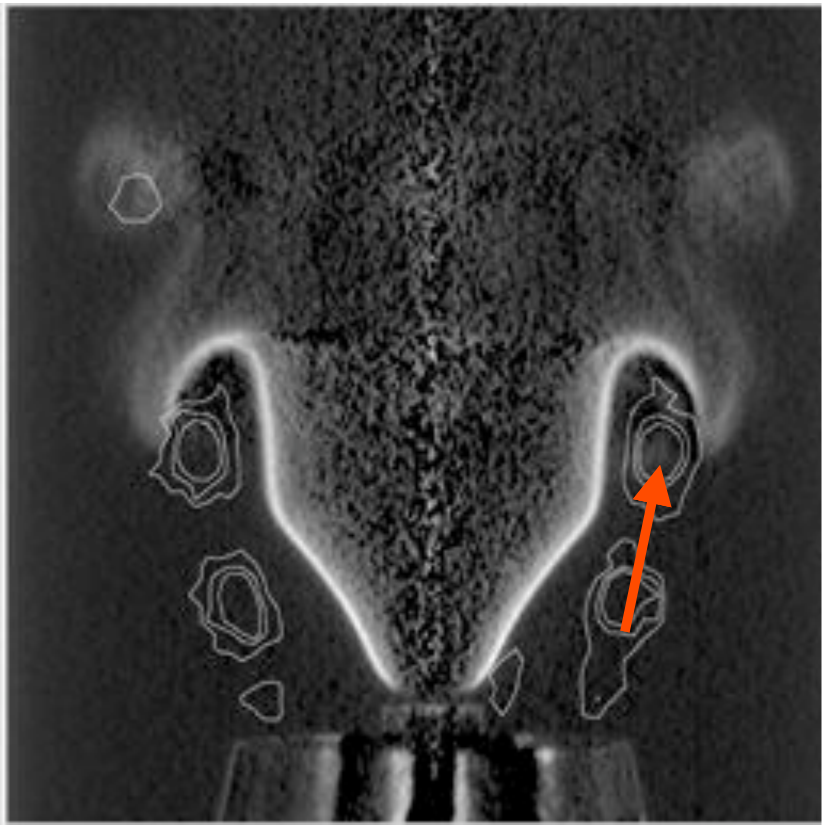
Phase :

- time lag $\varphi = \omega\tau$
- convective indepedent of the input level

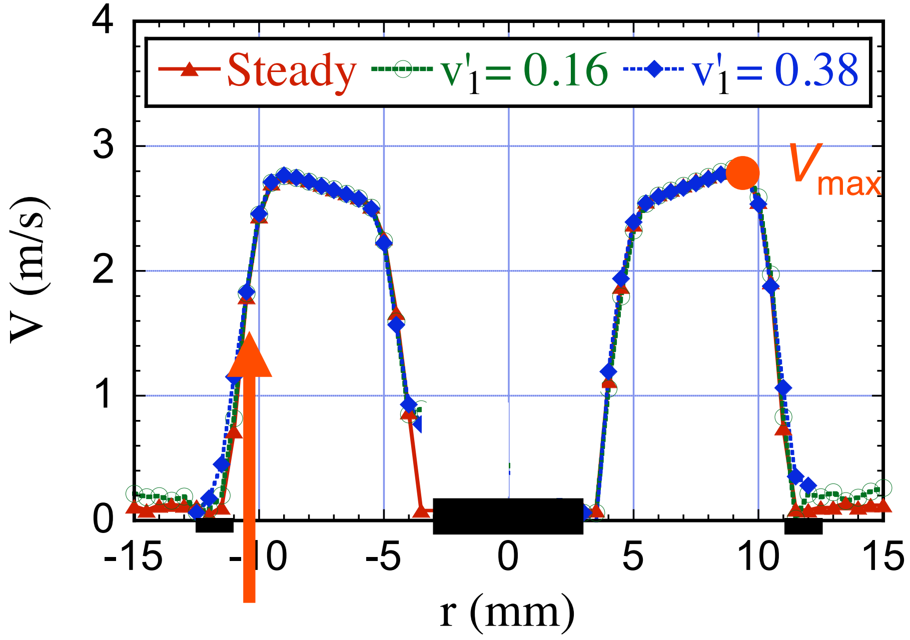


Durox et al. (2005)

Flame Transfer Function



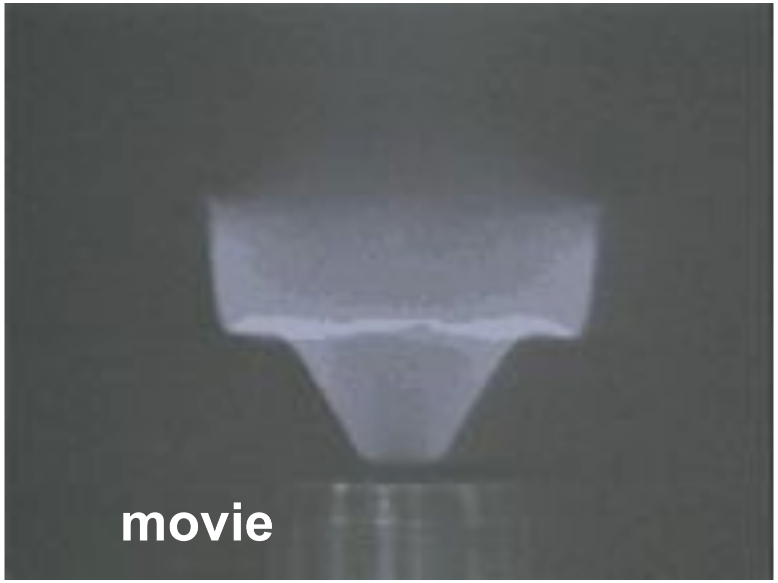
$\Phi = 0.8$, $V_d = 1.87$ m/s
$v'_1 = 0.15$ m/s , $f = 150$ Hz



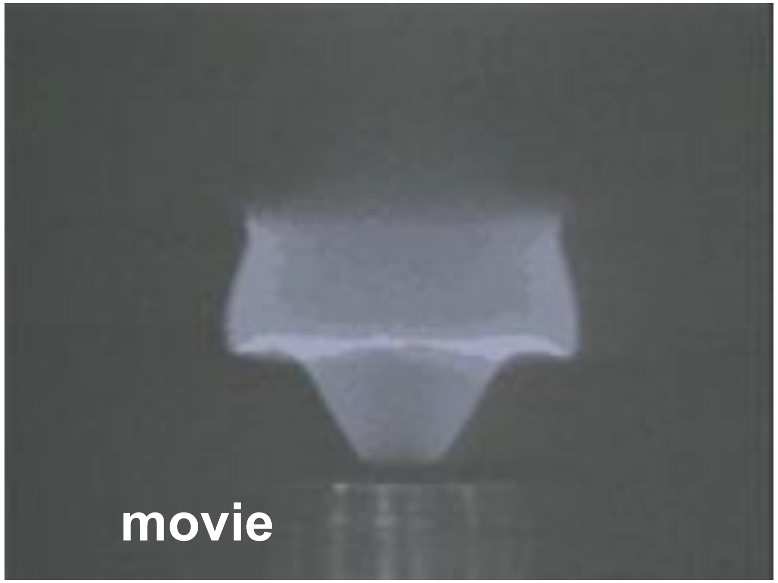
The flame front motion is controlled by the shear layer dynamics. The time lag corresponds to the travel time taken by a vortex to impinges the flame front (convected at about $V_{max}/2$). This time lag is barely affected by the input level.

Flame Transfer Function

Modulation level effect for the same mean flow operating condition



V_{rms}= 0.14 m/s



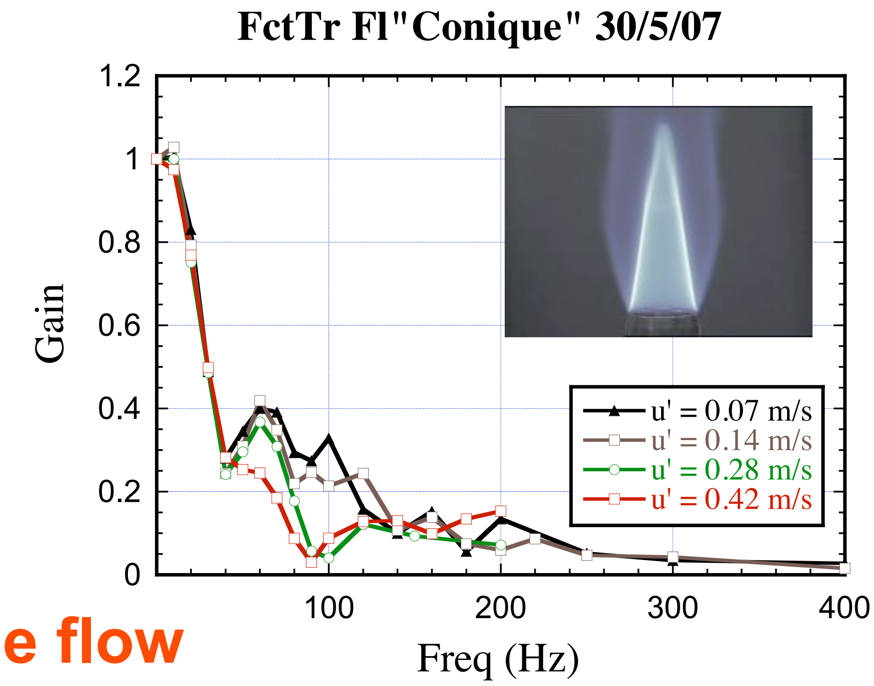
V_{rms}= 0.38 m/s

When the perturbation level increases saturation occurs : energy is transferred to higher harmonics and the gain examined at the forcing frequency drops.

FTF Gain

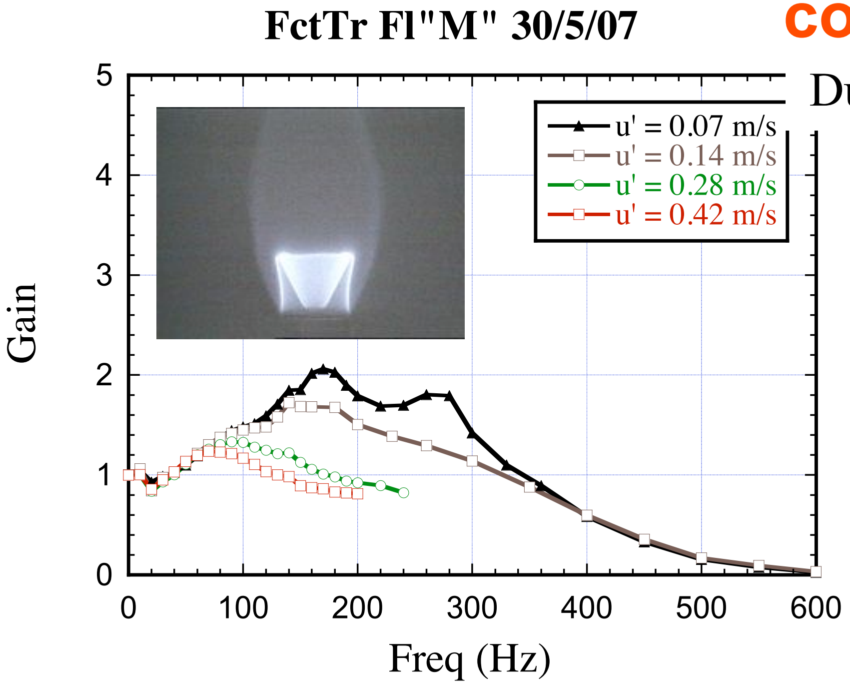
$$G = G(\omega, v_1)$$

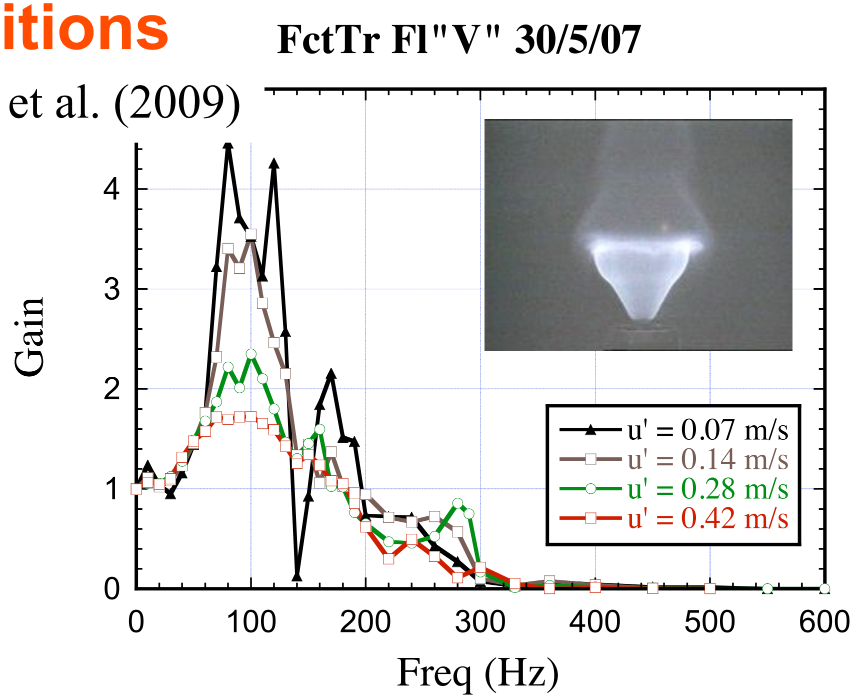
- Initial flame geometry
- Type of perturbation
- Frequency
- Perturbation level



Same flow conditions

Durox et al. (2009)



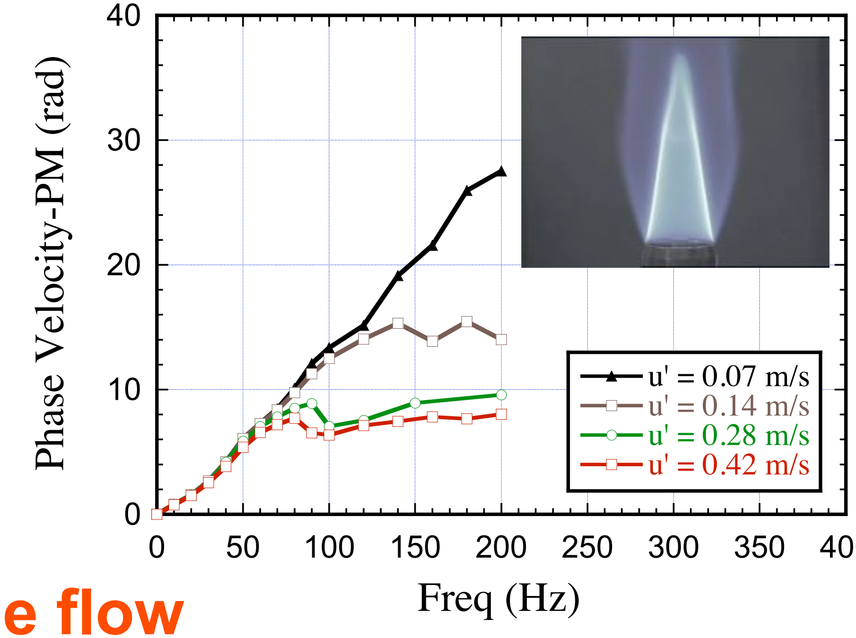


FTF phase

$$\varphi = \varphi(\omega, v_1)$$

- Initial flame geometry
- Type of perturbation
- Frequency
- Perturbation level

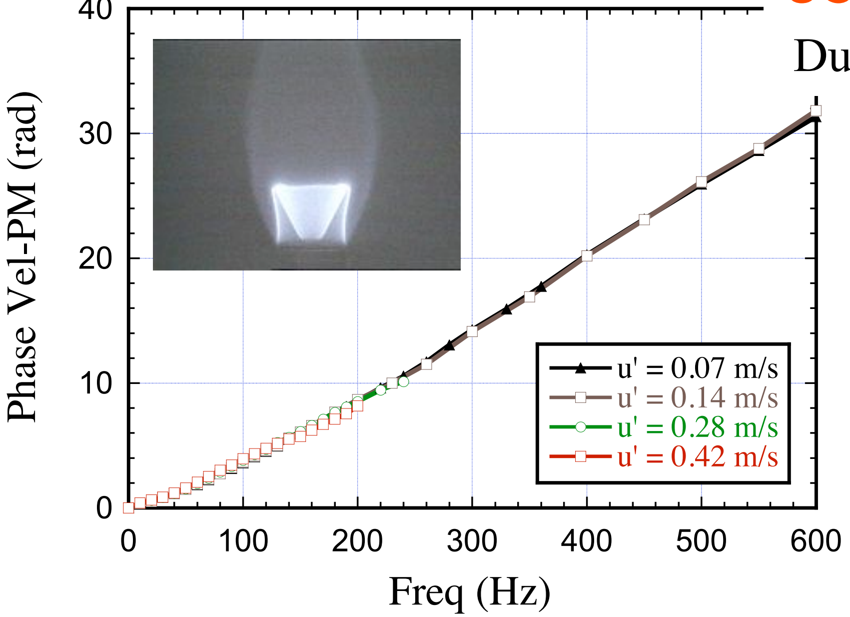
FctTr Fl"Conique" 30/5/07



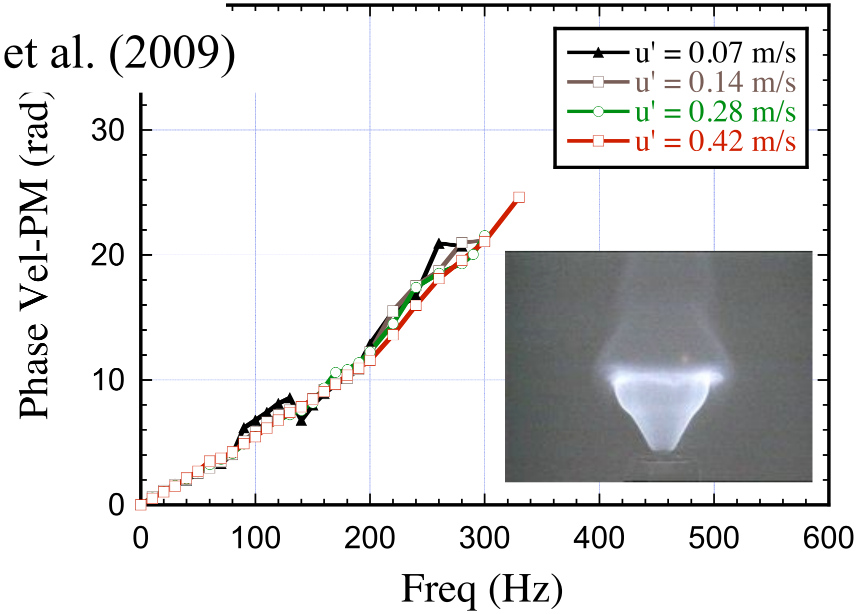
Same flow conditions

Durox et al. (2009)

FctTr Fl"M" 30/5/07



FctTr Fl"V" 30/5/07

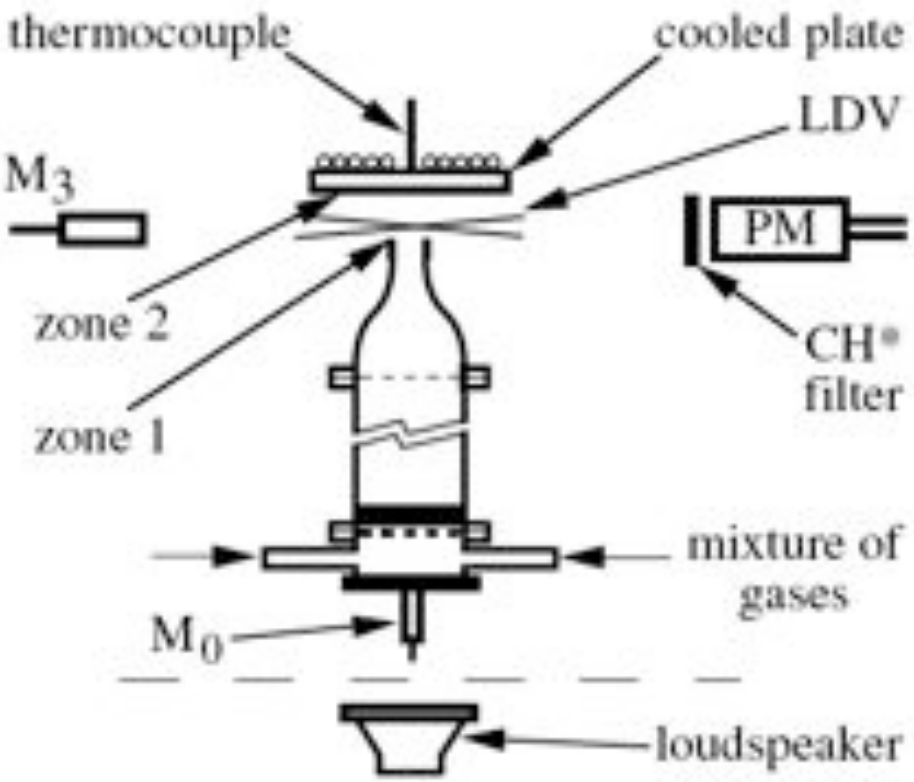


Roadmap

1. *Elementary mechanisms*
2. *Flame dynamics*
3. **Linear and nonlinear stability analysis**

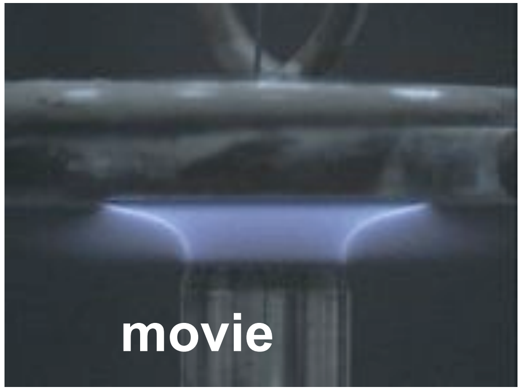
Linear stability analysis

Example 1: Self-sustained oscillation of a premixed jet flame impinging on a plate

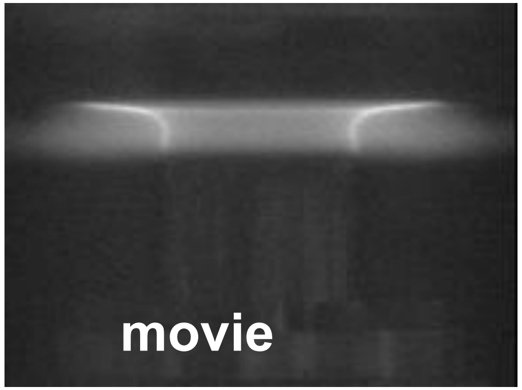


Schuller et al. (2002), Durox et al. (2002)

stable

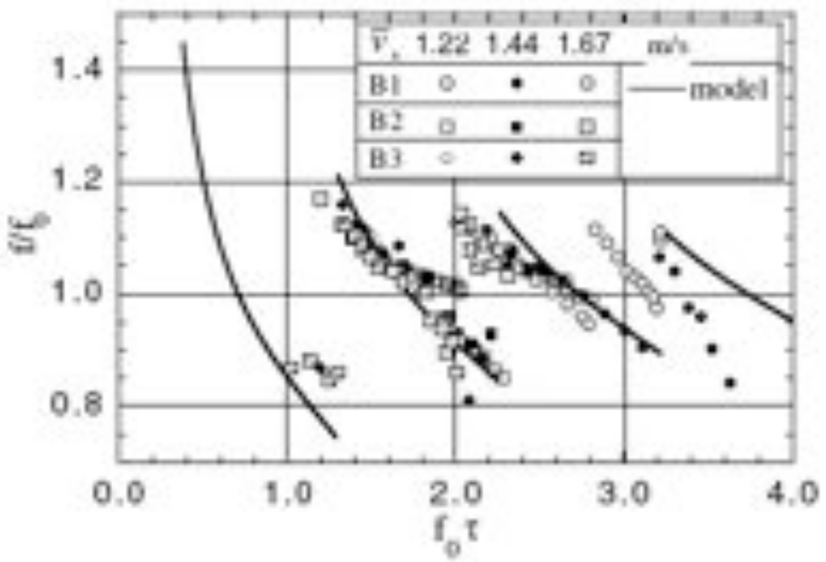


unstable



Linear stability analysis

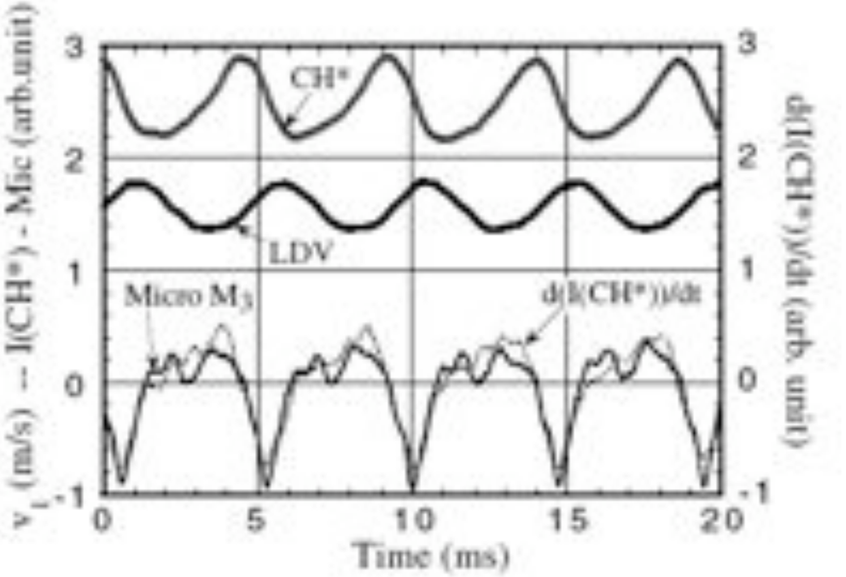
Instability frequency



Oscillation frequencies lie around one of the burner acoustic modes (here Helmholtz mode f_0) :

Acoustic-convective interaction

Signals time traces

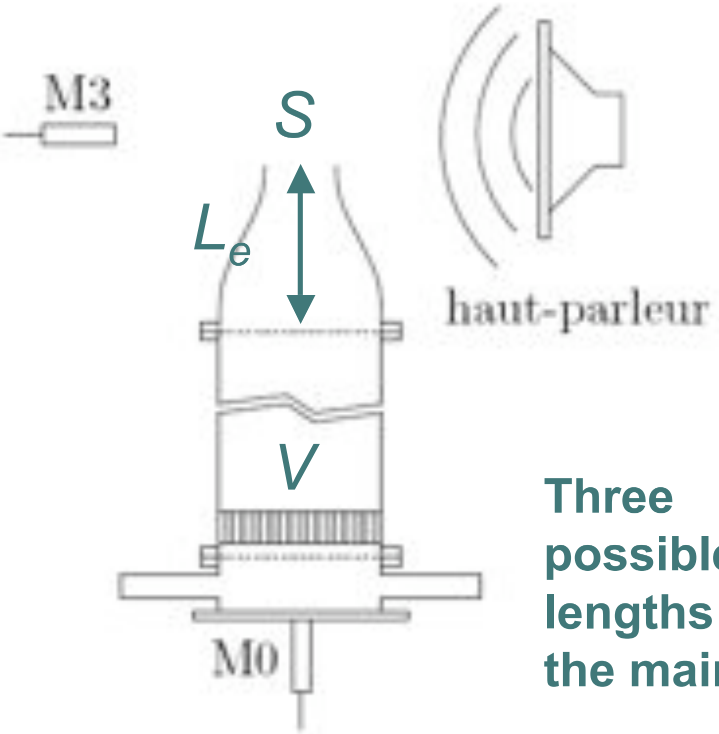


Heat release rate oscillations and velocity fluctuations at the base of the flame are nearly in phase opposition

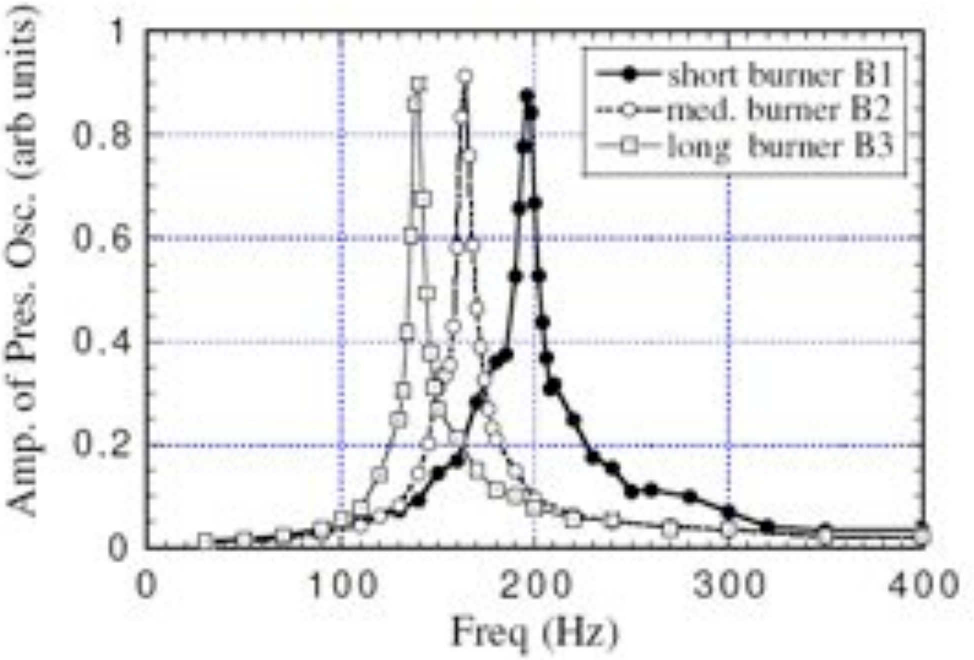
Necessity to have a knowledge of the burner eigenmodes

Linear stability analysis

Burner frequency response



Three possible lengths for the main body

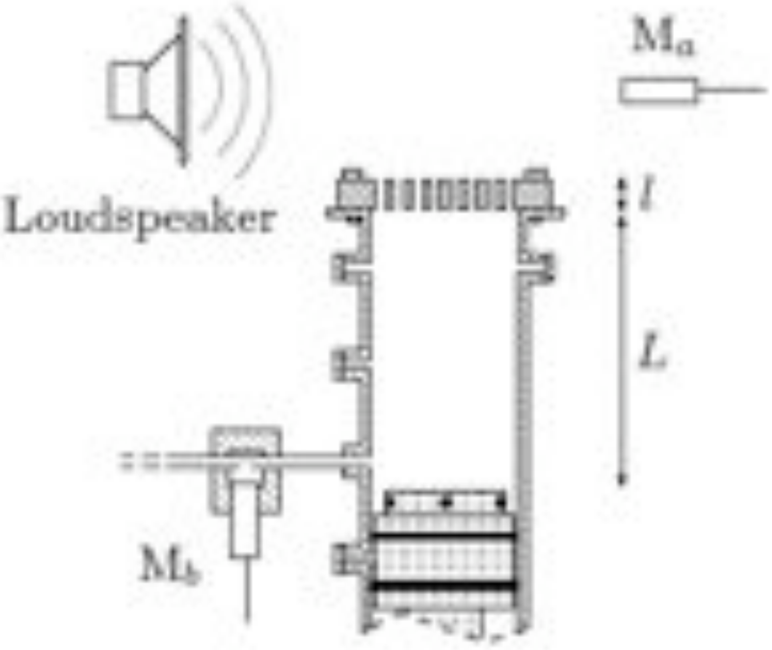


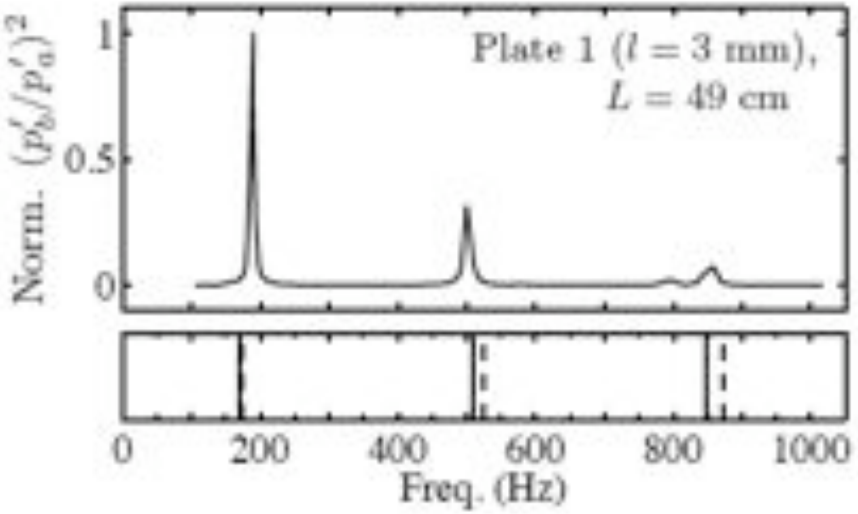
Helmholtz resonance = bulk oscillation

$$\omega_0^2 = \frac{c^2 S_1}{V L_e}$$

Linear stability analysis

Burner frequency response





λ/4 3λ/4 5λ/4

Quarter-wave resonances $$f_n = \frac{c}{4L}(2n+1),\ n = 0, 1, 2, \ldots$$

Jump conditions across a flame

How flow perturbations are affected by the flame?

Hypothesis

- Low Mach flow
- Compact 1D flame
- Composition changes neglected
- Perfect gas
- No thermal conductivity
- No viscosity

$$M \ll 1$$
$$\lambda \gg \delta$$
$$W = cte$$
$$p/\rho^{\gamma} = \exp(s/c_v)$$
$$k = 0$$
$$\nu = 0$$

$$c_v = \frac{r}{\gamma - 1} \qquad c_p = \frac{r\gamma}{\gamma - 1} \qquad r = \frac{R}{W}$$

Jump conditions across a flame

Transport equations for low Mach flow perturbations

$$\frac{\partial \rho_1}{\partial t} = -\rho_0 \boldsymbol{\nabla} \cdot \mathbf{v}_1$$

$$\frac{\partial \mathbf{v}_1}{\partial t} = -\frac{1}{\rho_0} \boldsymbol{\nabla} p_1$$

$$\frac{\partial s_1}{\partial t} = \frac{\dot{q}_1}{\rho_0 T_0}$$

reactants	flame	products
p_1^u		p_1^b
v_1^u		v_1^b
s_1^u		s_1^b

Jump conditions across a flame

Separation for density fluctuations

$$\rho = \rho(p, s) \qquad \left[\frac{\partial \rho}{\partial p}\right]_{s=s_0} = \frac{1}{c_0^2} \qquad \left[\frac{\partial \rho}{\partial s}\right]_{p=p_0} = -\frac{\rho_0}{c_p}$$

$$\rho_1 = \rho_1^{ac} + \rho_1^{en} = \frac{1}{c_0^2} p_1 - \frac{\rho_0}{c_p} s_1$$

$$\frac{1}{c_0^2}\frac{\partial p_1}{\partial t} + \rho_0 \boldsymbol{\nabla} \cdot \mathbf{v}_1 = 0$$

$$\rho_0 \frac{\partial \mathbf{v}_1}{\partial t} = -\boldsymbol{\nabla} p_1$$

$$s_1 = 0$$

$$-\frac{\rho_0}{c_p}\frac{\partial s_1}{\partial t} + \rho_0 \boldsymbol{\nabla} \cdot \mathbf{v}_1 = 0$$

$$p_1 = 0$$

$$\rho_0 \frac{\partial s_1}{\partial t} = \frac{\dot{q}_1}{T_0}$$

Jump conditions across a flame

$$-\frac{\rho_0}{c_p}\frac{\partial s_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}_1 = 0 \qquad (1)$$

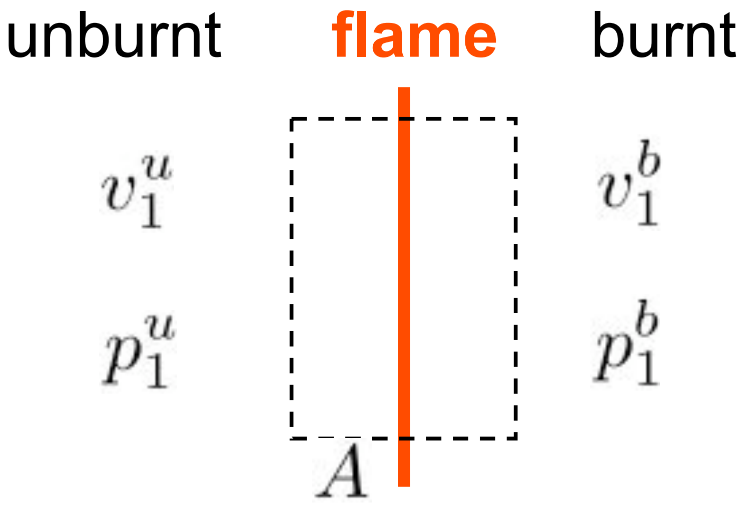
$$p_1 = 0 \qquad (2)$$

$$\rho_0 \frac{\partial s_1}{\partial t} = \frac{\dot{q}_1}{T_0} \qquad (3)$$

$$\nabla \cdot \mathbf{v}_1 = \frac{\dot{q}_1}{\rho_0 c_p T_0}$$

$$p_1 = 0$$

$$\rho_0 \frac{\partial s_1}{\partial t} = \frac{\dot{q}_1}{T_0}$$



$$\boxed{\begin{aligned} v_1^b - v_1^u &= \frac{\gamma - 1}{\gamma p_0 A}\dot{Q}_1 \\ p_1^b &= p_1^u \end{aligned}}$$

Dowling (1995), Candel et al. (1996)
Blackshear (1956)

Identification methods

Flame transfer matrix FTM

$$\begin{pmatrix} p_1^b \\ v_1^b \end{pmatrix} = \mathcal{T}_{\mathcal{F}} \begin{pmatrix} p_1^u \\ v_1^u \end{pmatrix} \quad \text{where} \quad \mathcal{T}_{\boldsymbol{F}} = \begin{bmatrix} \mathcal{T}_{11} & \mathcal{T}_{12} \\ \mathcal{T}_{21} & \mathcal{T}_{22} \end{bmatrix}$$

$$v_1^b - v_1^u = \frac{\gamma - 1}{\gamma p_0 A} \dot{Q}_1$$

$$p_1^b = p_1^u$$

Flame transfer function FTF

$$\begin{pmatrix} p_1^b \\ v_1^b \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 + KF \end{bmatrix} \begin{pmatrix} p_1^u \\ v_1^u \end{pmatrix}$$

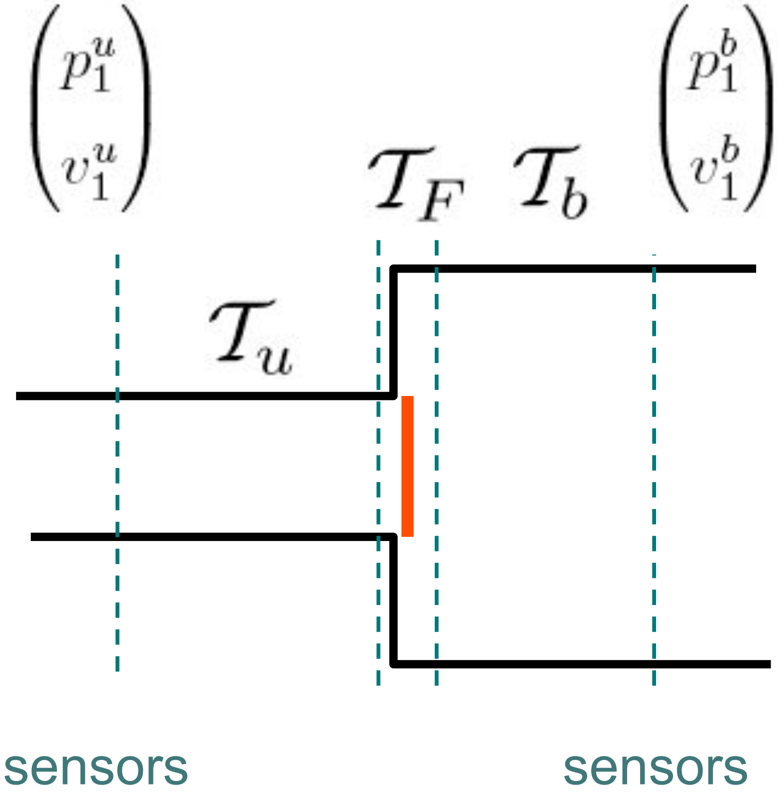
$$K = \frac{T_0^b}{T_0^u} - 1$$

$$F = \frac{\dot{Q}_1 / \dot{Q}_0}{v_1^u / v_0^u}$$

$$\dot{Q}_0 = \rho_0^u v_0^u A_0 c_p (T_0^b - T_0^u)$$

Identification methods

In practical systems, pressure and other flow perturbations are difficult to measure close enough from the reaction region : non compact



Additional terms that depend on the burner geometry and sensor locations

$$\begin{pmatrix} p_1^b \\ v_1^b \end{pmatrix} = \mathcal{T} \begin{pmatrix} p_1^u \\ v_1^u \end{pmatrix} \quad \text{where} \quad \mathcal{T} = \mathcal{T}_b \mathcal{T}_{\mathcal{F}} \mathcal{T}_u$$

Truffin & Poinsot (2005)

Identification methods

Experimental determination of FTM

Polifke et al. (2001)
Paschereit et al. (2002,2004)

4 unknowns

$$\mathcal{T} = \begin{bmatrix} \mathcal{T}_{11} & \mathcal{T}_{12} \\ \mathcal{T}_{21} & \mathcal{T}_{22} \end{bmatrix}$$

2 independent states are necessary

$$\begin{pmatrix} p^b_{1a} \\ v^b_{1a} \\ p^b_{1b} \\ v^b_{1b} \end{pmatrix} = \begin{bmatrix} T_{11} & T_{12} & 0 & 0 \\ T_{21} & T_{22} & 0 & 0 \\ 0 & 0 & T_{11} & T_{12} \\ 0 & 0 & T_{21} & T_{22} \end{bmatrix} \begin{pmatrix} p^u_{1a} \\ v^u_{1a} \\ p^u_{1b} \\ v^u_{1b} \end{pmatrix}$$

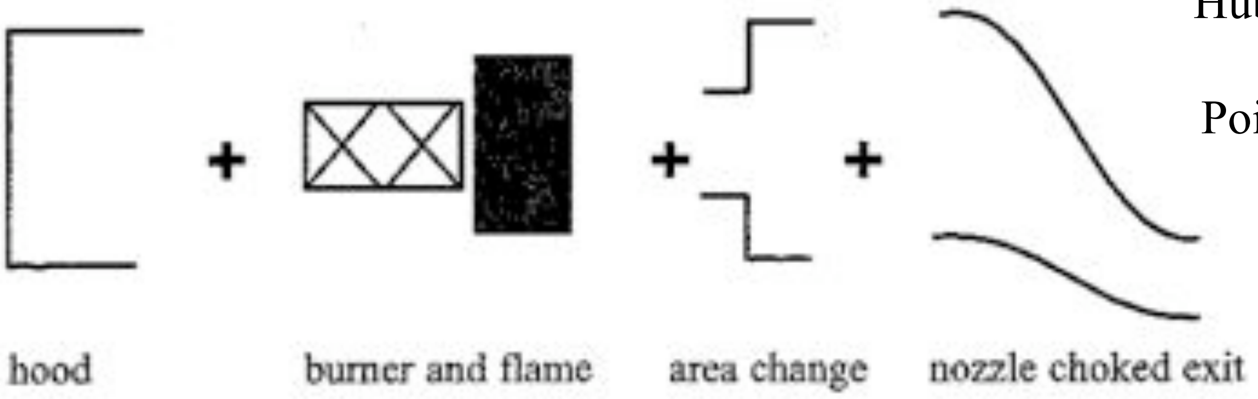
Difficulty

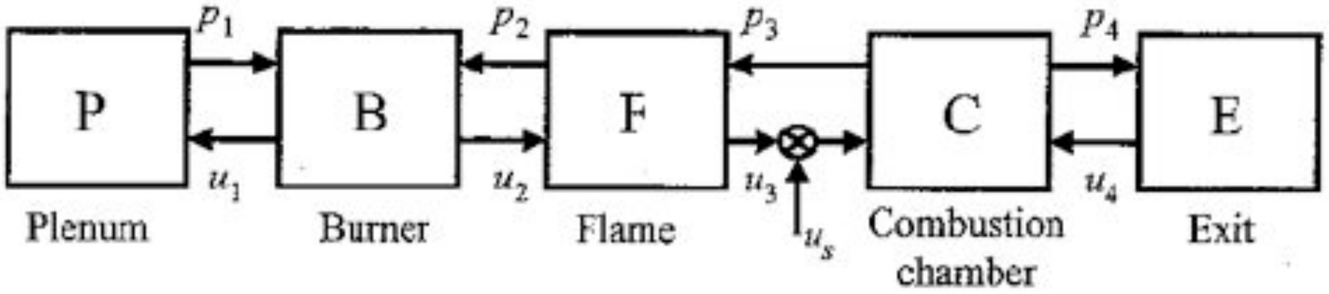
- Perturbation states should be well separated to obtain a well posed mathematical problem
- Flames are very sensitive to any slight modification
  - Boundary condition modifications
  - Perturbations from upstream or downstream

Linear stability analysis

Network of acoustic elements

Keller (1995)
Hubbard & Dowling (1998)
Paschereit et al. (1999)
Poinsot & Veynante (2001)





$$\det\left[\lambda I - M\right] = 0$$

Find the complex roots of the network and examine the imaginary component

Linear stability analysis

Modern developments

Nicoud and coworkers (2005,2006, 2007)

Helmholtz acoustic solver

$$\nabla \cdot (c_0^2 \nabla p_1) + \omega^2 p_1 = 0$$
$$-i\omega\rho_0 v_1 + \nabla p_1 = 0$$

↔

LES or experiments

$$\begin{pmatrix} p_1^b \\ v_1^b \end{pmatrix} = \mathcal{T}_{\mathcal{F}} \begin{pmatrix} p_1^u \\ v_1^u \end{pmatrix}$$

- Complex geometry - entire combustor + BC
- Mean field computed from RANS or LES
- Prediction of combustor 3D eigenmodes

- Real burner geometry
- Real operating conditions
- Extract FTM

Linear stability analysis of the modes of practical combustors

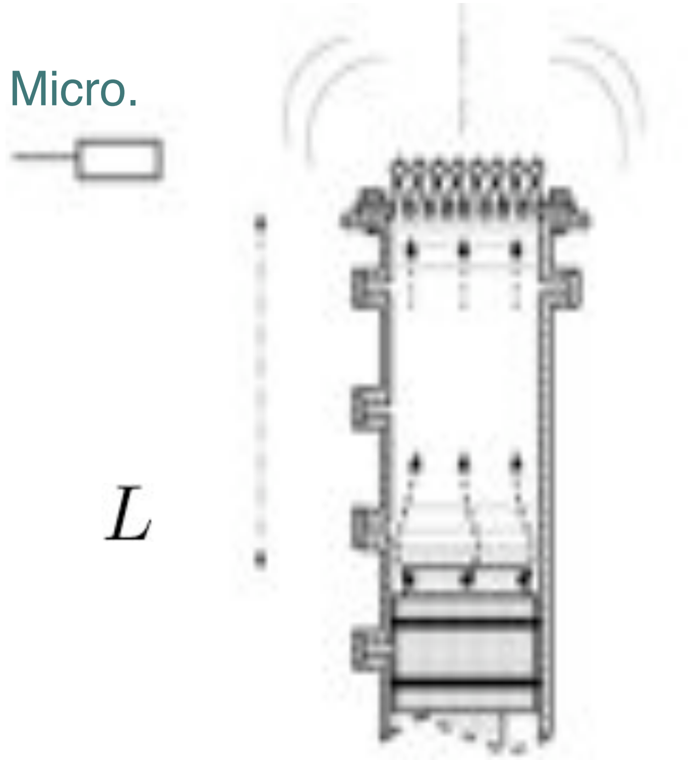
No interaction between mean flow and acoustics

Nonlinear stability analysis

Prediction of acoustic coupled combustion instabilities

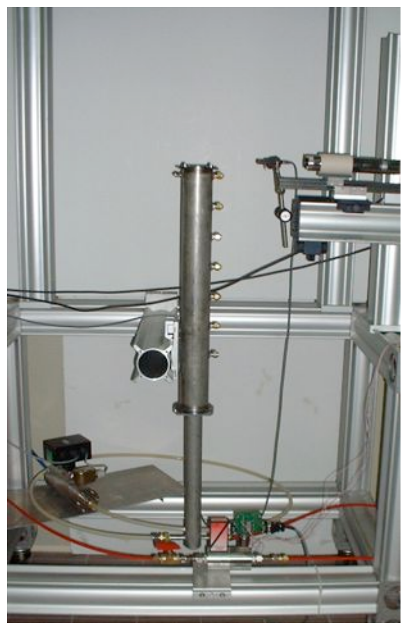
- limit cycles oscillation levels
- frequency shifting
- triggering
- mode switching
- hysteresis

Non linear stability analysis required



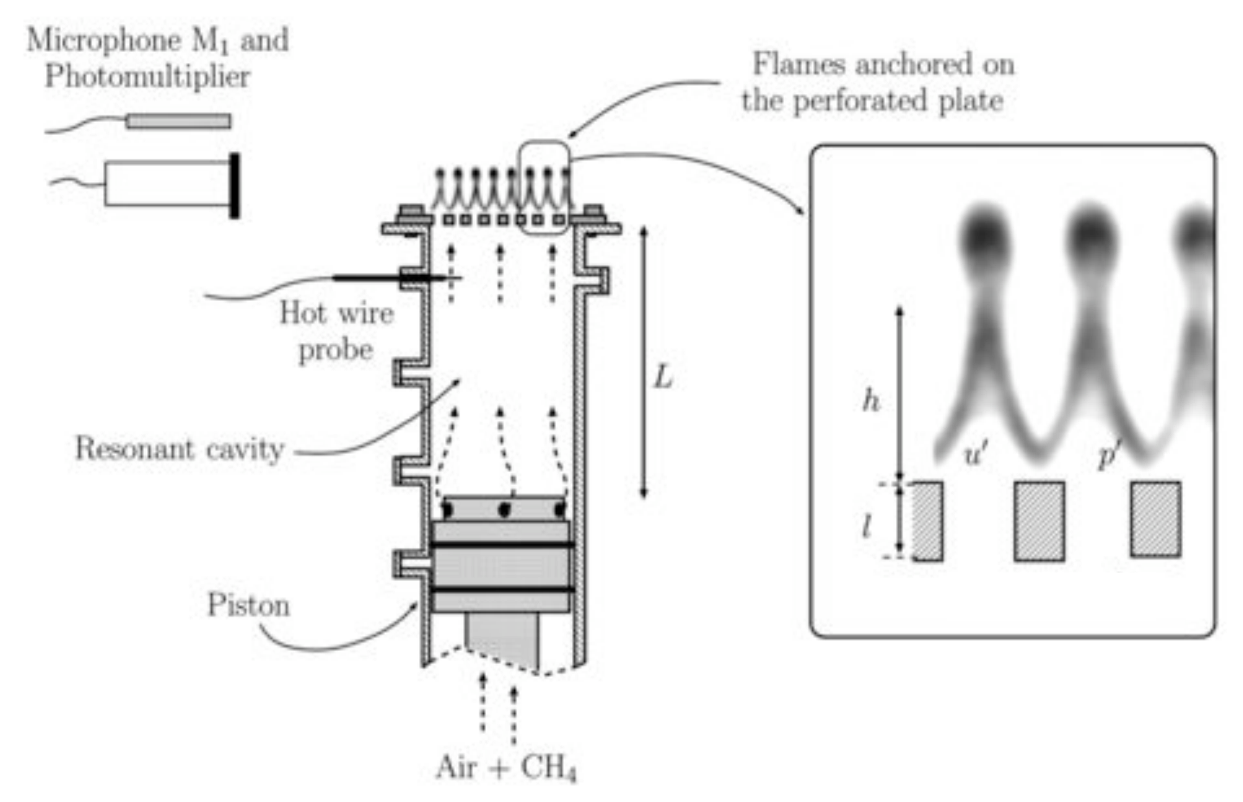
Noiray et al. (2008)

Experimental Setup



Perforated plate

Burner sketched



Equivalence ratio : $\Phi = 0.86$
Volumetric flow rate : $\dot{m} = 5.4\,10^{-3}\,\mathrm{kg\ s}^{-1}$
Thermal power : $14.4\ \mathrm{kW}$

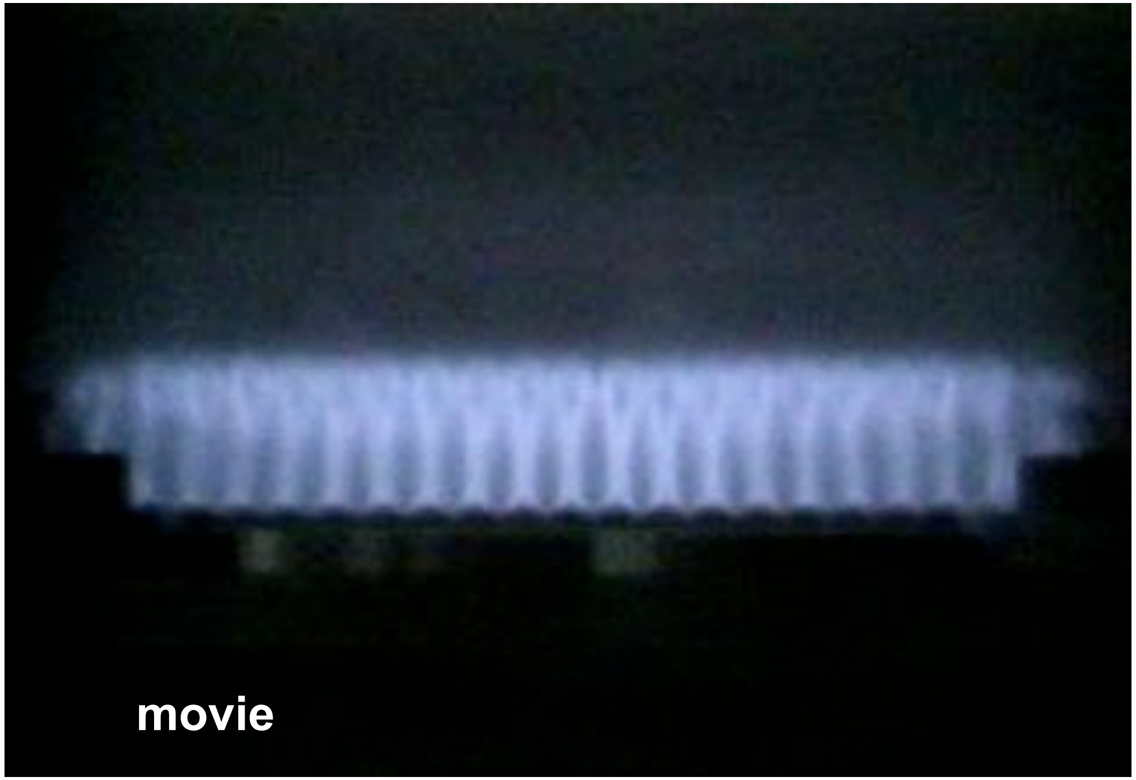
- Diameter D : 70 mm
- Depth L easily adjustable : from 90 to 750 mm
- non-confined reaction layer

Combustion regime

Depending on the burner depth *L* combustion can be **stable** or **unstable**.

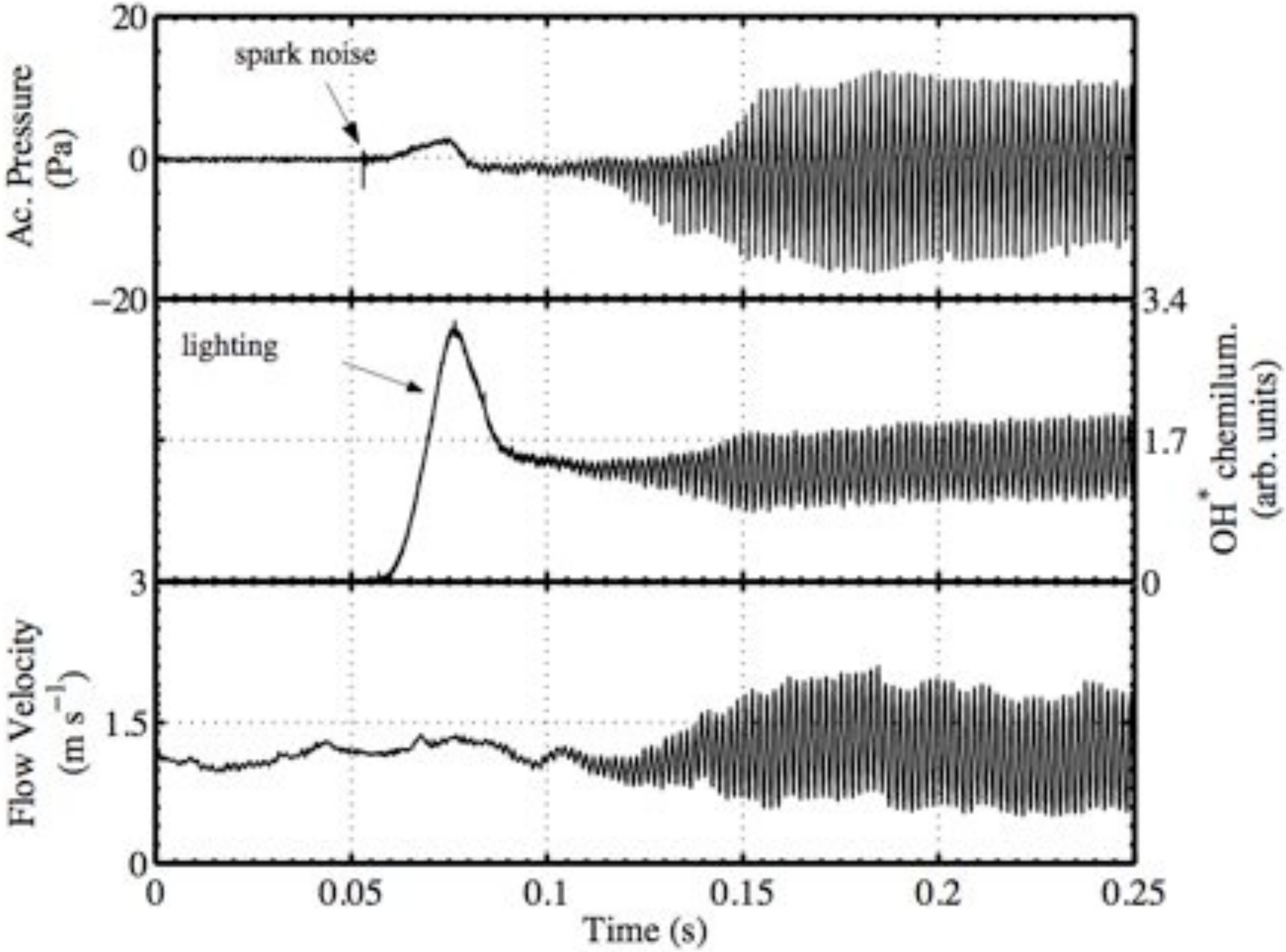
movie

Flames dynamics

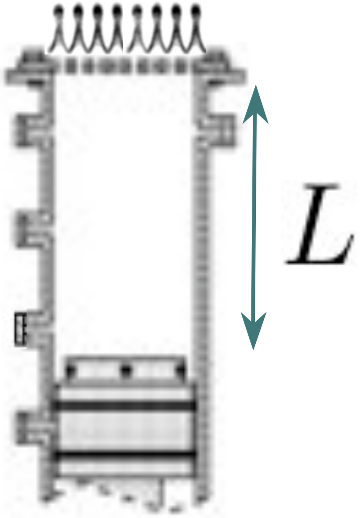


Oscillation cycle (f=530 Hz) in a typical **unstable situation**

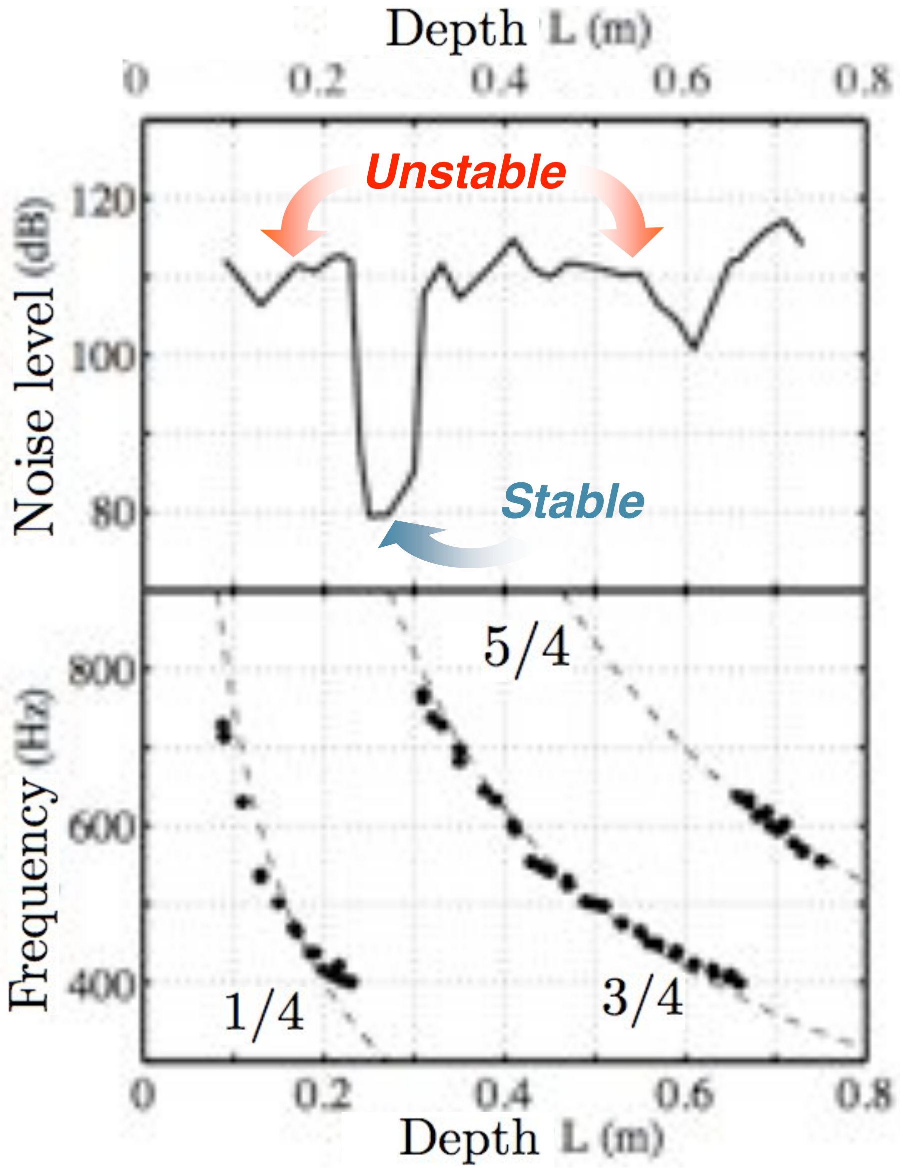
(SPL=110 dB, 40 cm away from the flames)



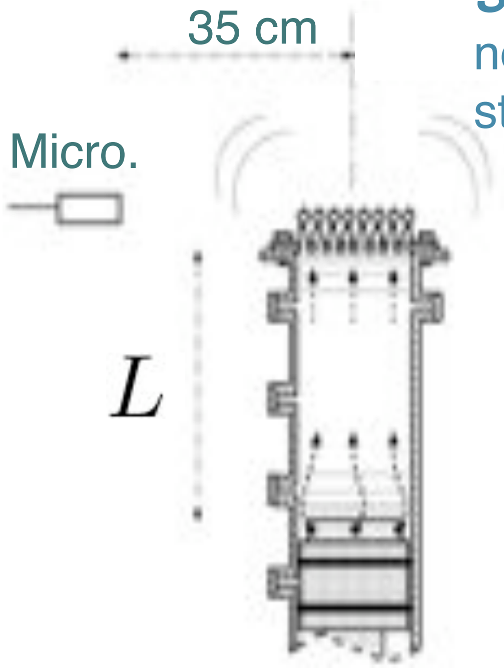
$L = 460\,\mathrm{mm}$



Effect of the cavity depth



Unstable regime : stable oscillatory equilibrium (nonzero amplitude)

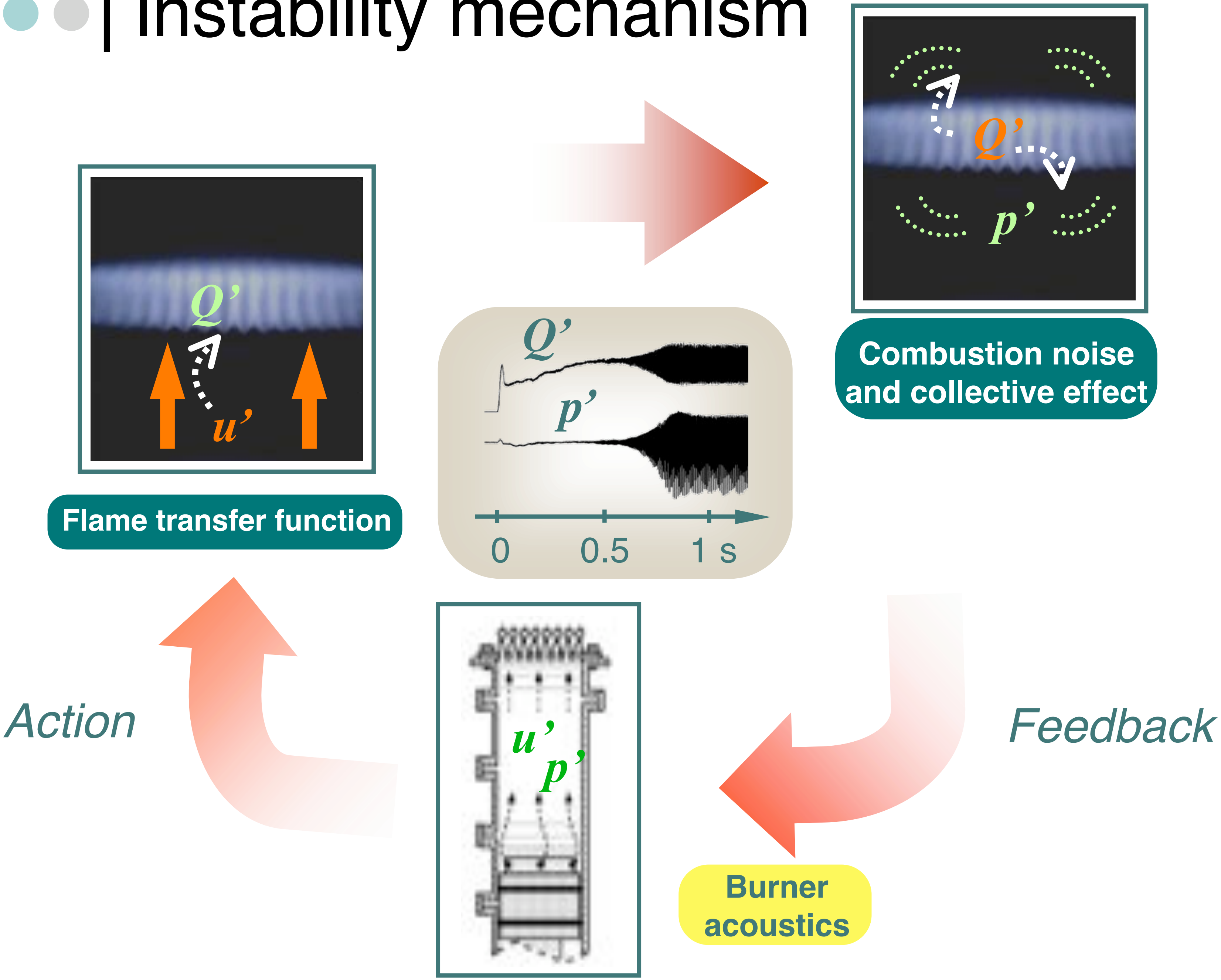


Stable regime : non oscillatory stable equilibrium

- Peak noise and flame oscillation frequency

– – Burner acoustic modes (quarter wave type resonator)

Instability mechanism



Flame transfer function

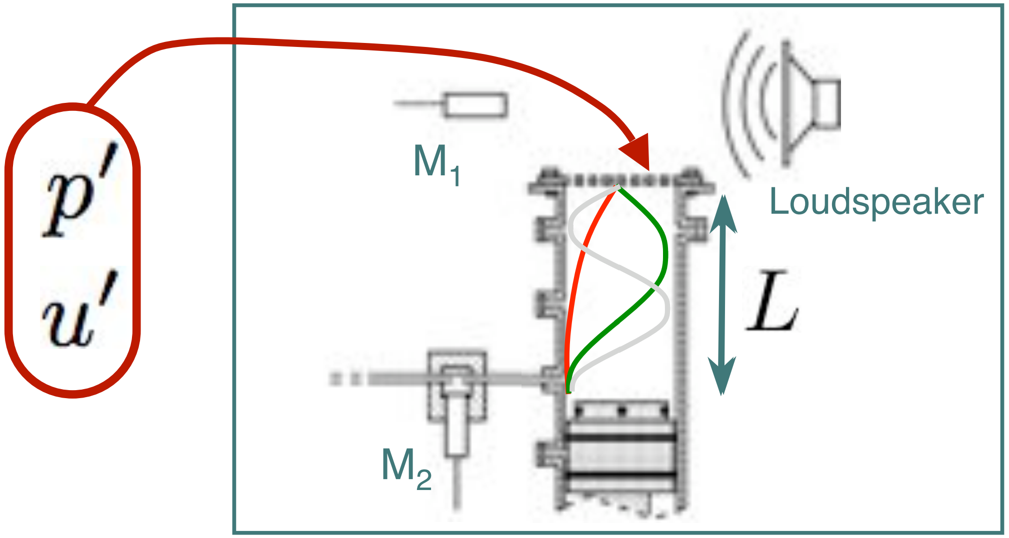
Combustion noise and collective effect

Burner acoustics

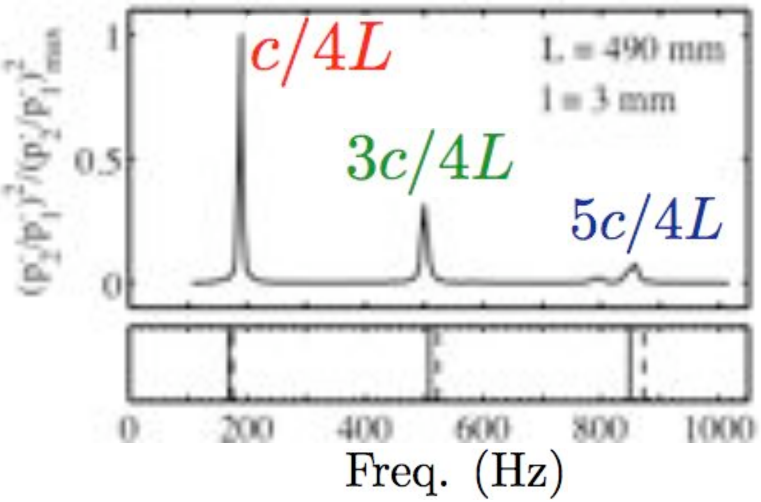
Action

Feedback

Burner acoustics



Experimental setup



Typical acoustic burner response

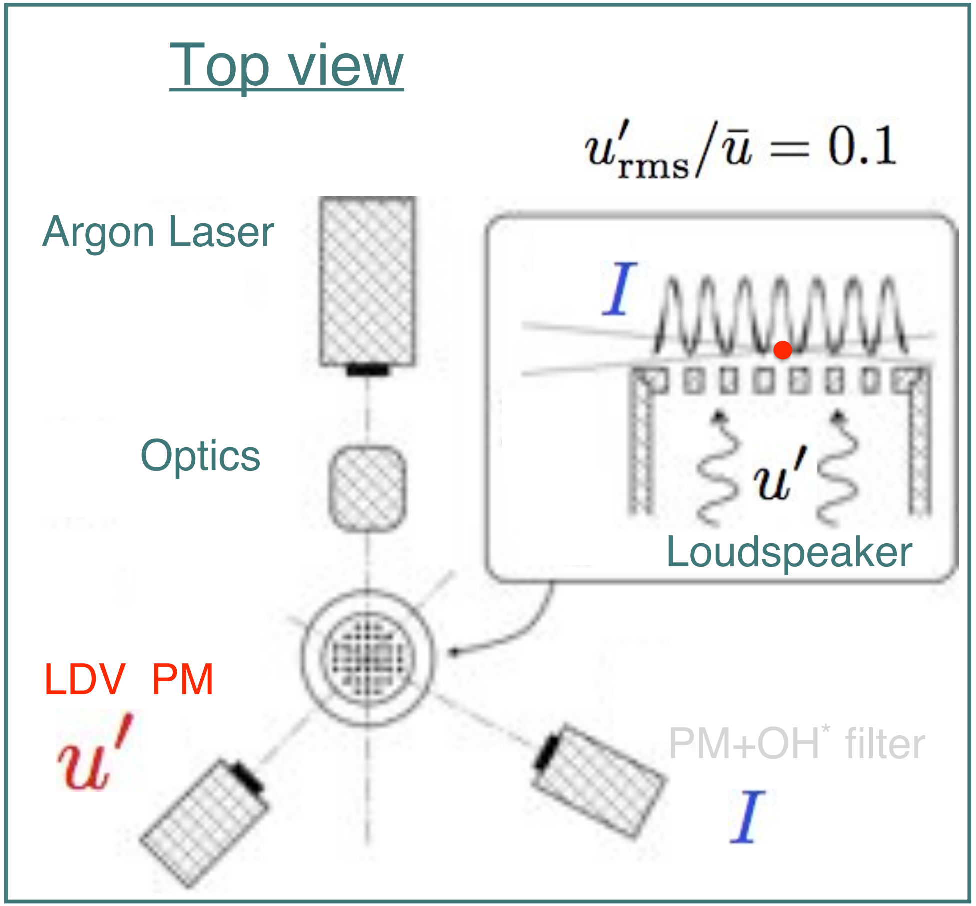
With perforated plate (Solid lines)

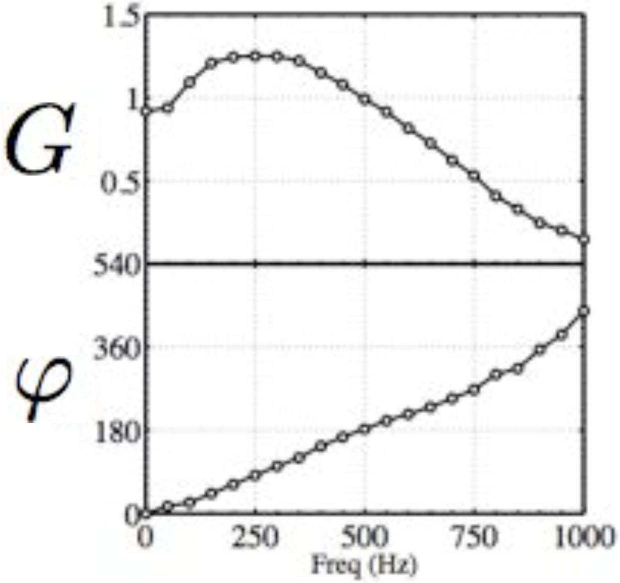
$$\frac{p'}{u'} = \overbrace{\underbrace{-i\rho c\mathcal{P}\tan^{-1}\left(\frac{\omega L}{c}\right)} + i\,\omega\rho l\left[1+\frac{l_\nu}{r_p}(1+i)\right]}$$

Without perforated plate, quarter wave type resonator (dashed lines) $\rightarrow$ $(2n-1)c/4L$

Flame Transfer Function

$$\boxed{\mathcal{F} = \frac{\dot{Q}'/\bar{\dot{Q}}}{u'/\bar{u}} = \frac{I'/\bar{I}}{u'/\bar{u}} = \frac{A'/\bar{A}}{u'/\bar{u}}}$$



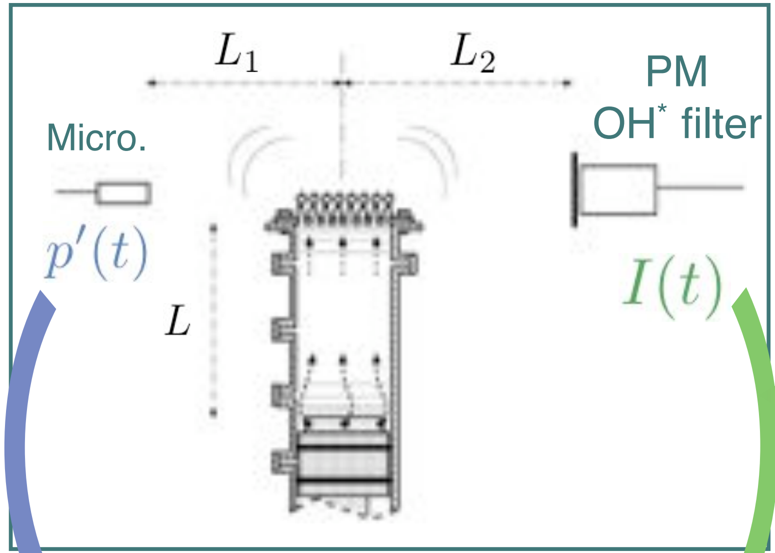


$$\mathcal{F} = G\,e^{i\varphi}.$$

$$\boxed{A' = \frac{\bar{A}}{\bar{u}} G\,e^{i\varphi} u'}$$

The flame response features a time lag which is favourable to self-sustained oscillations

Combustion noise

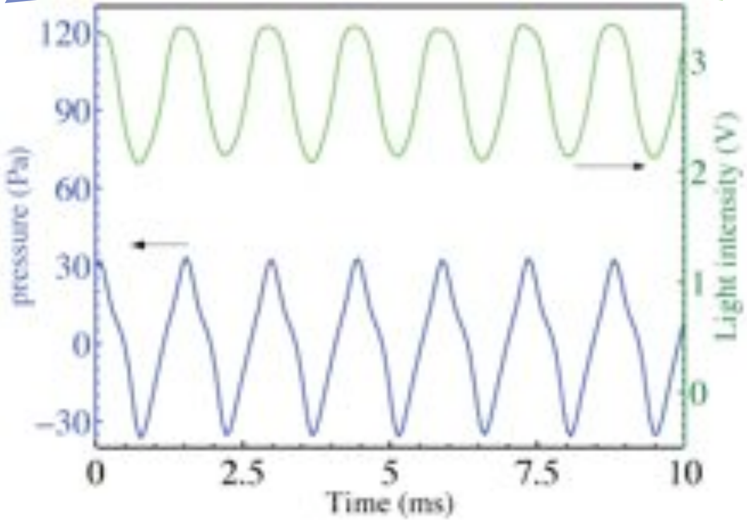


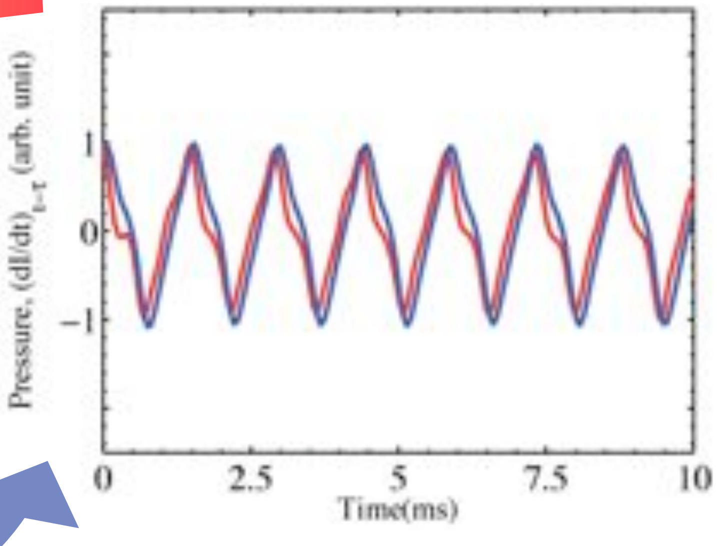
Experimental setup

$$p'(r,t) = \frac{\rho\,(E-1)\,S_L}{4\pi r}\left(\frac{dA}{dt}\right)_{t-\tau_{ac}}$$

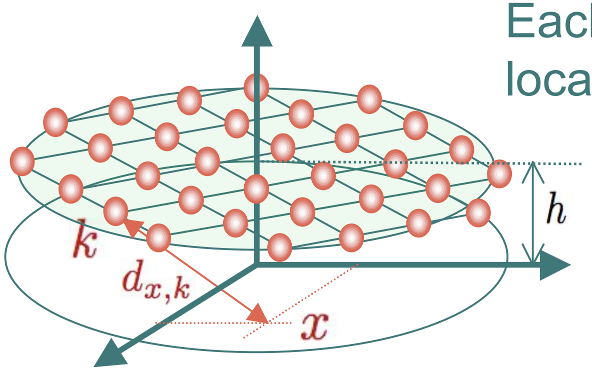
$$p'(r,t) \propto \left(\frac{dI'}{dt}\right)_{t-\tau_{ac}}$$

$$\left[\frac{d}{dt}\right]_{t-\tau_{ac}}$$

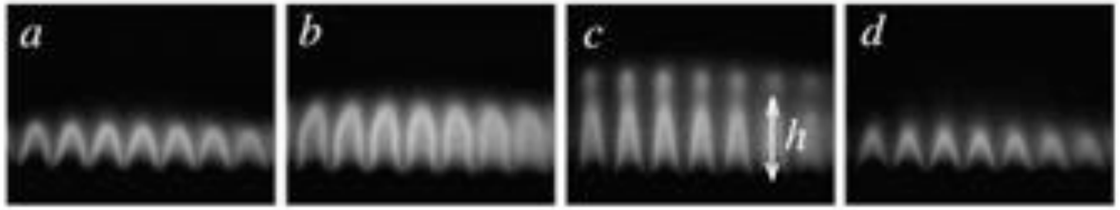




Combustion noise is responsible of the feedback of acoustic energy in the burner



Each of the N flames behaves like an acoustic monopole located at a distance h from the perforated plate

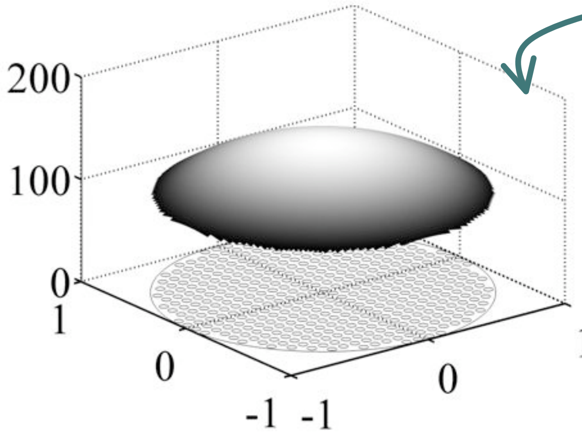


Acoustic pressure radiated on the perforated plate p' result from the contribution of each of the N flames

$$p'(x,t) = \sum_{k=1}^{N} p'(d_{x,k},t) = \sum_{k=1}^{N} \frac{\rho(E-1)S_L}{4\pi d_{x,k}} \left(\frac{d\mathcal{A}'_k}{dt}\right)_{t-\tau_{x,k}} \quad \Longrightarrow \quad p' = -i\omega\alpha\frac{\rho(E-1)S_L}{4\pi h}\mathcal{A}'$$

The coefficient α features the collective effect encountered : $\alpha = h\sum_{k=1}^{N}\frac{1}{d_{x,k}}$

Computation of the coefficient α :



$$h\sum_{k=1}^{N}\frac{1}{d_{x,k}}$$

$$\Longrightarrow \quad \alpha = 120$$

Collective effect in the radiated pressure field

Dispersion relation

Burner acoustics

$$\frac{p'}{u'} = -i\rho c\mathcal{P}\tan^{-1}\left(\frac{\omega L}{c}\right) + i\,\omega\rho l\left[1+\frac{l_\nu}{r_p}(1+i)\right]$$

Combustion noise

$$p' = -i\omega\alpha\frac{\rho\,(E-1)\,S_L}{4\pi h}\mathcal{A}'$$

Transfer function

$$\mathcal{A}' = \frac{\bar{\mathcal{A}}}{\bar{u}}G\,e^{i\varphi}u'$$

$$\rho c\tan^{-1}\left(\frac{\omega L}{c}\right) = \omega\alpha\frac{\rho\,(E-1)\,S_L}{4\pi h}\frac{\bar{\mathcal{A}}}{\bar{u}_p\mathcal{P}}G\,e^{i\varphi} + \frac{\omega\rho l}{\mathcal{P}}\left[1+\frac{l_\nu}{r_p}(1+i)\right]$$

Linear $\mathcal{H}(\omega_r + i\omega_i) = \mathcal{F}\left(\omega_r, |u'|\right)$ Nonlinear

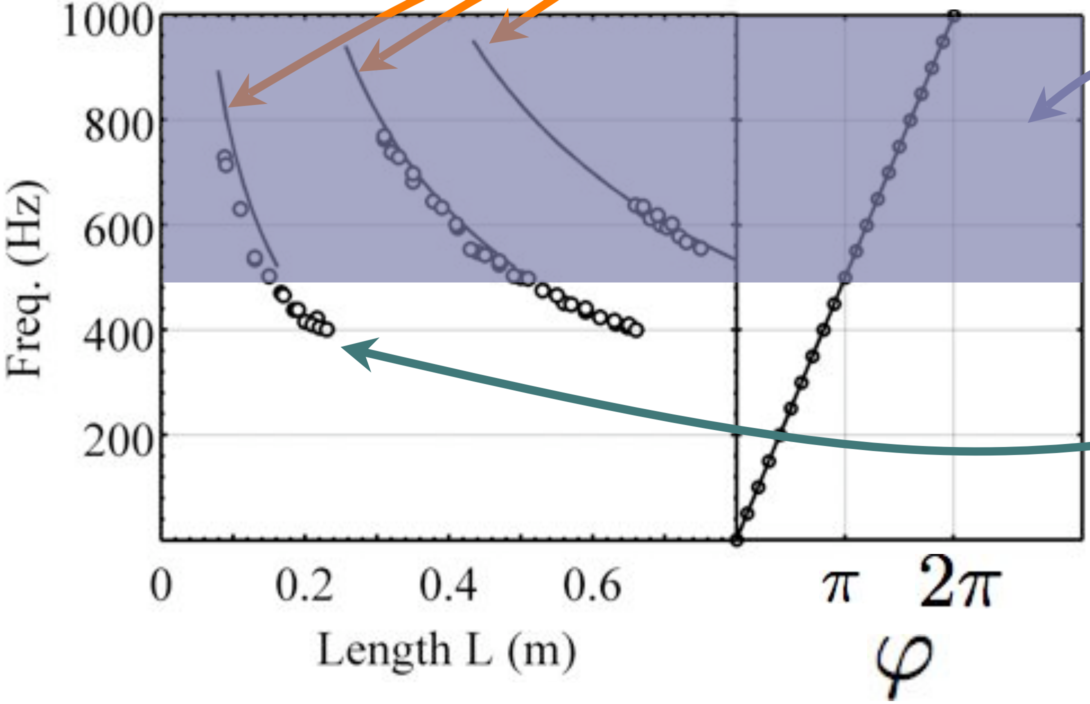
Linear stability analysis

Roots of the dispersion relation are sought for a single perturbation level for the FTF

$$a' = \tilde{a}e^{-i\omega t} \text{ where } \omega = \omega_r + i\omega_i$$
$$\omega_r = 2\pi f \text{ : frequency}$$
$$\omega_i \text{ : growth rate}$$

Noiray et al. (2006)
Noiray et al. (2007)

$(f, \omega_i > 0)$
Unstable



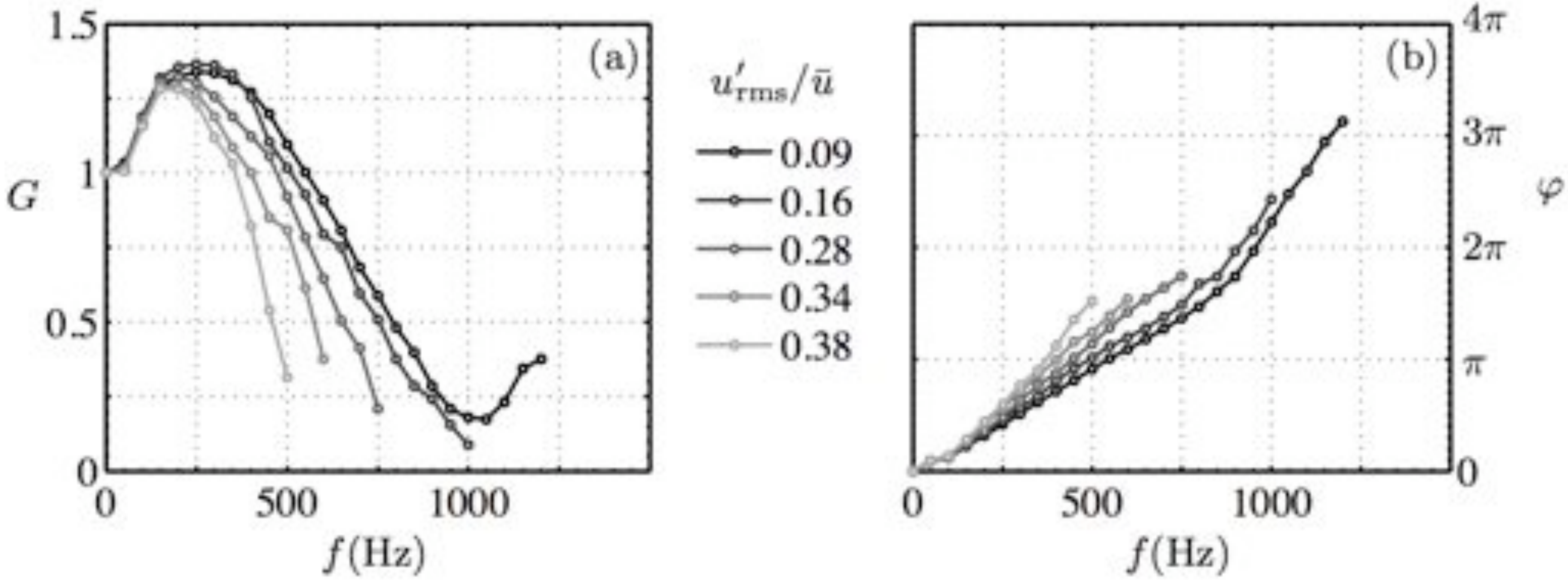
Unstable ranges $\pi < \varphi < 2\pi$
First order expansion in ω

Experimental data

Limits of the linear approach

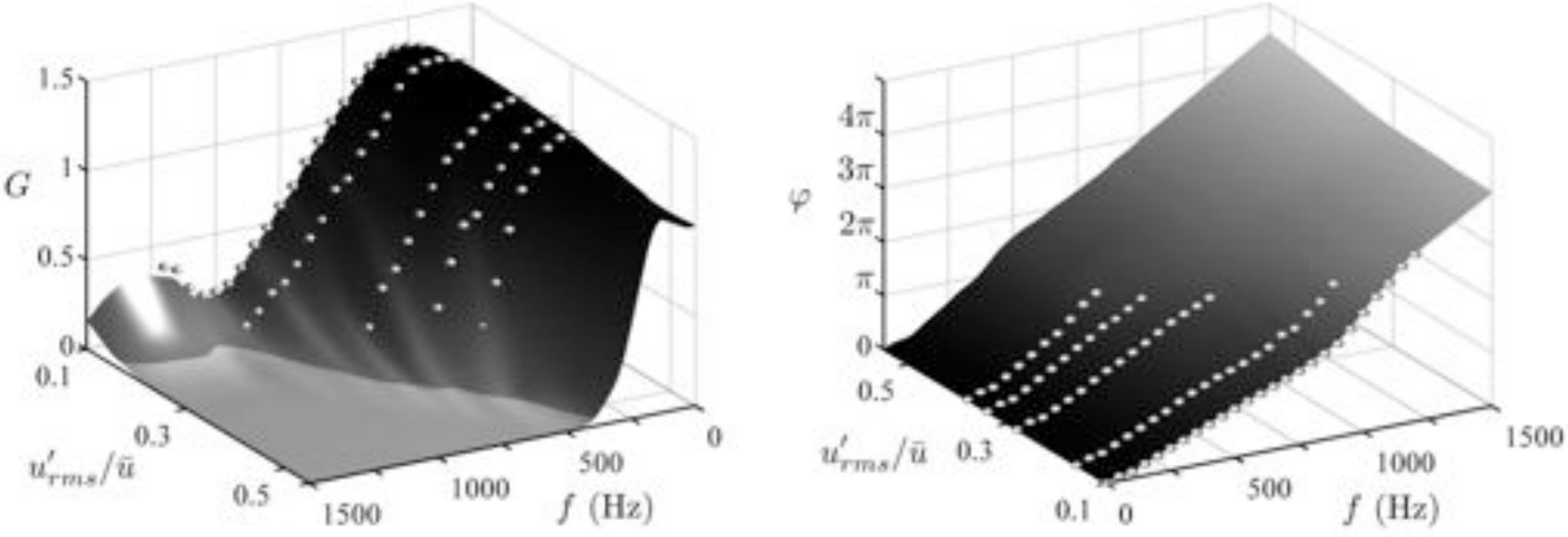
Predictions shifted in terms of frequency. Infinite growth of infinitesimal perturbations

Nonlinear flame response



$$\mathcal{F}\left(\omega_r, |u'|\right) = G\left(\omega_r, |u'|\right) \mathrm{e}^{i\varphi\left(\omega_r, |u'|\right)}$$

Flame Describing Function

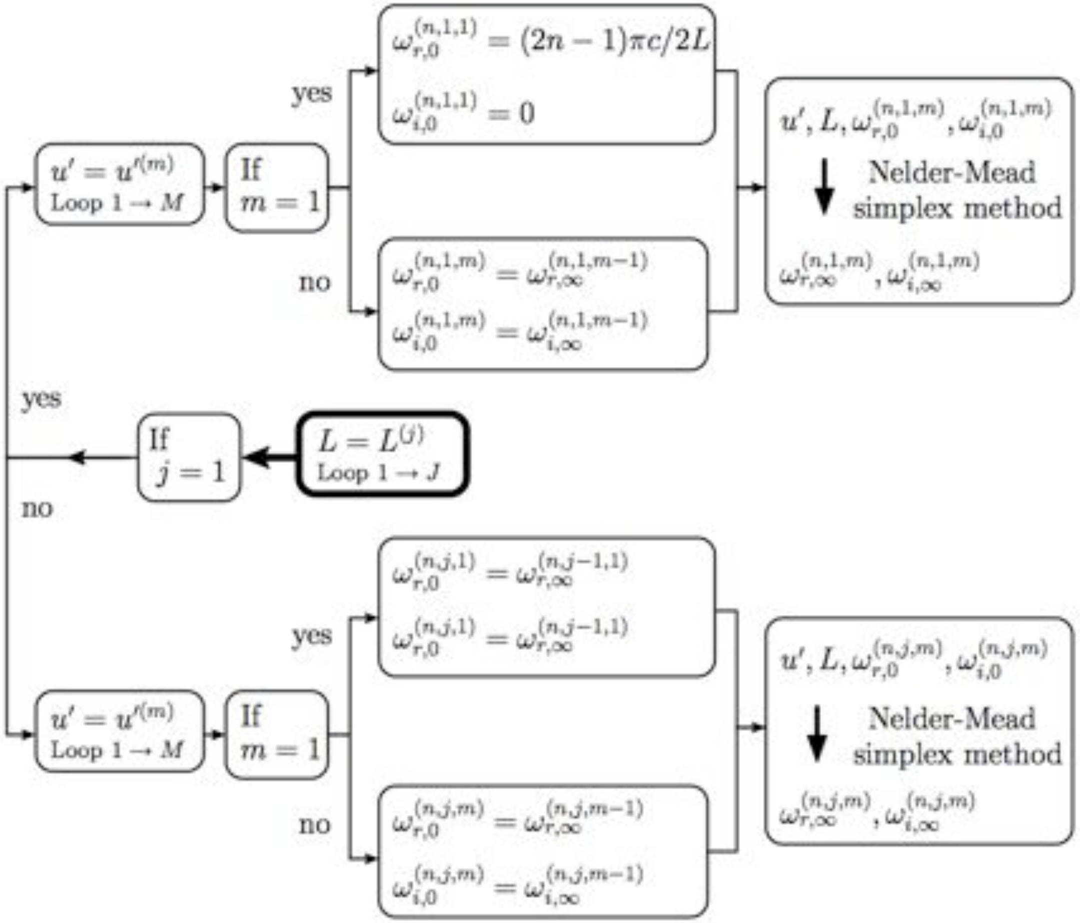


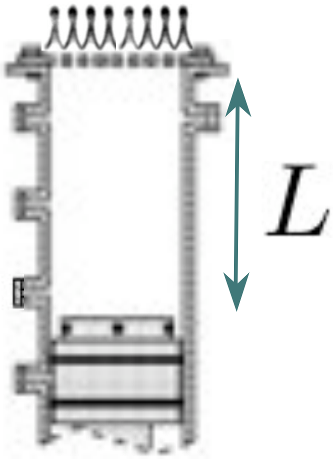
$$\mathcal{F}\left(\omega_r, |u'|\right) = G\left(\omega_r, |u'|\right) \mathrm{e}^{i\varphi\left(\omega_r, |u'|\right)}$$

Nonlinear algorithm

Roots of the dispersion relation are sought for increasing perturbation levels

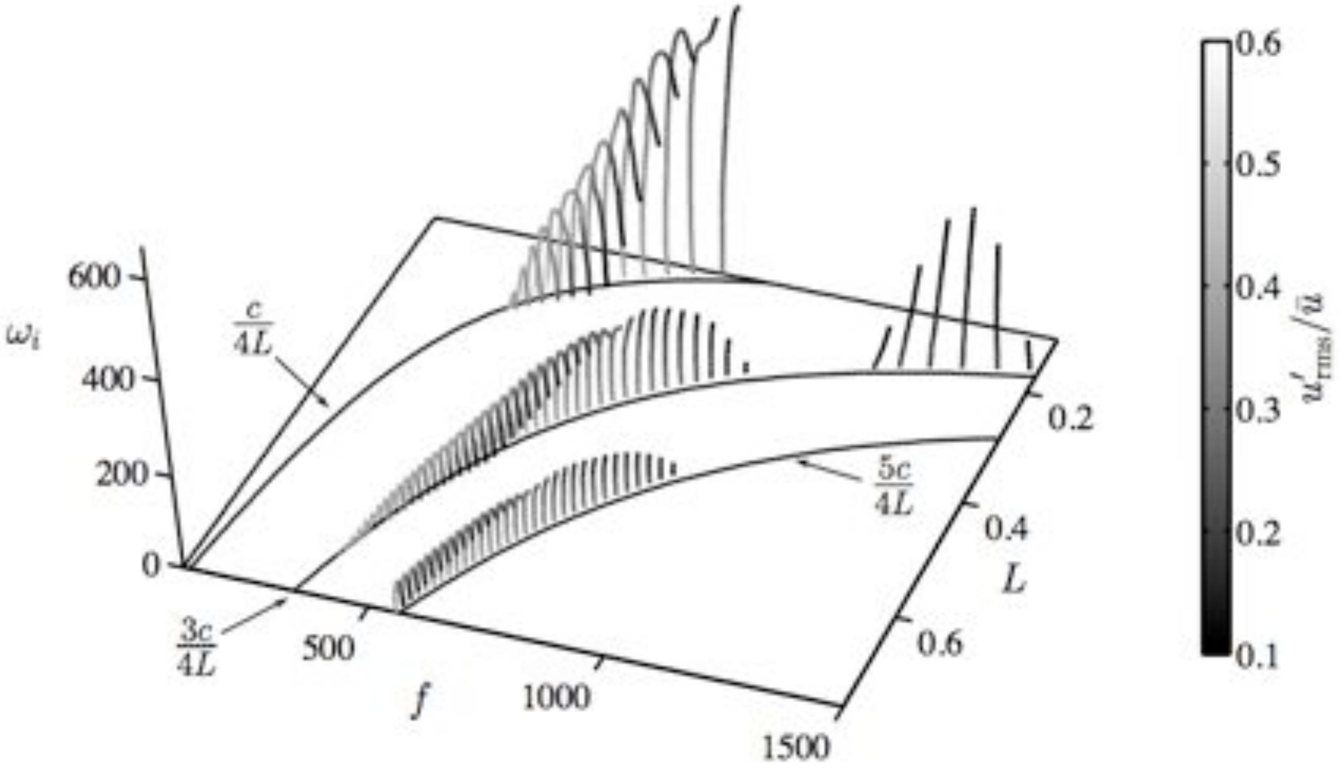
$$|\mathcal{H}(\omega_r + i\omega_i) - \mathcal{F}(\omega_r, |u'|)| = 0$$





For each length and for each mode (1/4,3/4 et 5/4), this yields a complex frequency solution of the dispersion relation as function of the input level

Solutions of the dispersion relation

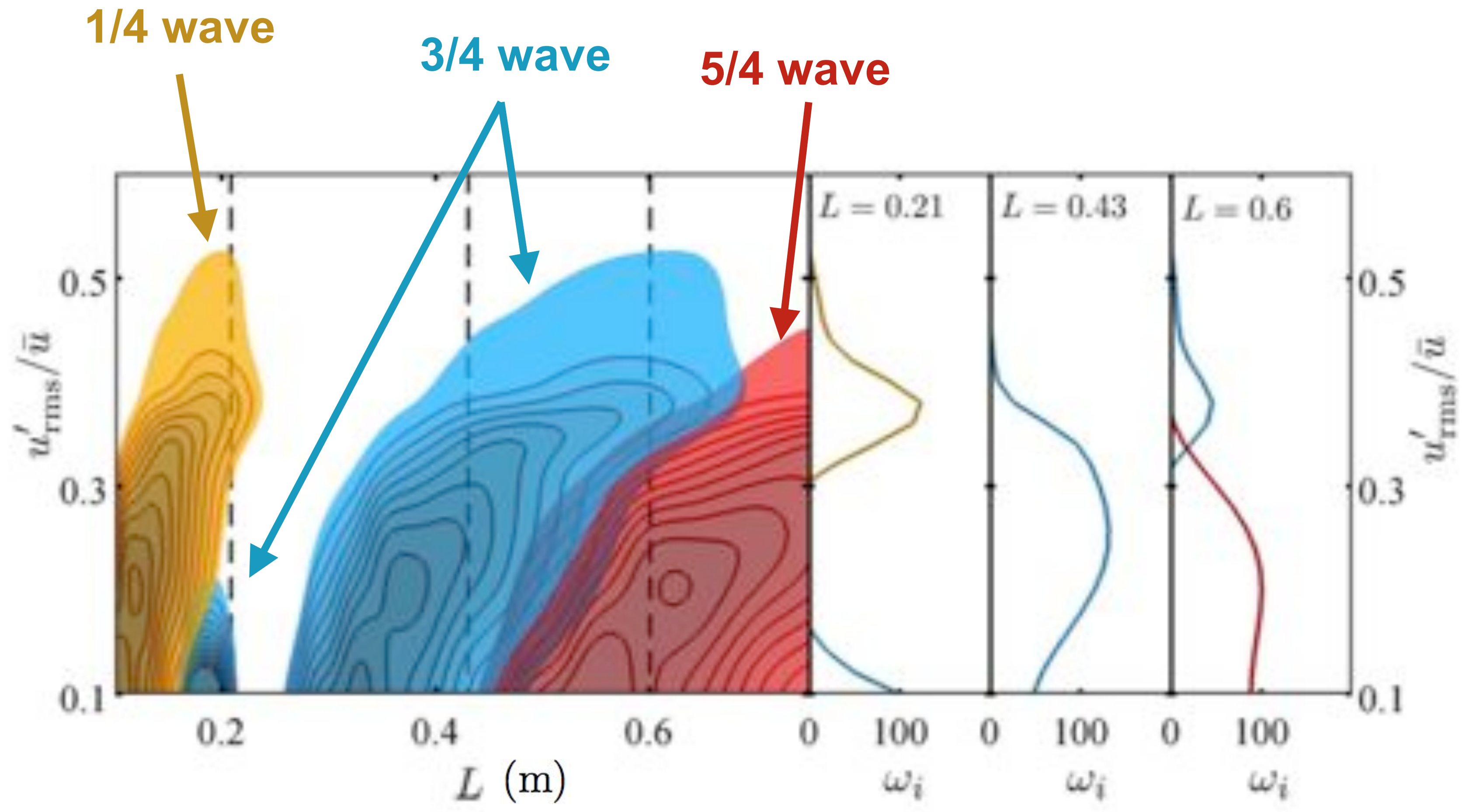


Set of solutions for each mode and for each length

$$\omega_r = \omega_r(|u'|)$$

$$\omega_i = \omega_i(|u'|)$$

Growth rates cartography



1/4 wave

3/4 wave

5/4 wave

Two types of trajectories can be identified

Linearly unstable mode

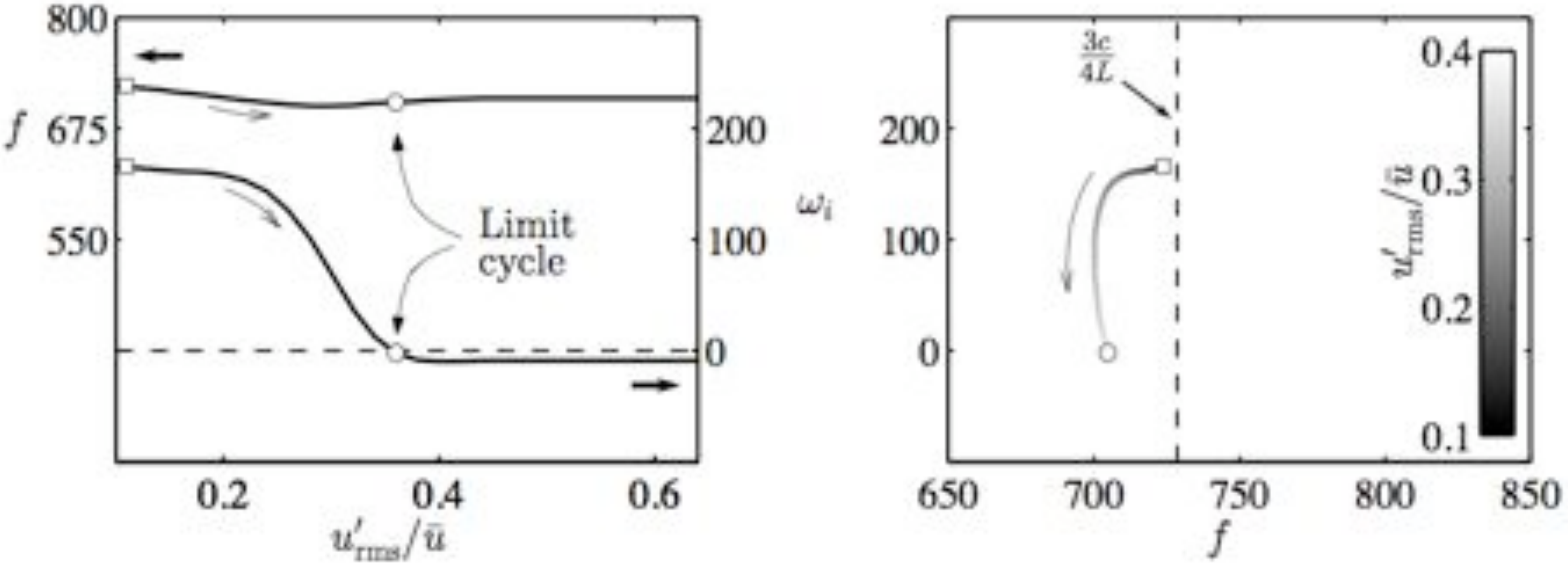
Type 1 trajectory in the state-space

$L = 35\ \mathrm{cm}$

mode 2

3/4 wave

Positive growth rate for small perturbations and saturation at a limit cycle when $\omega_i = 0$



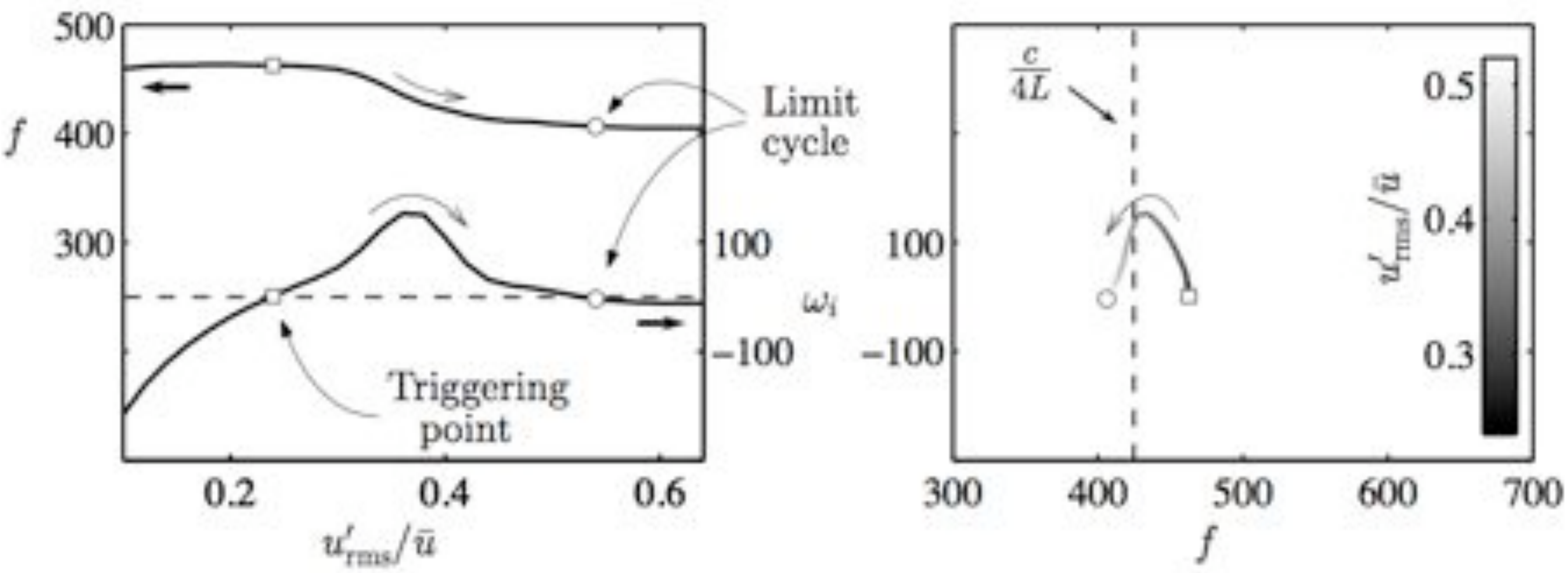
Nonlinearly unstable mode

Type 2 trajectory in the state-space

$L = 20$ cm
mode 1

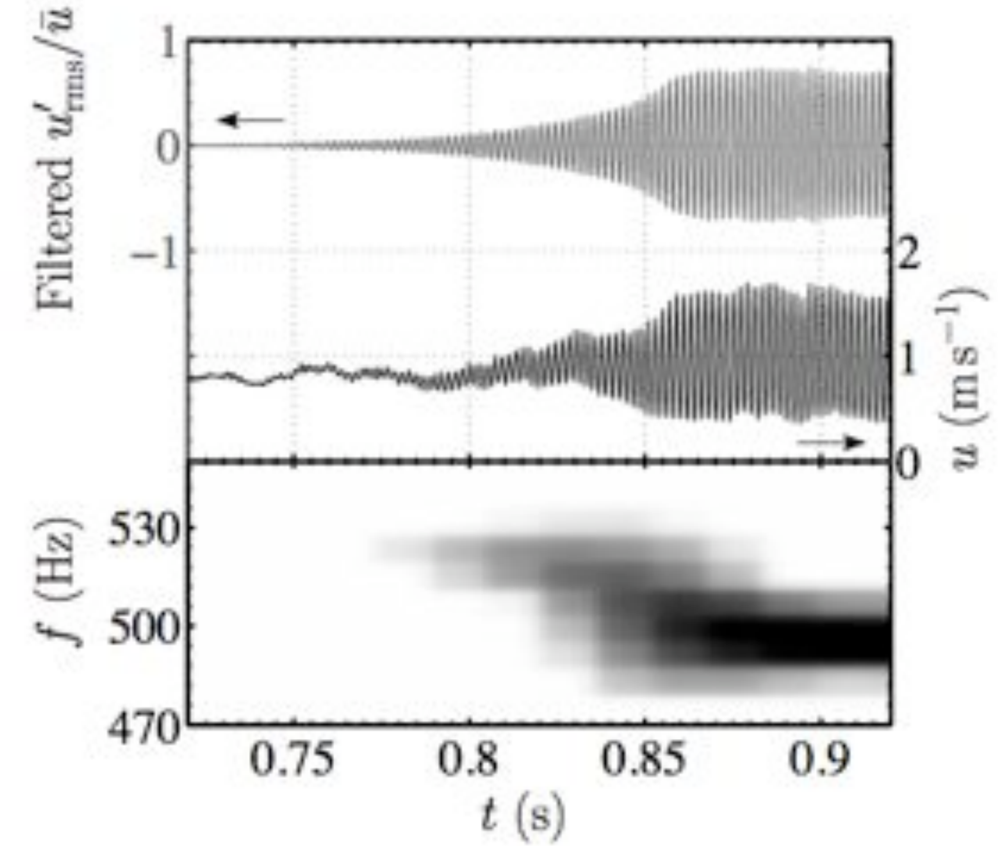
1/4 wave

Negative growth rate for an infinitesimal perturbation. Positive growth rate above a certain perturbation threshold and then saturation at a limit cycle when $\omega_i = 0$

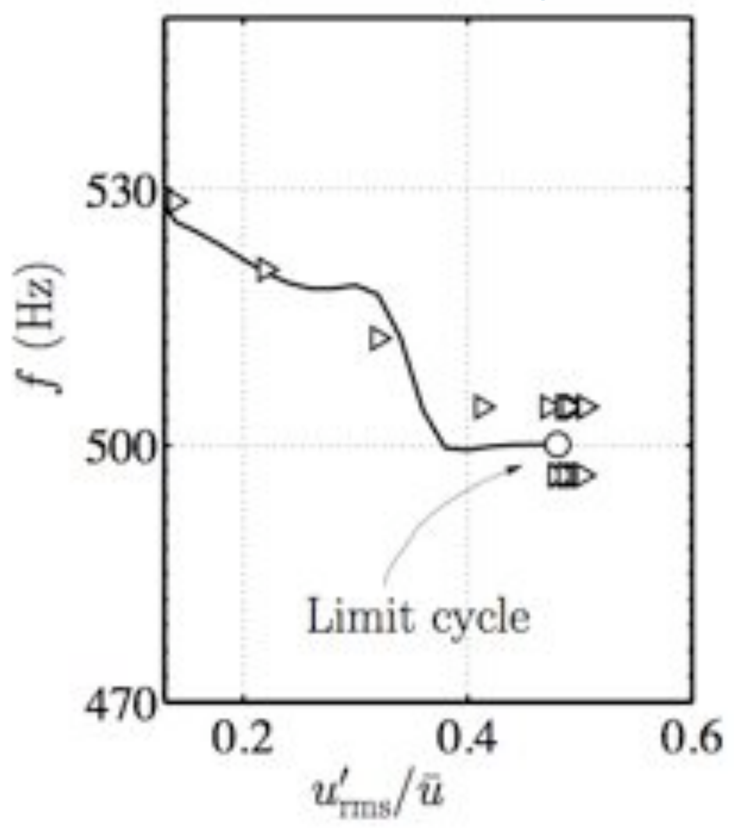


Frequency shift during growth of perturbation

Measurements



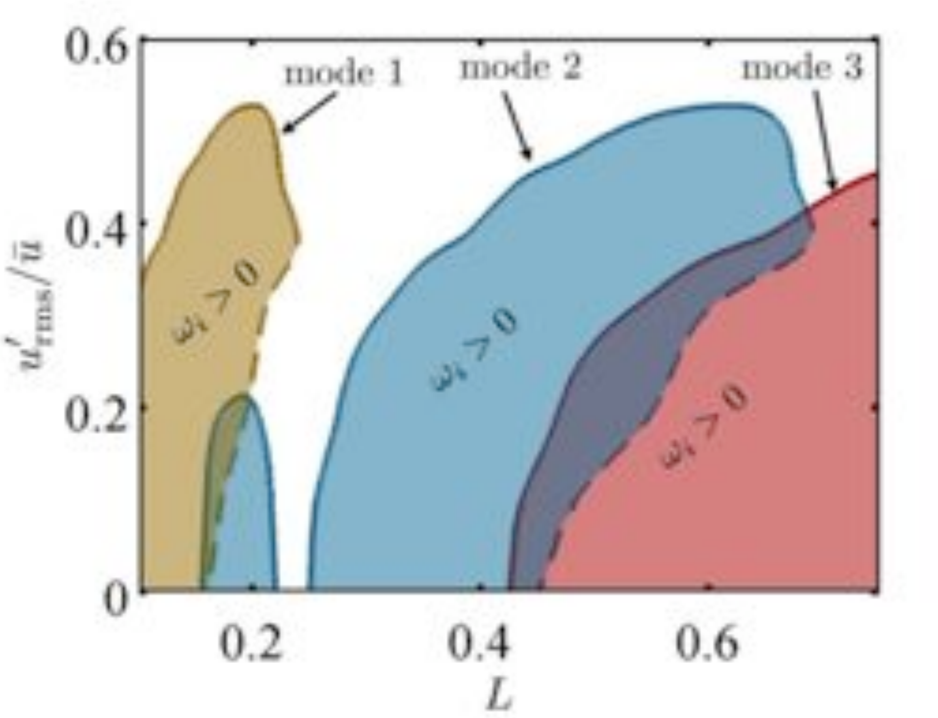
Modelling

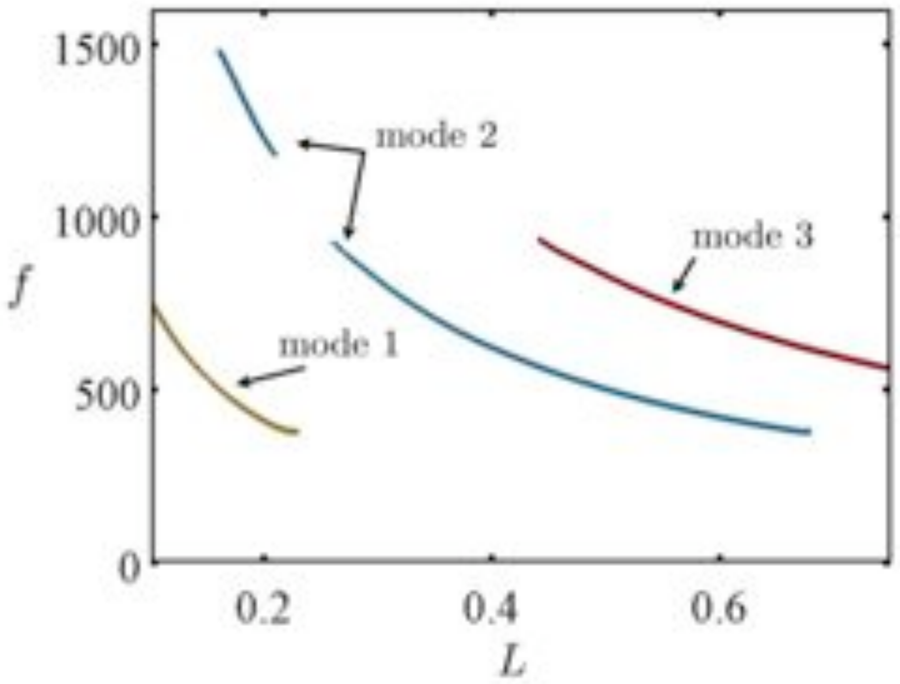


Good estimation of the limit cycle amplitude as well as the frequency shift during the phase of growth

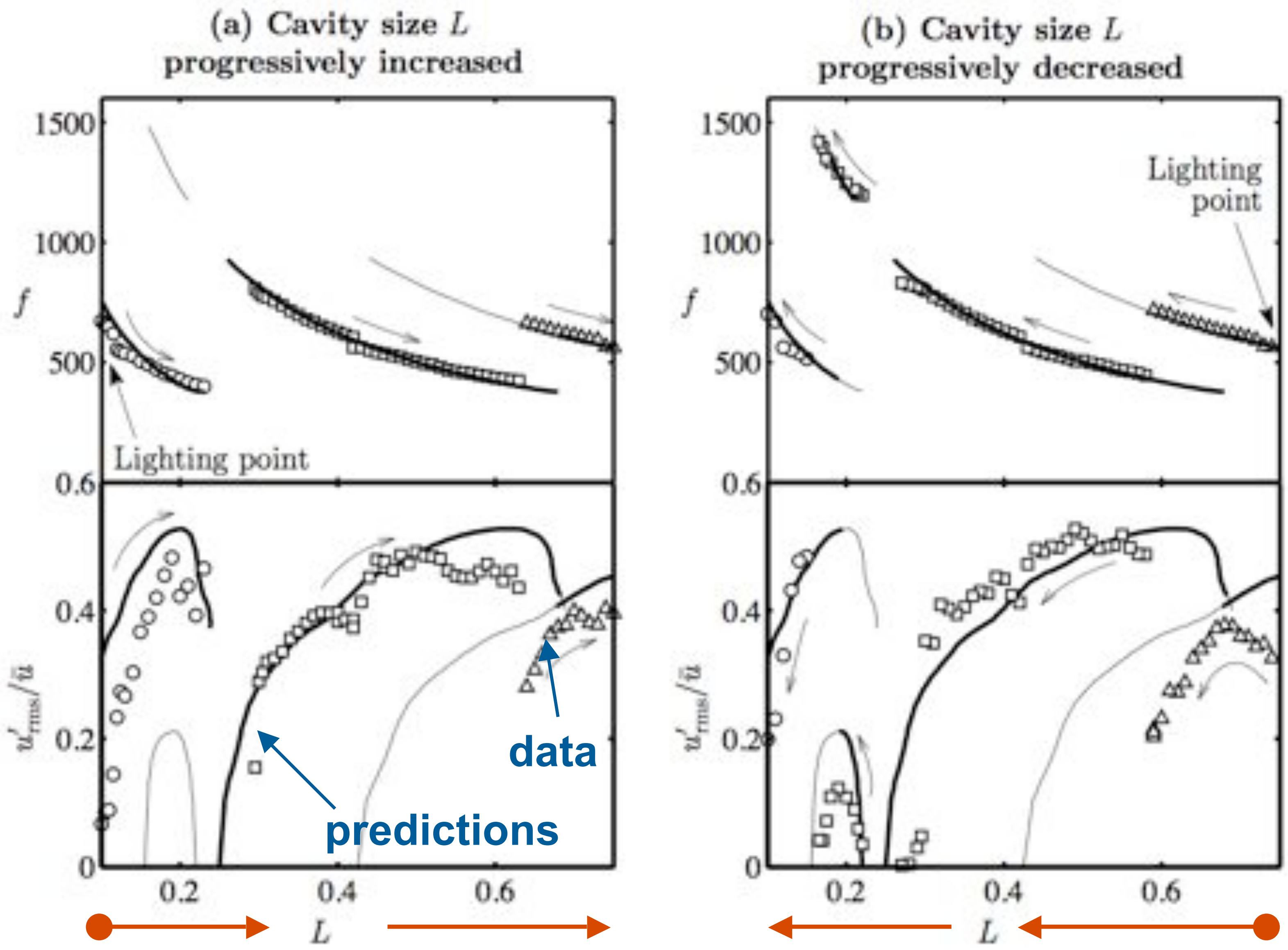
Hysteresis phenomena

Two ways read of the bifurcation diagram





Hysteresis prediction



(a) Cavity size L progressively increased

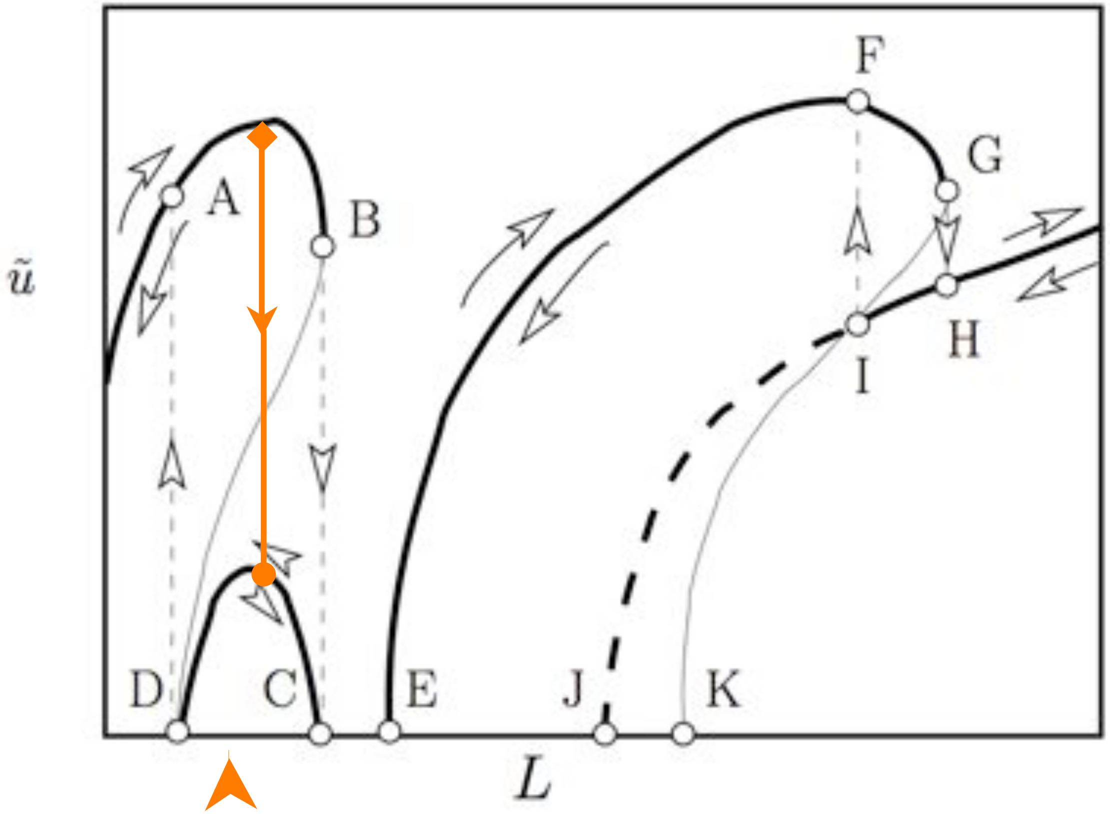
(b) Cavity size L progressively decreased

data

predictions

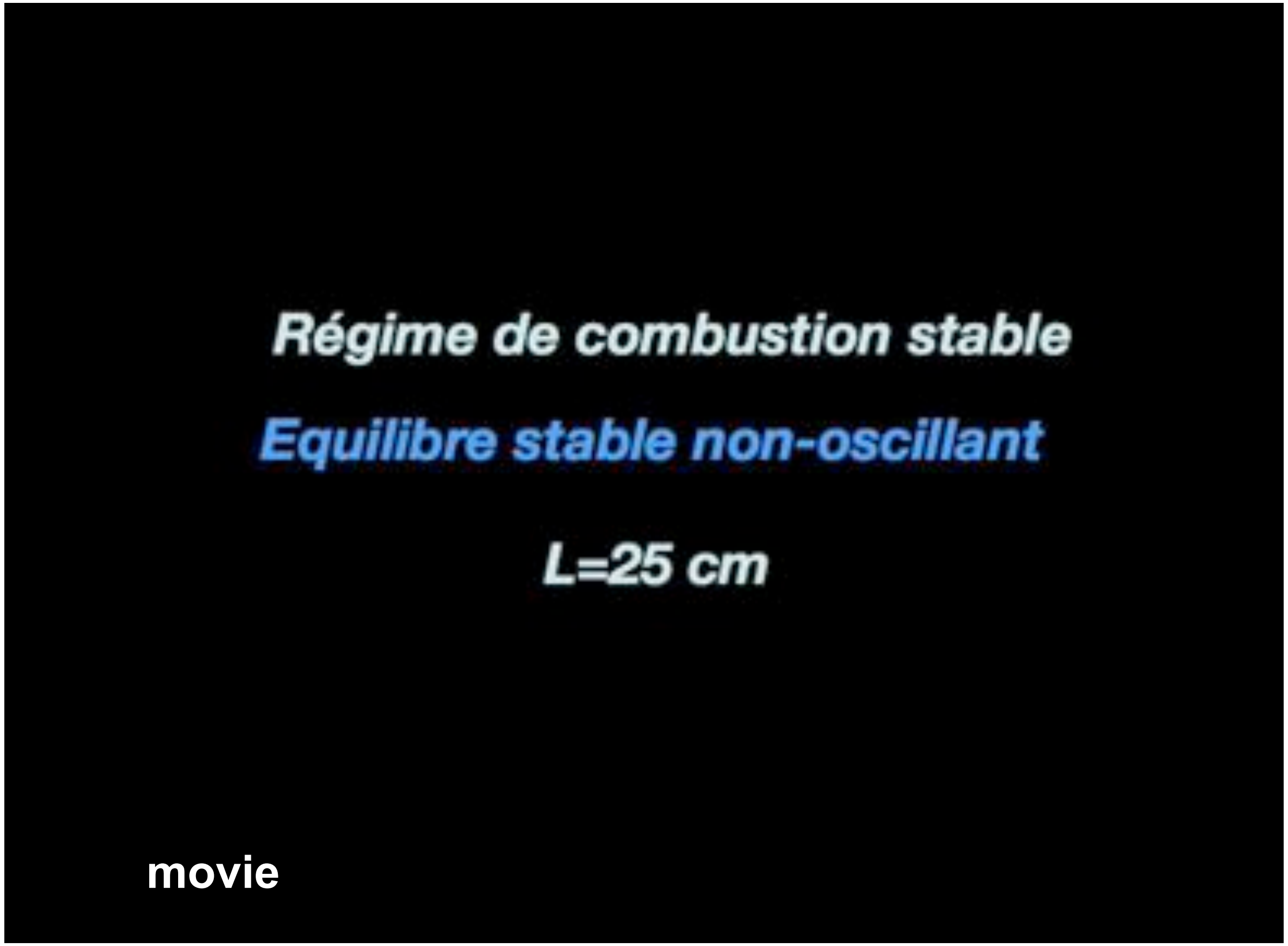
Hysteresis prediction

Length is fixed at a position where 2 stable equilibria coexist



The initial equilibrium is perturbed in a sufficiently jerk way in order to leave the system evolve to the second possible stable state

Hysteresis prediction



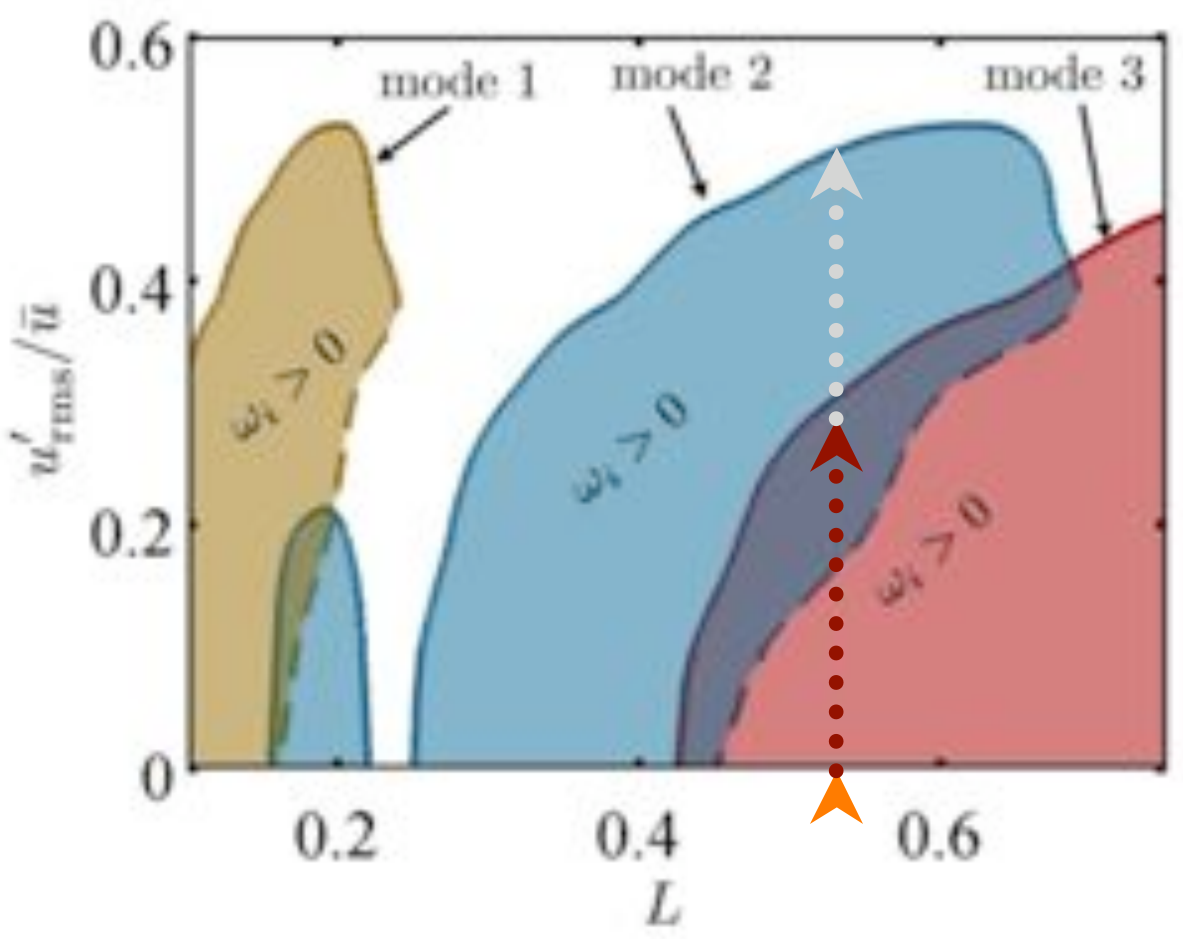
Régime de combustion stable

Equilibre stable non-oscillant

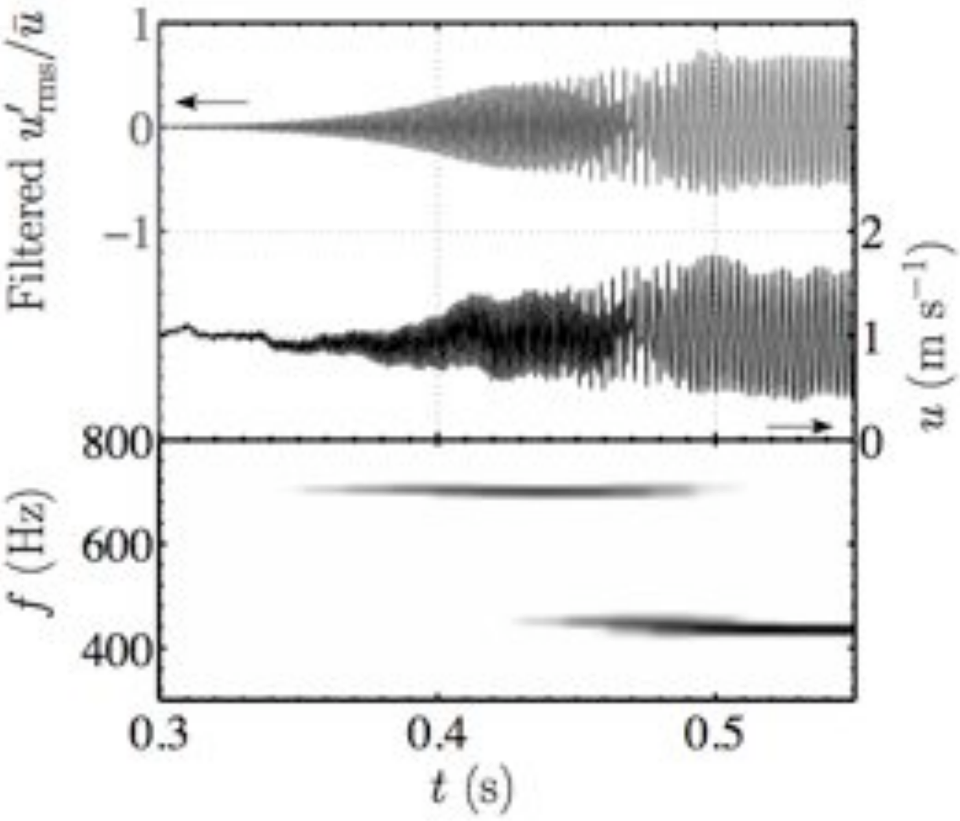
L=25 cm

movie

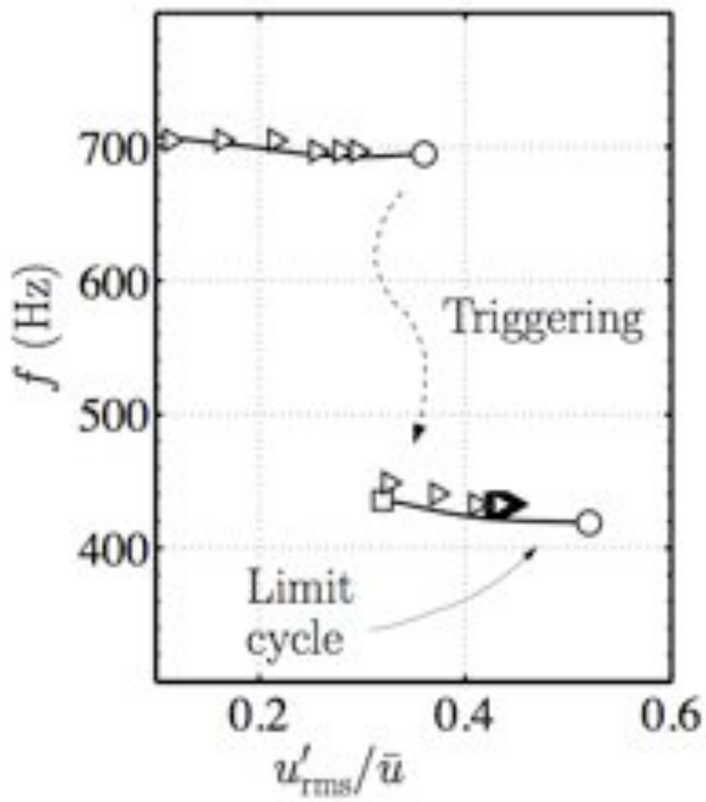
Mode hopping, triggering



Mode hopping, triggering



Line : predictions



Triangles : data

Conclusions

Conclusions

In progress