List Decoding for Arbitrarily Varying Multiple Access Channels with Conferencing Encoders

Holger Boche
Lehrstuhl für Theoretische Informationstechnik
Technische Universität München
80290 München, Germany

Rafael F. Schaefer
Department of Electrical Engineering
Princeton University
Princeton, NJ 08544, USA

***Abstract*—Research activities reveal a trend from an exclusive to a shared use of certain frequency bands. Then, uncoordinated interference will be unavoidable resulting in a channel that may vary in an arbitrary and unknown manner from channel use to channel use. This is the *arbitrarily varying channel (AVC)*, for which it has been shown that the classical deterministic approaches with pre-specified encoder and decoder fail if the AVC is symmetrizable. This necessitates more sophisticated strategies such as *common randomness (CR)* assisted strategies or *list decoding* which are capable to resolve the ambiguity induced by symmetrizable AVCs. Here, we study the *arbitrarily varying multiple access channel (AVMAC) with conferencing encoders*, which is motivated by cooperating base stations or access points in future communication systems. The capacity region of the AVMAC with conferencing encoders is established and it is shown that list decoding allows for reliable communication also for symmetrizable AVMACs. The list capacity region equals the CR-assisted capacity region for large enough list size. Finally, for fixed probability of decoding error the amount of resources, i.e., CR or list size, is shown to be finite.**

I. Introduction

A promising approach to increase the spectral efficiency of cellular systems, especially at the cell edges, is cooperation among neighboring base stations. High-speed backbones such as glass fiber will allow the base stations to exchange information about the channel state or the messages to transmit. First rigorous studies go back to Willems who studied the corresponding multiple access channel with conferencing encoders [1]. Not surprisingly, this is intensively discussed at the moment by the 3GGP LTE-Advanced group.

Another current research development reveals a paradigm shift from an exclusive to a shared use of certain frequency bands. While current systems such as cellular systems operate on exclusive frequency bands, there will be future systems such as sensor or ad-hoc networks which will operate on shared resources in a self-organizing and uncoordinated way. The major issue of this development is that interference will be ubiquitous making it the limiting factor of future wireless networks. Since there is no way to coordinate such induced interference, there is the need of new concepts.

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In particular, in such environments each receiver receives the signal he is interested in but also interfering signals from the other transmitters. As there is no coordination between the different transmitter-receiver pairs, there is no knowledge about the induced interference. Thus, all users have to be prepared for the worst, which is a channel that may vary in an arbitrary and unknown manner from channel use to channel use. This is the concept of *arbitrarily varying channels (AVCs)* [2–4] which provides a suitable and robust framework.

Unfortunately, it has been shown for AVCs that the traditional deterministic approach with pre-specified encoder and decoder fails if the AVC is symmetrizable resulting in zero capacity [3, 4]. Roughly speaking, such a channel can emulate a valid input so that it is impossible for the decoder to decide on the correct one. This necessitates more sophisticated strategies which overcome such channel conditions making reliable communication possible also for symmetrizable channels.

If *common randomness (CR)* is available at all users as a coordination resource, then they can use CR-assisted strategies allowing for reliable communication over symmetrizable channels [2, 3]. Here, encoder and decoder depend on the particular realization of the common randomness which has to be known at all users prior to the transmission. If such a coordination resource is not available, one is interested in alternatives that do not rely on such assumptions. It has been shown that *list decoding* might help to resolve the ambiguity of codewords caused by symmetrizable channels without the help of coordination resources. The capacity of the single-user AVC under list decoding is derived in [5, 6]. Bounds on the list sizes for the arbitrarily varying multiple access channel (AVMAC) are given in [7]. The broadcast channel with certain receiver side information under list decoding is studied in [8].

We study the *AVMAC with conferencing encoders* under list decoding for which we characterize the list capacity region. It either equals its CR-assisted capacity region or else is zero. This is done with the concept of *symmetrizability* and the list size at the decoder. In particular, if the list size is large enough, then a list strategy achieves the same performance as a CR-assisted strategy which requires coordination resources available at all users prior to transmission. In these cases the probability of error is required to vanish asymptotically and, usually, this requires an amount of common randomness which

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increases unbounded with increasing block length. Finally, it is shown that for fixed but non-vanishing probability of error, the amount of such resources needed to achieve the capacity is finite, i.e., in particular independent of the block length.[1]

II. System Model

The communication is affected by a channel which may vary in an unknown and arbitrary manner from channel use to channel use. To model such a behavior, we introduce a finite state set $\mathcal{S}$. Further, let $\mathcal{X}$, $\mathcal{Y}$, and $\mathcal{Z}$ be finite input and output sets. Then for a fixed state sequence $s^n \in \mathcal{S}^n$ of length n and input and output sequences $x^n \in \mathcal{X}^n$, $y^n \in \mathcal{Y}^n$, and $z^n \in \mathcal{Z}^n$, the discrete memoryless multiple access channel is given by $W^n(z^n|x^n,y^n,s^n) := \prod_{i=1}^n W(z_i|x_i,y_i,s_i)$.

Definition 1: The discrete memoryless *arbitrarily varying multiple access channel (AVMAC)* is the family

$$\mathfrak{W} := \big\{W^n(\cdot|\cdot,\cdot,s^n) : s^n \in \mathcal{S}^n\big\}.$$

Further, for any probability distribution $q \in \mathcal{P}(\mathcal{S})$, we denote the averaged multiple access channel by

$$\overline{W}_q(z|x,y) := \sum_{s\in\mathcal{S}} W(z|x,y,s)q(s). \tag{1}$$

In the classical AVMAC setup, none of the transmitters has any knowledge about the message the other one will transmit and the corresponding capacity region is studied in [9, 10]. Here, we study the case where both transmitters can cooperate in the sense that they can exchange limited information using Willems conferencing [1]. Such information can regard the messages to transmit but is not necessarily restricted to them.

Let $\mathcal{M}_1 := \{1,...,M_{1,n}\}$ and $\mathcal{M}_2 := \{1,...,M_{2,n}\}$ be the sets of messages of transmitters 1 and 2, respectively, and further $\mathcal{M} := \mathcal{M}_1 \times \mathcal{M}_2$. Then Willems conferencing is an iterative protocol which can be described as follows. In the first time slot, each transmitter sends some information to the other one. In the subsequent time slots, they send more information taking the information they received in the previous iterations into account. Such a Willems conference terminates after a fixed number of iterations I. Then, a pair (c_1,c_2) of functions is determined by functions $c_{i,1}, c_{i,2}, ..., c_{i,I}$, $i=1,2$ with

$$c_{i,1} : \mathcal{M}_i \to \mathcal{K}_{i,1} \tag{2}$$

and

$$c_{i,k} : \mathcal{M}_i \times \mathcal{K}_{\bar{i},1} \times ... \times \mathcal{K}_{\bar{i},k-1} \to \mathcal{K}_{i,k}$$

for $k = 2,3,...,I$ and $\bar{i} = 2$ if $i=1$ and $\bar{i}=1$ if $i=2$. The number of iterations I may be arbitrary but fixed and also $\mathcal{K}_{i,k}$ can be an arbitrary finite set. Thus, the pair (c_1,c_2) is defined by the concatenation of the individual $c_{i,k}$ as

$$(c_1,c_2) : \mathcal{M}_1 \times \mathcal{M}_2 \to \mathcal{K}_1 \times \mathcal{K}_2 \tag{3}$$

with $\mathcal{K}_1 := \mathcal{K}_{1,1} \times ... \times \mathcal{K}_{1,I}$ and $\mathcal{K}_2 := \mathcal{K}_{2,1} \times ... \times \mathcal{K}_{2,I}$.

[1] *Notation:* Discrete random variables are denoted by capital letters and their realizations and ranges by lower case and script letters; $\mathbb{N}$ and $\mathbb{R}_+$ denote the sets of positive integers and non-negative real numbers; $I(\cdot;\cdot)$ is the mutual information; $\mathbb{P}\{\cdot\}$ is the probability and $\mathcal{P}(\cdot)$ is the set of all probability distributions; $\mathcal{O}(\cdot)$ is the big-O notation.

If the Willems conferencing is unrestricted, an arbitrary amount of information can be exchanged such that both messages are available at both transmitters turning the AVMAC into a single-user AVC. Due to practical reasons we consider only limited exchange capabilities. Assuming a Willems conference pair $(c_1,c_2) : \mathcal{M}_1 \times \mathcal{M}_2 \to \mathcal{K}_1 \times \mathcal{K}_2$ as given in (3) is used for a block length n code. Then, the conference (c_1,c_2) has conferencing capacities $C_1, C_2 > 0$ if

$$\frac{1}{n}\log|\mathcal{K}_i| \le C_i, \quad i=1,2.$$

We call this an (n,C_1,C_2)-*Willems conference*, whose definition is independent of the number of iterations I.

III. Code Concepts and Coordination Resources

In general, for AVCs deterministic strategies do not suffice to establish reliable communication; in particular, symmetrizable channels require more sophisticated approaches [2–4].

A. Deterministic Codes

The deterministic approach relies on pre-specified encoders and decoder as specified in the following.

Definition 2: A *deterministic* $(n, M_{1,n}, M_{2,n}, C_1, C_2)$-*code* $\mathcal{C}$ for the AVMAC $\mathfrak{W}$ is a 5-tuple (c_1,c_2,f_1,f_2,ϕ) consisting of an (n,C_1,C_2)-Willems conference

$$(c_1,c_2) : \mathcal{M}_1 \times \mathcal{M}_2 \to \mathcal{K}_1 \times \mathcal{K}_2, \tag{4}$$

encoders at transmitters 1 and 2

$$f_1 : \mathcal{M}_1 \times \mathcal{K}_2 \to \mathcal{X}^n \quad \text{and} \quad f_1 : \mathcal{M}_2 \times \mathcal{K}_1 \to \mathcal{Y}^n, \tag{5a}$$

and a decoder at the receiver

$$\phi : \mathcal{Z}^n \to \mathcal{M}_1 \times \mathcal{M}_2. \tag{6}$$

Such a code implies the following system

$$\big\{(x_{jk}^n, y_{jk}^n, \mathcal{D}_{jk}) : j \in \mathcal{M}_1, k \in \mathcal{M}_2\big\} \tag{7}$$

with $x_{jk}^n = f_1(j, c_2(j,k))$, $y_{jk}^n = f_2(k, c_1(j,k))$, and disjoint decoding sets $\mathcal{D}_{jk} = \{z^n \in \mathcal{Z}^n : \phi(z^n) = (j,k)\}$.

Then for the deterministic code $\mathcal{C}$, the average probability of decoding error for state sequence $s^n \in \mathcal{S}^n$ is given by

$$\bar{e}_n(s^n|\mathcal{C}) := \frac{1}{|\mathcal{M}|}\sum_{(j,k)\in\mathcal{M}} W^n(\mathcal{D}_{jk}^c|x_{jk}^n, y_{jk}^n, s^n).$$

Definition 3: A rate pair $(R_1,R_2) \in \mathbb{R}_+^2$ is said to be *deterministically achievable* for the AVMAC $\mathfrak{W}$ with conferencing capacities $C_1, C_2 > 0$ if for any $\delta > 0$ there exists an $n(\delta) \in \mathbb{N}$ and a sequence of $(n, M_{1,n}, M_{2,n}, C_1, C_2)$-codes $\mathcal{C}$ such that for all $n \ge n(\delta)$ we have

$$\frac{1}{n}\log M_{i,n} \ge R_i - \delta, \qquad i=1,2,$$

while

$$\max_{s^n\in\mathcal{S}^n} \bar{e}_n(s^n|\mathcal{C}) \le \lambda_n$$

with $\lambda_n \to 0$ as $n \to \infty$. The *deterministic capacity region* $\mathcal{R}_{\text{det}}(C_1,C_2)$ of the AVMAC $\mathfrak{W}$ with conferencing encoders is the set of all achievable rate pairs.

We further need the concept of symmetrizability which is given in the following definition.

Definition 4: An AVMAC $\mathfrak{W}$ is called $(\mathcal{X},\mathcal{Y})$-*symmetrizable* if there exists a channel $\sigma : \mathcal{X}\times\mathcal{Y}\to\mathcal{P}(\mathcal{S})$ such that

$$\sum_{s\in\mathcal{S}} W(z|x,y,s)\sigma(s|x',y') = \sum_{s\in\mathcal{S}} W(z|x',y',s)\sigma(s|x,y)$$

holds for all $x,x'\in\mathcal{X}$, $y,y'\in\mathcal{Y}$, and $z\in\mathcal{Z}$. This means, the channel $\widetilde{W}(z|x,y,x',y') = \sum_{s\in\mathcal{S}} W(z|x,y,s)\sigma(s|x',y')$ is symmetric in (x,y) and (x',y') for all $x,x'\in\mathcal{X}$, $y,y'\in\mathcal{Y}$, and $z\in\mathcal{Z}$.

Roughly speaking, such a channel can emulate valid channel inputs making it impossible to decide on the correct one.

Remark 1: The capacity region of the classical AVMAC without conferencing encoders depend also on so-called $\mathcal{X}$-symmetrizability and $\mathcal{Y}$-symmetrizability conditions that operate only on one input. While these "marginal" conditions are important for the analysis of the AVMAC without conferencing encoders [11], it has been shown that only $(\mathcal{X},\mathcal{Y})$-symmetrizability is needed to completely characterize the capacity region of the AVMAC with conferencing encoders.

Let $\varPi$ be the set of all probability distributions $p\in\mathcal{P}(\mathcal{U}\times\mathcal{X}\times\mathcal{Y})$, where $\mathcal{U}$ is a finite (auxiliary) set and p further has the form $p(u,x,y) = P_U(u)P_{X|U}(x|u)P_{Y|U}(y|u)$. We define $\overline{\mathcal{R}}(p,q,C_1,C_2)$ consisting of all $(R_1,R_2)\in\mathbb{R}_+^2$ that satisfy

$$\begin{aligned} R_1 &\le I(X;\overline{Z}_q|Y,U) + C_1 \\ R_2 &\le I(Y;\overline{Z}_q|X,U) + C_2 \\ R_1+R_2 &\le \min\{I(X,Y;\overline{Z}_q), I(X,Y;\overline{Z}_q|U)+C_1+C_2\} \end{aligned}$$

where $\overline{Z}_q$ is the random variable associated with the output of the averaged channel $\overline{W}_q$, $q\in\mathcal{P}(\mathcal{S})$, cf. (1). Then, we set

$$\overline{\mathcal{R}}(C_1,C_2) := \bigcup_{p\in\varPi}\bigcap_{q\in\mathcal{P}(\mathcal{S})} \overline{\mathcal{R}}(p,q,C_1,C_2).$$

With this and the concept of symmetrizability, we are able to characterize the deterministic capacity region.

Theorem 1 ([12]): For the deterministic capacity region $\mathcal{R}_{\text{det}}(C_1,C_2)$ of the AVMAC $\mathfrak{W}$ with conferencing capacities $C_1,C_2>0$ we have

$$\mathcal{R}_{\text{det}}(C_1,C_2) = \overline{\mathcal{R}}(C_1,C_2)$$

if and only if the AVMAC $\mathfrak{W}$ is non-$(\mathcal{X},\mathcal{Y})$-symmetrizable. If the AVMAC $\mathfrak{W}$ is $(\mathcal{X},\mathcal{Y})$-symmetrizable, then $\mathcal{R}_{\text{det}}(C_1,C_2) = \{(0,0)\}$.

Remark 2: In addition, it turns out that a one-shot non-iterative Willems-conference is sufficient to achieve capacity, i.e., (3) consists only of the first conference round (2).

B. Common-Randomness-Assisted Codes

Since such a deterministic approach as discussed above with predetermined encoders and decoder fails if the channel is symmetrizable, one is interest in more sophisticated strategies that work well also in this case. This is where the common-randomness-assisted coding strategies come into play.

If the transmitters and receiver have access to a *common randomness (CR)*, then they can use this resource to coordinate their choice of encoders and decoder. This is modeled by a random variable $\varGamma$ on $\mathcal{G}_n$. Then, conference (3), encoders (5), and decoder (6) depend on the particular realization $\gamma\in\mathcal{G}_n$.

Definition 5: A *CR-assisted* $(n,M_{1,n},M_{2,n},C_1,C_2,\varGamma)$-*code* $\mathcal{C}_{\text{CR}}$ for the AVMAC $\mathfrak{W}$ is a family

$$\big\{\big(c_1(\gamma),c_2(\gamma),f_1(\gamma),f_2(\gamma),\phi(\gamma)\big):\gamma\in\mathcal{G}_n\big\}$$

with a random variable $\varGamma$ uniformly distributed on $\mathcal{G}_n$.

This means $\mathcal{G}_n$ defines a finite set of deterministic $(n,M_{1,n},M_{2,n},C_1,C_2)$-codes as given in Definition 2. The number of such codes contained in the CR-assisted code $\mathcal{C}_{\text{CR}}$ is then determined by $|\mathcal{G}_n|$. Thus, each realization $\gamma\in\mathcal{G}_n$ indicates which particular code is selected out of the whole ensemble.

Then for the CR-assisted code $\mathcal{C}_{\text{CR}}$, the average probability of decoding error for state sequence $s^n\in\mathcal{S}^n$ becomes

$$\begin{aligned} \bar{e}_{\text{CR},n}(s^n|\mathcal{C}_{\text{CR}}) &:= \frac{1}{|\mathcal{G}_n|}\sum_{\gamma\in\mathcal{G}_n}\bar{e}_n(s^n|\mathcal{C}(\gamma)) \\ &= \frac{1}{|\mathcal{G}_n|}\sum_{\gamma\in\mathcal{G}_n}\frac{1}{|\mathcal{M}|}\sum_{(j,k)\in\mathcal{M}} W^n\big((\mathcal{D}_{jk}(\gamma))^c|x_{jk}^n(\gamma),y_{jk}^n(\gamma),s^n\big). \end{aligned}$$

The definitions of a *CR-assisted achievable* rate pair and the *CR-assisted capacity region* $\mathcal{R}_{\text{CR}}(C_1,C_2,\varGamma)$ of the AVMAC $\mathfrak{W}$ with conferencing encoders follow accordingly.

Theorem 2 ([12]): The CR-assisted capacity region $\mathcal{R}_{\text{CR}}(C_1,C_2,\varGamma)$ of the AVMAC $\mathfrak{W}$ with conferencing capacities $C_1,C_2>0$ is

$$\mathcal{R}_{\text{CR}}(C_1,C_2,\varGamma) = \overline{\mathcal{R}}(C_1,C_2).$$

Thus, CR-assisted strategies achieve the same rates as the deterministic ones, but also for symmetrizable channels. An important observation for the analysis of list codes is the following.

Remark 3: We know from [12, Lemma 15] that the amount of common randomness that is needed for achieving the CR-assisted capacity region $\mathcal{R}_{\text{CR}}(C_1,C_2,\varGamma)$ of the AVMAC $\mathfrak{W}$ is quadratic in block length. This means for a transmission of block length n, it is sufficient to use a CR-assisted code which consists of n^2 deterministic codes, i.e., $\mathcal{G}_n := \{1,2,...,n^2\}$ and $|\mathcal{G}_n| = n^2$, cf. also Definition 5.

IV. Capacity Region under List Decoding

The previous discussion reveals the following dilemma: The traditional deterministic approach with pre-specified encoders and decoder only works for non-symmetrizable channels. And unfortunately, many channels of practical relevance fall in the category of symmetrizable channels resulting in zero capacity [4]. On the other hand, CR-assisted codes allow reliable communication also for such channel conditions. But the drawback of such more sophisticated approaches is the fact that they require a strong coordination between encoders and decoder based on common randomness. In particular, the actual realization has to be perfectly known at all users prior to transmission which might be hard to realize in practice especially for multi-user scenarios.

Thus, one is interested in strategies which work well in the case of symmetrizable channels but which do not rely on such coordination resources as common randomness. It has been shown for the single-user AVC that the concept of list decoding helps to resolve the ambiguity induced by symmetrizable channels [5, 6] without relying on additional coordination resources. In the following we want to analyze list decoding also for the AVMAC with conferencing encoders.

A. List Codes

While a deterministic decoder ϕ of Definition 2 decides on exactly one message pair $(m_1, m_2) \in \mathcal{M}_1 \times \mathcal{M}_2$ based on its received signal $z^n \in \mathcal{Z}^n$, a list decoder with list size L maps the received signal into up to L possible message pairs. The list code is specified as follows.

Definition 6: An $(n, M_{1,n}, M_{2,n}, C_1, C_2, L)$-*list code* $\mathcal{C}_{\text{list}}$ with list size L is a deterministic $(n, M_{1,n}, M_{2,n}, C_1, C_2)$-code of Definition 2 where the deterministic decoder (6) is replaced by a list decoder

$$\phi : \mathcal{Z}^n \to \mathfrak{P}_L(\mathcal{M}_1 \times \mathcal{M}_2)$$

where $\mathfrak{P}_L(\mathcal{M}_1 \times \mathcal{M}_2)$ is the set of all subsets of $\mathcal{M}_1 \times \mathcal{M}_2$ with cardinality at most L.

Similar to (7), such a code implies the system $\{(x_{jk}^n, y_{jk}^n, \mathcal{D}_{jk}) : j \in \mathcal{M}_1, k \in \mathcal{M}_2\}$ where the decoding sets are given by

$$\mathcal{D}_{jk} = \big\{z^n \in \mathcal{Z}^n : (j,k) \notin \phi(z^n)\big\}.$$

In particular, due to the list decoding, the decoding sets need not be disjoint and we have $|\{(j,k) : z^n \in \mathcal{D}_{jk}\}| \leq L$ so that $\phi(z^n) = \{(j,k) : z^n \in \mathcal{D}_{jk}\}$.

Then for the list code $\mathcal{C}_{\text{list}}$, the probability of decoding error for message pair $(j,k) \in \mathcal{M}_1 \times \mathcal{M}_2$ and $s^n \in \mathcal{S}^n$ is given by

$$\begin{aligned}\bar{e}_{L,n}\big((j,k), s^n|\mathcal{C}_{\text{list}}\big) &:= W^n(\mathcal{D}_{jk}^c|x_{jk}^n, y_{jk}^n, s^n) \\ &= \sum_{z^n:(j,k)\notin\phi(z^n)} W^n(z^n|x_{jk}^n, y_{jk}^n, s^n)\end{aligned}$$

and the average probability of decoding error for $s^n \in \mathcal{S}^n$ is

$$\bar{e}_{L,n}(s^n|\mathcal{C}_{\text{list}}) = \frac{1}{|\mathcal{M}|} \sum_{(j,k)\in\mathcal{M}} \bar{e}_{L,n}\big((j,k), s^n|\mathcal{C}_{\text{list}}\big).$$

Definition 7: A rate pair $(R_1, R_2) \in \mathbb{R}_+^2$ is said to be *list achievable* for the AVMAC $\mathfrak{W}$ with conferencing capacities $C_1, C_2 > 0$ if for any $\delta > 0$ there exists an $n(\delta) \in \mathbb{N}$ and a sequence of $(n, M_{1,n}, M_{2,n}, C_1, C_2, L)$-list codes $\mathcal{C}_{\text{list}}$ such that for all $n \geq n(\delta)$ we have

$$\frac{1}{n}\log\Big(\frac{M_{i,n}}{L}\Big) \geq R_i - \delta, \qquad i = 1, 2,$$

while

$$\max_{s^n\in\mathcal{S}^n} \bar{e}_{L,n}(s^n|\mathcal{C}_{\text{list}}) \leq \lambda_n$$

with $\lambda_n \to 0$ as $n \to \infty$. The *list capacity region* $\mathcal{R}_{\text{list}}(C_1, C_2, L)$ of the AVMAC $\mathfrak{W}$ with conferencing encoders is the set of all achievable rate pairs.

B. Symmetrizability

For the analysis of the list capacity region, we need a corresponding extension of the concept of symmetrizability given in Definition 4. We follow [5, 6] and introduce a refinement which distinguishes different degrees of symmetry.

We say a channel $\widetilde{W}(z|(x_1, y_1), (x_2, y_2), ..., (x_t, y_t))$ with input alphabet $(\mathcal{X} \times \mathcal{Y})^t$ and output alphabet $\mathcal{Z}$ is symmetric if for every permutation π on $\{1, 2, ..., n\}$ we have $\widetilde{W}(z|(x_1, y_1), (x_2, y_2), ..., (x_t, y_t)) = \widetilde{W}(z|(x_{\pi(1)}, y_{\pi(1)}), ..., (x_{\pi(t)}, y_{\pi(t)}))$ for all $(x_1, y_1), ..., (x_t, y_t) \in (\mathcal{X} \times \mathcal{Y})$ and $z \in \mathcal{Z}$. This leads to the following definition.

Definition 8: For any $t \geq 1$, an AVMAC $\mathfrak{W}$ is t-$(\mathcal{X}, \mathcal{Y})$-*symmetrizable* if there exists a channel $\sigma : (\mathcal{X} \times \mathcal{Y})^t \to \mathcal{P}(\mathcal{S})$ such that

$$\begin{aligned}&\widetilde{W}\big(z|(x_0, y_0), (x_1, y_1), ..., (x_t, y_t)\big) \\ &\quad := \sum_{s\in\mathcal{S}} W(z|x_0, y_0, s)\sigma(s|(x_1, y_1), ..., (x_t, y_t))\end{aligned} \tag{8}$$

is symmetric in $(x_0, y_0), (x_1, y_1), ..., (x_t, y_t)$ for all $(x_0, y_0), (x_1, y_1), ..., (x_t, y_t) \in \mathcal{X} \times \mathcal{Y}$ and $z \in \mathcal{Z}$. For convenience, we take all AVMACs to be 0-$(\mathcal{X}, \mathcal{Y})$-symmetrizable.

Intuitively, a t-$(\mathcal{X}, \mathcal{Y})$-symmetrizable channel can be interpreted as a channel where the state sequence can emulate t replicas of the channel input. In addition, from the definition it follows that if an AVMAC is t-$(\mathcal{X}, \mathcal{Y})$-symmetrizable, then it is also t'-$(\mathcal{X}, \mathcal{Y})$-symmetrizable for all $0 \leq t' \leq t$.

Similarly as for the deterministic approach, the $\mathcal{X}$-symmetrizability and $\mathcal{Y}$-symmetrizability conditions can also be extended to the list case as in (8). But again, the "joint" t-$(\mathcal{X}, \mathcal{Y})$-symmetrizability condition suffices to completely characterize the capacity region, cf. also Remark 1.

C. List Capacity Region

Now we are in the position to prove the list capacity region of the AVMAC with conferencing encoders.

Theorem 3: For the list capacity region $\mathcal{R}_{\text{list}}(C_1, C_2, L)$ for the AVMAC $\mathfrak{W}$ with conferencing capacities $C_1, C_2 > 0$ and list size L we have

$$\mathcal{R}_{\text{list}}(C_1, C_2, L) = \mathcal{R}_{\text{CR}}(C_1, C_2, \Gamma) \tag{9}$$

if and only if the AVMAC $\mathfrak{W}$ is non-L-$(\mathcal{X}, \mathcal{Y})$-symmetrizable. If the AVMAC $\mathfrak{W}$ is L-$(\mathcal{X}, \mathcal{Y})$-symmetrizable, then

$$\mathcal{R}_{\text{list}}(C_1, C_2, L) = \{(0, 0)\}. \tag{10}$$

Proof: We first prove the second part (10) by contradiction. Therefore, we assume that $\mathcal{R}_{\text{list}}(C_1, C_2, L) \neq \{(0, 0)\}$ so that there must be a rate pair $(R, 0)$ or $(0, R)$ that is achievable. If the rate is small in the sense it satisfies $R < \min\{C_1, C_2\}$, then the rate pair $(\frac{R}{2}, \frac{R}{2})$ is achievable as well. Indeed, if the rate is small enough (i.e. smaller than the conferencing capacities), each transmitter can completely inform the other transmitter about the own message making both messages at both transmitters available.

This allows interpreting both inputs x and y of the AVMAC as a joint input (x, y) so that we obtain a corresponding single-user AVC. Then, the L-$(\mathcal{X}, \mathcal{Y})$-symmetrizability becomes the classical L-symmetrizability of the single-user AVC [5, 6]. Moreover, as a result, the communication problem becomes an equivalent single-user AVC list coding problem whose single-user capacity is greater than zero. But we know that if a single-user AVC is L-symmetrizable, then the corresponding single-user list capacity is zero which contradicts the assumption of non-zero single-user capacity proving (10).

Next we prove the remaining first part (9). We assume $\mathcal{R}_{\text{CR}}(C_1, C_2, \Gamma) \neq \{(0, 0)\}$, since otherwise there is nothing to prove. We have to show that if the AVMAC $\mathfrak{W}$ is non-L-$(\mathcal{X}, \mathcal{Y})$-symmetrizable, then using list codes $\mathcal{C}_{\text{list}}$ we achieve the same rates as if we use CR-assisted codes $\mathcal{C}_{\text{CR}}$, cf. (9). This observation already suggests itself to incorporate CR-assisted codes within the list coding as done by the following protocol.

To make use of CR-assisted codes, there is the need of CR at transmitters and receiver, cf. Section III-B. As this coordination resource is not available a priori, we have to create it prior to the transmission of the messages. This has to be done carefully in such a way that we do not "waste" too much communication resources which would result in a loss of rates for the subsequent transmission of the actual messages.

Fortunately, from [12, Lemma 15], cf. also Remark 3, we know that the amount of CR needed for a capacity-achieving CR-assisted code is quadratic in block length. Thus, first transmitter 1 or 2 creates a $\gamma \in \mathcal{G}_n := \{1, 2, ..., n^2\}$ uniformly and informs the other transmitter about the particular realization $\gamma \in \mathcal{G}_n$ during the Willems-conference. As for transmission of block length n we need $|\mathcal{G}_n| = n^2$ common randomness, the conference resources spent for informing the other transmitter is of order $\mathcal{O}(\log n)$ so that $n/\mathcal{O}(\log n) \to 0$ as $n \to \infty$. Thus, the resources spent for informing are negligible and do not reduce the available conferencing resources.

After the conference, $\gamma \in \mathcal{G}_n$ is available at both transmitters and it remains to inform the receiver in a similar fashion by spending a negligible amount of resources. Therefore, prior to the transmission of the actual messages, the transmitters uses a list code $\mathcal{C}_{\text{list}}$ with list size L Again, as $|\mathcal{G}_n| = n^2$, the channel uses needed for transmission of $\gamma \in \mathcal{G}_n$ is of order $l_n = \mathcal{O}(\log n)$ and therewith negligible. That such a single-user code exists achieving positive rate is guaranteed by the fact that the AVMAC $\mathfrak{W}$ is non-L-$(\mathcal{X}, \mathcal{Y})$-symmetrizable. Concatenated on that list code, we use a CR-assisted code of block length n which is possible as CR, i.e., the particular realization $\gamma \in \mathcal{G}_n$, is now available at all users.

In the following we present the decoding error analysis for the protocol described above. To transmit the message pair $(j, k) \in \mathcal{M}_1 \times \mathcal{M}_2$ having $\gamma \in \mathcal{G}_n$ available, transmitters 1 and 2 transmit the concatenated codewords

$$x_{jk}^{l_n+n}(\gamma) = \big(x_\gamma^{l_n}, x_{jk}^n(\gamma)\big) \text{ and } y_{jk}^{l_n+n}(\gamma) = \big(y_\gamma^{l_n}, y_{jk}^n(\gamma)\big) \quad (11)$$

where $x_\gamma^{l_n}$ and $y_\gamma^{l_n}$ are codewords of a single-user list code to inform the receiver about $\gamma \in \mathcal{G}_n$. As $|\mathcal{G}_n| = n^2$, we have $l_n = \mathcal{O}(\log n)$ so that $l_n/n \to 0$ as $n \to \infty$ which means that there will be no loss in overall rate for transmission of messages $(j, k) \in \mathcal{M}_1 \times \mathcal{M}_2$.

The signal $z^{l_n+n} = (z^{l_n}, z^n) \in \mathcal{Z}^{l_n+n}$ is received and the receiver uses a list decoder with list size L to obtain a list of possible realizations $\gamma \in \mathcal{G}_n$ from the first part $z^{l_n} \in \mathcal{Z}^{l_n}$, i.e.,

$$\phi_1(z^{l_n}) = \big\{\gamma : z^{l_n} \in \mathcal{D}_\gamma\big\}.$$

Based on this list, the receiver creates a list decoder for the messages $(j, k) \in \mathcal{M}_1 \times \mathcal{M}_2$ transmitted in the second part $z^n \in \mathcal{Z}^n$, i.e.,

$$\phi_2(z^n) = \big\{(j, k) : \exists\, \gamma \in \phi_1(z^{l_n}) \text{ and } z^n \in \mathcal{D}_{jk}(\gamma)\big\}.$$

Thus, for any $z^{l_n+n} \in \mathcal{Z}^{l_n+n}$ let $\gamma_1, ..., \gamma_{L'} \in \mathcal{G}_n$ with $L' \leq L$ the list $\phi_1(z^{l_n}) = \{\gamma_1, ..., \gamma_{L'}\}$, the constructed list decoder

$$\begin{aligned}\phi(z^{l_n+n}) &= \big(\phi_1(z^{l_n}), \phi_2(z^n)\big)\\ &= \big\{(\gamma, j, k) : \gamma \in \{\gamma_1, ..., \gamma_{L'}\} \text{ and } z^n \in \mathcal{D}_{jk}(\gamma)\big\}\end{aligned}$$

defines a valid list decoder with list size not greater than L as required in Definition 6.

The channel $W(z|x, y, s)$ is non-L-$(\mathcal{X}, \mathcal{Y})$-symmetrizable by assumption. Then there exists a (small) rate $R < \min\{C_1, C_2\}$ so that this rate is achievable for the single-user interpretation of this channel with joint input $(x, y) \in \mathcal{X} \times \mathcal{Y}$, cf. also corresponding discussion of the proof of (10) in the beginning. In more detail, let $\lambda_1 \in (0, 1)$ arbitrary and set

$$l_n := \frac{2}{R} \log\Big(\frac{n}{L}\Big)$$

(since $R = \frac{1}{l_n} \log(\frac{n^2}{L})$). Then we know from the single-user AVC under list decoding [5, 6] that there exists a $n_1 = n_1(\lambda_1)$ such that for all $n \geq n_1$ there exists a list code

$$\big\{\big((x_\gamma^{l_n}, y_\gamma^{l_n}), \mathcal{D}_\gamma\big) : \gamma \in \{1, ..., n^2\}\big\}$$

of length l_n such that the average probability of error satisfies

$$\frac{1}{n^2} \sum_{\gamma \in \{1,...,n^2\}} W^{l_n}\big(\mathcal{D}_\gamma^c | (x_\gamma^{l_n}, y_\gamma^{l_n}), s^{l_n}\big) \leq \lambda_1 \quad (12)$$

for all $s^{l_n} \in \mathcal{S}^{l_n}$.

Next, we want to show that the error probability

$$\begin{aligned}\bar{e}_{L,n}(s^{l_n+n}|\mathcal{C}_{\text{list}}) &= \frac{1}{n^2} \frac{1}{|\mathcal{M}|} \sum_{\gamma \in \{1,...,n^2\}} \sum_{(j,k) \in \mathcal{M}}\\ &\times \sum_{\substack{z^{l_n+n}:\\ (\gamma,j,k) \notin \phi(z^{l_n+n})}} W^{l_n+n}\big(z^{l_n+n} | x_{jk}^{l_n+n}(\gamma), y_{jk}^{l_n+n}(\gamma), s^{l_n+n}\big)\end{aligned} \quad (13)$$

of the final concatenated list code is small as well for all $s^{l_n+n} \in \mathcal{S}^{l_n+n}$, i.e.,

$$\max_{s^{l_n+n} \in \mathcal{S}^{l_n+n}} \bar{e}_{L,n}(s^{l_n+n}|\mathcal{C}_{\text{list}}) \leq \lambda. \quad (14)$$

Now, if $(\gamma, j, k) \notin \phi(z^{l_n+n})$, then either a) $\gamma \notin \phi_1(z^{l_n})$ or b) $\gamma \in \phi_1(z^{l_n})$ and we have $z^n \notin \mathcal{D}_{jk}(\gamma_i)$ for all $\gamma_i \in \phi_1(z^{l_n})$. Accordingly, we define the error events

$$\begin{aligned}\mathcal{E}_1(\gamma) &:= \big\{z^{l_n+n} : \gamma \notin \phi_1(z^{l_n})\big\}\\ \mathcal{E}_2(\gamma, j, k) &:= \big\{z^{l_n+n} : \gamma \in \phi_1(z^{l_n}) \text{ and } \forall\, \gamma_i \in \phi_1(z^{l_n})\\ &\qquad\qquad \text{we have } z^n \notin \mathcal{D}_{jk}(\gamma_i)\big\}\end{aligned}$$

so that

$$\left\{z^{l_n+n} : (\gamma, j, k) \notin \phi(z^{l_n+n})\right\} \subset \mathcal{E}_1(\gamma) \cup \mathcal{E}_2(\gamma, j, k).$$

With this, the average probability of error in (13) can be bounded from above by

$$\bar{e}_{L,n}(s^{l_n+n}|\mathcal{C}_{\text{list}}) \leq \frac{1}{n^2}\frac{1}{|\mathcal{M}|}\sum_{\gamma\in\{1,\dots,n^2\}}\sum_{(j,k)\in\mathcal{M}} \times \Bigg(\sum_{z^{l_n+n}\in\mathcal{E}_1(\gamma)} W^{l_n+n}\big(z^{l_n+n}|x_{jk}^{l_n+n}(\gamma), y_{jk}^{l_n+n}(\gamma), s^{l_n+n}\big) + \sum_{z^{l_n+n}\in\mathcal{E}_2(\gamma,j,k)} W^{l_n+n}\big(z^{l_n+n}|x_{jk}^{l_n+n}(\gamma), y_{jk}^{l_n+n}(\gamma), s^{l_n+n}\big)\Bigg)$$

where we bound both terms individually. For the first term we observe that

$$\sum_{z^{l_n+n}\in\mathcal{E}_1(\gamma)} W^{l_n+n}\big(z^{l_n+n}|x_{jk}^{l_n+n}(\gamma), y_{jk}^{l_n+n}(\gamma), s^{l_n+n}\big) = \sum_{z^{l_n}:\gamma\notin\phi_1(z^{l_n})} W^{l_n}\big(z^{l_n}|x_\gamma^{l_n}, y_\gamma^{l_n}, s^{l_n}\big)$$

where the equality follows from the concatenated structure of the codewords (11) and the fact that the error event $\mathcal{E}_1(\gamma)$ only depends on the first part. Thus, by (12) we end up with

$$\frac{1}{n^2}\frac{1}{|\mathcal{M}|}\sum_{\gamma\in\{1,\dots,n^2\}}\sum_{(j,k)\in\mathcal{M}}\sum_{z^{l_n}:\gamma\notin\phi_1(z^{l_n})} W^{l_n}\big(z^{l_n}|x_\gamma^{l_n}, y_\gamma^{l_n}, s^{l_n}\big) = \frac{1}{n^2}\sum_{\gamma\in\{1,\dots,n^2\}} W^{l_n}\big(\mathcal{D}_\gamma^c|x_\gamma^{l_n}, y_\gamma^{l_n}, s^{l_n}\big) \leq \lambda_1.$$

For the second event we observe that if $z^{l_n+n} \in \mathcal{E}_2(\gamma, j, k)$, then we have $\gamma \in \phi_1(z^{l_n})$ and $z^n \notin \mathcal{D}_{jk}(\gamma)$ so that

$$\mathcal{E}_2(\gamma, j, k) \subset \left\{z^{l_n+n} : \gamma \in \phi_1(z^{l_n}) \text{ and } z^n \notin \mathcal{D}_{jk}(\gamma)\right\}.$$

With this we obtain for the second term

$$\begin{aligned}
&\sum_{z^{l_n+n}\in\mathcal{E}_2(\gamma,j,k)} W^{l_n+n}\big(z^{l_n+n}|x_{jk}^{l_n+n}(\gamma), y_{jk}^{l_n+n}(\gamma), s^{l_n+n}\big)\\
&\leq \sum_{\substack{z^{l_n+n}:\gamma\in\phi_1(z^{l_n})\\ \text{and } z^n\notin\mathcal{D}_{jk}(\gamma)}} W^{l_n+n}\big(z^{l_n+n}|x_{jk}^{l_n+n}(\gamma), y_{jk}^{l_n+n}(\gamma), s^{l_n+n}\big)\\
&\leq \sum_{z^n:z^n\notin\mathcal{D}_{jk}(\gamma)} W^n\big(z^n|x_{jk}^n(\gamma), y_{jk}^n(\gamma), s^n\big)\\
&= W^n\big(\mathcal{D}_{jk}^c(\gamma)|x_{jk}^n(\gamma), y_{jk}^n(\gamma), s^n\big)
\end{aligned}$$

where the last inequality follows from the concatenated structure of the codewords (11) and the last equality from the definition of the decoding sets of a CR-assisted code. Thus,

$$\frac{1}{n^2}\frac{1}{|\mathcal{M}|}\sum_{\gamma\in\{1,\dots,n^2\}}\sum_{(j,k)\in\mathcal{M}} W^n\big(\mathcal{D}_{jk}^c(\gamma)|x_{jk}^n(\gamma), y_{jk}^n(\gamma), s^n\big) \leq \lambda_1$$

since it is a "good" CR-assisted code according to Definition 5. Since $\lambda_1 \in (0,1)$ is arbitrary, we can choose λ_1 such that it satisfies $2\lambda_1 < \lambda$ which proves (14). Since the rate pair (R_1, R_2) in the CR-assisted code was arbitrary, the achievability is shown so that $\mathcal{R}_{\text{list}}(C_1, C_2, L) \supseteq \mathcal{R}_{\text{CR}}(C_1, C_2, \Gamma)$.

The converse, i.e., $\mathcal{R}_{\text{list}}(C_1, C_2, L) \subseteq \mathcal{R}_{\text{CR}}(C_1, C_2, \Gamma)$, follows immediately by observing that $\mathcal{R}_{\text{CR}}(C_1, C_2, \Gamma)$ remains the same for list decoding similarly as for the single-user AVC [5, 6] or the AVMAC (without conferencing encoders) [7]. ■

V. Finite Resources

The previous discussion showed that for any rate pair $(R_1, R_2) \in \mathcal{R}_{\text{CR}}(C_1, C_2, \Gamma)$ there exists a CR-assisted $(n, M_{1,n}, M_{2,n}, C_1, C_2, \Gamma)$-code $\mathcal{C}_{\text{CR}}$ whose average probability goes exponentially fast to zero [12], i.e.,

$$\max_{s^n\in\mathcal{S}^n}\frac{1}{|\mathcal{G}_n|}\sum_{\gamma\in\mathcal{G}_n}\bar{e}_n(s^n|\mathcal{C}(\gamma)) \leq e^{-n\epsilon}. \tag{15}$$

Unfortunately, we know that for this we need CR whose amount tends to infinity for increasing block length n, cf. Theorem 2, Remark 3, and [12]. Therefore we ask if it is possible to control the amount of needed resources and to achieve the same rates with a fixed amount of CR (i.e. independent of the block length n) when we allow for a fixed but non-vanishing probability of error.

Theorem 4: Let $\lambda \in (0,1)$ be arbitrary. Then for every $(R_1, R_2) \in \mathcal{R}_{\text{CR}}(C_1, C_2, \Gamma)$, there exists a fixed L such that

$$(R_1, R_2) \in \mathcal{R}_{\text{CR}}(\lambda, C_1, C_2, \Gamma)$$

with Γ is defined on $\mathcal{G}_n$ with $|\mathcal{G}_n| = L$. $\mathcal{R}_{\text{CR}}(\lambda, C_1, C_2, \Gamma)$ denotes the CR-assisted region $\mathcal{R}_{\text{CR}}(C_1, C_2, \Gamma)$ where we additionally allow for a non-vanishing probability of error λ.

Proof: Let $\lambda \in (0,1)$ and $\alpha > 0$ be arbitrary but fixed. Then for any $(R_1, R_2) \in \mathcal{R}_{\text{CR}}(C_1, C_2, \Gamma)$ we know from Theorem 2 that there is a CR-assisted code $\mathcal{C}_{\text{CR}}$ such that the error probability satisfies (15). Thus, the probability that for finite $|\mathcal{G}_n| = L$ and fixed $s^n \in \mathcal{S}^n$ this is greater than λ is

$$\begin{aligned}
&\mathbb{P}\Big\{\frac{1}{L}\sum_{i=1}^{L}\bar{e}_n(s^n|\mathcal{C}(i)) \geq \lambda\Big\}\\
&\leq \mathbb{P}\Big\{\exp\big(\alpha\sum_{i=1}^{L}\bar{e}_n(s^n|\mathcal{C}(i))\big) \geq \exp(\alpha\lambda L)\Big\}\\
&\leq \exp(-\alpha\lambda L)\prod_{i=1}^{L}\mathbb{E}\big[\exp\big(\alpha\bar{e}_n(s^n|\mathcal{C}(i))\big)\big].
\end{aligned}$$

By the fact that $\bar{e}_(s^n|\mathcal{C}(i)) \leq 1$ always holds and by standard arguments, cf. also [13], we obtain for the expectation $\mathbb{E}[\exp(\alpha\bar{e}_n(s^n|\mathcal{C}(i)))] < 1 + \exp(-n\epsilon + \alpha)$ so that

$$\mathbb{P}\Big\{\exp\big(\alpha\sum_{i=1}^{L}\bar{e}_n(s^n|\mathcal{C}(i))\big) \geq \exp(\alpha\lambda L)\Big\} \leq \exp(-\alpha\lambda L)\big(1 + \exp(-n\epsilon + \alpha)\big)^L.$$

Now, taking all state sequences $s^n \in \mathcal{S}^n$ into account yields

$$\mathbb{P}\Big\{\exp\big(\alpha\sum_{i=1}^{L}\bar{e}_n(s^n|\mathcal{C}(i))\big) \geq \exp(\alpha\lambda L) \text{ for some } s^n \in \mathcal{S}^n\Big\} \leq \exp\big(-n\epsilon\lambda\big(L - \big(\tfrac{\ln 2}{n\epsilon\lambda} + \tfrac{\ln|\mathcal{S}|}{\epsilon\lambda}\big)\big)\big) \tag{16}$$

where the second step follows with the choice $\alpha = n\epsilon$.

Now, if we choose $L > \underline{L} := \frac{1}{\epsilon\lambda} \ln|\mathcal{S}|$, then the probability that the average probability of error of the constructed code is smaller than the required λ is

$$\mathbb{P}\Big\{ \frac{1}{L} \sum_{i=1}^{L} \bar{e}_n(s^n|\mathcal{C}(i)) < \lambda \text{ for all } s^n \in \mathcal{S}^n \Big\} \underset{n\to\infty}{\longrightarrow} 1$$

exponentially fast as given by (16). ■

This establishes a *sufficient condition* on how large the finite amount of resources must be to achieve the whole region $\mathcal{R}_{\text{CR}}(\lambda, C_1, C_2, \varGamma)$ (with non-vanishing probability of error λ).

Corollary 1: For any $\lambda \in (0,1)$, there exists a CR-assisted $(n, M_{1,n}, M_{2,n}, C_1, C_2, \varGamma)$-code $\mathcal{C}_{\text{CR}}$ with $|\mathcal{G}_n| = L$ that achieves all rate pairs $(R_1, R_2) \in \mathcal{R}_{\text{CR}}(\lambda, C_1, C_2, \varGamma)$ if

$$L > \underline{L} = \frac{1}{\epsilon\lambda} \ln|\mathcal{S}|.$$

Next we want to establish also a *necessary* condition on the minimal amount of common randomness.

Theorem 5: Let $\lambda \in (0,1)$ arbitrary but fixed. Then for every rate pair $(R_1, R_2) \in \mathcal{R}_{\text{CR}}(\lambda, C_1, C_2, \varGamma)$, the amount of resources L has to satisfy

$$L > \frac{1}{2\lambda}.$$

Sketch of Proof: Let $\lambda \in (0,1)$ and $\tau > 0$ be arbitrary but fixed. Then for any $(R_1, R_2) \in \mathcal{R}_{\text{CR}}(\lambda, C_1, C_2, \varGamma)$ and sufficiently large $n \geq n(\tau)$, it holds for all deterministic codes $\mathcal{C}$ of rates (R_1, R_2) that $\max_{s^n \in \mathcal{S}^n} \bar{e}_n(s^n|\mathcal{C}) \geq \frac{1}{2} - \tau$, if the AVMAC $\mathfrak{W}$ is $(\mathcal{X}, \mathcal{Y})$-symmetrizable. This can be easily shown by extending the results from [4] and [12]. Then

$$\lambda \geq \max_{s^n \in \mathcal{S}^n} \frac{1}{L} \sum_{i=1}^{L} \bar{e}_n(s^n|\mathcal{C}(i)) \geq \max_{s^n \in \mathcal{S}^n} \frac{1}{L} \bar{e}_n(s^n \mathcal{C}(1)) \geq \frac{1}{L}\Big(\frac{1}{2} - \tau\Big).$$

Thus, $L \geq (\frac{1}{2} - \tau)\frac{1}{\lambda}$ for all $\tau > 0$ which finally implies $L > \frac{1}{2\lambda}$ proving the theorem since $\tau > 0$ arbitrary. ■

From Theorem 4 we know that if $L > \frac{1}{\epsilon\lambda} \ln|\mathcal{S}|$ then there exist many CR-assisted codes $\mathcal{C}_{\text{CR}}$ with finite L that achieve the desired performance for symmetrizable channels. From Theorem 5 we further know that $L > \frac{1}{2\lambda}$ is necessary meaning that for $L \leq \frac{1}{2\lambda}$ no $\mathcal{C}_{\text{CR}}$ with finite L is possible. Unfortunately, there is a gap between these two bounds so that it is open to characterize the minimal CR in general.

Remark 4: We want to note that such a CR-assisted code $\{(x_{jk}^n(i), y_{jk}^n(i), \mathcal{D}_{jk}(i)) : i \in \{1, ..., L\}, (j,k) \in \mathcal{M}\}$, cf. (7) and Definition 5, can be converted into a deterministic list code $\{(x_{jk}^n(i), y_{jk}^n(i), \mathcal{D}_{jk}(i)) : (i,j,k) \in \{1, ..., L\} \times \mathcal{M}\}$ with list size L, cf. [13] for further details for the single-user case.

This further reveals the following interesting observation. The list sizes determined by Theorems 4 and 5 might be greater than the actually symmetrizability of the channel and the resulting list size due to the strategy of Theorem 3. Thus, a list code solely created by a CR-assisted code as in Remark 4 might not suffice to achieve the optimal performance. Interestingly, on the other hand, the strategy of Theorem 3 given by a "bad" list code with arbitrary small rate in combination with a "good" CR-assisted code is optimal in the sense that it achieves the minimal needed list size.

VI. Conclusion

We studied the AVMAC with conferencing encoders for which we derived the list capacity region. It can be completely characterized using the L-$(\mathcal{X}, \mathcal{Y})$-symmetrizability condition and it is shown that for large enough list size, i.e., larger than the symmetrizability of the channel, the capacity region under list decoding equals the one for highly involved CR-assisted strategies based on coordination resources which have to be available at all users. Thus, a large enough list size allows to overcome the need of coordination resources. Allowing for a small but non-vanishing probability of error, the amount of resources (i.e. common randomness or list size) can be shown to be finite and independent of the block length. This is in contrast to the approach with vanishing error requiring an increasing and unbounded amount of such resources.

The fact that both transmitters are able to cooperate using their conferencing links further reveals the following noteworthing observations. Investments in infrastructure immediately yield gains in spectral efficiency as an increase in conferencing capacities results in higher transmission rates. Moreover, the ability of conferencing makes the communication more robust as the capacity region is solely characterized by the L-$(\mathcal{X}, \mathcal{Y})$-symmetrizability condition while the capacity of the classical AVMAC (without conferencing encoders) depends on its marginal symmetrizability conditions as well.

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