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Erratum

Erratum to: Entanglement Transmission and Generation under Channel Uncertainty: Universal Quantum Channel Coding

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There is a missing square root in Lemma 3 in the original paper. We state the corrected version of Lemma 3 and indicate the changes that have to be made in the proof of Lemma 3 as well as in the rest of the paper due to this correction.

The changes are stated in separate sections, with section titles given by the numbers of the pages where the changes have to be made.

Also, thanks to the comments of an unknown referee, Lemma 14 is replaced by a variant that is independent of the output dimension $\mathcal{F}_l'$.

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Lemma 3. *Let $\rho \in \mathcal{S}(\mathcal{H})$ for some Hilbert space $\mathcal{H}$. Let, for some other Hilbert space $\mathcal{K}$, $\mathcal{A} \in \mathcal{C}(\mathcal{H}, \mathcal{K})$, $\mathcal{D} \in \mathcal{C}(\mathcal{K}, \mathcal{H})$, $q \in \mathcal{B}(\mathcal{K})$ be an orthogonal projection.*

1. *Denoting by $\mathcal{Q}^\perp$ the completely positive map induced by $q^\perp := \mathbf{1}_\mathcal{K} - q$ we have*

$$F_e(\rho, \mathcal{D} \circ \mathcal{A}) \geq F_e(\rho, \mathcal{D} \circ \mathcal{Q} \circ \mathcal{A}) - 2\sqrt{F_e(\rho, \mathcal{D} \circ \mathcal{Q} \circ \mathcal{A}) F_e(\rho, \mathcal{D} \circ \mathcal{Q}^\perp \circ \mathcal{A})}. \tag{18}$$

2. *If for some $\epsilon > 0$ the relation $F_e(\rho, \mathcal{D} \circ \mathcal{Q} \circ \mathcal{A}) \geq 1 - \epsilon$ holds, then*

$$F_e(\rho, \mathcal{D} \circ \mathcal{Q}^\perp \circ \mathcal{A}) \leq \epsilon,$$

and (18) implies

$$F_e(\rho, \mathcal{D} \circ \mathcal{A}) \geq 1 - 3\sqrt{\epsilon}. \tag{19}$$

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3. *If for some $\epsilon > 0$ merely the relation $tr\{q\mathcal{A}(\rho)\} \geq 1 - \epsilon$ holds then we can conclude that*

$$F_e(\rho, \mathcal{D} \circ \mathcal{A}) \geq F_e(\rho, \mathcal{D} \circ \mathcal{Q} \circ \mathcal{A}) - 2\sqrt{\epsilon}. \tag{20}$$

Inequality (25) on page 72 has to be replaced by

$$X \overset{\mathbf{c}}{\leq} \sqrt{\sum_{i=1}^{\kappa \cdot h} \lambda_i |\alpha_i|^2 \langle \psi, \tilde{\mathcal{D}}(|x_i\rangle\langle x_i|)\psi\rangle \sum_{j=1}^{\kappa \cdot h} \lambda_j |\beta_j|^2 \langle \psi, \tilde{\mathcal{D}}(|y_j\rangle\langle y_j|)\psi\rangle}. \tag{25}$$

Accordingly, inequality (28) changes to

$$F_e(\rho, \mathcal{D} \circ \mathcal{A}) \geq F_e(\rho, \mathcal{D} \circ \mathcal{Q} \circ \mathcal{A}) - 2\sqrt{F_e(\rho, \mathcal{D} \circ \mathcal{Q} \circ \mathcal{A}) \cdot F_e(\rho, \mathcal{D} \circ \mathcal{Q}^{\perp} \circ \mathcal{A})}. \tag{28}$$

Please, replace equation (27) on page 73 by

$$F_e(\rho, \mathcal{D} \circ \mathcal{Q}^{\perp} \circ \mathcal{A}) = \sum_{j=1}^{\kappa \cdot h} \lambda_j |\beta_j|^2 \langle \psi, \tilde{\mathcal{D}}(|y_j\rangle\langle y_j|)\psi\rangle. \tag{1}$$

Lines 20 to 23 on page 73 have to be replaced by

$$\begin{aligned} X &\leq \sqrt{\sum_{i=1}^{\kappa \cdot h} \lambda_i |\alpha_i|^2 \sum_{j=1}^{\kappa \cdot h} \lambda_j |\beta_j|^2} \\ &\leq \sqrt{\epsilon}, \end{aligned}$$

thus by Eq. (23) we have

$$F_e(\rho, \mathcal{D} \circ \mathcal{A}) \geq F_e(\rho, \mathcal{D} \circ \mathcal{Q} \circ \mathcal{A}) - 2\sqrt{\epsilon}.$$

Lines 3 to 6 on page 74 should be replaced by:

From our assumption $F_e(\rho, \mathcal{D} \circ \mathcal{Q} \circ \mathcal{A}) \geq 1 - \epsilon$, it follows $F_e(\rho, \mathcal{D} \circ \mathcal{Q} \circ \mathcal{A}) \geq 1 - \sqrt{\varepsilon}$ and together with (28) we obtain that

$$\begin{aligned} F_e(\rho, \mathcal{D} \circ \mathcal{A}) &\geq F_e(\rho, \mathcal{D} \circ \mathcal{Q} \circ \mathcal{A}) - 2\sqrt{F_e(\rho, \mathcal{D} \circ \mathcal{Q}^{\perp} \circ \mathcal{A})}) \\ &\geq (1 - \sqrt{\epsilon} - 2\sqrt{\epsilon}) \\ &\geq 1 - 3\sqrt{\epsilon}, \end{aligned}$$

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In accordance with the changes in Lemma 3 Equation (35) has to be changed to

$$\min_{\mathcal{N}_j \in \mathfrak{I}} F_e(\pi_{\mathcal{F}_l}, \mathcal{R}^l \circ \mathcal{N}_j^{\otimes l} \circ \mathcal{W}^l) \geq 1 - \sqrt{3 \cdot N \cdot \epsilon_l} \quad \forall l \in \mathbb{N}. \tag{35}$$

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Line 27 has to be changed to the following:

$$F_e(\pi_{\mathcal{E}_l}, \mathcal{R}^l \circ \mathcal{N}_j^{\otimes l} \circ \mathcal{W}_j^l) \geq 1 - 3\sqrt{N\epsilon_l} \quad \forall j \in \{1, \ldots, N\}.$$

Line 32 has to be changed to the following:

2. $\min_{\mathcal{N}_j \in \mathfrak{I}} F_e(\pi_{\mathcal{E}_l}, \mathcal{R}^l \circ \mathcal{N}_j^{\otimes l} \circ \mathcal{W}_j^l) \geq 1 - 3\sqrt{N\epsilon_l}$.

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Inequality (43) has to be replaced by

$$\min_{i \in \{1,\ldots,N_{\tau_l}\}} F_e(\pi_{\mathcal{F}_l}, \mathcal{R} \circ \mathcal{N}_i^{\otimes l} \circ \mathcal{W}^l) \geq 1 - \sqrt{3 \cdot N_{\tau_l} \cdot \epsilon_l}, \tag{43}$$

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The forelast line of the proof of Lemma 9 has to be replaced by:

$$\inf_{\mathcal{N} \in \mathfrak{I}} F_e(\pi_{\mathcal{F}_l}, \mathcal{R} \circ \mathcal{N}^{\otimes l} \circ \mathcal{W}^l) \geq 1 - \sqrt{3N_{\tau_l}\epsilon_l} - l\tau_l, \tag{2}$$

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Line 11 has to be replaced by the following line:

$$F_e(\pi_{\mathcal{E}_l}, \mathcal{R}^l \circ \mathcal{N}'^{\otimes l} \circ \mathcal{W}_{\mathcal{N}'}^l) \geq 1 - 3\sqrt{N_{\tau_l} \cdot \varepsilon_l} \qquad \forall\, \mathcal{N}' \in \mathfrak{I}_{\tau_l}^{\circ},$$

Line 15 has to be replaced by the following line:

$$F_e(\pi_{\mathcal{E}_l}, \mathcal{R}^l \circ \mathcal{N}^{\otimes l} \circ \mathcal{W}_{\mathcal{N}}^l) \geq 1 - 3\sqrt{N_{\tau_l} \cdot \varepsilon_l} - \frac{2}{l} \quad \forall l \in \mathbb{N},\ \mathcal{N} \in \mathfrak{I}.$$

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For the arguments to follow, the additional reference [34] is needed. It can be found at the end of this erratum.

The proofs of Theorem 9 and equation (85) can then be carried out with the help of the version of Lemma 14 that is stated in this erratum and which is due to [34]. Please, replace the last eight lines of page 87 by the follwowing:

For the converse part in the case of a finite compound channel we need the following lemma that is due to Alicki and Fannes [34]:

Lemma 14 (cf. [34]). *For two states $\sigma, \rho \in \mathcal{S}(\mathcal{H}_1 \otimes \mathcal{H}_2)$ with trace distance $f = \|\sigma - \rho\|_1$,*

$$|\Delta S(\rho) - \Delta S(\sigma)| \leq 4f \log \dim \mathcal{H}_1 + 2\eta(f) + 2\eta(1-f),$$

where

$$\Delta S(\,\cdot\,) := S(\mathrm{tr}_{\mathcal{H}_1}[\,\cdot\,]) - S(\,\cdot\,)$$

and $\eta : [0,1] \to \mathbb{R}$ is given by the formula $\eta(x) := -x \log x$.

PAGE 88 and 89

Line 23 on page 88 should, by application of the well-known estimate $\|a-b\|_1 \leq 2\sqrt{1-F(a,b)}$ for $a, b \in \mathcal{S}(\mathcal{E}_l \otimes \mathcal{F}_l')$ be rewritten as

$$\|\psi^l - \sigma_k^l\|_1 \leq 2\sqrt{N\varepsilon_l}.$$

In the following lines, including the first 9 lines on page 89, please replace the factor $\frac{2}{e}$ by $4\max_{x\in[0,1]}(-x\log(x))$ and $\sqrt{N\varepsilon_l}$ by $2\sqrt{N\varepsilon_l}$.

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Please, proceed as on pages 88 and 89 by turning the first equality in line 8 into the estimate

$$\|\psi^l - \sigma_k^l\|_1 \leq 2\sqrt{N\varepsilon_l},$$

then replace every term $\frac{2}{e}$ by $4\max_{x\in[0,1]}(-x\log(x))$ and the terms $\sqrt{N\varepsilon_l}$ by $2\sqrt{N\varepsilon_l}$.

Reference

[34] Alicki, R., Fannes, M.: Continuity of quantum conditional information. J. Phys. A **37**, L55 (2004)

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