Throughput Maximization for Energy Harvesting Nodes Transmitting over Time-Varying Channels

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***Abstract*—We investigate in this work the throughput maximizing transmission strategies of an energy harvesting transmitter which communicates over a time-varying channel. Two system models are considered to this end, the *generic model* for which the energy harvesting node is treated as a continuous-time control system, and the *MQAM model* for which practical modulation schemes are employed for transmission. A generalized circuit power function dependent on the transmit power is formulated for the generic model. We apply optimal control theory with this model and propose a modified water-filling algorithm to obtain the optimal transmission policy. The circuit power modelling with MQAM transmissions, on the other hand, is based on studies on the power dissipation of various functional components of the transmitter circuitry. Energy-efficient modulation orders can be found and the throughput maximization becomes a linear optimization problem in this case. A heuristic algorithm with near-optimal performance is also developed to conquer the difficulty of problems with large number of variables. Besides, the *dynamic programming* (DP) technique is applied with both system models which not only verifies the optimality of our obtained power allocations, but also provides an online solution to the situation that the transmitter does not have in advance the full channel state information (CSI) of the entire time slot of interest.**

I. Introduction

The development of energy harvesting technology has enabled the implementation of communication devices which are capable of converting different forms of ambient energy to electrical energy and using it for communications [1]. From an energy exploitation perspective, such devices should be designed to make efficient use of the harvested energy to optimize the performance metric specified by the communication applications. Due to the inconstancy and randomness of the energy that becomes available at the energy harvesting node, the optimal transmission strategy it should take differs from that for a regular device powered by fixed utilities. A number of recent publications have addressed this issue under various scenarios and with different optimization frameworks, *e.g.*, [2][3][4]. The uniqueness of our work [5] lies in the inclusion of power dissipation within the circuitry of the energy harvesting node to the system energy consumption model, and the investigation of the impact of such considerations. The motivation here is the fact that for short-range communications, which is the typical application scenario of energy harvesting nodes, the circuit power consumption of the transmitter is comparable to the radiated transmit power which allows for a sufficiently good receive signal-to-noise ratio (SNR) [6]. To this end, we study two system models in this work, a generic one with a generalized circuit model, and the MQAM model which takes a more practical perspective.

The paper focuses on the throughput maximization problem of an energy harvesting node transmitting over a time-varying channel. It is an extension and generalization of our previous work [5] where the communication channel is assumed constant. A similar problem without circuit power consideration is discussed in [7]. Here we highlight the difference in optimal transmit power functions with the generic model, and propose modulation adaptation algorithms for the MQAM model.

The rest of the paper is organized as follows: in Sec. II, we introduce first the generic system model and then give the mathematical formulation of the throughput maximization problem. The optimal transmission strategy with this model is derived using optimal control theory in Sec. III. Then we turn to the more practical scenario and describe the MQAM model in Sec. IV, followed by elaborations on the design of optimal as well as heuristic algorithms for the power allocation strategies in Sec. V. Simulation results are included in the respective sections. The paper is finally concluded in Sec. VI.

II. Generic System Model and Throughput Maximization

We consider an energy harvesting node which transmits data to a single receiver via a time-varying channel during the time slot $[0, T]$. With the generic model we assume that the node is able to adapt its transmit power continuously both in time and quantity, *i.e.*, denoting the transmit power at the time instance t with $p_{\mathrm{tx}}(t)$, we have $p_{\mathrm{tx}}(t) \in [0, +\infty)$, $t \in [0, T]$.

A. Data Transmission Model

The instantaneous data rate, denoted with $r(t)$, is modelled with the function $f\left(t, p_{\mathrm{tx}}(t)\right) = \log\left(1 + |h(t)|^2 p_{\mathrm{tx}}(t)\right)$, where $h(t)$ stands for the channel coefficient at time t and is piecewise continuous. The important property of f for the later derivations is its monotonicity and strict concavity in p_{tx}.

B. Circuit Power Model

We formulate the power consumption of the transmitter circuitry, denoted with p_{c}, as a function of the transmit power, *i.e.*, $p_{\mathrm{c}}(t) = g_{\mathrm{c}}(p_{\mathrm{tx}}(t))$, and obtain the total power dissipation formula as $p(t) = g(p_{\mathrm{tx}}(t)) = p_{\mathrm{tx}}(t) + p_{\mathrm{c}}(t)$. The function g_{c} is assumed nondecreasing on $[0, +\infty)$, continuous and continuously differentiable on $(0, +\infty)$. Note that it is necessary to allow for a discontinuous point of $g_c(p_{tx})$ at $p_{tx} = 0$ due to the different modes that the transmitter could be operating on. When the transmitter is not sending any signal, it can be turned into *sleep* mode for which the circuit power consumption is low enough to be negligible, *i.e.*, we assume $g_c(0) = 0$. Otherwise, the transmitter is considered as in *active* mode and its circuit incurs additional energy consumption, which means $g_c(p_{tx}) > 0$ for $p_{tx} > 0$. An example of such g_c functions is the linear circuit power model given by

$$g_c(p_{tx}) = \begin{cases} b \cdot p_{tx} + c, & p_{tx} > 0, \\ 0, & p_{tx} = 0, \end{cases} \tag{1}$$

where b and c are both nonnegative constants.

Furthermore, we have seen in [5] that the convexity of g_c plays an important role in preserving the convex structure of the throughput maximization problem. It is also assumed here that g_c is convex on $(0, +\infty)$, the condition of which, fortunately, is met by practical system models including the MQAM model we are going to introduce in Sec. IV.

C. Energy Harvesting and Expenditure

An energy harvesting node gathers energy from the environment and stores them in its storage medium. Although usually not fully predictable, the energy arrivals in the time slot $[0, T]$ are assumed completely known in advance at the transmitter, so that the performance limit of the system can be evaluated.

We utilize the *cumulative* model to describe the energy arrival as well as the energy expenditure of the transmitter. Let the nondecreasing functions $A(t)$ and $W(t)$ represent the total amount of energy that is available by time t and the total energy consumption of the node by time t, respectively. Due to causality, $W(t) \leq A(t)$ must be satisfied, $\forall t \in [0, T]$. Moreover, physical limitations on the energy storage that the node is equipped with give rise to a function $D(t)$ which represents the minimal amount of energy that has to be consumed by time t in order to avoid energy loss caused by storage overflow. Let E_{max} be the maximum amount of energy that the node can store. Assuming E_{max} constant, we have $D(t) = \max(0, A(t) - E_{max})$, $\forall t \in [0, T]$. No continuity requirement is imposed on $A(t)$ or $D(t)$, yet at a point of discontinuity on $A(t)$, let us denote it with t_0, we assume that $A(t_0^+) - A(t_0^-) < E_{max}$, *i.e.*, there is no energy overflow caused by a very large instantaneous energy input.

In order to maximize the throughput over $[0, T]$, all available energy should be used for data transmission and energy overflow should be avoided as much as possible, given that the data rate is an increasing function of the power consumption. With the continuous power model and no transmit power constraint, energy overflows can be avoided altogether. This means, $W(t) \geq D(t)$ has to be satisfied for $W(t)$ to be optimal. Since the energy consumption function $W(t)$ is bounded by $A(t)$ from above and by $D(t)$ from beneath, we refer to the functions $A(t)$ and $D(t)$ as the *boundary curves*.

D. Throughput Maximization

The energy harvesting transmitter aims at maximizing the throughput achieved from time 0 to T by properly adapting its transmit power without violating the causality constraint. Such a design goal can be expressed by the optimization

$$\begin{aligned} \max_{p_{tx}(t) \in \mathcal{P}} \quad & I = \int_0^T r(t) \mathrm{d}t \\ \text{s.t.} \quad & W(t) = \int_0^t p(\tau) \mathrm{d}\tau \leq A(t), \\ & W(0) = 0, \end{aligned} \tag{2}$$

where $\mathcal{P}$ is the set of finite, nonnegative, piecewise continuous functions defined over $[0, T]$. The conditions $W(t) \geq D(t)$ and $W(T) = A(T)$ are only necessary for optimality and are therefore not explicitly included in (2). We refer to the optimal solution to (2) as the *optimal transmission strategy* and denote it with $p_{tx}^*(t)$. If the energy storage of the node has an nonempty initial state A_0 with $A_0 \leq E_{max}$, and there is no energy arrival during $[0, T]$, *i.e.*, $A \equiv A_0$, $D \equiv 0$ for $t \in [0, T]$, the optimization as in (2) attains a much simpler structure and this special case will be referred to as the *basic problem*. Optimization (2) without these particular conditions on $A(t)$ is called the *general problem* correspondingly.

III. Optimal Transmission Strategy for the Generic Model

We have studied the throughput-maximizing transmission strategy for the generic model in [5], in the static scenario that the channel condition does not change over $[0, T]$. With a time-varying channel we could distinguish between two scenarios: first, the transmitter has full CSI of the whole time slot before transmission, and second, with a block-fading channel, the transmitter has only the statistics of the channel which does not change over time. Moreover, via the reciprocity of the channel or some reliable feedback link, it learns the CSI of the next block perfectly before it starts. In any case, we begin from ease with analysing the basic problem.

A. With Full CSI

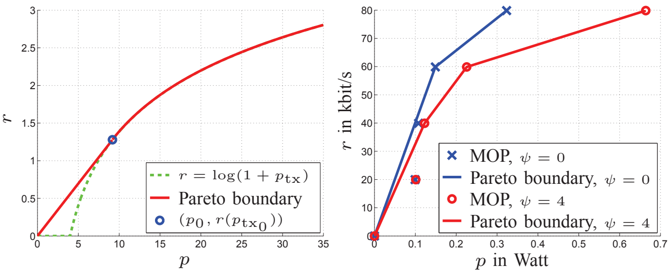
For the basic problem, the optimal power allocation in case that no circuit power is considered is the well-known water-filling solution [8]. When g_c is convex and continuous over $[0, +\infty)$, the same derivation can be applied and we only need to recover p_{tx}^* from the optimal total power dissipation p^* with the inverse function g^{-1}. When g_c is discontinuous at $p_{tx} = 0$ and the channel stays constant during $[0, T]$, the optimal transmission strategy is to use constantly

$$p_{tx}^* = \begin{cases} p_{tx_0}, & \text{if } \frac{A_0}{g(p_{tx_0})} \leq T, \\ g^{-1}\left(\frac{A_0}{T}\right), & \text{otherwise} \end{cases} \tag{3}$$

for a time period of $\frac{A_0}{g(p_{tx_0})}$ in the former case and of T in the latter, where p_{tx_0} denotes the *energy-efficient* transmit power which is the solution to the equation [5]

$$(f_{p_{tx}} g - f g_{p_{tx}})(p_{tx}) = 0. \tag{4}$$

In (4) and the following, a function with a subscript of one of its variables refers to the partial derivative of the function w.r.t. this variable. We know from (3) that employing a transmit power smaller than p_{tx_0} is suboptimal, which can be explained from a geometric point of view with Figure 1(a). Due to the discontinuity of g_{c} at $p_{\mathrm{tx}} = 0$, the rate-power curve is shifted away from the origin to the right. As a result, the tangent line from the origin to the rate-power curve lies above the corresponding part of the curve, and therefore contributes to the Pareto boundary of the r-p relation. The transmit power at the tangent point is exactly p_{tx_0}, and any data rate between 0 and $r(p_{\mathrm{tx}_0})$ should be achieved via time-sharing of the zero transmit power and the energy-efficient transmit power.



(a) Generic model (b) MQAM model, $d = 20$ m

Figure 1. Pareto boundaries of the rate-power dependency

With a time-varying channel, p_{tx_0} changes over time as the function f depends on t, yet it is still true that p^*_{tx} is always no smaller than p_{tx_0} at the same point. We show this by viewing the energy harvesting transmitter as a control system and applying the *Pontryagin Maximum Principle* (PMP) [9][10]. Firstly, we write the basic throughput maximization problem in the standard form of a control problem [10] as

$$\begin{aligned} \max_{p_{\mathrm{tx}} \in \mathcal{P}} \quad & \int_0^T f(t, p_{\mathrm{tx}}) \mathrm{d}t \\ \text{s.t.} \quad & \dot{W} = g(p_{\mathrm{tx}}), \qquad (5) \\ & W(0) = 0, \quad W(T) = A_0, \end{aligned}$$

where $\dot{W}$ stands for the derivative of W w.r.t. time t, and the t-argument in the control function p_{tx} is omitted. The optimization (5) is a fixed time, fixed end-point control problem with a discontinuous state equation for which the PMP can not be applied directly. We assume for the moment that the channel power gain $|h(t)|^2$ is monotonically decreasing in time. It can be easily seen that for such a case there exists a time instance $t_1 \in (0, T]$, such that $p^*_{\mathrm{tx}} > 0$ for $t \in [0, t_1]$, and $p^*_{\mathrm{tx}} = 0$ for $t \in (t_1, T]$. Therefore, we could replace the terminal time T with t_1 and reformulate (5) into a fixed end-point, free time control problem with a continuous state equation. The Hamiltonian of the reformulated control problem is given by

$$H(t, p_{\mathrm{tx}}, \lambda) = -f(t, p_{\mathrm{tx}}) + \lambda \cdot g(p_{\mathrm{tx}}),$$

where λ is an auxiliary variable associated with the state equation. Since H does not explicitly depend on W, the co-state equation suggests that $\dot{\lambda}^* = H_W = 0$, *i.e.*, λ^* is constant. Applying PMP and the transversality condition to the reformulated problem, we have

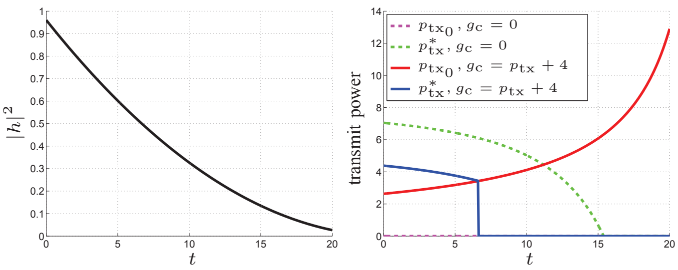
$$H_{p_{\mathrm{tx}}}(t, p^*_{\mathrm{tx}}, \lambda^*) = -f_{p_{\mathrm{tx}}}(t, p^*_{\mathrm{tx}}) + \lambda^* g_{p_{\mathrm{tx}}}(p^*_{\mathrm{tx}}) = 0, \qquad (6)$$

$$\begin{aligned} H(t_1, p^*_{\mathrm{tx}}(t_1), \lambda^*) &= -f(t_1, p^*_{\mathrm{tx}}(t_1)) + \lambda^* g(p^*_{\mathrm{tx}}(t_1)) \\ &= 0. \qquad (7) \end{aligned}$$

It follows from (6) that $\lambda^* = \frac{f_{p_{\mathrm{tx}}}(t, p^*_{\mathrm{tx}})}{g_{p_{\mathrm{tx}}}(p^*_{\mathrm{tx}})}$. Evaluating the right-hand side at $t = t_1$ and plugging the result into (7), we have

$$f_{p_{\mathrm{tx}}}(t_1, p^*_{\mathrm{tx}}(t_1)) g(p^*_{\mathrm{tx}}(t_1)) - f(t_1, p^*_{\mathrm{tx}}(t_1)) g_{p_{\mathrm{tx}}}(p^*_{\mathrm{tx}}(t_1)) = 0,$$

which means the optimal transmit power at the end-point t_1 equals the corresponding energy-efficient transmit power. Moreover, it can be shown from (6) that $p^*_{\mathrm{tx}}(t) > p^*_{\mathrm{tx}}(t_1)$, and from the definition of energy-efficient transmit power that $p_{\mathrm{tx}_0}(t) < p_{\mathrm{tx}_0}(t_1)$, $\forall t < t_1$, which leads to the conclusion that $p^*_{\mathrm{tx}}(t) \geq p_{\mathrm{tx}_0}(t)$, $\forall t \in [0, t_1]$. Due to the concavity and convexity we have imposed on f and g, conditions (6) and (7) are sufficient to determine p^*_{tx}. We show an example of the basic problem with $A_0 = 80$ and the monotonically decreasing channel $|h(t)|^2 = (t - 24)^2/600$ in Figure 2, where the linear circuit power model with $b = 1$, $c = 4$ is employed. Compared to the case that $g_{\mathrm{c}} = 0$ which has constantly zero energy-efficient transmit power, the node should operate in active mode for a shorter time when the circuit power is considered.



(a) Channel gain (b) p_{tx_0} and p^*_{tx}

Figure 2. Optimal transmit power with monotonically decreasing channel

Since with the basic problem there is no energy arrival during $[0, T]$, we can extend the result above to those channel gain functions that can be sorted in strict descending order. To be more specific, for a fixed λ, one computes p_{tx} that satisfies (6) at each point within $[0, T]$, compare it with the corresponding p_{tx_0}, and set it to zero if p_{tx_0} is larger. The optimal auxiliary variable λ^* can then be found iteratively, at which all the available energy A_0 is consumed. There is one particular case that should be treated with care, that is, the channel gain function being constant for some period of time. For example, with block-fading channel the regular water-filling procedure does not give the correct p^*_{tx}. We propose a modified version in Algorithm 1, where N_{b} is the total number of blocks and $T_{\mathrm{b}}(n)$ represents the length of block n. Going from the block with the best channel condition to that with the worst, the algorithm sets the value of λ according to the energy-efficient transmit power of the block currently under inspection, and determines whether there is enough energy to support this λ. If so, the algorithm goes on to examine more blocks; otherwise, energy should be allocated according to the water-filling principle on all inspected blocks, and for the remaining blocks the transmitter should stay in sleep mode.

Algorithm 1 Modified Water-filling Algorithm

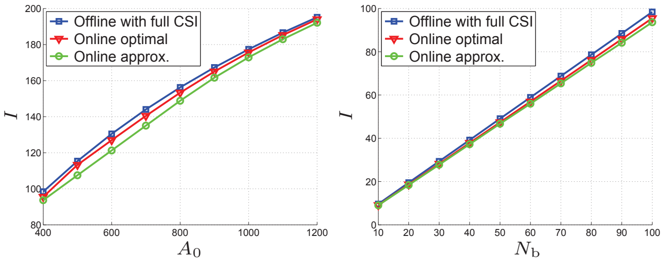
```
Sort and index all blocks in descending order of their
channel gains: |h(1)|^2 > |h(2)|^2 > ··· > |h(N_b)|^2
Compute p_tx0(n) and set p_tx(n) ← 0, n = 1,...,N_b
for n = 1,...,N_b do
   Compute λ that corresponds to p_tx0(n) from (6)
   Compute p_tx(i), i = 1,...,n−1 from (6) with λ
   W ← energy consumption of blocks 1,...,n−1
   if W ≥ A_0 then
      Compute p_tx(i), i = 1,...,n−1 with regular WF
      return
   end if
   p_tx(n) ← p_tx0(n)
   Active time on the block T_a ← (A_0−W)/g(p_tx(n))
   return if T_a ≤ T_b(n)
end for
```

The optimal transmission strategy for the general problem can be obtained with the *critical marginal gain* based algorithm, which is in analogy with the *critical slope* based algorithm proposed in [8][5]. The idea is to translate the constant slope property to the constant marginal gain (*i.e.*, λ) requirement. Algorithm details are omitted due to similarity.

B. With Partial CSI

In the scenario that the transmitter only has partial CSI, we maximize the expected throughput with the dynamic programming (DP) technique, which is commonly used to deal with dynamic systems where control decisions are made in stages and certain system parameters are random [11]. The algorithm executes first an offline part where the *cost-to-go* function J is evaluated at each block for each possible energy input, which represents the expected throughput from that block till T, and then an online part where the actual optimization of transmit power is performed based on J sequentially. Due to the high complexity we also propose a simple approximation of the DP, where in the calculation of J we replace $\mathrm{E}\left[\log(1+|h|^2 p_{\mathrm{tx}})\right]$ over all channel realizations with $\log\left(1+\mathrm{E}\left[|h|^2\right]p_{\mathrm{tx}}\right)$, *i.e.*, we neglect the random variations in channel gain for the offline part of the algorithm, and simply take the mean channel gain for every block in the time slot.

Finally we show some numerical results for the generic model in Figure 3. Proposed algorithms for the basic problem with a block-fading channel are simulated where the channel coefficients are generated with a standard Gaussian distribution. Here we use again the linear circuit power model with $b = 1$ and $c = 4$. In Figure 3(a), the achievable throughput with increasing input energy is shown while in Figure 3(b), the slot length T is varied in terms of N_{b} where T_{b} is fixed to 1. The performance achieved with the DP algorithm and its approximation is almost as good as the optimal transmission strategy obtained with full CSI, according to both figures.



(a) $N_{\mathrm{b}} = 100$ (b) $A_0 = 4N_{\mathrm{b}}$

Figure 3. Throughput achieved with full or partial CSI

IV. MQAM System Model

In practice, the information bits are modulated with discrete modulation levels and transmitted symbol by symbol. We consider that the energy harvesting transmitter uses the common MQAM schemes and set up the *MQAM model* to capture the characteristics of such a system. We assume from now discrete energy arrivals and introduce the channel, data transmission and circuit power models which are different from before.

For the wireless communication channel we consider both path loss and shadow fading for the radio propagation effect. Let p_{tx} be the transmit power and p_{rx} be the corresponding receive power. Assuming that the distance d between transmitter and receiver is constant, we write the receive SNR as

$$\gamma = \frac{p_{\mathrm{rx}}}{N_0 B} = \frac{1}{N_0 B M_{\mathrm{l}} G_1} \cdot \frac{p_{\mathrm{tx}}}{10^{\psi/10} d^{\kappa}},$$

where $\frac{N_0}{2}$ is the noise power spectrum density, B stands for the transmission bandwidth which is approximately equal to the inverse of the symbol duration, κ is the path loss exponent, and G_1 is the the power gain factor at the reference distance of 1 meter [6][12]. The effect of shadowing in dB is characterized by the random variable ψ which is Gaussian distributed with zero mean and variance σ_{ψ}^2. Interference, other background noise, and internal hardware loss are compensated with the link margin M_{l}. An upper bound on the uncoded bit error probability for MQAM is given by [6]

$$\pi_{\mathrm{b}} \leq \frac{4}{\log_2 M} \cdot \left(1 - \frac{1}{\sqrt{M}}\right) \cdot \mathrm{e}^{-\frac{3}{M-1} \cdot \frac{\gamma}{2}}.$$

When a target BER $\pi_{\mathrm{b}}^{(\mathrm{rq})}$ is predefined and fixed, the minimal required receive SNR and consequently, the minimal required transmit power can be computed, where the latter depends completely on d and M. We refer to a modulation level as one *mode of operation* (MOP) in the sequel. The achieved data rate in bit/sec is given by $r = B(1-\pi_{\mathrm{b}}^{(\mathrm{rq})})\log_2 M$.

The total power consumption p of the transmitter in active mode consists of 3 parts given by $p = p_{\mathrm{tx}} + p_{\mathrm{amp}} + p_{\mathrm{ct}} = (1+\alpha)p_{\mathrm{tx}} + p_{\mathrm{ct}}$ [6]. Besides the transmit power p_{tx}, an important part of the total power consumption comes from the power amplifier given as $p_{\mathrm{amp}} = \alpha \cdot p_{\mathrm{tx}}$, where $\alpha = \frac{\xi}{\eta} - 1$ with η the drain efficiency of the amplifier and $\xi = 3 \cdot \frac{\sqrt{M}-1}{\sqrt{M}+1}$ the peak-to-average ratio which depends on the MQAM constellation size. The power consumptions of the DAC, the transmit filters, the mixer, and the frequency synthesizer are included as a whole in p_{ct} and they constitute the constant part in p. The notations and values of the system parameters are summarized in Table I.

Table I
SYSTEM PARAMETERS

$f_c = 2.5$ GHz	$B = 10$ kHz	$\frac{N_0}{2} = -174$ dBm/Hz
$\kappa = 3.5$	$\sigma_\psi = 3$	$T_b = 4$ seconds
$\eta = 0.35$	$M_l = 30$ dB	$G_1 = 40$ dB
$\pi_b^{(\text{rq})} = 10^{-3}$	$P_{\text{ct}} = 98.2$ mW	$M \in \{4, 16, 64, 256\}$

Table II
SIMULATION PARAMETERS

Parameter set index	$E_{\max}$ in Joule	Maximal energy per arrival
PAR. I	40	$0.4 \times E_{\max}$
PAR. II	40	$0.6 \times E_{\max}$
PAR. III	100	$0.4 \times E_{\max}$
PAR. IV	100	$0.6 \times E_{\max}$

V. TRANSMISSION STRATEGIES FOR MQAM MODEL

Similar to what has been done with the generic model, we first take a look at the Pareto boundary of the rate-power dependency which is shown in Figure 1(b) for $d = 20$ meters. The MOPs are represented by a few discrete points on the rate-power graph, where the origin corresponds to the sleep mode. Any point on the straight line connecting two points can be achieved via time-sharing of the two MOPs. The Pareto boundary of the r-p relation is a concave function which may not pass through all points representing the MOPs. The distance d and the shadow fading parameter ψ both have an influence on which MOPs are *energy-efficient*, *i.e.*, are on the Pareto boundary. The energy-inefficient MOPs should in general not be employed when throughput is to be maximized, given that the symbol duration is much smaller compared to T and T_b, in which case we can view the system as operating approximately with continuous-time and assume any required time-sharing solution feasible. For the MQAM model with constant channel, the optimal solution to the basic problem is given by the time-sharing of the two energy-efficient MOPs, which are responsible for producing the power value $\frac{A_0}{T}$ [13].

A. Optimal solution with full CSI

As the number of energy-efficient MOPs for each transmission distance and each channel realization is very limited, we formulate the MOP adaptation for the general throughput maximization as a linear optimization problem, where each variable corresponds to the time-share of one energy-efficient MOP on one stage. A stage is defined as a time period that ends either with a channel change or with an energy arrival event. This implies that, over one stage, the channel condition stays constant and there is no energy arrival. A maximal and a minimal energy consumption constraint are added for each energy arrival, which correspond to the limitations on W imposed by the jumps in curves A and D. The linear optimization can be solved by any standard solver to optimality. Note that since we have a highest modulation order now, the transmitter is limited in the maximal power it could consume which may lead to an overflow at the energy storage. In such an occasion, the boundary curves should be adjusted accordingly.

B. Suboptimal solution with full CSI

With a large number of blocks and energy arrivals on $[0, T]$, the number of variables in the LP could become intractably large. We therefore propose the following heuristic algorithms to achieve a fast solution with low complexity. For the basic problem, we first compute the bit/Joule values of all energy-efficient MOPs, sort them in descending order, and select in loop the next best MOP until the total energy consumption exceeds A_0. During this process, the blocks corresponding to the selected MOPs are assumed to use one MOP exclusively. If a block is already active when another MOP on the block is selected, the new MOP should replace the old one, representing a raise in the modulation order on the block. Then, in order to curtail the extraneous energy consumption, for all active blocks we compute the throughput reduction caused by replacing the respective MOP with a time-share of it and its next lower energy-efficient MOP. In the end, the block with the smallest throughput reduction is chosen for fitting the energy to A_0. The whole procedure is termed as the *basic heuristic algorithm* and is summarized in Algorithm 2.

Algorithm 2 Basic Heuristic Algorithm for MQAM Model

```
Obtain energy-efficient MOPs for all blocks
W ← total energy consumption when the highest MOP is
chosen for every block
return if  W ≤ A_0
Sort and index all energy-efficient MOPs in descending
order of their bit/Joule values: r_1/p_1 ≥ r_2/p_2 ≥ ···
W ← 0, i ← 1
while W < A_0 do
    n ← block index of MOP i, W_n ← p_i · T_b
    W ← Σ_{n=1}^{N_b} W_n, i ← i + 1
end while
if W > A_0 then
    B ← {n : W − W_n < A_0}
    for each n ∈ B do
        Find the current active MOP on block n
        Find the next lower energy-efficient MOP on block n
        Compute the time-shares of the two MOPs to consume
        the energy A_0 − W + W_n
        ΔR(n) ← reduction in throughput
    end for
    b ← argmin_n ΔR(n)
    Replace the exclusive MOP assignment on block b with
    the corresponding time-sharing solution
end if
```

The intuition of the basic heuristic algorithm comes from observations on the optimal solutions, which are obtained via

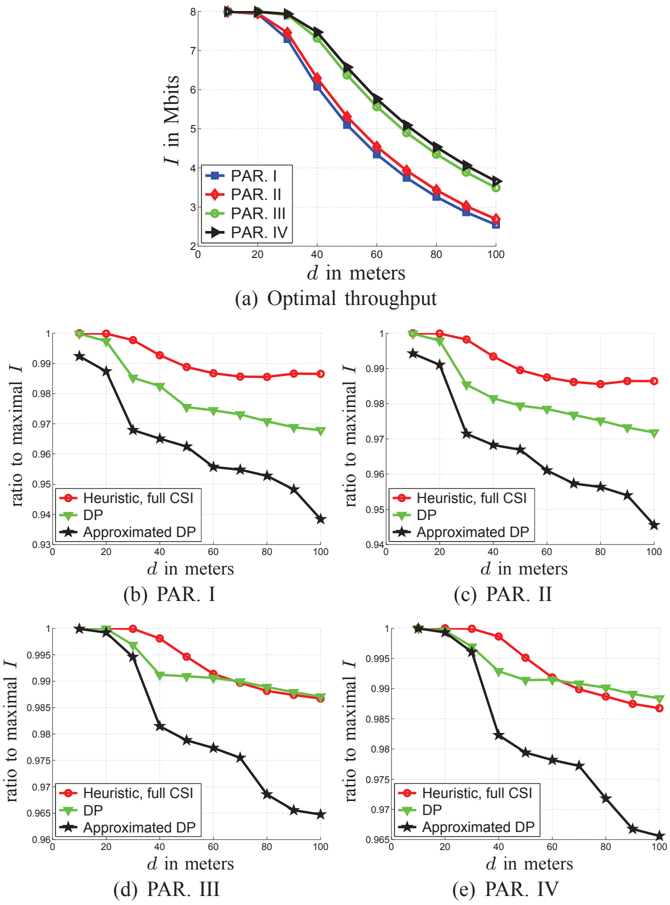
LP, that in most cases, there is only one block with a time-sharing of two modulation orders while all other blocks on $[0, T]$ employ a single modulation order. We employ the basic heuristic algorithm recursively to solve the general throughput maximization problem. Starting globally, we use the algorithm to find a MOP adaptation without considering the restrictions imposed by the boundary curves A and D, but only the energy constraint at the end-point $W(T) = A(T)$. If the solution happens to fulfil all energy constraints defined by the energy arrivals, the algorithm terminates; otherwise, we formulate a subproblem from time 0 to the time instance that a constraint violation happens, and the end-point of the subproblem is determined by whether the obtained solution violates the constraint of A or of D. In this way, we break the whole problem into several smaller ones which could eventually be solved with the basic heuristic algorithm.

C. Simulation results

Simulations of the general throughput maximization problem for the MQAM model are conducted with 4 parameter sets listed in Table II. The time slot is taken as 100 seconds and the energy arrivals are at least 5 seconds apart from each other. After 5 seconds some more inter-arrival time following the negative exponential distribution with a mean of 10 seconds is expected. The amount of energy in each arrival is uniformly distributed from 0 to the maximal value specified for each parameter set. 1000 simulation trials are performed, each with randomly, independently generated channel coefficients and energy arrivals. Besides the optimal and heuristic algorithms that apply to the full CSI case, we also test the partial CSI case with DP and approximated DP algorithms which are designed in the same way as with the generic model. Numerical results averaged over all simulation trials are shown in Figure 4, where the comparisons clearly demonstrate the near-optimal performance of the heuristic algorithm, as well as the very small loss due to the lack of full CSI.

VI. Conclusion

The throughput-maximizing transmission strategies of an energy harvesting node which communicates over a time-varying channel are discussed in this paper. The optimization is formulated and studied with different system models and under altered scenarios, and various mathematical tools have been used to tackle the problem. We see from a theoretical point of view the difference in the optimal transmission strategies between the cases where circuit power consumption is considered or assumed zero, and compare via numerical simulations the performance of optimal and heuristic algorithms for both cases that the transmitter has full or only partial CSI.



(a) Optimal throughput

(b) PAR. I (c) PAR. II

(d) PAR. III (e) PAR. IV

Figure 4. Average throughput and algorithm comparisons

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