

# Coupled flow large deformation porous media interaction and applications to biomechanics



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## Introduction to Poromechanics

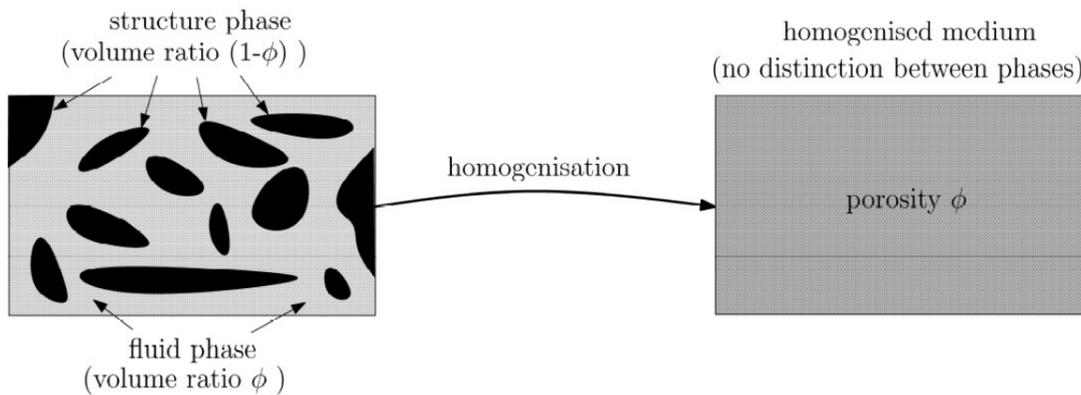


Figure 1: Conceptual sketch of homogenization process

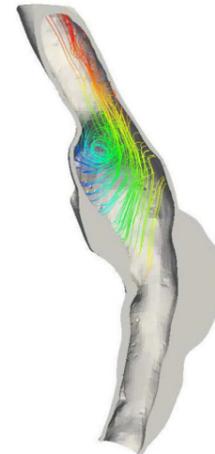
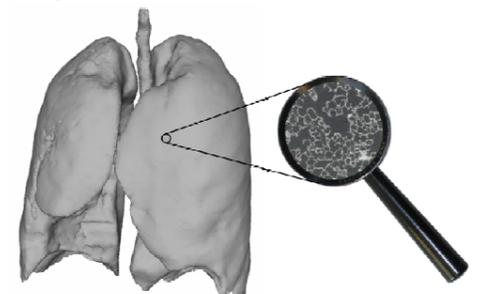


Figure 2: Porous media in biomechanics

a) Abdominal Aortic Aneurism



b) lung tissue

- Modelling deformation and flow in porous media is of great interest due to its possible application areas in various fields of engineering, like geophysics, civil engineering, physical chemistry, material sciences and biomechanics.

- Based on the mathematical homogenization theory, so called "Reference Volume Averaging" has been hugely applied in engineering science (see e.g. [1]).

- This method leads to a continuous description of the porous medium, where fluid and solid are modelled as overlapping continua and, hence, the actual interface is not resolved explicitly.

- Finally, a volume coupled fluid-structure-interaction problem is derived, which enables modelling of a porous medium on a macroscopic scale without presuming detailed knowledge of the pore geometry.

## Governing Macroscopic System Equations

- Darcy's Law models the interstitial flow through the porous medium

$$\rho_0^F \frac{\partial \mathbf{u}^F}{\partial t} \Big|_{\mathbf{x}} - \rho_0^F \left( \frac{\partial \mathbf{d}^S}{\partial t} \Big|_{\mathbf{x}} \cdot \nabla \right) \mathbf{u}^F + \nabla p^F - \rho_0^F \hat{\mathbf{b}} + \mu^F \mathbf{k}^{-1} \phi \mathbf{u}^c = \mathbf{0},$$

- The continuity equation depends on the change of porosity. Thus, although the fluid flow is assumed to be incompressible, the macroscopic equation shows characteristics of compressible flow:

$$\frac{\partial \phi}{\partial t} \Big|_{\mathbf{x}} + \phi \nabla \cdot \mathbf{u}^F + \nabla \phi \cdot \mathbf{u}^c = 0,$$

- The porous skeleton equation contains volumetric coupling terms due to its interaction with the interstitial flow:

$$\rho_0^S (1 - \phi_0) \frac{\partial^2 \mathbf{d}^S}{\partial t^2} \Big|_{\mathbf{x}} - \nabla_0 \cdot (\mathbf{F} \cdot \mathbf{S}^S) - \rho_0^S (1 - \phi_0) \hat{\mathbf{b}}^S + J p^F \mathbf{F}^{-T} \nabla_0 \phi - \mu^F \mathbf{k}^{-1} J \phi^2 \mathbf{u}^c = \mathbf{0}$$

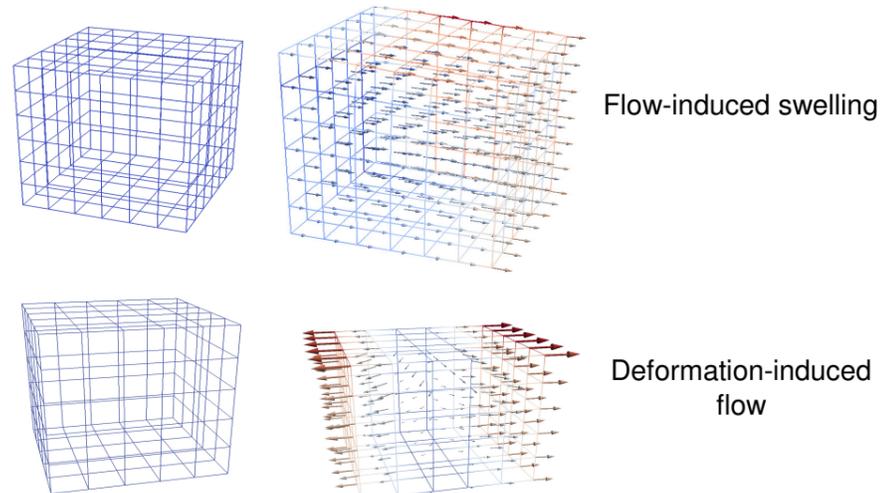


Figure 3: Numerical examples of simple configurations

## Extensions

- For application to more sophisticated physical problems we developed multiple extensions to the existing model.

- Passive scalar transport within the porous medium.

- Fluid-Porous-Structure-Interaction includes special conditions at the porous medium fluid interface (Beavers-Joseph-Saffmann condition)

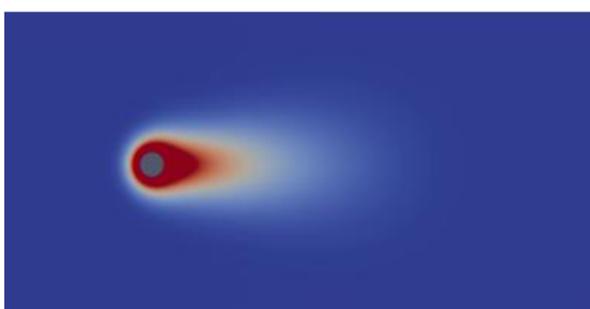


Figure 4: transport of scalar quantity through porous medium passing an obstacle

## Future work: Cell Mechanics

### Project Idea

- Development of advanced experimental and computational models and methods enabling the investigation of 3D cell migration

- Analysis of role of

- Extracellular matrix (ECM) as porous medium
- Interstitial flow
- Cell-ECM adhesion
- Intracellular signaling

### Aims

- Realistic simulation of health and disease for the first time
- Improvement of general understanding of cell migration
- Support of novel diagnostic and therapeutic methods (e.g., cancer treatment)

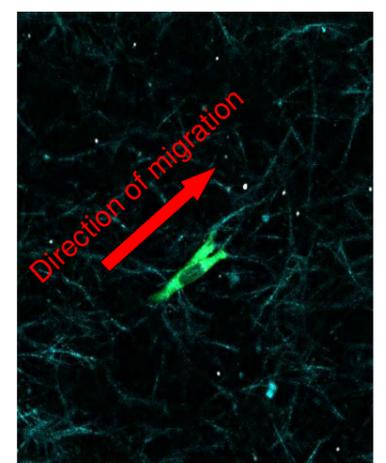


Figure 5: real time tracking of migrating cell. Good contrast due to expression of Green Fluorescent Protein

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