First steps towards computational image reconstruction for photoacoustic tomography

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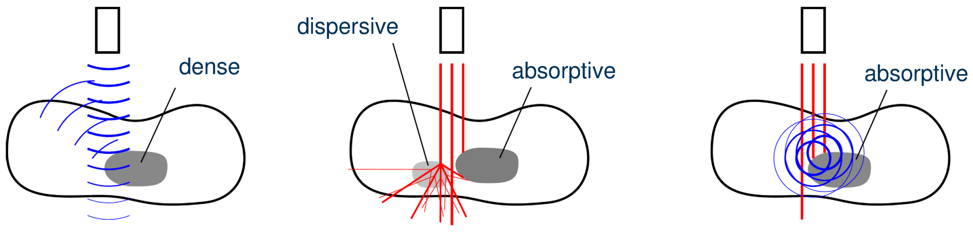
Introduction

Medical Imaging

- Several medical imaging techniques exist to deliver insight on the properties of an underlying object of interest.
- Photoacoustic tomography is a promising new method combining optical tomography and ultrasonography [1].
- Optical energy is transferred to pressure waves via the photoacoustic effect.



Ultrasonography: Generation of sound waves and subsequent detection of echoes

Diffuse optical tomography: Illumination of an object and detection of traversed light

Photoacoustic tomography: Illumination of an object and detection of sound waves

- Common reconstruction algorithms are based on the assumption of mechanically homogeneous material and a uniform illumination.

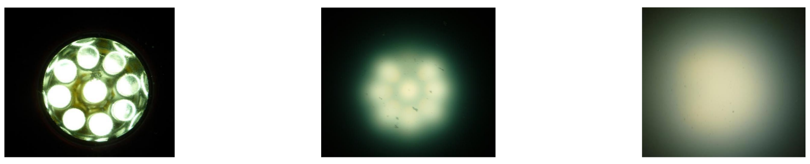
Aim

- The aim is to improve the quality of image reconstruction by allowing for mechanically heterogeneous materials for the simulation of the pressure propagation.

Light propagation

Physics

- Light is considered as a concentration of optical energy instead of an electromagnetic wave.
- For strongly scattering matter, the directionality of the light gets lost after traversing a specific distance [2].



Light source behind a layer of colored but nonscattering material

Light source behind a layer of colored and scattering material

Light source behind a thicker layer of colored and scattering material

- The light propagation is described by a steady-state reaction-diffusion system complemented by a Dirichlet and a Robin boundary condition

$$\mu_a \phi(\boldsymbol{x}) - \nabla \cdot (D \nabla \phi(\boldsymbol{x})) = 0 \text{ on } \Omega$$
$$\phi(\boldsymbol{x}) = d(\boldsymbol{x}) \text{ on } \Gamma_D \qquad (1)$$
$$\phi(\boldsymbol{x}) + 2D\nabla\phi(\boldsymbol{x}) \cdot \hat{\boldsymbol{n}}(\boldsymbol{x}) = 0 \text{ on } \Gamma_R$$

Numerical treatment

- The system of equations (1) is discretized with a standard finite element approach. For a stable formulation Nitsche's method is applied to the Robin type boundary condition.

Sound propagation

Physics

- The wave equation for the pressure propagation neglecting any shear phenomena is derived from the conservation of mass, the conservation of linear momentum and a linearized pressure-density relation and reads as

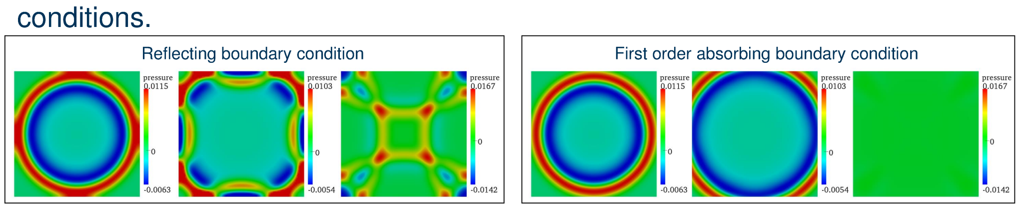
$$\frac{1}{\rho_0 c^2} \frac{\partial^2 p}{\partial t^2} = \nabla \cdot \left(\frac{1}{\rho_0} \nabla p \right)$$

Numerical treatment

- The wave equation is rewritten as a first order system and a mixed finite element approach with mass-lumping is applied [3].
- The resulting ordinary differential equation in matrix form is

$$\begin{bmatrix} \boldsymbol{D} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{P}} \\ \dot{\boldsymbol{V}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{R} \\ -\boldsymbol{R}^{\mathrm{T}} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{P} \\ \boldsymbol{V} \end{bmatrix}$$

- Due to the particular approach, the mass matrices $\boldsymbol{D}$ and $\boldsymbol{B}$ are diagonal and block-diagonal, respectively, and, hence, the inversion is cheap.
- An explicit time integration scheme is applied.
- In acoustics, special attention must be given to non-reflecting boundary conditions.



Pressure distribution in a square domain for two different boundary conditions for three points in time

Photoacoustics

Photoacoustic effect

- The optical energy is absorbed in places according to the optical properties.
- Subsequently, the optical energy is converted to heat, via the thermal expansion, the local temperature rises initiating a pressure wave

$$p_0(\boldsymbol{x}) = -\Gamma \eta_{th} \tau_p \mu_a(\boldsymbol{x}) \phi(\boldsymbol{x}) \qquad (2)$$

Numerical representation

- The light propagation problem is solved corresponding to the given boundary conditions.
- An initial pressure distribution can be calculated with equation (2) given above, however, the absorption coefficient, the optical flux, and the initial pressure are discrete quantities which are defined at elements and nodes of a mesh. Furthermore, the mesh for the light propagation and for the sound propagation are nonconforming due to differing refinement requirements.
- A mapping has to be performed in order to calculate the initial pressure for the sound propagation depending on the light propagation solution.

Reconstruction

Inverse analysis

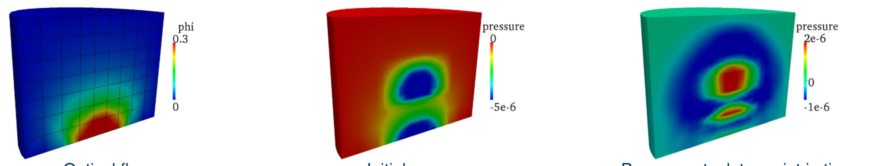
- Here, optimization is performed with respect to the optical absorption coefficient.
- A modified Levenberg-Marquardt algorithm is utilized

$$\left(\boldsymbol{J}_f(\boldsymbol{\varpi})^{\mathrm{T}} \boldsymbol{J}_f(\boldsymbol{\varpi}) + \mu \boldsymbol{I}\right) \boldsymbol{h}_L = -\boldsymbol{J}_f(\boldsymbol{\varpi}) \boldsymbol{f}(\boldsymbol{\varpi})$$

The Jacobian $\boldsymbol{J}_f$ is calculated with a finite difference approach.

Numerical example

- A cylindrical geometry is exposed to a laser source. A strongly absorbing region is located in the middle of the cylinder.

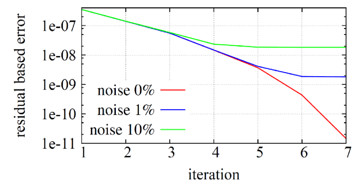


Optical flux

Initial pressure

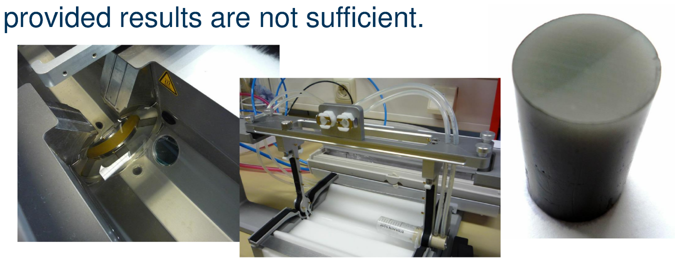
Pressure at a later point in time

- Reconstruction is performed with recorded pressure curves at the boundary of the cylinder to validate the interaction of forward and inverse model.
- Further reconstruction is performed with overlain noise to study the robustness of the algorithm.



Experimental validation

- For an experimental validation, three phantoms with heterogeneous optical and acoustical properties are created.
- Data is generated with the multispectral optoacoustic tomograph MSOT inVision256-TF (iThera Medical GmbH, Neuherberg, Germany).
- Reconstruction is performed with varying numbers of free parameters.
- So far, the provided results are not sufficient.



Conclusion and Outlook

- An algorithm to simulate diffusive light propagation and pressure propagation in heterogeneous media is developed which allows for the reconstruction of optical absorption maps.
- The algorithm is not susceptible to overlaied noise. However, reconstruction with experimental data sets is not yet sufficient.
- Several simplifications and approximations deteriorate the quality of the results.

Future work

- The hybridizable discontinuous Galerkin method will be used for the wave equation to improve accuracy and efficiency.
- The simplifications will be released to improve the quality of the results.

References

[1] V. Ntziachristos. Going deeper than microscopy: The optical imaging frontier in biology. *Nature methods*, 7(8):603-614, 2010.
[2] S.L. Jacques and B.W. Pogue. Tutorial on diffuse light transport. *Journal of Biomedial Optics*, 14(4):041302-041302-19, 2008.
[3] G. Cohen and S. Fauqueux. Mixed finite elements with mass-lumping for the transient wave equation. *Journal of Computational Acoustics*, 8(1):171-188, 2000.