Throughput Maximization for Energy Harvesting Receivers

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***Abstract*—We investigate in this work the optimal control strategies of a receiver that is powered by the ambient energy it harvests over time. Two scenarios are mainly considered, the single-antenna system with non-fading channel, and the multi-antenna system with fading channel. The throughput achieved on the limited time slot $[0, T]$ is maximized with respect to the resolution of the A/D converter and the transmission bandwidth/number of antennas. To this end, the receiver is assumed to have full, non-causal knowledge about the energy arriving process, and the transmitter is assumed to be in full cooperation with the receiver. Convexity of the formulated optimizations is discussed, and the optimal control theory is applied for finding the optimal state trajectories of the system. Numerical simulations are performed for the analysis of the optimal control variables, as well as for the comparison of throughput achieved with different channel gains and optimizations.**

I. Introduction

Energy-efficiency of communication and communication networks have drawn a lot of research attention over the past years, driven by the desire to reduce operation cost as well as to address environmental issues. The energy consumption of different components of a communication device is studied in [1][2], based on which optimizations on various system parameters can be formulated. Different from this hardware initiated investigation, the potential for energy-efficiency improvement is also studied from the viewpoint of communication scheme and protocol, network deployment and operation, *etc.* As an example, [3] addresses the fundamental tradeoff between power consumption and communication delay, which can be utilized to reduce the energy consumption while preserving certain QoS requirement for data transmission.

With the energy harvesting technology, transceivers can be powered by the energy they harvest from the environment [4]. The design and control of such devices are naturally connected with the investigations on energy-efficient communications. In fact, we face more complexity as the energy input is now time-variant and random. The exploitation of energy by an energy harvesting transmitter has been studied in several works. In [5] and [6], the authors formulate the throughput maximization problem with fading and non-fading channels, and find the optimal adaptations of transmit power over time. In our work [7], the circuit power of the transmitter is taken into the power consumption model, which gives rise to an *energy-efficient* transmit power below which one should not operate. To this end, the shape of the optimal state trajectory, which has one-to-one correspondence with the transmit power function in time, can be significantly changed. We present in this work, our recent investigations on how an energy harvesting receiver should be controlled in order to achieve the same optimization goal, *i.e.*, to maximize throughput. The parameters to be adapted, which connect the instantaneous data rate with power consumption, include the resolution of the A/D converter, the transmission bandwidth, and the number of antennas. While working on a different scenario as before, we try to exploit the similarity in problem structure as much as possible.

The rest of the paper is organized as follows: in Section II we introduce the system model from the aspects of power consumption, achievable rates, and energy harvesting, and formulate the throughput maximization as a standard control problem by the end. Section III and Section IV are devoted to the analysis on the optimal solutions of the most basic form of the throughput maximization problem, for the single-antenna system and for the multi-antenna system, respectively. Construction of the optimal state trajectory for the general throughput maximization problem is discussed in Section V. Simulation results are shown in Section VI before we summarize and conclude the paper with Section VII.

II. System Model

We consider the point-to-point communication between a transmitter and a receiver on a limited time slot $[0, T]$. The receiver is an energy harvesting device which has non-causal information about the energy arrivals, *i.e.*, it knows perfectly before the communication starts, when and how much energy will be harvested during $[0, T]$. Although such an assumption is quite unrealistic, it enables the theoretical investigation on the performance limit of the system. In the following we study the achievable rate/ergodic capacity and the power consumption of a singe-antenna and a multi-antenna receiver, respectively, as dependent on different system parameters.

A. Single-antenna system with non-fading channel

The receiver consumes energy in order to perform analog-to-digital conversion, decoding, and other signal processing tasks. Among all, the energy consumption of the A/D converter (ADC) contributes a significant part. Assuming uncoded data transmission between the transmitter and the receiver which suggests zero decoding power, we take energy consumption of the ADC as the total energy consumption of the receiver

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for simplicity. The power dissipation of the ADC, depending on its *operation modes*, can be calculated as [8]

$$P^{(\mathrm{s})} = \begin{cases} c \cdot BN_0 \cdot 2^b, & b > 0, \\ 0, & b = 0, \end{cases} \tag{1}$$

where B is the transmission bandwidth, b is the resolution of the ADC which we assume a real number, c is a constant determined by the specific design of the ADC, and N_0 is the noise power spectral density. The superscript of P is used to distinguish from the multi-antenna case which we will discuss later. In the *active* mode, which is indicated by the positive resolution, the power dissipation grows linearly with the bandwidth and exponentially with the resolution. In the *sleep* mode for which $b = 0$, *i.e.*, the receiver does not receive any signal, the power consumption is much lower than in the active mode and we assume it to be zero.

Let $\gamma = \frac{\alpha}{BN_0}$ denote the receive signal-to-noise ratio (SNR) where α is the combined power gain of the transmit power and the transmission channel, which is assumed constant and perfectly known by both the transmitter and the receiver. A capacity lower bound achieved at the receiver, as dependent on the transmission bandwidth B, the ADC resolution b, and the receive SNR γ, is given by [2]

$$R^{(\mathrm{s})} = B \log_2 \left(\frac{1+\gamma}{1+\gamma \cdot 2^{-2b}} \right). \tag{2}$$

B. Multi-antenna system with fading channel

Now suppose the receiver is equipped with $M \geq 1$ antennas, and the transmitted signal goes through a random fading channel. For simplicity we assume i.i.d. channel coefficients which are Gaussian distributed with zero mean and variance $\frac{1}{M}$. As one ADC is required for each antenna, the total power consumption of the receiver reads

$$P^{(\mathrm{m})} = \begin{cases} cBN_0 \cdot 2^b \cdot M, & b > 0, \\ 0, & b = 0. \end{cases} \tag{3}$$

Here the same resolution b is applied for all ADC, since the channel exhibits the same property across the antennas.

We further assume that the transmitter in this scenario is equipped with the same number of antennas as the receiver. This not only allows for a closed-form capacity expression, but is also reasonable in that the nodes of a wireless sensor network are usually equal. Given uniform power allocation at the transmitter for all antennas, the ergodic capacity of the channel can be approximated by [9][10]

$$R^{(\mathrm{m})} = BM \cdot \left[2 \log_2 \left(1 + \chi - \frac{1}{4} \left(\sqrt{4\chi + 1} - 1 \right)^2 \right) - \frac{\log_2 \mathrm{e}}{4\chi} \cdot \left(\sqrt{4\chi+1} - 1 \right)^2 \right], \tag{4}$$

where χ is the effective SNR after the ADC expressed as

$$\chi = \frac{\left(1 - 2^{-2b}\right) \cdot \frac{\gamma}{M}}{1 + 2^{-2b} \cdot \frac{\gamma}{M}} = \frac{\left(1-2^{-2b}\right)\gamma}{M + \gamma \cdot 2^{-2b}}. \tag{5}$$

Note that although the same symbol is used, $R^{(\mathrm{s})}$ and $R^{(\mathrm{m})}$ have distinct physical meanings. We want to emphasize here, that $R^{(\mathrm{s})}$ is a capacity lower bound for the single-antenna system with given receive SNR, while $R^{(\mathrm{m})}$ stands for the ergodic capacity of a MIMO fading channel with the same number of transmit and receive antennas. We simply use the terms *single-antenna system* and *multi-antenna system* later to refer to the two system models, yet it should be noted that the number of antennas is not the only underlying difference.

C. Energy harvesting and expenditure

We utilize the *cumulative* model to describe the energy arrival as well as the energy expenditure of the receiving node. Let $P(t)$ denote the instantaneous power consumption of the receiver, which is calculated according to (1) for the single-antenna system and to (3) for the multi-antenna system. The cumulative energy consumption by time t, denoted by the function $W(t)$, follows as

$$W(t) = \int_0^t P(\tau)\, \mathrm{d}\tau, \quad t \in [0, T]. \tag{6}$$

The receiver harvests energy from the environment and stores them in its storage medium. We let $E_{\max}$ be the maximum amount of energy that the node can store and assume $E_{\max}$ to be constant. Let the function $A(t)$ represent the cumulative available energy at the receiver, which increases its value when energy is harvested from the ambience. Obviously, $W(t) \leq A(t)$ must be satisfied for all $t \in [0, T]$ due to causality. Furthermore, we define the function $D(t)$ to indicate the minimal amount of energy that has to be consumed by time t in order to avoid *energy miss* caused by storage overflow, *i.e.*, mathematically, we have

$$D(t) = \max(0, A(t) - E_{\max}), \quad t \in [0, T]. \tag{7}$$

In other words, if at time instance t_o, the relation $W(t_\mathrm{o}) < D(t_\mathrm{o})$ stands, then not all the energy available from the environment at that time instance can be captured by the node. As a result, the value of $A(t)$ for $t \in [t_\mathrm{o}, T]$ is reduced by $A(t_\mathrm{o}) - (W(t_\mathrm{o}) + E_{\max})$, which corresponds to the amount of missed energy. The function $D(t)$ should then be adjusted accordingly for $t \in [t_\mathrm{o}, T]$.

We impose no continuity requirement on $A(t)$ or $D(t)$, yet at a point of discontinuity on $A(t)$, let us denote it with t_d, we assume that $A(t_\mathrm{d}^+) - A(t_\mathrm{d}^-) < E_{\max}$, *i.e.*, there is no energy overflow caused by a very large instantaneous energy input.

D. Throughput maximization

The general form of the throughput maximization problem is given in the standard form of a control problem as

$$\begin{aligned} \max \quad & \int_0^T R \, \mathrm{d}t \\ \text{s.t.} \quad & \dot{W} = P, \\ & W \leq A, \quad W(0) = 0, \end{aligned} \tag{8}$$

where W serves as the state variable of the system and is initialized with 0. The optimization is on different system parameters for single-antenna and multi-antenna systems, which

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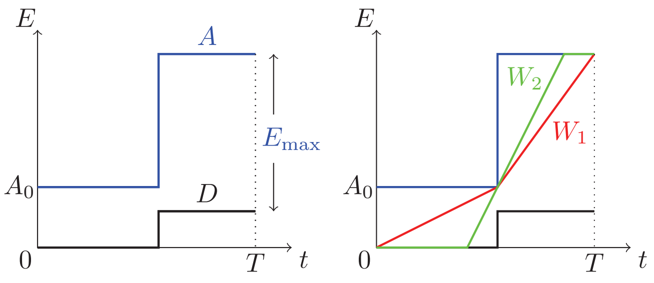

Figure 1. Boundary curves and admissible trajectories

will be discussed in detail in the next section. Note that we have omitted the time indices in (8) for simplicity and conformation with the convention, yet we should keep in mind that the optimization variables as well as the state variable are all functions of time defined on $[0, T]$, and the causality constraint $W \leq A$ must be satisfied for each $t \in [0, T]$. The optimal solution to (8) is also referred to as the *optimal control* of the receiver, and the corresponding state trajectory is denoted with W^*.

In order to maximize the throughput over $[0, T]$, all available energy should be used by time T and energy overflow should be avoided as much as possible, given that the data rate is an increasing function of the power consumption. This condition can be easily verified for both the single-antenna and the multi-antenna system models. Moreover, as we assume that the system parameters, such as the ADC resolution, can be adapted continuously and are not bounded from above, energy overflows can be avoided altogether. This means, $W \geq D$ has to be satisfied for all $t \in [0, T]$, for W to be optimal. Taken these optimality considerations into account, (8) can be equivalently formulated as

$$\begin{aligned} \max \quad & \int_0^T R\,\mathrm{d}t \\ \text{s.t.} \quad & \dot{W} = P, \\ & D \leq W \leq A, \\ & W(0) = 0,\ W(T) = A(T). \end{aligned} \tag{9}$$

Since the optimal state trajectory W^* is bounded by A from above and by D from beneath, we refer to the functions A and D as the *boundary curves*. Geometrically, W^* is a trajectory that lies between A and D, and adjoins the points $(0, 0)$ and $(T, A(T))$ on the time-energy graph. We term all non-decreasing curves that satisfy the above conditions as *admissible trajectories*, and illustrate the concept in Figure 1. An example of boundary curves A and D is shown on the left-hand side, and on the right-hand side, W_1 and W_2 are two admissible trajectories under this setup.

In the next two sections, we discuss optimal solutions to the *basic problem*, which has the simple form that the energy storage at $t = 0$ is $A_0 \leq E_{\max}$, and there is no energy arrival during $[0, T]$, *i.e.*, $A \equiv A_0$, $D \equiv 0$, $t \in [0, T]$. In correspondence to this, (8) without such additional restrictions on the boundary curves is referred to as the *general problem*.

III. Optimizations of the Single-antenna System

For single-antenna systems, the transmission bandwidth B and ADC resolution b are the two key parameters that determine the power consumption and the achievable rate at the receiver, according to (1) and (2). Besides the assumption that B and b can be adapted continuously in time at the transmitter and the receiver, respectively, we also assume that the transmitter and receiver are perfectly synchronized and are in full cooperation. This means, both parties share the information on channel states and energy arrivals, and know how the other is going to adapt its parameter before the transmission takes place. In this section we first discuss the separate optimizations of bandwidth and resolution, and then come to the joint optimization of the two parameters.

A. Optimizing bandwidth with fixed resolution

With fixed positive resolution, the power consumption $P^{(\mathrm{s})}$ becomes a linear function of B. The achievable rate $R^{(\mathrm{s})}$ in this case is a monotonically increasing and strictly concave function in B. Therefore, the throughput maximization (8) on bandwidth B is a convex optimization problem. Consequently, the *Pontryagin Maximum Principle* (PMP) [11][12], which is a first-order necessary condition for optimality of control problems, is sufficient in this case to determine the global optimum. The Hamiltonian of the basic problem is given by

$$H(B, W, \lambda) = -R^{(\mathrm{s})}(B) + \lambda \cdot P^{(\mathrm{s})}(B), \tag{10}$$

where λ is an auxiliary variable associated with the state equation. Since H does not explicitly depend on the state trajectory W, the co-state equation suggests that

$$\dot{\lambda}^* = -H_W(B^*, W^*, \lambda^*) = -H_W(B^*, \lambda^*) = 0, \tag{11}$$

i.e., λ^* is constant. Note that the overhead dot denotes the derivative of the function with respect to time, and the function with a variable as its subscript stands for the partial derivative of the function with respect to that variable. The PMP requires

$$\begin{aligned} H_B(B^*, \lambda^*) &= -R_B^{(\mathrm{s})}(B^*) + \lambda^* \cdot P_B^{(\mathrm{s})}(B^*) \\ &= 0, \qquad \forall t \in [0, T]. \end{aligned} \tag{12}$$

As $P_B^{(\mathrm{s})}$ is constant and $R_B^{(\mathrm{s})}$ is monotonically decreasing, the pointwise condition (12) can only be satisfied if B^* is constant. This means, with fixed ADC resolution, using constant transmission bandwidth leads to the maximal throughput, and the corresponding optimal trajectory W^* is a straight line segment with slope B^*. The end-point condition $W(T) = A_0$ gives the optimal bandwidth as

$$B^* = \frac{A_0}{T \cdot cN_0 \cdot 2^b}, \quad t \in [0, T]. \tag{13}$$

B. Optimizing resolution with fixed bandwidth

When transmission bandwidth is fixed, the receiver still has the degree of freedom to adapt the ADC resolution to achieve the maximum throughput. From a mathematical point of view, the main difference between this scenario and the one we discussed in the last subsection lies in the discontinuity of

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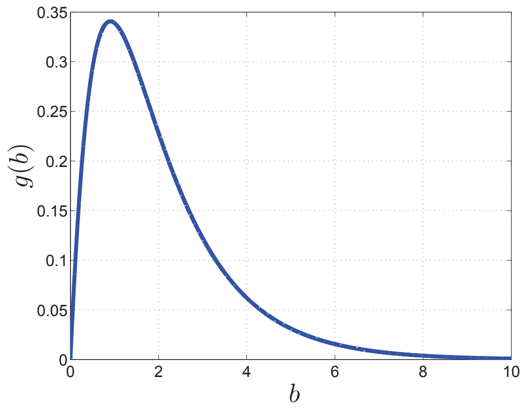

Figure 2. Function g as dependent on b

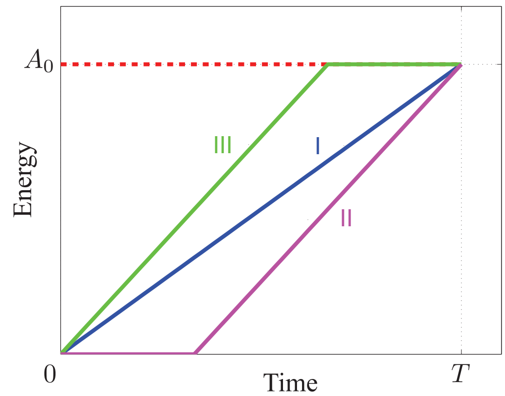

Figure 3. Optimal trajectories for resolution optimization

$P^{(\mathrm{s})}$ at zero resolution. With fixed B and variable resolution $b > 0$, the power consumption $P^{(\mathrm{s})}$ is strictly convex in b, while the achievable rate $R^{(\mathrm{s})}$ is strictly concave in b. Since for positive resolution the throughput maximization problem is convex, we obtain similar result as in the last subsection, that the optimal resolution b^* in the active mode is constant. As the sleep mode does not incur any power consumption, all available energy is consumed during the active period in order to achieve the maximum throughput. Therefore, to determine the value of b^*, the following one-dimensional optimization needs to be solved:

$$\begin{aligned} \max_{b>0} \quad & R^{(\mathrm{s})}(b) \cdot \frac{A_0}{P^{(\mathrm{s})}(b)} = A_0 \cdot \frac{R^{(\mathrm{s})}(b)}{P^{(\mathrm{s})}(b)} \\ \text{s.t.} \quad & \frac{A_0}{P^{(\mathrm{s})}(b)} \leq T. \end{aligned} \tag{14}$$

Let us define the function

$$g(b) \triangleq \frac{R^{(\mathrm{s})}(b)}{P^{(\mathrm{s})}(b)}, \quad b > 0, \tag{15}$$

which represents the number of received information bits per Joule of energy consumption. Obviously, g is an energy-efficiency measure, and the objective of (14) is equivalent to the maximization of g. We illustrate the shape of g in Figure 2, where all constant system parameters have been normalized.

In order to obtain the maximizer of g, we set its first-order derivative to 0 and obtain the relation

$$\left(R_b^{(\mathrm{s})} P^{(\mathrm{s})} - R^{(\mathrm{s})} P_b^{(\mathrm{s})}\right)(b) = 0. \tag{16}$$

Due to the concavity of $R^{(\mathrm{s})}$ and the convexity of $P^{(\mathrm{s})}$ in b, the function $R_b^{(\mathrm{s})} P^{(\mathrm{s})} - R^{(\mathrm{s})} P_b^{(\mathrm{s})}$ is found monotonically decreasing. Also taken into account the fact that

$$\left(R_b^{(\mathrm{s})} P^{(\mathrm{s})} - R^{(\mathrm{s})} P_b^{(\mathrm{s})}\right)(0^+) > 0, \tag{17}$$

we see that (16) has a unique solution, which we denote with b_0. The constraint in (15) provides a lower bound on b. When b_0 falls below this lower bound, then b^* must satisfy the inequality constraint with equality, because the function g decreases monotonically after it reaches its stationary point. To sum up, the optimal solution to (14) is given by

$$b^* = \begin{cases} b_0, & {P^{(\mathrm{s})}}^{-1}\left(\frac{A_0}{T}\right) < b_0, \\ {P^{(\mathrm{s})}}^{-1}\left(\frac{A_0}{T}\right), & \text{otherwise.} \end{cases} \tag{18}$$

The optimal receive strategy is thus the following: when ${P^{(\mathrm{s})}}^{-1}\left(\frac{A_0}{T}\right) < b_0$, the resolution b_0 should be employed until the available energy is exhausted. The receiver is turned into sleep mode for the rest of the time slot. The optimal trajectory W^* in this case is illustrated by Curve II and III in Figure 3. Note that the placement of the sleeping period has no influence on the achieved throughput, and therefore we say that Curve II and Curve III are *equivalent*. When ${P^{(\mathrm{s})}}^{-1}\left(\frac{A_0}{T}\right) \geq b_0$, the receiver should use the resolution ${P^{(\mathrm{s})}}^{-1}\left(\frac{A_0}{T}\right)$ for the whole time slot. Curve I in Figure 3 indicates the optimal state trajectory in this case. Put simply, ADC resolution lower than b_0 should be avoided altogether, and a duty cycle is required when the energy-to-time ratio $\frac{A_0}{T}$ is relatively low. To this end, we refer to b_0 as the *energy-efficient resolution*.

Plugging (1) and (2) into (16), we obtain the relation

$$\frac{2\gamma}{2^{2b_0} + \gamma} = \ln\left(\frac{1+\gamma}{1 + \gamma \cdot 2^{-2b_0}}\right). \tag{19}$$

Obviously, b_0 depends explicitly only on γ, and we show this dependency in Figure 4. In the asymptotic case, we have

$$\begin{aligned} b_0 &\to \frac{1}{2}\log_2 3 \approx 0.7925, \quad \gamma \to 0, \\ b_0 &\to \frac{1}{\ln 2} \approx 1.4427, \quad \gamma \to +\infty. \end{aligned} \tag{20}$$

C. Joint optimization of bandwidth and resolution

The rate function $R^{(\mathrm{s})}$, as given by (1), is concave in both B and b, yet it is not jointly concave in both variables, as the associated Hessian matrix is not always negative definite. This means, the convexity of the optimization problem (8) on both B and b can not be guaranteed, and thus we can not presume the optimality of a local maximum, or even the existence of an optimal solution. To this end, we perform a variable transform

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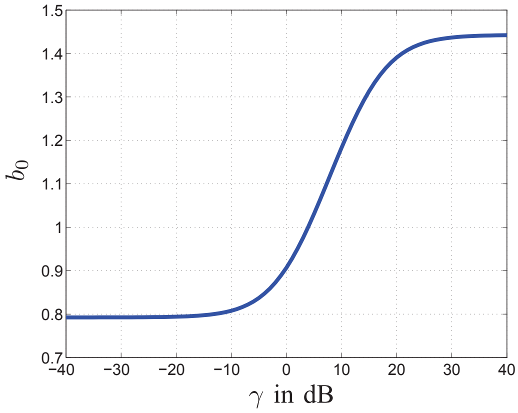

Figure 4. Energy-efficient resolution

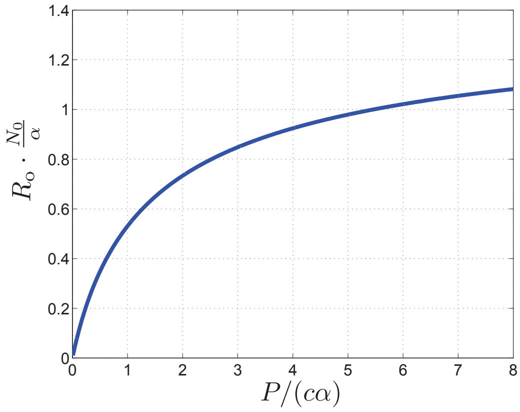

Figure 5. Concavity of R_o in P

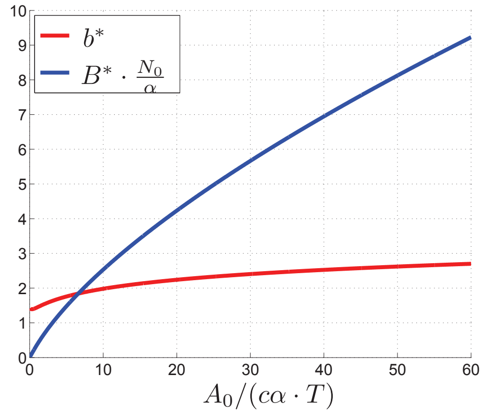

Figure 6. Optimized control parameters b^* and B^*

by writing the resolution b as a function of bandwidth B and power consumption P, which leads to

$$R^{(s)}(B,P) = \begin{cases} B \cdot \log_2\left(\dfrac{1+\gamma}{1+(cN_0)^2 \cdot \frac{\gamma B^2}{P^2}}\right), & P > 0, \\ 0, & P = 0, \end{cases}$$
$$P^{(s)}(B,P) = P. \tag{21}$$

The throughput maximization is then formulated as an optimization on B and P, both as functions of time. The change of variable from b to P provides us with an optimization of better structure and more tractability, as the constraint in (8) now involves only P but not B. To this end, we can decompose the maximization into inner, one-dimensional optimizations of B with given P, and the outer optimization of P as a function of time, which is mathematically given by

$$\max_{B,P} \int_0^T R^{(s)}(B,P)\,\mathrm{d}t = \max_P \int_0^T \max_B R^{(s)}(B,P)\,\mathrm{d}t \triangleq \max_P \int_0^T R_o(P)\,\mathrm{d}t. \tag{22}$$

Since $R^{(s)}$ is concave in B, the inner maximization of (22) is convex and can be solved by setting $R_B^{(s)}$ to zero. Numerical results show that the optimal bandwidth as dependent on the fixed power consumption, denoted with $B^*(P)$, satisfies $cN_0 \cdot B^*(P) < P$, *i.e.*, a positive ADC resolution is allowed. The resulting function $R_o = R^{(s)}(B^*(P), P)$ can be found continuous on $[0, +\infty)$ and strictly concave in P, as plotted in Figure 5 with the proper scaling. Consequently, the outer optimization on function P is convex, and by applying the PMP we have

$$P^* = \frac{A_0}{T}, \quad t \in [0, T], \tag{23}$$

which suggests that B^* is also constant. To this end, the joint optimization of bandwidth and resolution is reduced to solving a one-dimensional problem of finding $B^*\left(\frac{A_0}{T}\right)$.

In conclusion, the optimal control strategy for joint adaptation of B and b is simply to keep both parameters constant throughout the time slot of interest, and the optimal value B^* yields the maximal instantaneous data rate under power consumption $\frac{A_0}{T}$. The optimal solutions with different $\frac{A_0}{T}$ values are shown in Figure 6, with some constant scaling for generality. It can be seen that with increasing energy-to-time ratio, the increment in the optimal bandwidth B^* is much more significant than that of the optimal resolution b^*. Yet the latter also goes to infinity with infinitely large energy-to-time ratio.

IV. Optimizations of the Multi-antenna System

For the multi-antenna system where equal number of antennas at the transmitter and the receiver is required, we assume fixed transmission bandwidth, and focus on the optimizations of ADC resolution and the number of antennas. The same steps are followed as for single-antenna systems: we discuss first the separate optimizations of one parameter while the other is kept constant, and then deal with the joint optimization of the two. The methods used and the results obtained here resemble a lot of similarity with those from Section III.

The power consumption $P^{(m)}$, according to (3), is apparently linear in M and convex in b for $b > 0$. It can be verified with numerical experiments, that $R^{(m)}$ as expressed in (4) is strictly concave in b, given fixed $M > 0$, and is also strictly concave in M, given fixed $b > 0$. However, the joint concavity of the function in both b and M is hard to see. Therefore, for

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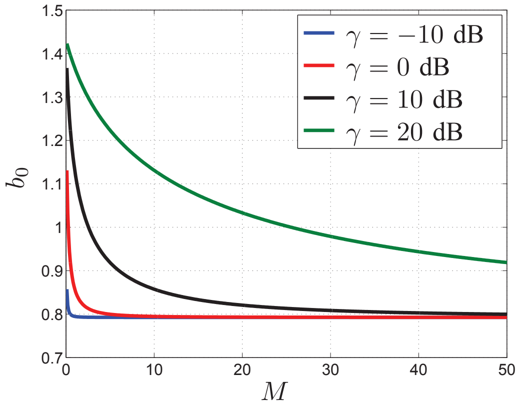

Figure 7. Energy-efficient resolution as dependent on M and γ

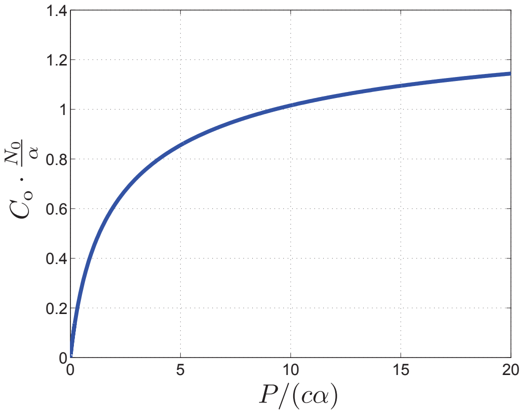

Figure 8. Concavity of C_o in P

the joint optimization we again need a variable transform to guarantee convexity of the problem.

A. Optimizing the number of antennas with fixed resolution

Given $b > 0$, the throughput maximization problem is convex in M. The PMP implies that for the basic problem, M^* is constant, and in order to utilize all energy, we have

$$M^* = \frac{A_0}{T \cdot cBN_0 \cdot 2^b}, \quad t \in [0, T]. \tag{24}$$

This is to say, the optimal control strategy is to use constantly the lowest number of antennas possible, such that all available energy can be exhausted at the end of the time slot.

B. Optimizing resolution with fixed number of antennas

Due to the discontinuity of $P^{(m)}$ at $b = 0$, duty cycles might be required for the optimal receive strategy. When the receiver is in active mode, *i.e.*, $b > 0$, using constant resolution maximizes the throughput due to the convex structure of the problem. Consequently, the basic throughput maximization problem is equivalent to the one-dimensional optimization

$$\max_{b>0} \; R^{(m)}(b) \cdot \frac{A_0}{P^{(m)}(b)} \quad \text{s.t.} \quad \frac{A_0}{P^{(m)}(b)} \leq T. \tag{25}$$

Define the function that measures bits per Joule by

$$\eta(b) \triangleq \frac{R^{(m)}(b)}{P^{(m)}(b)}. \tag{26}$$

For each given fixed M and γ, the function η has a unique maximum point, denoted with b_0 and refer to as the *energy-efficient* resolution, which satisfies

$$\left(R_b^{(m)} P^{(m)} - R^{(m)} P_b^{(m)}\right)(b_0) = 0. \tag{27}$$

In Figure 7, the variation of b_0 with different values of M and γ is illustrated. The monotonicity of b_0 as a function of M can be observed, suggesting that with more antennas employed, lower ADC resolution should be used for better energy efficiency. The asymptotic value of b_0, with $M \to +\infty$, is approximately 0.79 and appears to be independent of γ.

With the same idea we find when optimizing b for the single-antenna system, any resolution lower than $b_0(M)$ should not be employed for the sake of maximizing throughput. To be more specific, the optimal receive strategy is to use the resolution $b_0(M)$ for a time period of $\frac{A_0}{P^{(m)}(b_0(M))}$, when $P^{(m)}(b_0(M)) \geq \frac{A_0}{T}$, and turn the receiver into sleep mode for the rest of the time slot. When $P^{(m)}(b_0(M)) < \frac{A_0}{T}$, the resolution $\frac{A_0}{T \cdot cBN_0 \cdot M}$ should be used for the whole time slot.

C. Joint optimization of resolution and the number of antennas

Since the objective functional of (8) for the multi-antenna case is not always jointly concave in M and b, the optimization on the two variables is nonconvex. We try to circumvent this problem by performing a variable transform, and work with M and the power consumption P instead of with M and b. The dependency of b on P and M is given by

$$b = \log_2\left(\frac{P}{c \cdot BN_0 \cdot M}\right). \tag{28}$$

After the transformation, the constraint involves a single variable P. The joint optimization can now be divided into an inner optimization on M for given P, and an outer optimization on P, as described by the following equation.

$$\begin{aligned} \max_{P,M} \int_0^T R^{(m)}(P, M)\,\mathrm{d}t &= \max_P \int_0^T \max_M R^{(m)}(P, M)\,\mathrm{d}t \\ &\triangleq \max_P \int_0^T C_o(P)\,\mathrm{d}t \end{aligned} \tag{29}$$

The fulfilment of the inequality $M^*(P) < P$ can be verified via simulations, where $M^*(P)$ stands for the optimal solution to the inner optimization. Hence, a positive b can always be computed given P and $M^*(P)$. The dependency of C_o on P is shown in Figure 8, where the parameter $\gamma = 0$ dB has been taken. We observe that C_o is continuous on $[0, +\infty)$ and is strictly concave in P, which means that the outer optimization problem in (29) is convex. As a result, we find the optimal power consumption function as

$$P^* = \frac{A_0}{T}, \quad t \in [0, T], \tag{30}$$

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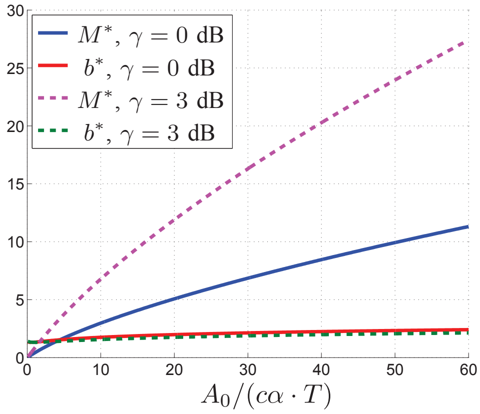

Figure 9. Optimized control parameters b^* and M^*

and correspondingly, the optimal number of antennas M^* and the optimal ADC resolution b^* are all constant. Their variations due to different $\frac{A_0}{T}$ values are shown in Figure 9.

V. General Throughput Maximization Problem

Recall that the basic problem with $A \equiv A_0$, $D \equiv 0$ is a special case of the throughput maximization problem (8). Based on the results we obtain in Sections III and IV, the *critical slope* based construction algorithm proposed in [7] can be applied for finding the optimal state trajectory of (8).

Depending on the optimal solution to the basic problem, the optimizations we have discussed can be classified into two categories. The first category, including:

- optimizing B with fixed b for the single-antenna system,
- optimizing jointly B and b for the single-antenna system,
- optimizing M with fixed b for the multi-antenna system,
- optimizing jointly M and b for the multi-antenna system,

has in common that the optimal solution gives constant power consumption over $[0, T]$. The other two optimizations, namely

- optimizing b with fixed B for the single-antenna system,
- optimizing b with fixed M for the multi-antenna system,

belong to the second category for which there exists a lower bound on the optimal power consumption, thus giving rise to possible duty cycles in the optimal control strategy.

For the first category, the optimal state trajectory W^* is unique, and the following *optimality criterion* is established: *there do not exist any two points on W^* that can be connected by a distinct admissible straight line segment.* This statement can be easily verified considering the optimal state trajectory of the basic solution. As direct consequence of the optimality criterion, it has been found that *the points at which W^* changes slope are either on A or on D*. Moreover, *the slope change at a point on D is negative, whereas the slope change at a point on A is positive.* These criteria suggest that in the construction of W^*, it is crucial to determine the slope of each segment according to the intersection point with one of the boundary curves. To this end, at any admissible point, the *critical slope* is defined as the slope, above which renders the line segment to intersect with A first, and below which renders the line segment to intersect with D first. Starting from the origin of the time-energy graph, line segments with critical slopes can be found iteratively, which go until intersection with the boundary curves. Such a construction method gives a unique trajectory that satisfies the optimality criterion, and is therefore the optimal one W^* [13]. As an easy example, the trajectory W_1 in Figure 1 is optimal for this category of optimizations with the given boundary curves. It has two straight line segments with respective critical slopes, and after the intersection point with A, the slope increases.

Unlike those in the first category, optimizations in the second category have infinitely many optimal solutions. Yet they are all equivalent in the sense that they lead to the same maximal throughput. In the construction of one of the optimal trajectories, the critical slope based construction procedure serves as the first step. As a second step, the slope of each segment of the constructed trajectory is compared to the power consumption corresponding to the energy-efficient value of the control variable, in our case the energy-efficient ADC resolution b_0. If the critical slope is smaller, the current segment needs to be replaced by a horizontal line segment, which corresponds to the sleeping period, and a straight line segment with its slope equal to the power consumption corresponding to the energy-efficient control, *e.g.*, $P(b_0)$. A third step might be necessary, where the obtained trajectory is made smoother by adjusting the placement of the sleeping period, such that the number of switches between active mode and sleep mode could be reduced. The trajectory W_2 in Figure 1 illustrates one of the possible W^* for the second category of optimizations, where the energy-efficient power consumption is higher than the slope of the second line segment of W_1. With other systems or system parameters, the energy-efficient power consumption might be smaller than the slope of the first line segment of W_1. In that case W_1 is the unique optimal state trajectory.

VI. Simulation Results

For simulations we set $T = 1000$ seconds, $\frac{N_0}{2} = 174$ dBm/Hz, $c = 10^9$, $E_{\max} = 1$ Joule, and assume discrete, statistically independent energy arrivals. The interarrival time follows the exponential distribution with a mean of 60 seconds. The amount of energy in each arrival is uniformly distributed on $[0, 0.4 \times E_{\max}]$. We first illustrate the optimal state trajectories for the two categories of optimizations in Figure 10 and Figure 11. The trajectory consists of 4 segments, marked with different colors, and the optimized values of the control variables are shown above each segment.

We give the simulation 1000 runs and show the average optimal throughput for both systems in Figure 12 and Figure 13, as dependent on the channel gain α. Depending on the value of the fixed parameter, the performance gap between the separate optimizations and the joint optimization could be larger or smaller, in different channel gain regimes.

VII. Conclusions

The achievable rate and power consumption of a receiver are dependent on and related by the transmission bandwidth

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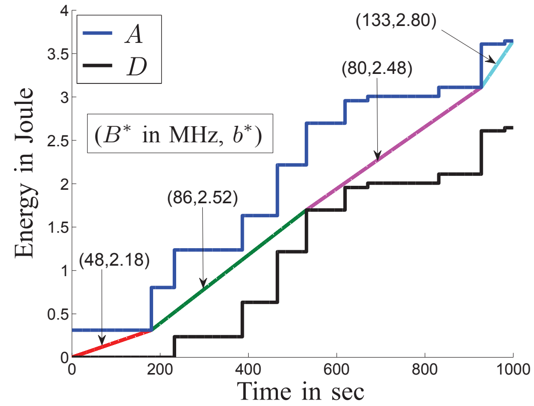

Figure 10. W^* for the joint optimization of B and b of a single-antenna system, $\alpha = -100$ dBW

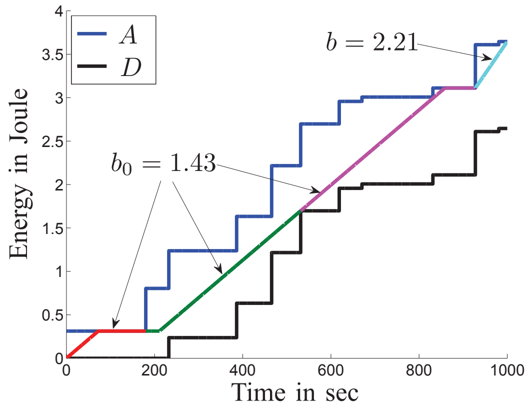

Figure 11. W^* for optimized b of a single-antenna system, $\alpha = -60$ dBW, $B = 200$ MHz

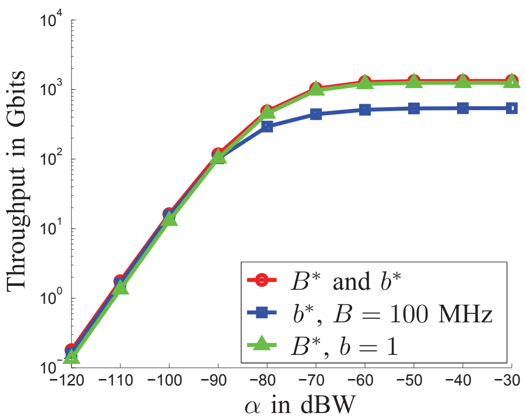

Figure 12. Average throughput of single-antenna system

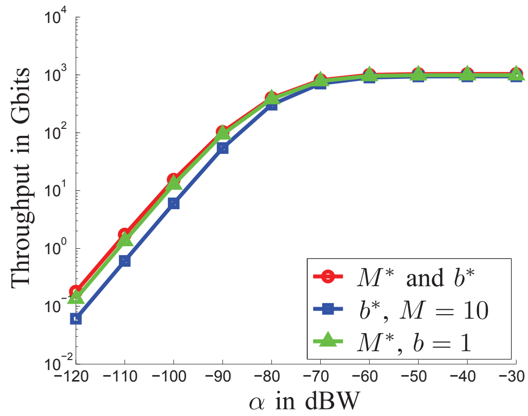

Figure 13. Average throughput of multi-antenna system, $B = 100$ MHz

B, the ADC resolution b, and the number of antennas M it employs. We consider in this work the throughput maximization problem at an energy harvesting receiver, where the optimal temporal adaptation of B and/or b for the single-antenna system, and of M and/or b for the multi-antenna system have been explored. For the optimization of b with the other parameter fixed, there exists an energy-efficient resolution b_0 below which the ADC should not work. For the other optimizations, the control variables are always constant functions determined by the desired power consumption.

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