Interference Temperature Analysis in a Large Scale Cellular System

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***Abstract*—We consider the downlink of a cellular network with cooperative multiple antenna base stations and single antenna user terminals. Based on a limited number of measured interference channels and an interference prediction strategy, we address the implementation of interference temperatures in a large scale system. With interference temperatures, each base station has linear constraints on its transmit covariance matrix. These constraints limit the intercell interference at mobile devices in other cells to a certain level.**

I. Introduction

In this paper, we concentrate on interference coordination, where each MD is served only by its associated *base station* (BS). But, the BSs try to mitigate the *intercell interference* (ICI) they create at *mobile devices* (MDs) in other cells. Interference coordination includes many known techniques next to interference temperatures, where the caused interference is limited with linear constraints to the transmit covariances [1]. For network sum rate optimizations, this results in a weighted sum rate optimization with multiple linear constraints and can be solved with the minimax duality from [2] as shown in [3].

Other interference coordination techniques include interference alignment, where the BSs try to spare subspaces of the received signals from interference [4]. Interference pricing is an iterative technique for finding a balance between the signal power at the served MDs and the interference caused at other MDs [5]. Interference leakage power based optimizations try to maximize the ratio between the received power at the served MDs and the caused interference power at other MDs [6].

It is still an open topic, if cellular systems will benefit from cooperation. The gain of cooperation is very promising, but the drawbacks are often neglected. With interference alignment, the possible degrees of freedom can be given to characterize the behavior of networks with ICI [4]. But the costs for measuring all the additional channels and communicating the strategic information between the transmitters are not taken into account. A fair comparison to networks without cooperation requires to include the benefits and the drawbacks. We will take a look at the gain of the interference temperature technique, where we reduce the transmission efficiency according to the number of measured interference channels. We used this method already to give an upper bound on interference coordination [7]. Similar upper bounds are given for network MIMO, where each MD can be served by multiple BSs, in [8] and [9].

As we limit the number of measured interference channels, we have to care about the interference over the unknown interference channels. When the BSs update their precoding simultaneously, the problem of interference awareness arises [10]. The precoder optimizations cannot be based on the interference power during the transmission, as this information will not be available before the precoding is selected. To match the cost function of the optimization and the performance measure, we consider the expectation of the rates with respect to the ICI [11]. The partitioning of the ICI into interference over measured channels and over unknown channels, and the described strategies for both types of ICI allow us to implement interference coordination in a large scale cellular system.

II. System Model

A cellular network with 19 three faced sites and, therefore, 57 BSs is considered. Each BS serves the MDs of the hexagonal shaped cell it covers. An MD in the set $\mathcal{K}$ of all MDs is specified by the tuple $(b,k) \in \mathcal{K}$, where $b \in \mathcal{B}$ identifies the BS in the set $\mathcal{B}$ of all BSs and $k \in \mathcal{K}_b$ the MD in the set $\mathcal{K}_b$ of all MDs in the cell of BS b. The wrap-around method is used to treat all cells equally and the channels are found with the 3GPP MIMO urban macro cell model [12].

A. Supported Rate

In this paper, each BS has N antennas and serves $K = |\mathcal{K}_b|$ single antenna MDs, respectively. The vectors $\boldsymbol{h}_{\hat{b},b,k} \in \mathbb{C}^N$ contain the channel coefficients between the antennas of BS $\hat{b}$ and MD (b,k). With $(\bullet)^{\mathrm{T}}$ and $(\bullet)^{\mathrm{H}}$ the transposition and the complex conjugate transposition are denoted, respectively. With the pre-log factor ξ, which takes the signaling overhead into account, the achievable, normalized rate of MD (b,k) in bits per channel usage can be expressed as

$$r_{b,k} = \xi \log_2 \left(1 + \frac{|\boldsymbol{h}_{b,b,k}^{\mathrm{T}} \boldsymbol{p}_{b,k}|^2}{\sum_{\hat{k}>k} |\boldsymbol{h}_{b,b,k}^{\mathrm{T}} \boldsymbol{p}_{b,\hat{k}}|^2 + \theta_{b,k} + \sigma_{b,k}^2} \right), \quad (1)$$

$$\theta_{b,k} = \sum_{\hat{b} \in \mathcal{B} \setminus b} \boldsymbol{h}_{\hat{b},b,k}^{\mathrm{H}} \boldsymbol{Q}_{\hat{b}} \boldsymbol{h}_{\hat{b},b,k}, \quad (2)$$

where $\boldsymbol{p}_{b,k} \in \mathbb{C}^N$ is the beamforming vector of BS b intended for serving MD (b,k). The sum transmit covariance matrix of BS b can be found as $\sum_k \boldsymbol{p}_{b,k} \boldsymbol{p}_{b,k}^{\mathrm{H}} = \boldsymbol{Q}_b \in \mathbb{C}^{N \times N}$. All BSs have to satisfy the transmit power constraint $\mathrm{tr}(\boldsymbol{Q}_b) \le P$.

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$\sum_{\hat{k}>k} |\boldsymbol{h}_{b,b,k}^{\mathrm{T}} \boldsymbol{p}_{b,\hat{k}}|^2$ is the variance of the intracell interference with dirty paper coding. Only the signals from BS b to MDs with an index $\hat{k} > k$ contribute to the intracell interference at MD (b, k). The signals to MDs with an index $\hat{k} < k$ can be considered as known interference at the transmitter. Costa showed that interference, which is known to the transmitter, does not reduce the rate of the transmission [13].

$\theta_{b,k}$ is the variance of the received ICI at MD (b, k), which will be investigated further in the following sections. The worst case Gaussian noise approximation $\sigma_{b,k}^2 = \sigma_\eta^2 + \sigma_{\mathrm{od},b,k}^2 + \theta_{\mathrm{bg}}$ is the sum variance of the thermal noise σ_η^2, the *channel state information* (CSI) outdating $\sigma_{\mathrm{od},b,k}^2$, and the background ICI θ_{bg}. The background ICI models the BSs further away than the closest 57 BSs for a signal variance per transmit antenna of P/N.

B. Outdating of the Channel Measurements

Due to outdating, the measured channel $\boldsymbol{h}_{\hat{b},b,k}$ differs from the actual channel $\hat{\boldsymbol{h}}_{\hat{b},b,k}$ at a different time and frequency instance. With $\mathrm{E}[\bullet]$ for the expectation and $\|\bullet\|_2$ for the Euclidean norm, respectively, the measurement error variance

$$\sigma_{\mathrm{e},\hat{b},b,k}^2 = \mathrm{E}\left[\|\hat{\boldsymbol{h}}_{\hat{b},b,k} - \boldsymbol{h}_{\hat{b},b,k}\|_2^2\right] = \sigma_{\mathrm{e}}^2 \, \mathrm{E}\left[\|\hat{\boldsymbol{h}}_{\hat{b},b,k}\|_2^2\right] \tag{3}$$

can be found as the variance of the actual channel $\mathrm{E}\left[\|\hat{\boldsymbol{h}}_{\hat{b},b,k}\|_2^2\right]$ scaled down with the normalized mean error variance of the outdating σ_{e}^2. This outdating variance is calculated from the correlation between channels at different time and frequency instances and depends on the block length for which the channel is assumed to be constant as described in [7].

The CSI outdating variance at MD (b, k) is computed from the sum over all measurement error variances, which are associated with measured channels that link a BS with MD (b, k). Not all channels are assumed to be measured and only measured channels are used for serving users and mitigating interference. With a signal variance per transmit antenna of $\frac{P}{N}$, the CSI outdating variance reads as

$$\sigma_{\mathrm{od},b,k}^2 = \frac{P}{N} \sum_{\hat{b} \in \mathcal{C}_{b,k}} \sigma_{\mathrm{e},\hat{b},b,k}^2, \tag{4}$$

where $\mathcal{C}_{b,k}$ is the set containing all BSs, which measured the channel to MD (b, k).

$\hat{\boldsymbol{h}}_{\hat{b},b,k}$ is generated from the channel model in the simulations. The according measured channel is set to

$$\boldsymbol{h}_{\hat{b},b,k} = \sqrt{1 - \sigma_{\mathrm{e}}^2} \hat{\boldsymbol{h}}_{\hat{b},b,k}. \tag{5}$$

With this selection, the variance of the generated channel is preserved in the sum of the variance of the measured channel and the measurement error variance. If the channel is not assumed to be measured, the generated channel will be used directly in the simulations. Pilot contamination and other errors during the channel measurements are neglected. Except for the outdating, the channel measurements are assumed to be perfect.

C. Signaling Overhead

We employ a time division duplex system, where the reciprocity of the propagation channels is exploited. The channels are measured in the uplink and the gained information is then utilized in the downlink. The number of channels a BS can measure is equivalent to the length of the pilot sequences $T_{\text{pilots}} = K + L$. Each BS can measure the channels to its own K MDs and L interference channels, additionally. With the block length T_{block} between two channel measurements and neglecting other overhead contributions, we find the efficiency of the signaling as $\xi = \frac{T_{\text{block}} - (K+L)}{T_{\text{block}}}$. $T_{\text{block}} - (K + L)$ are the symbols remaining for data transmission.

III. Prediction of the Unknown Interference

The BSs are assumed to calculate their beamforming in a distributed manner, but all BSs update their beamforming at the same time. The ICI at each MD will change the moment the beamforming is applied and cannot be known in advance. Therefore, the BSs compute their beamforming based on an assumed ICI $\tilde{\theta}_{b,k}$. The BSs are blind to the ICI change and take the risk, that the actual ICI $\hat{\theta}_{b,k}$ increases and the MD cannot decode the transmitted symbols or that $\hat{\theta}_{b,k}$ decreases and valuable resources are wasted [10]:

$$\hat{r}_{b,k} = \begin{cases} \tilde{r}_{b,k} = r_{b,k}|_{\theta_{b,k}=\tilde{\theta}_{b,k}}, & \text{for } \tilde{\theta}_{b,k} \geq \hat{\theta}_{b,k} \\ 0, & \text{for } \tilde{\theta}_{b,k} < \hat{\theta}_{b,k}. \end{cases} \tag{6}$$

If the transmission is successful, the rate $\tilde{r}_{b,k}$ will only depend on the assumed ICI and not on the actual ICI.

Most optimizations in the literature utilize the expectation of the ICI or an ICI realization from a previous step as the assumed ICI. This results in a mismatch between the cost function of the optimization and the actual performance measure. To counteract this problem, we consider the expectation of the rate with respect to the random ICI variance $\hat{\Theta}_{b,k}$ [11]:

$$E_{\hat{\Theta}_{b,k}}[\hat{r}_{b,k}] = \tilde{r}_{b,k} F_{\hat{\Theta}_{b,k}}\left(\tilde{\theta}_{b,k}\right), \tag{7}$$

where $F_{\hat{\Theta}_{b,k}}(\theta)$ is the *cumulative distribution function* (CDF) of $\hat{\Theta}_{b,k}$. By taking the expectation, the cost function and the performance measure become the same.

To perform the described procedure, the CDFs of the ICI at each associated MD need to be available at the serving BS. The CDFs can be approximated with long term measurements at the MDs. To reduce the feedback, a probability distribution can be matched to the measurements at the MDs. Then, only the parameters of the distribution function need to be transmitted to the BS. It could also be possible to estimate a rough CDF directly based on the channel measurements. This would not require any additional measurements and feedback for the CDF. The actual ICI cannot be known in advance in the regarded scenario, because the transmit covariances at all BS change at the same time, while the channels are assumed to be constant for the block of transmission. The CDFs of the ICI for this scenario are different from the CDFs, where the channels also change over time. Although the later scenario with changing channels is more realistic,

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especially for CDFs, which are approximated with long term measurements, the scenario with fixed channels is assumed to reduce the simulation complexity. The cost of acquiring the CDFs is neglected in the following.

Additionally, it is possible to measure $\hat{\theta}_{b,k}$ with a second pilot, which removes the uncertainty in the ICI afterwards but increases the overhead [7].

IV. Upper Bound to Interference Coordination

The expectation of the sum rate of all MDs in the system is used as cost function. With L measured interference channels per BS, an upper bound to interference coordination can be given, as described in [7]. The ICI can be split into the interference over the measured interference channels and the interference over the unknown interference channels.

The result of a joint optimization with respect to all beamforming vectors R_{coop} is always smaller than the result of the upper bound R_{upper}, where all measured interference channels are set to zero:

$$R_{\text{coop}} \leq R_{\text{upper}} = \max_{\left\{\boldsymbol{p}_{b,k}, \tilde{\theta}_{b,k} | \forall (b,k) \in \mathcal{K}\right\}} \sum_{(b,k)\in\mathcal{K}} \mathrm{E}_{\breve{\Theta}_{b,k}}\left[\breve{r}_{b,k}\right],$$
$$\text{s.t. } \operatorname{tr}(\boldsymbol{Q}_b) \leq P \ \forall b, \tag{8}$$
$$\breve{r}_{b,k} = \hat{r}_{b,k}\big|_{\hat{\theta}_{b,k}=\breve{\theta}_{b,k}} \tag{9}$$
$$\breve{\theta}_{b,k} = \hat{\theta}_{b,k}\Big|_{\boldsymbol{h}_{\hat{b},b,k}=\boldsymbol{0}\ \forall \hat{b}\in\mathcal{C}_{b,k}\setminus b}, \tag{10}$$

$\mathcal{C}_{b,k}$ is the set containing all BSs, which know the channel to the MD (b,k). If some interference channels are set to zero, the statistics of the ICI will chance accordingly.

By using (7) in problem (8), the rate of an MD only depends on the precoding of the associated BS and the CDF of the ICI. Therefore, the joint problem breaks down into a distributed problem at each BS, respectively:

$$R_{\text{sum},b} = \max_{\left\{\boldsymbol{p}_{b,k}, \tilde{\theta}_{b,k} | \forall (b,k) \in \mathcal{K}_b\right\}} \sum_{(b,k)\in\mathcal{K}_b} \tilde{r}_{b,k} F_{\breve{\Theta}_{b,k}}\left(\tilde{\theta}_{b,k}\right),$$
$$\text{s.t. } \operatorname{tr}(\boldsymbol{Q}_b) \leq P. \tag{11}$$

The optimal solution of problem (11) can be found with an alternating optimization. For a fixed assumed ICI, a weighted sum rate optimization has to be solved. For fixed precoders, the optimal assumed ICI can be found with numerical algorithms [11].

The sum rate is improved in every step and the algorithm converges. The behavior of the sum rate per cell along the iterations can be seen in Figure 1 for a low mobility scenario with a common MD device speed of $v = 3\,\mathrm{km/h}$, which is used in all following figures. The actually supported sum rate $\sum_{(b,k)\in\mathcal{K}_b} r_{b,k}|_{\theta_{b,k}=\breve{\theta}_{b,k}}$ can only be reached, if the actual ICI $\breve{\theta}_{b,k}$ values are known through a second pilot. It can be seen that the expected sum rate $R_{\text{sum},b}$ and the actually achieved sum rate $\sum_{(b,k)\in\mathcal{K}_b} \breve{r}_{b,k}$ converge to almost the same value. The remaining error can be explained with the finite resolution and imprecision of the CDFs, which are approximated with Monte Carlo simulations. The actually acheived sum rate is calculated with (6) and includes the cases of outage. At odd iteration steps the transmit beamforming is optimized and at even iterations steps the optimal assumed interference is calculated.

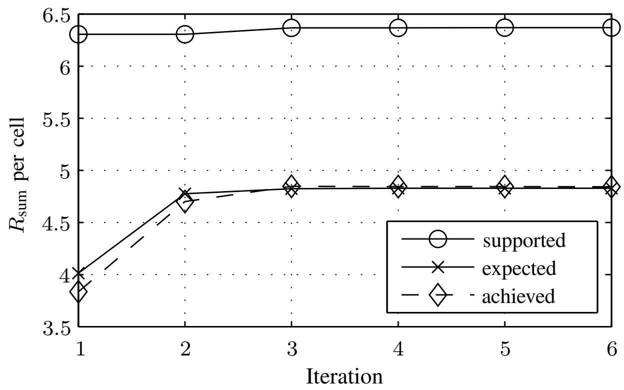

Figure 1. Convergence, $K = 4$, $N = 4$, $L = 0$, $T_{\text{block}} = 120$

This upper bound is not achievable, because the costs of nulling the L interference channels per BS are neglected. As it was shown in [7], this loose upper bound strongly limits the possible gain of cooperation. The upper bound is plotted in Figure 2 for different values of L. All curves ascend in the beginning for longer block lengths, because the efficiency of the system improves. At some point, each curve starts to descend, because the outdating of the channel degrades the possible rates. The upper bound reaches higher values for increasing L in the beginning, because more and more interference is suppressed. But, increasing L above the optimal value of $L = 35$ reduces the possible rates. The eliminated interference cannot compensate the reduced efficiency due to the increased overhead.

The possible improvements through cooperation are substantial in this scenario, but vanish for increasing mobility. The upper bound does not show how much improvement is possible with cooperation. It gives a loose limit to the possible improvement. It can be seen, that the gain from $L = 0$ to $L = 10$ is almost of factor two. But the additional gain of the optimum with $L = 35$ is not much higher.

For this upper bound, we do not make any restrictions to the backbone network, which connects all BSs. Despite the amount of exchanged information between the BSs, this upper bound will hold, if each MD is only served by its associated BS.

V. Lower Bounds to Interference Coordination

At the first glance, it seems that interference coordination demands from the BSs to achieve two contradicting goals. On the one hand, the BSs try to serve their own MDs with all their degrees of freedom. On the other hand, the BSs try to spend their degrees of freedom to limit the interference they create at MDs in other cells. The previously described upper bound achieves both goals perfectly. The measured interference channels are set to zero. No interference is caused

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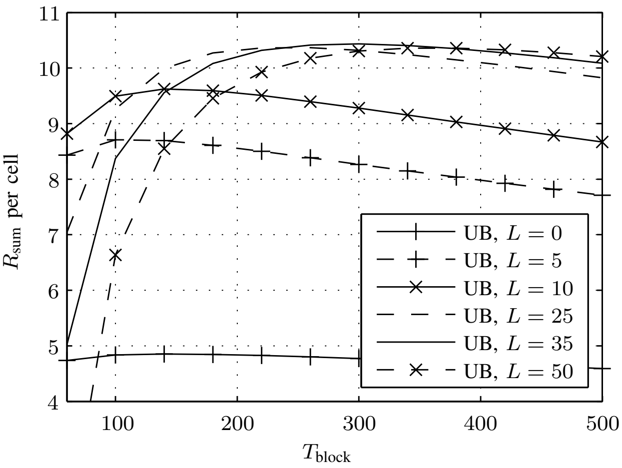

Figure 2. Upper bound, $K = 4$, $N = 4$

over these channels and not a single degree of freedom has to be spend for mitigating the interference. The BS can still use all their degrees of freedom to serve their own MDs.

With interference coordination the BSs pursue the higher goal of maximizing the sum rate of the network. This will inherently maximize the sum rate of the cell. A compromise between serving the associated MDs and limiting the ICI at other MDs has to be found. The optimal compromise will always be at least as good as if only one of the goals is followed. For a given L, two lower bounds can be defined by looking at the two extremes.

A. No Cooperation

Obviously, no cooperation is always an option. The BSs use all their degrees of freedom to serve their own MDs egoistically. This rate can be found with problem (11), where no channels are set to zero. L has still an influence on the efficiency ξ.

B. Zero Forcing of the Interference

The other extreme is to use the degrees of freedom to set the interference caused over measured channels to zero. The number of degrees of freedom at a BS are assumed to be equal to the number of antennas N. If the number of measured interference channels at a BS is smaller than the number of antennas, $N - L$ degrees of freedom will be left for serving the associated MDs. This can be implemented by transmitting in the nullspace of the measured interference channels as described in [14]. The transmit covariance matrices are decomposed:

$$\boldsymbol{Q}_b = \boldsymbol{V}_b \hat{\boldsymbol{Q}}_b \boldsymbol{V}_b^{\mathrm{H}}, \tag{12}$$

where $\boldsymbol{V}_b \in \mathbb{C}^{N,N-L}$ is the nullspace of the measured interference channels. The lower bound rates can be found by plugging (12) into the sum rate optimization (11) and optimizing the remaining reduced transmit covariance matrices $\hat{\boldsymbol{Q}}_b \in \mathbb{C}^{N-L,N-L}$.

If L is equal to or larger than N, zero forcing of the interference will force the BS to shut down.

C. Simulation

In Figure 3 the lower bounds are plotted over the block-length T_{block}. For $L = 0$, the lower bound with zero forcing of the interference (LB ZF) and the lower bound without cooperation (LB NC) are equal. These curves are also equal to the upper bound with $L = 0$. For increasing L, the LB NC decreases according to the overhead efficiency. The LB ZF increases at first, but degrades at $L = 3$ dramatically and hits zero for $L > 4$.

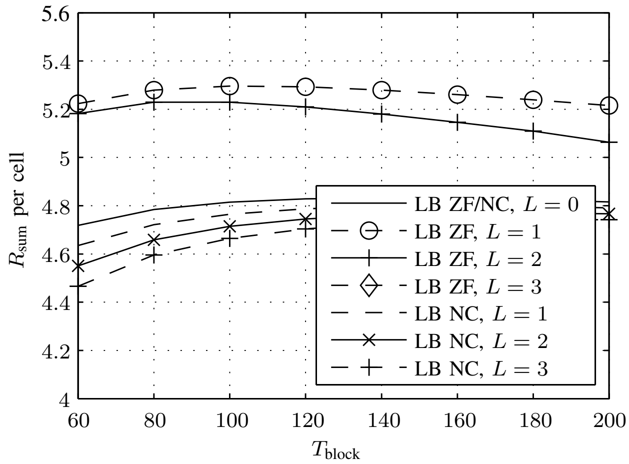

Figure 3. Lower Bounds, $K = 4$, $N = 4$, $L = 0$, $T_{\text{block}} = 120$

VI. Interference Temperatures

It is not possible to set the interference channels to zero as described for the upper bound and it is not advisable to ignore or zero force the interference as discussed for the lower bounds. But, the BSs can limit the ICI they cause over the L measured interference channels to a certain level, the interference temperatures [1].

The problem at each BS can be formulated as a weighted sum rate optimization with linear constraints:

$$\begin{aligned} &\max_{\left\{\boldsymbol{p}_{b,k}, \tilde{\theta}_{b,k} | \forall (b,k) \in \mathcal{K}_b\right\}} \sum_{(b,k)\in\mathcal{K}_b} \tilde{r}_{b,k} F_{\tilde{\Theta}_{b,k}}\left(\tilde{\theta}_{b,k}\right), \\ &\text{s.t. } \operatorname{tr}(\boldsymbol{Q}_b) \le P \\ &\qquad \theta_{b,\hat{b},\hat{k}} \le \gamma_{b,\hat{b},\hat{k}}, \forall (\hat{b},\hat{k}) \in \mathcal{L}_b, \end{aligned} \tag{13}$$

where $\mathcal{L}_b$ is the set of MDs, which are not associated to BS b, but the channel between these MDs and BS b is known. $\theta_{b,\hat{b},\hat{k}} = \boldsymbol{h}^{\mathrm{H}}_{b,\hat{b},\hat{k}} \boldsymbol{Q}_b \boldsymbol{h}_{b,\hat{b},\hat{k}}$ is the interference BS b causes at MD $(\hat{b},\hat{k})$ and $\gamma_{b,\hat{b},\hat{k}}$ is the corresponding interference limit.

For a fixed assumed ICI, the weighted sum rate maximization with multiple linear constraints can be solved via the minimax duality presented in [2] as shown in [1], [3]. The optimization of the assumed ICI is independent of the linear constraints and can be solved as discussed before.

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A. Sum Rate Maximization with a Single Linear Constraint

To get an insight to the sum rate maximization with linear constraints, the minimax duality is revised for a single constraint. The weighted sum rate maximization over the beamforming vectors and for a fixed assumed ICI in the *downlink* (DL),

$$\max_{\{\boldsymbol{p}_{b,k}|\forall(b,k)\in\mathcal{K}_b\}} \sum_{(b,k)\in\mathcal{K}_b} \tilde{r}_{b,k}^{\mathrm{DL}} F_{\check{\Theta}_{b,k}}\left(\tilde{\theta}_{b,k}\right),$$
$$\text{s.t. } \operatorname{tr}(\boldsymbol{\Phi}_{b,0}\boldsymbol{Q}_b) \le \gamma_{b,0}, \tag{14}$$

with the single constraint $\operatorname{tr}(\boldsymbol{\Phi}_{b,0}\boldsymbol{Q}_b) \le \gamma_{b,0}$ is regarded, where $\boldsymbol{\Phi}_{b,0}$ is the full-rank constraint direction matrix and $\gamma_{b,0}$ the constraint limit. For $\boldsymbol{\Phi}_{b,0}$ equal to the identity matrix and $\gamma_{b,0} = P$, this constraint is equal to the power constraint. The rate (1) is rewritten as

$$\tilde{r}_{b,k}^{\mathrm{DL}} = \xi \log_2\left(\frac{\sum_{\hat{k}\ge k} |\boldsymbol{h}_{b,b,k}^{\mathrm{T}}\boldsymbol{p}_{b,\hat{k}}|^2 + \phi_{b,k}}{\sum_{\hat{k}> k} |\boldsymbol{h}_{b,b,k}^{\mathrm{T}}\boldsymbol{p}_{b,\hat{k}}|^2 + \phi_{b,k}}\right), \tag{15}$$

where the ICI and the noise are combined in the substitute noise $\phi_{b,k} = \tilde{\theta}_{b,k} + \sigma_{b,k}^2$.

By switching the role of the transmitter and the receivers, the dual *uplink* (UL) problem can be formulated [2]:

$$\max_{\{q_{b,k}|\forall(b,k)\in\mathcal{K}_b\}} \sum_{(b,k)\in\mathcal{K}_b} \tilde{r}_{b,k}^{\mathrm{UL}} F_{\check{\Theta}_{b,k}}\left(\tilde{\theta}_{b,k}\right),$$
$$\text{s.t. } \sum_{(b,k)\in\mathcal{K}_b} \phi_{b,k} q_{b,k} \le \gamma_{b,0}, \tag{16}$$

where the evaluated CDFs of the ICI are regarded as fixed weights for the optimization. The rate of an MD in the UL reads as

$$\tilde{r}_{b,k}^{\mathrm{UL}} = \log_2\left(\frac{\left|\sum_{\hat{k}\le k} \boldsymbol{h}_{b,b,\hat{k}}^{*}\boldsymbol{h}_{b,b,\hat{k}}^{\mathrm{T}} q_{b,\hat{k}} + \boldsymbol{\Phi}_{b,0}\right|}{\left|\sum_{\hat{k}< k} \boldsymbol{h}_{b,b,\hat{k}}^{*}\boldsymbol{h}_{b,b,\hat{k}}^{\mathrm{T}} q_{b,\hat{k}} + \boldsymbol{\Phi}_{b,0}\right|}\right). \tag{17}$$

Not only did the role of the transmitter and the receivers change, also the role of the constraint and the noise changed. Problem (16) can be solved with standard weighted sum rate maximization codes [15]. The optimal transmit covariances in the uplink can then be transformed to the optimal transmit covariances in the downlink.

B. Sum Rate Maximization with Multiple Linear Constraints

The weighted sum rate maximization with an arbitrary number of constraints $X = |\mathcal{X}_b|$ is given as

$$\max_{\{\boldsymbol{p}_{b,k}|\forall(b,k)\in\mathcal{K}_b\}} \sum_{(b,k)\in\mathcal{K}_b} \tilde{r}_{b,k}^{\mathrm{BC}} F_{\check{\Theta}_{b,k}}\left(\tilde{\theta}_{b,k}\right),$$
$$\operatorname{tr}(\boldsymbol{\Phi}_{b,x}\boldsymbol{Q}_b) \le \gamma_{b,x}\ \forall x \in \mathcal{X}_b, \tag{18}$$

where the constraint matrices $\boldsymbol{\Phi}_{b,x}$ do not need to be of full rank, individually. But the concatenation of all constraint matrices needs to be of full rank.

The constraints in problem (13) can be mapped to the constraints in problem (18). For index $x = 0$, $\boldsymbol{\Phi}_{b,0}$ is set to the identity matrix and $\gamma_{b,0} = P$. For indices $x > 0$, the constraint matrix is $\boldsymbol{\Phi}_{b,x} = \boldsymbol{h}_{b,x}\boldsymbol{h}_{b,x}^{\mathrm{H}}$, where x is taken from the set $\mathcal{L}_b$ of MDs, which profit from the interference limitation. Therefore, the set of all constraint indices $\mathcal{X}_b = 0 \cup \mathcal{L}_b$ has $X = L + 1$ elements.

Problem (18) is relaxed to the weighted sum rate maximization

$$\max_{\{\boldsymbol{p}_{b,k}|\forall(b,k)\in\mathcal{K}_b\}} \sum_{(b,k)\in\mathcal{K}_b} \tilde{r}_{b,k}^{\mathrm{BC}} F_{\check{\Theta}_{b,k}}\left(\tilde{\theta}_{b,k}\right),$$
$$\text{s.t. } \operatorname{tr}\left(\left(\sum_{x\in\mathcal{X}} \lambda_x \boldsymbol{\Phi}_{b,x}\right)\boldsymbol{Q}_b\right) \le \sum_{x\in\mathcal{X}} \lambda_x \gamma_{b,x} = \varLambda, \tag{19}$$

where the constraints are combined with a weighted sum to a single constraint. For fixed weights $\lambda_x \ge 0$, problem (19) can be solved as described before. If the individual constraints of problem (18) are met, the sum constraint of problem (19) will hold as well and the sum constraint cannot be stricter than all the individual constraints together. Therefore, problem (18) is an upper bound to problem (19). By taking a look at the Karush–Kuhn–Tucker conditions of problem (18) and (19), it can be shown that the upper bound becomes tight for the weights $\hat{\lambda}_x$, which minimize the sum [3].

Scaling all λ_x jointly does not change problem (19). Therefore, $\sum_{x\in\mathcal{X}} \lambda_x\gamma_{b,x}$ can be set to any positive value $\varLambda$. The minimization of the sum rate with respect to the λ_x reads as

$$\min_{\{\lambda_x\ge 0|\forall X\in\mathcal{X}_b\}} \sum_{(b,k)\in\mathcal{K}_b} \tilde{r}_{b,k}^{\mathrm{MAC}} F_{\check{\Theta}_{b,k}}\left(\tilde{\theta}_{b,k}\right),$$
$$\text{s.t. } \sum_{x\in\mathcal{X}_b} \lambda_x \gamma_{b,x} = \varLambda. \tag{20}$$

An iterative algorithm is proposed to tackle Problem (20). The individual λ_x are optimized independently while keeping the others fixed:

$$\check{\lambda}_x^i = \min_{\{\lambda_x\ge 0\}} \sum_{(b,k)\in\mathcal{K}_b} \tilde{r}_{b,k}^{\mathrm{MAC}} F_{\check{\Theta}_{b,k}}\left(\tilde{\theta}_{b,k}\right),$$
$$\text{s.t. } \sum_{x\in\mathcal{X}_b} \lambda_x \gamma_{b,x} = \varLambda, \tag{21}$$

which can be solved with standard numerical techniques. The solution to $\hat{\lambda}_x^i$ at iteration i is an update of the previous solution with the newly found $\check{\lambda}_x^i$, which needs to be scaled back to the sum constraint:

$$\hat{\lambda}_x^i = (1-d)\hat{\lambda}_x^{i-1} + d\check{\lambda}_x^i \frac{\varLambda}{\sum_{x\in\mathcal{X}_b} \lambda_x\gamma_{b,x}}, \tag{22}$$

where $0 \le d \le 1$ is the step size parameter.

The optimal solution to problem (18) can be found by alternatingly solving problem (19) for fixed λ_x and problem (20) for fixed precoding vectors. The convergence speed depends on the number of the constraints. For scenarios with only one additional constraints, no more than 4 iteration steps are needed. But, the number of required iterations rapidly grows with the number of constraints.

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C. Selection of $\gamma_{b,\hat{b},\hat{k}}$

The interference constraint limits are selected heuristically. $\gamma_{b,\hat{b},\hat{k}}$ is set to a scaled version of the mean ICI plus noise, which remains at the MD $(\hat{b}, \hat{k})$ after the ICI over the measured channels is subtracted.

$$\gamma_{b,\hat{b},\hat{k}} = \alpha \left(\mathrm{E}\left[\check{\theta}_{\hat{b},\hat{k}} \right] + \sigma^2_{\hat{b},\hat{k}} \right) \tag{23}$$

For the two extreme values of the scaling $\alpha \geq 0$, the solution of the described algorithm converges to the solution with the earlier described lower bounds:

1) $\gamma_{b,\hat{b},\hat{k}} = 0$, $\forall(\hat{b}, \hat{k}) \in \mathcal{L}_b$.: If all interference constraint limits are set to zero, the rates will converge to the solution, where the BSs transmit in the nullspace of the interference channels. For $L \geq N$ all rates are zero.

2) $\gamma_{b,\hat{b},\hat{k}} \to \infty$, $\forall(\hat{b}, \hat{k}) \in \mathcal{L}_b$.: By setting the interference constraint limits to a very large value, the solution of the sum rate maximization with multiple linear constraints is equal to the lower bound, where the ICI is simply ignored.

The influence of the common scaling α can be seen in Figure 4 for $L = \{1, \ldots, 5\}$. The lower bound with zero forcing of the interference is marked with an $\times$ at the left border of the plot for the different selections of L and the lower bound which ignores the interference is marked with an $\circ$ at the right border of the plot. All curves converge to the corresponding solution of the lower bounds for $\alpha \to 0$ and $\alpha \to \infty$. For any L, the optimal value for α can be found around $\alpha = 1$, where the optimal α is larger for larger L. The influence of choosing α correctly increased for increasing L. For $L = 1$, the gain of the best α compared to the lower bounds is rather small, while the correct selection of α for $L = 5$ is crucial. Although the lower bounds decrease for increasing L, the optimal value increases. But, the gain from $L = 3$ to $L = 5$ is very small. A further increase of L will result in worse sum rates at the optimal α.

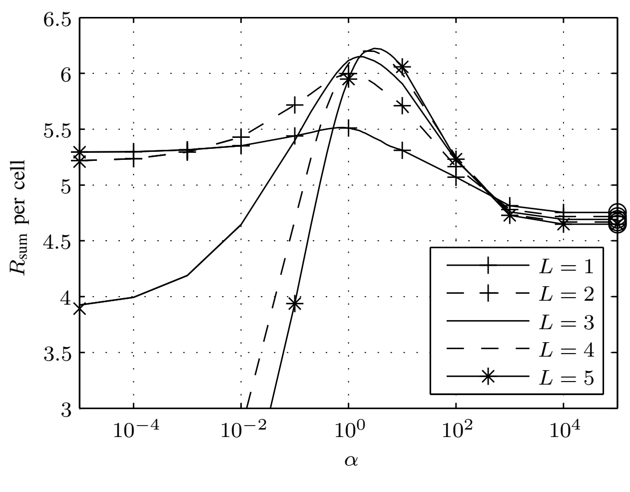

Figure 4. Lower Bounds, $K = 4$, $N = 4$, $L = 0$, $T_{\text{block}} = 120$

The described selection of $\gamma_{b,\hat{b},\hat{k}}$ requires that the BSs communicate. The BSs, which measured the interference channel to an MD, need to know the ICI, which remains at the regarded MD after the ICI over the measured channels is subtracted. The CDFs need to be adapted to the changed ICI situation.

VII. Conclusion

Our main contribution is the combination of the weighted sum rate maximization with multiple linear constraints with the restriction to a limited set of measured interference channels and the interference prediction. This allows us to use the interference temperature methods in a large scale system.

We compare the interference temperature algorithm with an upper bound and two lower bounds. A heuristic is given for selecting the interference temperatures.

References

[1] Hoon Huh, H.C. Papadopoulos, and G. Caire, "Multiuser MISO Transmitter Optimization for Intercell Interference Mitigation," *IEEE Transactions on Signal Processing*, vol. 58, no. 8, pp. 4272–4285, Aug. 2010.

[2] Wei Yu, "Uplink-Downlink Duality via Minimax Duality," *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 361–374, Feb. 2006.

[3] Lan Zhang, Rui Zhang, Ying-Chang Liang, Yan Xin, and H.V. Poor, "On Gaussian MIMO BC-MAC Duality With Multiple Transmit Covariance Constraints," *IEEE Transactions on Information Theory*, vol. 58, no. 4, pp. 2064–2078, Apr. 2012.

[4] V.R. Cadambe and S.A. Jafar, "Interference Alignment and Degrees of Freedom of the k-User Interference Channel," *IEEE Transactions on Information Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.

[5] Jianwei Huang, R.A. Berry, and M.L. Honig, "Distributed Interference Compensation for Wireless Networks," *Selected Areas in Communications, IEEE Journal on*, vol. 24, no. 5, pp. 1074–1084, May 2006.

[6] M. Sadek, A. Tarighat, and A.H. Sayed, "A Leakage-Based Precoding Scheme for Downlink Multi-User MIMO Channels," *IEEE Transactions on Wireless Communications*, vol. 6, no. 5, pp. 1711–1721, May 2007.

[7] H.H. Brunner, M.T. Ivrlac, and J.A. Nossek, "Upper Bound to Interference Coordination with Channel State Information Outdating," in *EW 2011, 11th European Wireless Conference*, Apr. 2011, pp. 1–5.

[8] Hoon Huh, A.M. Tulino, and G. Caire, "Network MIMO with Linear Zero-Forcing Beamforming: Large System Analysis, Impact of Channel Estimation, and Reduced-Complexity Scheduling," *IEEE Transactions on Information Theory*, vol. 58, no. 5, pp. 2911–2934, May 2012.

[9] M.K. Karakayali, G.J. Foschini, R.A. Valenzuela, and R.D. Yates, "On the Maximum Common Rate Achievable in a Coordinated Network," in *ICC '06, IEEE International Conference on Communications*, June 2006, vol. 9, pp. 4333–4338.

[10] M. T. Ivrlač and J. A. Nossek, "Intercell-Interference in the Gaussian MISO Broadcast Channel," in *GLOBECOM '07, IEEE Global Telecommunications Conference*, Nov. 2007.

[11] H.H. Brunner and J.A. Nossek, "Handling Unknown Interference in Cellular Networks with Interference Coordination," in *WSA 2012, International ITG Workshop on Smart Antennas*, Mar. 2012, pp. 133–139.

[12] "Spatial Channel Model for Multiple Input Multiple Output (MIMO) Simulations," Tech. Rep. 25.996 V9.0.0, 3rd Generation Partnership Project, Technical Specification Group Radio Access Network, Dec. 2009.

[13] M. Costa, "Writing on Dirty Paper (Corresp.)," *IEEE Transactions on Information Theory*, vol. 29, no. 3, pp. 439–441, May 1983.

[14] H.H. Brunner and J.A. Nossek, "Mitigation of Intercell Interference without Base Station Cooperation," in *WSA 2010, International ITG Workshop on Smart Antennas*, Feb. 2010, pp. 1–7.

[15] R. Hunger, D.A. Schmidt, M. Joham, and W. Utschick, "A General Covariance-Based Optimization Framework Using Orthogonal Projections," in *SPAWC 2008, IEEE 9th Workshop on Signal Processing Advances in Wireless Communications*, July 2008, pp. 76–80.

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