A Block Markov Encoding Scheme for Broadcasting Nested Message Sets

Shirin Saeedi, Vinod Prabhakaran and Suhas Diggavi
shirin.saeedi@tum.de, vinodmp@tifr.res.in, suhasdiggavi@ucla.edu

A Block Markov Encoding Scheme

- Extend the channel by introducing a "virtual resource" in $E_j$.
- Rate Pair $(R_1, R_2) = (0, 1)$ is achievable over this extended channel using the basic linear superposition coding. E.g., for $W_1^*/w_1 = [1, 2, 3, 4]$, the following code achieves rate pair $(R_1 = 1, R_2 = 0)$:

$$X_1 = [0, 1, 2, 3, 4]$$

- Can we use the above code to achieve rate pair $(R_1 = 1, R_2 = 3)$ over the original channel?
- We utilize the virtual signal using a block Markov encoding scheme.
- In the 1<sup>st</sup> block, encoding done as suggested by the code in [1]. To provide receiver 1 and the private receivers with the information of $X_1(j_1)$ (as promised by the virtual resource in $E_1$), we use information symbol $w_1^{j_1}$ in the next block, to convey $X_2(j_1)$. This symbol is ensured to be decoded at receiver 1 and the private receivers and it indeed emulates $E_1$.

- In the 2<sup>nd</sup> block, we simply encode $X_1(j_1)$ and directly send it to receiver 1 and the private receivers.
- Decoding is via backward decoding.
- This encoding technique can be applied more generally and results in an achievable rate-region which is strictly larger than those addressed in [1, 2].

**Theorem 1.** The rate pair $(R_1, R_2)$ is achievable if there exist parameters $T, S \subseteq I_1$, such that

- Relaxed non-negativity constraints: $\sum_{\gamma \in T} X_1(j) \geq 0$ and $\sum_{\gamma \in T} X_2(j) \geq 0$.
- Decodability at public receiver $i$: $R_i \leq \sum_{j \in I_1 : j \in \gamma} X_1(j) + \sum_{j \in I_1 : j \notin \gamma} X_2(j)$.
- Decodability at private receiver $p$: $R_2 \leq \sum_{j \in I_1 : j \in \gamma} X_1(j) + \sum_{j \in I_1 : j \notin \gamma} X_2(j)$.

The General BC

Similarly, superposition coding can be enhanced via a block Markov scheme and achieve the following rate-region:

**Theorem 2.** The rate pair $(R_1, R_2)$ is achievable if there exist parameters $S \subseteq I_1$, and auxiliary random variables $Y_T$, $0 \leq T \leq 2^{|S|}$ (with joint pmf $P_{Y_T}(y_T) = \prod_{t \in T} P_{Y_t}(y_t, y_t)$), such that

- Relaxed non-negativity constraints: $\sum_{\gamma \in S} X_1(j) \geq 0$ and $\sum_{\gamma \in S} X_2(j) \geq 0$.
- Decodability at public receiver $i$: $R_i \leq \sum_{\gamma \in S} X_1(j) + \sum_{\gamma \in S} X_2(j)$.
- Decodability at private receiver $p$: $R_2 \leq \sum_{\gamma \in S} X_1(j) + \sum_{\gamma \in S} X_2(j)$.

References:

