Rate Balancing in the Vector BC with Erroneous CSI at the Receivers

Michael Joham*, Andreas Gründinger*, Adriano Pastore†, Javier R. Fonollosa†, and Wolfgang Utschick*
* Asscociate Institute for Signal Processing, Technische Universität München, 80290 München, Germany
Email: {joham,gruendinger,utschick}@tum.de
† Signal Processing and Communications Group, Universitat Politècnica de Catalunya, 08034 Barcelona, Spain
Email: {adriano.pastore,javier.fonollosa}@upc.edu

***Abstract*—For the vector broadcast channel (BC), the case of erroneous channel state information (CSI) at the receiver is considered. Employing a well established lower bound for the mutual information with Gaussian signaling, a rate balancing problem is formulated where the rates of the different users are maximized under a transmit power constraint, but the rates of the different users have fixed ratios. A duality w.r.t. the signal-to-interference-and-noise ratio (SINR) between the vector BC with erroneous receiver CSI and an appropriately constructed vector multiple access channel (MAC) is established. Based on the observation that an interference function can be defined in the dual vector MAC that is standard, an iterative algorithm can be found for an appropriately formulated quality-of-service (QoS) optimization that is used for solving the balancing problem.**

I. Introduction

For the case of perfect CSI, the vector BC is well researched and understood. Based on the SINR duality of [1], the sum capacity based on dirty paper coding (DPC, [2]) could be identified. Additionally, the optimality of DPC to reach the whole capacity region was shown in [3]. The rate balancing problem for the vector BC with error-free CSI was investigated in [4]. In [5], a QoS formulation with DPC was considered where the transmit power is minimized under constraints for the minimum SINRs of the different users. The QoS problem with linear precoding was solved in [6]. Additionally, an algorithm to obtain the solution for the balancing of the SINRs under a transmit power constraint was proposed in [6]. In [7], the framework of interference functions from [8] was applied to solve the QoS problem with linear precoding.

In contrast, the case of erroneous CSI at the receiver side is considered in this paper where we concentrate on the design of linear precoding in this scenario. The CSI is obtained via a training channel and the channel estimate is fed back from the receivers to the transmitter. However, we do not explicitly formulate the estimation step. Instead, we use a general CSI error model with the assumption that the estimation error is independent of the estimate, that holds for minimum mean square error estimation of Gaussian channels for example.

The theoretical basis of this paper is the lower bound for the mutual information introduced in [9] for single-input single-output (SISO) channels with erroneous receiver CSI. In [10], [11], this bound has been generalized to the multiple-input multiple-output (MIMO) case. The main conclusion from this lower bound is that the CSI errors lead to additional noise terms which depend on the statistical properties of the channel estimation errors and, interestingly, on the beamformers used for data transmission. Due to these properties, the application of existing schemes developed for the case of error-free CSI is not straightforward.

Our contributions are as follows. First, we construct a vector MAC that is dual to the considered vector BC in the sense that values for the mutual information lower bound can be achieved in the BC iff they are achievable in the dual MAC. Thus, the BC problems can be solved equivalently in the dual MAC. Second, we construct an interference function in the dual MAC whose entries constitute the SINRs which are connected to the mutual information lower bounds via a one-to-one map. As it can be shown that the interference function is standard, the framework of [8], [7] can also be applied to the case of imperfect CSI at the receivers. Third, the rate balancing problem, i.e., the ratios of the mutual information lower bounds and the rate targets of the different users are maximized and set to the same value using the given transmit power, is solved for the case of imperfect CSI at the receivers.

II. System Model & Assumptions

We consider the transmission over a vector BC with N transmit antennas and K single-antenna receivers. The data signal $s_k \sim \mathcal{N}_{\mathbb{C}}(0,1)$ for user k is linearly precoded with the beamformer $\boldsymbol{t}_k \in \mathbb{C}^N$. The resulting transmitted signal $\boldsymbol{x} = \sum_{k=1}^K \boldsymbol{t}_k s_k \in \mathbb{C}^N$ propagates over the vector channel $\boldsymbol{h}_k^{\mathrm{H}} \in \mathbb{C}^{1\times N}$ and is perturbed by the noise $n_k \sim \mathcal{N}_{\mathbb{C}}(0,\sigma_k^2)$. Therefore, the received signal of user k reads as

$$y_k = \boldsymbol{h}_k^{\mathrm{H}} \sum_{i=1}^K \boldsymbol{t}_i s_i + n_k.$$

We do not make the standard assumption that the channel state $\boldsymbol{h}_k, k = 1,\ldots,K$, is available error-free. Instead, the k-th receiver estimates $\boldsymbol{h}_k$, e.g., via a common pilot channel, and feeds back the estimate to the transmitter. For decoding, the knowledge of $\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{t}_k$ is necessary at the k-th receiver, that can be obtained with a dedicated pilot channel for example. Nevertheless, we assume that the knowledge about both, $\boldsymbol{h}_k$ and $\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{t}_k$, is erroneous at the receiver.

After the feedback, the transmitter has obtained the quantized channel estimates $\hat{\boldsymbol{h}}_k, k = 1,\ldots,K$. Additionally, we assume that the transmitter knows the covariance matrix $\boldsymbol{C}_{\tilde{\boldsymbol{h}}_k} = \mathrm{E}[\tilde{\boldsymbol{h}}_k \tilde{\boldsymbol{h}}_k^{\mathrm{H}}]$ of the zero-mean CSI error $\tilde{\boldsymbol{h}}_k = \boldsymbol{h}_k - \hat{\boldsymbol{h}}_k$.

Since we assume that the channel estimates $\hat{\boldsymbol{h}}_k$ are instantly available at the transmitter, the beamforming vectors $\boldsymbol{t}_k$ can be designed as functions of the matrix $\hat{\boldsymbol{H}} \triangleq [\hat{\boldsymbol{h}}_1, \ldots, \hat{\boldsymbol{h}}_K]$. Similarly, $\boldsymbol{T} = [\boldsymbol{t}_1, \ldots, \boldsymbol{t}_K]$ denotes the precoding matrix, i.e., the collection of beamforming vectors $\boldsymbol{t}_k, k = 1, \ldots, K$.

III. Mutual Information Lower Bound

The optimal signaling for the case of erroneous CSI at the receiver is unknown even for the SISO case [9]. Additionally, the standard expression for the mutual information for the Gaussian vector BC with Gaussian signaling, e.g., $\mathrm{E}[\log_2(1+\mathrm{SINR}_k)]$ for user k, is not applicable for erroneous receiver CSI. Thus, we have to resort to a lower bound that is valid for Gaussian signaling (e.g., [10], [11], [12])

$$\begin{aligned} I(s_k; y_k) &= h(s_k) - h(s_k|y_k) \\ &\geq \mathrm{E}\left[\log_2\left(1 + \sigma_{\mathrm{eff},k}^{-2} \left|\hat{\boldsymbol{h}}_k^{\mathrm{H}} \boldsymbol{t}_k\right|^2\right)\right] \triangleq \mathrm{E}\left[\underline{I}_k^{\mathrm{BC}}\right] \end{aligned} \tag{1}$$

with the variance $\sigma_{\mathrm{eff},k}^2$ of the effective noise n_{eff} received by user k, $n_{\mathrm{eff}} = n_k + \sum_{i=1}^K \tilde{\boldsymbol{h}}_k^{\mathrm{H}} \boldsymbol{t}_i s_i + \sum_{i=1, i\neq k}^K \hat{\boldsymbol{h}}_k^{\mathrm{H}} \boldsymbol{t}_i s_i$, viz., the signal portions in the received signal y_k that are either due to noise, interference, or channel estimation errors. The variance of the effective noise can be written as

$$\sigma_{\mathrm{eff},k}^2 = \sigma_k^2 + \sum_{i=1}^K \boldsymbol{t}_i^{\mathrm{H}} \boldsymbol{C}_{\tilde{\boldsymbol{h}}_k} \boldsymbol{t}_i + \sum_{i=1, i\neq k}^K \left|\hat{\boldsymbol{h}}_k^{\mathrm{H}} \boldsymbol{t}_i\right|^2.$$

The bound (1) is found by assuming that $s_k \sim \mathcal{N}_{\mathbb{C}}(0,1)$, bounding $h(s_k|y_k)$ with $\mathrm{E}[\log_2(\pi\,\mathrm{e}\,\mathrm{E}[|s_k - \mathrm{E}[s_k|y_k]|^2|y_k])]$, replacing $\mathrm{E}[s_k|y_k]$ by the linear minimum mean square error estimate $\mathrm{E}[s_k y_k^*](\mathrm{E}[|y_k|^2])^{-1} y_k$, and applying Jensen's inequality.

Note that the effective noise n_{eff} of user k, and thus $\sigma_{\mathrm{eff},k}^2$, depends on the respective beamformer $\boldsymbol{t}_k$. With the definition of the "signal-to-interference-and-noise ratio"

$$\mathrm{SINR}_k^{\mathrm{BC}} = \sigma_{\mathrm{eff},k}^{-2} \left|\hat{\boldsymbol{h}}_k^{\mathrm{H}} \boldsymbol{t}_k\right|^2 \tag{2}$$

the lower bound is expressible in the usual form, i.e., $\underline{I}_k^{\mathrm{BC}} = \log_2(1 + \mathrm{SINR}_k^{\mathrm{BC}})$ and $I(s_k; y_k) \geq \mathrm{E}[\log_2(1 + \mathrm{SINR}_k^{\mathrm{BC}})]$. In this paper, we use $\underline{I}_k^{\mathrm{BC}}$ as the figure of merit for user k.

IV. Mutual Information Upper Bound

Following a similar procedure as in [13] to obtain an upper bound $I(s_k; y_k) \leq \mathrm{E}\big[\overline{I}_k^{\mathrm{BC}}\big]$, we expand the mutual information into the difference of entropies

$$\begin{aligned} I(s_k; y_k) &= h(y_k) - h(y_k|s_k) \\ &\leq \log_2(\pi\,\mathrm{e}\,\sigma_{y_k}^2) - h(y_k|s_1, \ldots, s_K) \triangleq \mathrm{E}\big[\overline{I}_k^{\mathrm{BC}}\big]. \end{aligned}$$

The inequality results on one hand from upper bounding $h(y_k)$ by the differential entropy of a Gaussian variable with the same variance $\sigma_{y_k}^2 = \mathrm{E}[|y_k|^2]$, and on the other hand, by lower bounding $h(y_k|s_k)$ through additional conditioning. In fact, when conditioned on $s_1, \ldots, s_K$, the variable y_k is Gaussian, whereby we obtain an analytic upper bound $\overline{I}_k^{\mathrm{BC}}$. After some algebra, we find

$$\overline{I}_k^{\mathrm{BC}} = \underline{I}_k^{\mathrm{BC}} + \Delta_k^{\mathrm{BC}}. \tag{3}$$

If the $\tilde{\boldsymbol{h}}_k$ are Gaussian, the non-negative bound difference Δ_k^{BC} is given by

$$\Delta_k^{\mathrm{BC}} = \mathrm{E}\log_2 \frac{\sigma_k^2 + \sum_{i=1}^K \boldsymbol{t}_i^{\mathrm{H}} \boldsymbol{C}_{\tilde{\boldsymbol{h}}_k} \boldsymbol{t}_i + \sum_{i\neq k}^K \left|\hat{\boldsymbol{h}}_k^{\mathrm{H}} \boldsymbol{t}_i\right|^2}{\sigma_k^2 + \sum_{i=1}^K \boldsymbol{t}_i^{\mathrm{H}} \boldsymbol{C}_{\tilde{\boldsymbol{h}}_k} \boldsymbol{t}_i |s_i|^2}, \tag{4}$$

where the expectation is over all inputs $s_i, \forall i$. Using methods similar to [14], one can show that

$$\Delta_k^{\mathrm{BC}} \leq \frac{1}{\ln(2)} \cdot \theta\left(1 + \frac{\sum_{i\neq k}^K \left|\hat{\boldsymbol{h}}_k^{\mathrm{H}} \boldsymbol{t}_i\right|^2}{\sum_{i=1}^K \boldsymbol{t}_i^{\mathrm{H}} \boldsymbol{C}_{\tilde{\boldsymbol{h}}_k} \boldsymbol{t}_i^{\mathrm{H}}}, \frac{\sum_{i=1}^K \boldsymbol{t}_i^{\mathrm{H}} \boldsymbol{C}_{\tilde{\boldsymbol{h}}_k} \boldsymbol{t}_i^{\mathrm{H}}}{\sigma_k^2}\right)$$

where $\theta(x,y) = \ln(1+xy) - \mathrm{e}^{\frac{1}{y}}\,\mathrm{E}_1\!\left(\frac{1}{y}\right)$, with the exponential integral $\mathrm{E}_1(z) = \int_z^\infty t^{-1}\,\mathrm{e}^{-t}\,\mathrm{d}t$. It is shown in [14] that $\theta(x,y) < \gamma + \ln(x)$ with the Euler-Mascheroni constant $\gamma \approx 0.577$. Note in particular that when only user k is active, i.e., $\boldsymbol{t}_i = \boldsymbol{0}, \forall i \neq k$, we have the single-user bound $\overline{\Delta}_{k,\mathrm{su}}^{\mathrm{BC}} < \gamma$.

V. Problem Formulation

We consider the balancing optimization which is well established for the case of error-free CSI and transpose it to the case of erroneous CSI. The average rates relative to the quality-of-service (QoS) requirements, expressed as minimum rates $\bar{\tau}_k, k = 1, \ldots, K$, are balanced under an average transmit power constraint, i.e.,

$$\max_{\beta, \hat{\boldsymbol{H}} \mapsto \boldsymbol{T}(\hat{\boldsymbol{H}})} \beta \qquad \text{s.t.:} \quad \begin{aligned} &\mathrm{E}\left[\underline{I}_k^{\mathrm{BC}}\right] \geq \beta\bar{\tau}_k, \quad \forall k \\ &\mathrm{E}\left[\|\boldsymbol{T}(\hat{\boldsymbol{H}})\|_{\mathrm{F}}^2\right] \leq \bar{P}_{\mathrm{tx}} \end{aligned} \tag{5}$$

where $\|\boldsymbol{T}(\hat{\boldsymbol{H}})\|_{\mathrm{F}}^2 = \sum_{k=1}^K \|\boldsymbol{t}_k(\hat{\boldsymbol{H}})\|_2^2$ denotes the squared Frobenius norm of the precoding matrix $\boldsymbol{T}$, and thus, the average total transmit power is $\mathrm{E}[\|\boldsymbol{T}(\hat{\boldsymbol{H}})\|_{\mathrm{F}}^2]$. Problem (5) belongs to the class of variational optimization problems, where the optimum is taken over a family of $\mathbb{C}^{N\times K} \to \mathbb{C}^{N\times K}$ functions that map the current CSI $\hat{\boldsymbol{H}}$ to the precoding matrix $\boldsymbol{T}$. It turns out that (5) can be equivalently formulated as

$$\begin{aligned} &\max_{\hat{\boldsymbol{H}} \mapsto P_{\mathrm{tx}}(\hat{\boldsymbol{H}}), \hat{\boldsymbol{H}} \mapsto \boldsymbol{\tau}(\hat{\boldsymbol{H}})} \mathrm{E}\big[\beta^\star(P_{\mathrm{tx}}(\hat{\boldsymbol{H}}), \boldsymbol{\tau}(\hat{\boldsymbol{H}}))\big] \\ &\text{s.t.:} \quad \mathrm{E}\big[P_{\mathrm{tx}}(\hat{\boldsymbol{H}})\big] \leq \bar{P}_{\mathrm{tx}}, \quad \mathrm{E}\big[\beta^\star \tau_k(\hat{\boldsymbol{H}})\big] \geq \mathrm{E}\big[\beta^\star\big]\bar{\tau}_k, \ \forall k \end{aligned} \tag{6}$$

with $\beta^\star = \beta^\star(P_{\mathrm{tx}}(\hat{\boldsymbol{H}}), \boldsymbol{\tau}(\hat{\boldsymbol{H}}))$ in the last constraint, and

$$\beta^\star(P_{\mathrm{tx}}, \boldsymbol{\tau}) = \max_{\beta, \{\boldsymbol{t}_1, \ldots, \boldsymbol{t}_K\}} \beta \qquad \text{s.t.:} \quad \begin{aligned} &\sum_{k=1}^K \|\boldsymbol{t}_k\|_2^2 \leq P_{\mathrm{tx}} \\ &\underline{I}_k^{\mathrm{BC}} \geq \beta\tau_k, \quad \forall k. \end{aligned} \tag{7}$$

In (5) and (6), an average transmit power constraint is imposed. However, if the system doesn't allow an instantaneous power above $\bar{P}_{\mathrm{tx}}$, it is necessary to change the transmit power constraint to $\|\boldsymbol{T}(\hat{\boldsymbol{H}})\|_{\mathrm{F}}^2 \leq \bar{P}_{\mathrm{tx}}$ in (5) and to set $P_{\mathrm{tx}}(\hat{\boldsymbol{H}}) = \bar{P}_{\mathrm{tx}}$ in (6). Since the variational problems arising in (5) and (6) appear to be difficult, we avoid finding the solution to the full problem (5). Instead, in the nested two-stage formulation (6)–(7), we will only solve optimally the inner problem (7) and choose heuristics for the variational outer part (6). For example, the instantaneous rate targets may be chosen to be constant, i.e., $\boldsymbol{\tau}(\hat{\boldsymbol{H}}) = \bar{\boldsymbol{\tau}}$. Hence, we will concentrate on (7) in the following. However, note that, even in the case of an instantaneous power constraint, i.e., $P_{\text{tx}} = \bar{P}_{\text{tx}}$, and where all $\tau_k = \bar{\tau}_k, \forall k$ are chosen to be constant, $\underline{I}_k^{\text{BC}}$ still depends on $\hat{\boldsymbol{H}}$ and, thus, also the maximum balancing factor $\beta^\star(\bar{P}_{\text{tx}}, \bar{\boldsymbol{\tau}})$ found via (7). The resulting optimum of (5) is $\mathrm{E}[\beta^\star(\bar{P}_{\text{tx}}, \bar{\boldsymbol{\tau}})]$.

Note that the constraints in (7) are active in the optimum due to the monotonicity properties of $\text{SINR}_k^{\text{BC}}$ that is, $\text{SINR}_k^{\text{BC}}$ increases with $\|\boldsymbol{t}_k\|_2^2$ but decreases with $\|\boldsymbol{t}_i\|_2^2, i \neq k$ [see (2)]. Therefore, the transmit power is P_{tx} in the optimum and the resulting rates are balanced, i.e., the rate of user k is $\beta^\star \tau_k$. We can conclude that the appropriate value for the balancing factor β must be found and the resulting non-linear equation system evolving from the constraints must be solved to obtain the solution to (7). As will be described in Section VIII, the following QoS formulation is useful for solving this equation system. Given QoS requirements, expressed as minimum rates $\tau_k, k = 1, \ldots, K$, shall be fulfilled using minimum resources, expressed as total transmit power $\|\boldsymbol{T}\|_{\text{F}}^2$, i.e.,

$$P^\star(\boldsymbol{\tau}) \triangleq \operatorname{argmin} \|\boldsymbol{T}\|_{\text{F}}^2 \qquad \text{s.t.:} \quad \underline{I}_k^{\text{BC}} \geq \tau_k, \ \forall k. \tag{8}$$

Note that $\underline{I}_k^{\text{BC}} = \tau_k, \forall k$ in the optimum of (8) due to the monotonicity properties of $\text{SINR}_k^{\text{BC}}$. As has already been discussed in [6], [15], (8) and (7) are inverse problems. It holds that [15]

$$P^\star(\beta^\star(P_{\text{tx}}, \boldsymbol{\tau})\tau_1, \ldots, \beta^\star(P_{\text{tx}}, \boldsymbol{\tau})\tau_K) = P_{\text{tx}}. \tag{9}$$

In other words, the optimal precoders for (8) coincide with those of (7) if the appropriate $\beta^\star$ is known. This connection of (8) and (7) will be exploited in Section VIII. For example, (7) could be solved via a bisection where β is ajusted such that $P^\star(\beta\tau_1, \ldots, \beta\tau_K)$ is equal to P_{tx} since $\beta^\star(P_{\text{tx}})$ is monotonically increasing in P_{tx} (e.g., [15]). However, the difficulty of such a bisection procedure is that (8) might be infeasible contrary to (7). In [16], a simple feasibility test was proposed for the case of error-free CSI, i.e., $\boldsymbol{C}_{\tilde{\boldsymbol{h}}_k} = \boldsymbol{0}, \ \forall k$.

Additionally note that (8) can equivalently be formulated as a power minimization with minimum SINR requirements $\gamma_k = 2^{\tau_k} - 1, \forall k$, since a bijective map connects the mutual information lower bound $\underline{I}_k^{\text{BC}}$ and $\text{SINR}_k^{\text{BC}}$.

VI. SINR Duality for Erroneous Receiver CSI

In the following, an SINR duality between the vector BC with erroneous receiver CSI and an appropriately constructed vector MAC is presented. This SINR duality is a generalization of that proposed in [17] since the channels, and therefore the respective estimation errors, need not have the same covariance matrix.

Let the channels of the dual vector MAC be defined as $\boldsymbol{b}_k = \sigma_k^{-1}\boldsymbol{h}_k \in \mathbb{C}^N$. Accordingly, the estimates and the errors are $\hat{\boldsymbol{b}}_k = \sigma_k^{-1}\hat{\boldsymbol{h}}_k$ and $\tilde{\boldsymbol{b}}_k = \sigma_k^{-1}\tilde{\boldsymbol{h}}_k$, respectively. With the noise $\boldsymbol{\eta} \sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{0}, \mathbf{I})$, the received signal in the dual vector MAC is

$$\boldsymbol{r} = \sum_{k=1}^{K} \boldsymbol{b}_k \sqrt{p_k}\xi_k + \boldsymbol{\eta} \in \mathbb{C}^N.$$

Here, $\xi_k \sim \mathcal{N}_{\mathbb{C}}(0, 1)$ denotes the data signal for user k and $p_k \in \mathbb{R}_+$ is the power of the k-th user in the MAC. To be able to find a connection to the original vector BC, the receiver applies the equalizer $\boldsymbol{f}_k \in \mathbb{C}^N$ to the received signal $\boldsymbol{r}$ to get the data signal estimate $\hat{\xi}_k = \boldsymbol{f}_k^{\text{H}}\boldsymbol{r}$. Also for the dual MAC, a lower bound for the mutual information of user k with Gaussian signaling can be found. With

$$\text{SINR}_k^{\text{MAC}} = \frac{\left|\boldsymbol{f}_k^{\text{H}}\hat{\boldsymbol{b}}_k\right|^2 p_k}{\|\boldsymbol{f}_k\|_2^2 + \sum_i \boldsymbol{f}_k^{\text{H}}\boldsymbol{C}_{\tilde{\boldsymbol{b}}_i}\boldsymbol{f}_k p_i + \sum_{i\neq k}\left|\boldsymbol{f}_k^{\text{H}}\hat{\boldsymbol{b}}_i\right|^2 p_i} \tag{10}$$

this lower bound is $\underline{I}_k^{\text{MAC}} = \log_2(1+\text{SINR}_k^{\text{MAC}})$. In the following, we will show that there exists a one-to-one relationship between $\text{SINR}_k^{\text{BC}}$ and $\text{SINR}_k^{\text{MAC}}$.

Assume that some SINRs are achievable in the vector MAC. If we set

$$\boldsymbol{t}_k = \alpha_k \boldsymbol{f}_k \qquad \forall k. \tag{11}$$

we get from $\text{SINR}_k^{\text{BC}} = \text{SINR}_k^{\text{MAC}}$ [see (2) and (10)] that

$$\begin{aligned}\Bigg(\|\boldsymbol{f}_k\|_2^2 + \sum_i \boldsymbol{f}_k^{\text{H}}\boldsymbol{C}_{\tilde{\boldsymbol{b}}_i}\boldsymbol{f}_k p_i + \sum_{i\neq k}\left|\boldsymbol{f}_k^{\text{H}}\hat{\boldsymbol{b}}_i\right|^2 p_i\Bigg)\alpha_k^2 = \\ = p_k + \sum_i \boldsymbol{f}_i^{\text{H}}\boldsymbol{C}_{\tilde{\boldsymbol{b}}_k}\boldsymbol{f}_i p_k \alpha_i^2 + \sum_{i\neq k}\left|\hat{\boldsymbol{b}}_k^{\text{H}}\boldsymbol{f}_i\right|^2 p_k \alpha_i^2\end{aligned}$$

for $k \in \{1, \ldots, K\}$. The K scalar equations can be comprised in

$$\boldsymbol{\Phi}\boldsymbol{a} = \boldsymbol{p} \tag{12}$$

where $\boldsymbol{a} = [\alpha_1^2, \ldots, \alpha_K^2]^{\text{T}}$ and $\boldsymbol{p} = [p_1, \ldots, p_K]^{\text{T}}$ with $\boldsymbol{a}, \boldsymbol{p} \in \mathbb{R}_+^K$. Since

$$[\boldsymbol{\Phi}]_{k,\ell} = \begin{cases} \|\boldsymbol{f}_k\|_2^2 + \sum\limits_{i\neq k} \boldsymbol{f}_k^{\text{H}}\boldsymbol{C}_{\tilde{\boldsymbol{b}}_i}\boldsymbol{f}_k p_i + \sum\limits_{i\neq k}\left|\boldsymbol{f}_k^{\text{H}}\hat{\boldsymbol{b}}_i\right|^2 p_i & \ell = k \\ -\boldsymbol{f}_\ell^{\text{H}}\boldsymbol{C}_{\tilde{\boldsymbol{b}}_k}\boldsymbol{f}_\ell p_k - \left|\hat{\boldsymbol{b}}_k^{\text{H}}\boldsymbol{f}_\ell\right|^2 p_k & \text{else} \end{cases}$$

the matrix $\boldsymbol{\Phi}$ is column-wise diagonally dominant with positive diagonal entries and non-positive off-diagonal elements. Therefore, $\boldsymbol{\Phi}^{-1}$ exists and has non-negative entries. The resulting solution $\boldsymbol{a} = \boldsymbol{\Phi}^{-1}\boldsymbol{p}$ is non-negative, i.e., all α_k^2's are non-negative. In other words, any SINRs achievable in the dual MAC are also achievable in the original BC. From the multiplication of (12) with the all-ones vector, i.e.,

$$\mathbf{1}^{\text{T}}\boldsymbol{\Phi}\boldsymbol{a} = [\|\boldsymbol{f}_1\|_2^2, \ldots, \|\boldsymbol{f}_K\|_2^2]\boldsymbol{a} = \mathbf{1}^{\text{T}}\boldsymbol{p} = \sum_{k=1}^{K} p_k$$

we infer that, based on (11), $\text{SINR}_k^{\text{BC}} = \text{SINR}_k^{\text{MAC}}$ for $k = 1, \ldots, K$, is always possible using the same transmit power $\sum_k \|\boldsymbol{t}_k\|_2^2$ in the BC as in the dual MAC.

Starting from given beamformers $\boldsymbol{t}_k$ in the BC and using $\boldsymbol{f}_k = \boldsymbol{t}_k, \forall k$, it can be shown with similar steps as above that $\text{SINR}_k^{\text{MAC}} = \text{SINR}_k^{\text{BC}}$ can be accomplished for all $k \in \{1, \ldots, K\}$ by an appropriate choice for the MAC powers p_k.

The resulting power allocation $p_1, \ldots, p_K$ always exists and is non-positive. Moreover, $\sum_k p_k = \sum_k \|\boldsymbol{t}_k\|_2^2$ holds. This proves also the converse of following theorem.

Theorem 1. *Any SINRs* (2) *are achievable in the vector BC with erroneous CSI at the transmitter and the receivers using some total transmit power, iff the same SINRs* (10) *are achievable in the dual vector MAC with erroneous CSI at the transmitters and the receiver employing the same total transmit power.*

Due to Theorem 1, we will formulate and solve the problems in the dual vector MAC in the following. The BC solution can then be obtained with (11) and (12).

VII. Interference Function for Erroneous Receiver CSI

Whereas an SINR based formulation is essentially a precoder design in the BC, in the dual MAC it reformulates to a joint optimization of the transmit power allocation $\boldsymbol{p} = [p_1, \ldots, p_K]^{\mathrm{T}}$ and the adaptive receive strategies $\boldsymbol{f}_k$, $k \in \{1, \ldots, K\}$. For the joint power allocation and equalizer design in the dual MAC based on the individual SINRs, a generic framework with general interference functions of [8], [7], [18] can be applied. To this end, let us define the effective interference of user k

$$\mathcal{Z}_k(\boldsymbol{p}, \boldsymbol{f}_k) = \frac{\|\boldsymbol{f}_k\|_2^2 + \sum_i \boldsymbol{f}_k^{\mathrm{H}} \boldsymbol{C}_{\tilde{\boldsymbol{b}}_i} \boldsymbol{f}_k p_i + \sum_{i \neq k} \left|\boldsymbol{f}_k^{\mathrm{H}} \hat{\boldsymbol{b}}_i\right|^2 p_i}{\left|\boldsymbol{f}_k^{\mathrm{H}} \hat{\boldsymbol{b}}_k\right|^2}$$

such that $\mathrm{SINR}_k^{\mathrm{MAC}} = p_k / \mathcal{Z}_k(\boldsymbol{p}, \boldsymbol{f}_k)$ [cf. (10)]. As can be easily shown, the interference function

$$\boldsymbol{\mathcal{Z}}(\boldsymbol{p}, \boldsymbol{F}) = [\mathcal{Z}_1(\boldsymbol{p}, \boldsymbol{f}_1), \ldots, \mathcal{Z}_K(\boldsymbol{p}, \boldsymbol{f}_K)]^{\mathrm{T}} \tag{13}$$

is standard (see [8]) for fixed equalizers $\boldsymbol{F} = [\boldsymbol{f}_1, \ldots, \boldsymbol{f}_K]$ (see [7]), i.e., it satisfies the following three axioms for $\boldsymbol{p} \geq \boldsymbol{p}' \geq \boldsymbol{0}$ and $\mu > 1$:

A1. Positivity: $\boldsymbol{\mathcal{Z}}(\boldsymbol{p}, \boldsymbol{F}) > \boldsymbol{0}$

A2. Monotonicity: $\boldsymbol{\mathcal{Z}}(\boldsymbol{p}, \boldsymbol{F}) \geq \boldsymbol{\mathcal{Z}}(\boldsymbol{p}', \boldsymbol{F})$

A3. Scalability: $\mu \boldsymbol{\mathcal{Z}}(\boldsymbol{p}, \boldsymbol{F}) > \boldsymbol{\mathcal{Z}}(\mu \boldsymbol{p}, \boldsymbol{F})$.

As has already been demonstrated in [8], choosing the optimum equalizer also leads to a standard interference function, that is,

$$\boldsymbol{\mathcal{I}}(\boldsymbol{p}) = [\min_{\boldsymbol{f}_1} \mathcal{Z}_1(\boldsymbol{p}, \boldsymbol{f}_1), \ldots, \min_{\boldsymbol{f}_K} \mathcal{Z}_K(\boldsymbol{p}, \boldsymbol{f}_K)]^{\mathrm{T}} \tag{14}$$

is standard. This observation led to the algorithmic solutions in [7]. Note that the optimizer for the k-th element in (14) can be written as

$$\boldsymbol{f}_{\mathrm{opt},k}(\boldsymbol{p}) = \alpha_k \left(\mathbf{I} + \sum\nolimits_i \boldsymbol{C}_{\tilde{\boldsymbol{b}}_i} p_i + \sum\nolimits_{i \neq k} \hat{\boldsymbol{b}}_i \hat{\boldsymbol{b}}_i^{\mathrm{H}} p_i\right)^{-1} \hat{\boldsymbol{b}}_k.$$

The choice for $\alpha_k \in \mathbb{C} \setminus \{0\}$ is arbitrary. Therefore, we can set $\alpha_k = 1$ for all $k \in \{1, \ldots, K\}$.

Algorithm 1 Rate Balancing

Require: $\boldsymbol{p}^{(0)} > \boldsymbol{0}$ with $\mathbf{1}^{\mathrm{T}} \boldsymbol{p}^{(0)} = P_{\mathrm{tx}}$, $n = 0$

repeat
- $n \leftarrow n + 1$ — increase iteration counter
- $\beta^{(n)} = \max\{\beta \in \mathbb{R} : \boldsymbol{p}^{(n-1)} \geq \mathrm{diag}(\boldsymbol{\gamma}(\beta)) \boldsymbol{\mathcal{I}}(\boldsymbol{p}^{(n-1)})\}$ — increase balancing factor
- $\tilde{\boldsymbol{p}} = \mathrm{diag}(\boldsymbol{\gamma}(\beta^{(n)})) \boldsymbol{\mathcal{I}}(\boldsymbol{p}^{(n-1)})$ — fixed point step
- $\boldsymbol{p}^{(n)} = \frac{P_{\mathrm{tx}}}{\mathbf{1}^{\mathrm{T}} \tilde{\boldsymbol{p}}} \tilde{\boldsymbol{p}}$ — use full power

until $\|\boldsymbol{p}^{(n)} - \boldsymbol{p}^{(n-1)}\|_1 \leq \epsilon$

VIII. Algorithmic Solution

Based on the duality of Section VI, the balancing formulation (7) can be equivalently formulated in the dual MAC

$$\max_{\beta, \boldsymbol{p} \geq \boldsymbol{0}} \beta \qquad \text{s.t.:} \quad \mathbf{1}^{\mathrm{T}} \boldsymbol{p} \leq P_{\mathrm{tx}} \qquad \boldsymbol{p} \geq \mathrm{diag}(\boldsymbol{\gamma}(\beta)) \boldsymbol{\mathcal{I}}(\boldsymbol{p}) \tag{15}$$

with the interference function $\boldsymbol{\mathcal{I}}(\boldsymbol{p})$ introduced in (14) and $\boldsymbol{\gamma}(\beta) = [2^{\beta\tau_1} - 1, \ldots, 2^{\beta\tau_K} - 1]^{\mathrm{T}}$. Note that the rates are balanced in (15). Hence, the algorithmic solution for SINR balancing from [6], [7] is not applicable.

As has already been mentioned in Section V, (15) can be solved with the help of the QoS optimization (8) that leads to

$$\min_{\boldsymbol{p} \geq \boldsymbol{0}} \mathbf{1}^{\mathrm{T}} \boldsymbol{p} \qquad \text{s.t.:} \quad \boldsymbol{p} \geq \mathrm{diag}(\boldsymbol{\gamma}(\beta)) \boldsymbol{\mathcal{I}}(\boldsymbol{p}) \tag{16}$$

in the dual MAC. For example, based on the observation (9), a bisection may be performed on β. The bisection is converged when (16) has the given transmit power P_{tx} as its optimum.

Instead, we focus on (iterative) approaches for directly solving (15). To this end, we remark that all balancing constraints and the power constraint are jointly active in the optimal point $(\beta^\star, \boldsymbol{p}^\star)$, i.e.,

$$\mathbf{1}^{\mathrm{T}} \boldsymbol{p}^\star = P_{\mathrm{tx}} \qquad \text{and} \tag{17a}$$

$$\boldsymbol{p}^\star = \mathrm{diag}(\boldsymbol{\gamma}(\beta^\star)) \boldsymbol{\mathcal{I}}(\boldsymbol{p}^\star). \tag{17b}$$

Due to the monotonicity property of $\mathcal{I}(\boldsymbol{p})$ together with the scalability property (cf. Axioms A2 and A3, respectively), the (vector) constraint in (16) is always active. Moreover, since (16) and (15) are inverse problems, this is also true for the rate balancing problem, which results in (17a) and (17b).

Unfortunately, there exist no closed-form expressions for the tuples $(\beta, \boldsymbol{p})$ jointly satisfying (17a) and (17b). However, the naturally arising fixed point iteration

$$\boldsymbol{p}^{(n)} = \mathrm{diag}(\boldsymbol{\gamma}(\beta)) \boldsymbol{\mathcal{I}}(\boldsymbol{p}^{(n-1)}) \tag{18}$$

was proven in [8] to globally converge to the unique fixed point. Existence of such a fixed point is ensured for any $\beta > 0$ with $\boldsymbol{p}^{(n-1)} \geq \mathrm{diag}(\boldsymbol{\gamma}(\beta)) \boldsymbol{\mathcal{I}}(\boldsymbol{p}^{(n-1)})$. Then, the sequence $\{\boldsymbol{p}^{(n)}\}$ is monotonically decreasing. This motivates the procedure given in Algorithm 1.

Note that the procedure in Algorithm 1 converges. In the first step, the balancing factor $\beta^{(n)}$ is chosen as the maximum feasible one for the given power allocation. Then, a fixed point step according to (18) is performed that reduces the transmit power, i.e., $\tilde{\boldsymbol{p}} \leq \boldsymbol{p}^{(n-1)}$, while $\tilde{\boldsymbol{p}} \geq \operatorname{diag}(\boldsymbol{\gamma}(\beta^{(n)}))\boldsymbol{\mathcal{I}}(\tilde{\boldsymbol{p}})$ still holds. By normalizing the power allocation to use full transmit power, the ($\geq$) becomes a strict inequality. Thus, the balancing factor will be increased in the following step, i.e., $\beta^{(n+1)} \geq \beta^{(n)}$. That is, the balancing factor β is increased (or remains constant) in each iteration. Furthermore, as its value is bounded above by $\beta^\star$, convergence is ensured. Being converged, the duality result of Section VI can be used to compute the optimizers of (7).

Simulations have shown that Algorithm 1 requires a large number of iterations until convergence in many scenarios. Therefore, we used the iteration proposed in [7] for the fixed point step instead (due to its faster convergence [18]). To this end, we reformulate (13) in matrix-vector notation, i.e.,

$$\boldsymbol{\mathcal{Z}}(\boldsymbol{p}, \boldsymbol{F}) = \boldsymbol{\Psi}(\boldsymbol{F})\boldsymbol{p} + \boldsymbol{\xi}(\boldsymbol{F})$$

with $\boldsymbol{\xi}(\boldsymbol{F}) = [\xi_1, \ldots, \xi_K]^{\mathrm{T}}$, $\xi_k = \|\boldsymbol{f}_k\|_2^2 / |\boldsymbol{f}_k^{\mathrm{H}} \hat{\boldsymbol{b}}_k|^2$, and

$$[\boldsymbol{\Psi}(\boldsymbol{F})]_{k,\ell} = \frac{1}{\left|\boldsymbol{f}_k^{\mathrm{H}} \hat{\boldsymbol{b}}_k\right|^2} \begin{cases} \boldsymbol{f}_k^{\mathrm{H}} \boldsymbol{C}_{\tilde{\boldsymbol{b}}_k} \boldsymbol{f}_k & \ell = k \\ \boldsymbol{f}_k^{\mathrm{H}} \boldsymbol{C}_{\tilde{\boldsymbol{b}}_\ell} \boldsymbol{f}_k + \left|\boldsymbol{f}_k^{\mathrm{H}} \hat{\boldsymbol{b}}_\ell\right|^2 & \text{else.} \end{cases}$$

We have $\operatorname{diag}(\boldsymbol{\gamma})(\boldsymbol{\Psi}(\boldsymbol{F})\boldsymbol{p} + \boldsymbol{\xi}(\boldsymbol{F}) = \boldsymbol{p}$ in the optimum of (16). Rearranging this equation, leads to the fixed point iteration

$$\boldsymbol{p}^{(n)} = \left(\mathbf{I}_K - \operatorname{diag}(\boldsymbol{\gamma})(\boldsymbol{\Psi}(\boldsymbol{F})\right)^{-1} \operatorname{diag}(\boldsymbol{\gamma})(\boldsymbol{F}),$$

where $\boldsymbol{F} = [\boldsymbol{f}_{\text{opt},1}(\boldsymbol{p}^{(n-1)}), \ldots, \boldsymbol{f}_{\text{opt},K}(\boldsymbol{p}^{(n-1)})]$. As shown in [7], this iteration leads also to an decreasing sequence $\{\boldsymbol{p}^{(n)}\}$ and converges to the unique fixed point when starting a tuple $(\beta^{(0)}, \boldsymbol{p}^{(0)})$ that is feasible for (16).

Unfortunately, above algorithm is still slow in the high SNR regime. Therefore, we adopt also the fixed point iteration of [19, Section III.]. Based on the discussion above and the requirements in (17a) and (17b), when we adapt β in each step of the fixed point iteration in (18) such that $\mathbf{1}^{\mathrm{T}}\boldsymbol{p}^{(n)} = P_{\text{tx}}$ is fulfilled, the power constraint will be fulfilled after convergence and the target based rate constraints will be satisfied with equality as required. That is, the resulting β will be the global optimum of (15). With the right hand side of (18), this sum power requirement for the update rule can be written as

$$\mathbf{1}^{\mathrm{T}} \operatorname{diag}(\boldsymbol{\gamma}(\beta))\boldsymbol{\mathcal{I}}(\boldsymbol{p}^{(n-1)}) - P_{\text{tx}} = 0. \tag{19}$$

This implicit requirement, gives us the alternative update rule for the balancing factor, i.e., β can be chosen as the single positive real root of (19) in each iteration, e.g., via the Newton-Raphson method (e.g., see [20]). The resulting two step procedure is shown in Algorithm 2.

Note that Algorithm 2 has shown to have a much faster convergence speed, independent of the considered SNR regime. However, unlike for Algorithm 1, the sequence $\{\beta^{(n)}\}$ has a non-monotonic behavior and may even lie above the optimal $\beta^\star$ for some iterations n.

Algorithm 2 Rate Balancing

Require: $\boldsymbol{p}^{(0)} > \mathbf{0}$ with $\mathbf{1}^{\mathrm{T}}\boldsymbol{p}^{(0)} = P_{\text{tx}}$, $n = 0$

1: **repeat**
2: $\quad n \leftarrow n + 1$ — increase iteration counter
3: $\quad \beta^{(n)} = \max\{\beta \in \mathbb{R} : \mathbf{1}^{\mathrm{T}} \operatorname{diag}(\boldsymbol{\gamma}(\beta))\boldsymbol{\mathcal{I}}(\boldsymbol{p}^{(n-1)}) = P_{\text{tx}}\}$ — find balancing factor
4: $\quad \boldsymbol{p}^{(n)} = \operatorname{diag}(\boldsymbol{\gamma}(\beta^{(n)}))\boldsymbol{\mathcal{I}}(\boldsymbol{p}^{(n-1)})$ — fixed point step
5: **until** $\|\boldsymbol{p}^{(n)} - \boldsymbol{p}^{(n-1)}\|_1 \leq \epsilon$

IX. Numerical Results

For a numerical verification of the achieved performance with erroneous receiver CSI, we performed numerical simulations for a system with $K = 4$ users, $N = 4$ antennas at the transmitter, and relative rate targets $\tau_1 = \tau_3 = 1$ and $\tau_2 = \tau_4 = 2$. For the simulations, channel realizations $\boldsymbol{h}_k$, $k \in \{1, \ldots, 4\}$ were generated from the zero-mean complex Gaussian distribution with identity covariance matrices $\boldsymbol{C}_{\boldsymbol{h}_k} = \mathbf{I}_N$. The channel estimates $\hat{\boldsymbol{h}}_k$ and the channel error covariances $\boldsymbol{C}_{\tilde{\boldsymbol{h}}_k}$ were determined via MMSE channel estimation from noisy pilot-based training observations of the channel realizations. For the pilot-based training, the same transmit power P_{tx} was used as for the data transmission.

In Fig. 1, the average of the achieved optimal common balancing level $\beta^\star$ is plotted versus the SNR $P_{\text{tx}}\sigma^{-2}$, with $\sigma_1^2 = \cdots = \sigma_4^2 = \sigma^2$. For the figure, we used 1000 channel realizations and calculated the channel estimates for each realization. Then, the optimal common balancing levels for perfect CSI and erroneous CSI were determined by switching to the dual MAC and applying Algorithms 1 and 2. Finally, we averaged over the results of $\beta^\star$ for drawing the perfect and the erroneous CSI curves.

In the figure, two observations can be made. First, the erroneous CSI curves lie close to the perfect CSI curves over the whole SNR range. The difference between the respective balancing factors is approximately only 0.5. Second, the slopes of all curves seem to be the same at high SNR. This effect stems from the dependence of the training SNR on the data SNR.

In Fig. 2, the achievable average rate regions are drawn for the SNR values 0dB, 15dB, and 30dB in a two user system with $N = 2$ transmit antennas. For 0dB SNR, erroneous receiver CSI suffers from both, high noise during data transmission and channel estimation. Thus, its average rate region boundary is far below the perfect CSI boundary. The shape of rate region boundaries explain why time-sharing between the single-user points would be optimal. With increasing SNR, the relative distance between the perfect and erroneous CSI rate region boundaries decreases. Moreover, there appears some locally convex part around the bisecting line. Therefore, no time-sharing is necessary for sum rate maximization.

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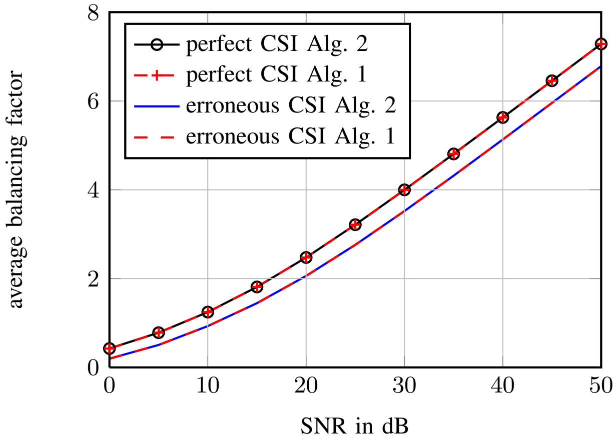

Fig. 1. Average achievable balancing level in a 4 by 4 vector BC

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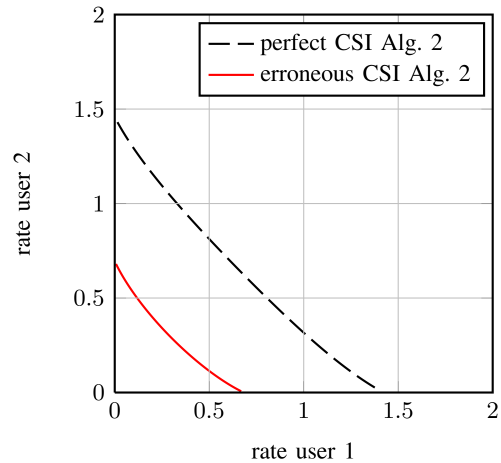
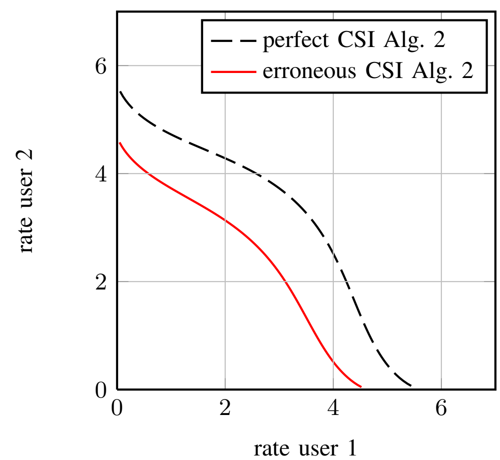
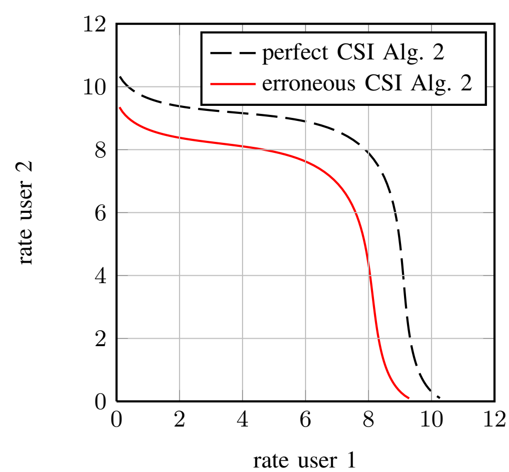


(a) SNR 0dB



(b) SNR 15dB



(c) SNR 30dB

Fig. 2. Achievable rate region for weak, medium, and strong SNR