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Abstract

Even though parallel multiple-input multiple-output (MIMO) broadcast channels are known to be separable from an information theoretic point of view, performing separate encoding and decoding on each of the parallel channels has been shown to be potentially suboptimal in broadcast channels with linear transceivers. In this work, we show that suboptimality of such a carrier-noncooperative transmission also occurs in broadcast channels with zero-forcing and quality of service constraints if time-sharing is not allowed. The proof is given by constructing a minimal example and identifying a rate tuple that is achievable using carrier-cooperative zero-forcing with a certain sum power, but requires a higher sum power with carrier-noncooperative zero-forcing. This observation is of practical relevance since zero-forcing without time-sharing is a popular assumption in the design of low-complexity optimization algorithms.

I. Introduction

There are many practical communication systems where a base station serves a set of users with individual data streams by transmitting over a set of orthogonal resources, such as (groups of) subcarriers of a frequency selective channel or time intervals in a fading channel. Assuming Gaussian noise at the receivers, such systems can be modeled as parallel Gaussian broadcast channels. In such a setting,

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transmission can either be performed separately on each resource, which is called *carrier-noncooperative* (CN), or jointly by spreading data streams across several resources, which is called *carrier-cooperative* (CC) transmission [1], [2]. A mathematical definition of CN and CC transmission is provided together with the system model in Section II.

The concept of parallel broadcast channels shall not be confused with the case of interfering broadcast channels (e.g., [3]). The latter describes the case where several base stations have to share the available resources when communicating with their respective sets of users. Such a setting is an interference channel scenario, where techniques such as interference alignment [4], [5] are needed to exploit all possible degrees of freedom, and where separate transmission on each carrier (i.e., CN transmission) is known to be suboptimal [6]. By contrast, the model of parallel broadcast channels includes only one base station, and the term *parallel* only refers to the fact that transmission is carried out on several orthogonal resources (e.g., carriers) in a parallel manner. In such a classical broadcast scenario, interference alignment is not a feasible strategy, and results concerning the suboptimality of CN transmission in interference channels (cf. [6]) do not apply. Also note that the considerations in this paper differ from the recent results in interference channels in that they are not based on a degrees of freedom analysis, but hold for the case of finite signal-to-noise ratio.

Even though it is the less general concept, CN transmission is known to be capacity achieving in parallel single-antenna [7], [8] and multiantenna broadcast channels [9], [10]. Therefore, many algorithms to optimize transmit strategies in parallel MIMO broadcast channels (e.g., [11]–[14]) are based on the assumption of CN transmission. However, if the transmit strategy is restricted to linear transceivers, CN transmission is no longer optimal, i.e., a gain in system performance can be achievable with CC transmission [15].

In this paper, we consider systems employing zero-forcing beamforming without time-sharing (ZFBF), which is a popular technique to design efficient optimization algorithms (e.g., [11], [12], [16]–[18]). With *time-sharing*, we refer to the technique of dividing the total transmission time into an *arbitrary* number of intervals with *arbitrary* length and applying different transmit strategies one after another. Then, only the time averages of the per-user data rates and the sum transmit power are considered instead of the instantaneous values. This can lead to a better performance, but, on the other hand, implementing a system that allows *arbitrary numbers of variable-length time slots* is difficult and involves high signaling overhead. The high practical relevance of linear precoding without time-sharing can also be seen from the fact that this assumption is made in a high number of publications (e.g., [1], [11], [12], [19]–[23]).

In this paper, we compare CC zero-forcing strategies without time-sharing to CN zero-forcing strategies without time-sharing, i.e., both types of transmission are restricted to not apply time-sharing. For the case where time-sharing is allowed in both types of transmission, the question whether or not CN transmission is optimal cannot be answered by means of the minimal example used in this work and is left open for future research.

From the existing literature, it is not possible to answer the question whether CN transmission is optimal or suboptimal in broadcast channels that are constrained to zero-forcing strategies. In particular, the answer to this question cannot be derived from the result presented in [15]: therein, it was shown that the optimal CN strategy out of the set of all linear strategies (i.e., without zero-forcing constraints and without a restriction to transmission without time-sharing) might be outperformed by some linear CC strategy. However, the set of ZFBF strategies is only a subset of the set of all linear strategies. Therefore, we cannot conclude whether there can exist CC strategies out of the ZFBF subset which outperform the optimal CN ZFBF strategy. Also from our previous work [2] and our companion work [24], it is not possible to draw conclusions about the optimality or suboptimality of CN transmission in the ZFBF case since these works only consider the question of algorithm design and perform numerical studies, but do not contain analytical results about the suboptimality of CN transmission.

The main contribution of this work is presented in Section III, where we show that suboptimality of CN transmission can also occur in the ZFBF case if problems with quality of service (QoS) constraints are considered. The proof is given by constructing a minimal example and identifying per-user rates that are achievable with a certain sum transmit power using carrier-cooperative (CC) ZFBF, but require a higher transmit power using carrier-noncooperative (CN) ZFBF.

To get more intuition about why this suboptimality occurs, we continue the study of the minimal example in Section IV, where we briefly sketch a method to compute the optimal CC ZFBF strategy in the considered scenario. The considerations in that section reveal that for certain channel realizations, which we call *spectrally similar* channels, the transmit power needed for one user must be traded off against the power needed for the other user. The optimal trade-off might require carrier cooperation. These observations are very helpful to also draw conclusions about larger scenarios as can be seen in our companion work [24], where numerical simulations show significant gains for CC ZFBF in larger systems if the channels have a certain spectral similarity.

The paper is rounded off by Section V, in which we show that suboptimality of CN transmission can also occur if zero-forcing combined with dirty paper coding (DPC-ZF, e.g., [10]–[12], [16]) is considered, and by some concluding remarks in Section VI.

Notation: We use $\bullet^{\mathrm{T}}$ to denote the transpose, $\bullet^{*}$ for the conjugate, $\bullet^{\mathrm{H}}$ for the conjugate transpose, $\mathbf{I}_L$

for the identity matrix of size L, $\mathbf{0}$ for the zero vector, and $\boldsymbol{e}_i$ for the ith canonical unit vector (1 as ith entry and 0 elsewhere).

II. System Model

A set of C parallel MIMO broadcast channels with M transmit antennas, K users, and N_k receive antennas for user k can be described by

$$\hat{\boldsymbol{x}}_k = \boldsymbol{G}_k^{\mathrm{H}} \underbrace{\begin{bmatrix} \boldsymbol{H}_k^{(1)} & & \\ & \ddots & \\ & & \boldsymbol{H}_k^{(C)} \end{bmatrix}}_{\boldsymbol{H}_k} \sum_{j=1}^{K} \boldsymbol{B}_j \boldsymbol{x}_j + \underbrace{\begin{bmatrix} \boldsymbol{\eta}_k^{(1)} \\ \vdots \\ \boldsymbol{\eta}_k^{(C)} \end{bmatrix}}_{\boldsymbol{\eta}_k} \tag{1}$$

where $\boldsymbol{H}_k^{(c)} \in \mathbb{C}^{N_k \times M}$ is the channel matrix of user k on carrier c, and $\boldsymbol{\eta}_k^{(c)} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{C}_k^{(c)})$ is additive circularly symmetric complex Gaussian noise. In this setting, $S_k \leq \min\{N_k C, MC\}$ streams of Gaussian data symbols are intended for user k, i.e., $\boldsymbol{x}_k = [x_{k,1}, \ldots, x_{k,S_k}] \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{S_k})$.

If all beamforming matrices $\boldsymbol{B}_k \in \mathbb{C}^{MC \times S_k}$ and receive filters $\boldsymbol{G}_k^{\mathrm{H}} \in \mathbb{C}^{S_k \times N_k C}$ can be decomposed as

$$\boldsymbol{B}_k = \operatorname{blockdiag}\left(\boldsymbol{B}_k^{(1)}, \ldots, \boldsymbol{B}_k^{(C)}\right) \tag{2}$$

$$\boldsymbol{G}_k^{\mathrm{H}} = \operatorname{blockdiag}\left(\boldsymbol{G}_k^{(1),\mathrm{H}}, \ldots, \boldsymbol{G}_k^{(C),\mathrm{H}}\right) \tag{3}$$

matching the block-diagonal structure of the channel matrices $\boldsymbol{H}_k$, the transmission is carrier-noncooperative. However, if the matrices $\boldsymbol{B}_k$ and $\boldsymbol{G}_k^{\mathrm{H}}$ have arbitrary structure, data streams might be spread across several resources c, which is a carrier-cooperative transmit scheme (e.g., [1], [2]).

If we define zero-forcing in a strict manner as in [12], an estimate $\hat{x}_{k,s}$ of the sth stream of user k may not contain interference of any other data stream, including streams of the same user, i.e.,

$$\boldsymbol{g}_{k,s}^{\mathrm{H}} \boldsymbol{H}_k \boldsymbol{b}_{\ell,t} = 0 \quad \forall k, s, \ell, t : (\ell, t) \neq (k, s) \tag{4}$$

where $\boldsymbol{g}_{k,s}^{\mathrm{H}}$ is the sth row of $\boldsymbol{G}_k^{\mathrm{H}}$ and $\boldsymbol{b}_{\ell,t}$ is the tth column of $\boldsymbol{B}_\ell$. A weaker zero-forcing constraint requiring only suppression of interference between streams of different users, i.e.,

$$\boldsymbol{g}_{k,s}^{\mathrm{H}} \boldsymbol{H}_k \boldsymbol{b}_{\ell,t} = 0 \quad \forall k, s, \ell, t : \ell \neq k \tag{5}$$

was used, e.g., in [17]. The results of this paper hold for both definitions of zero-forcing.

In this paper, we only consider systems that do not employ time-sharing, i.e., we do not allow averaging transmit powers and data rates over several transmit strategies. Thus, the transmit strategy is completely described by the choice of the matrices $\boldsymbol{G}_k^{\mathrm{H}}$ and $\boldsymbol{B}_k$ for all k, and the sum transmit power is given by

$$P = \sum_{k=1}^{K} \mathrm{tr}[\boldsymbol{B}_k \boldsymbol{B}_k^{\mathrm{H}}]. \tag{6}$$

The QoS constraints can be formulated in terms of minimum rate constraints as

$$\sum_{s=1}^{S_k} \log_2 \left(1 + \gamma_{k,s} \boldsymbol{b}_{k,s}^{\mathrm{H}} \boldsymbol{b}_{k,s}\right) \geq \varrho_k \tag{7}$$

where ϱ_k is a constant specifying the required rate of user k, the product $\gamma_{k,s} \boldsymbol{b}_{k,s}^{\mathrm{H}} \boldsymbol{b}_{k,s}$ is the signal-to-noise ratio (SNR) of stream s of user k, and

$$\gamma_{k,s} = \frac{\left| \boldsymbol{g}_{k,s}^{\mathrm{H}} \boldsymbol{H}_k \boldsymbol{b}_{k,s} \right|^2}{\boldsymbol{g}_{k,s}^{\mathrm{H}} \boldsymbol{C}_k \boldsymbol{g}_{k,s} \boldsymbol{b}_{k,s}^{\mathrm{H}} \boldsymbol{b}_{k,s}} \tag{8}$$

with $\boldsymbol{C}_k = \mathrm{blockdiag}(\boldsymbol{C}_k^{(1)}, \ldots, \boldsymbol{C}_k^{(C)})$ is the subchannel gain, which is invariant to scaling of $\boldsymbol{b}_{k,s}$ and $\boldsymbol{g}_{k,s}^{\mathrm{H}}$. Since we assume that the QoS constraints are to be fulfilled without time-sharing, a user selection (e.g., as in [18]) is not possible and all users have to be served simultaneously.

III. Suboptimality of Carrier-Noncooperative Transmission in Broadcast Channels with Zero-Forcing

The main result of this paper is stated in the following theorem.

Theorem 1: In parallel MIMO broadcast channels with zero-forcing constraints (4) or (5) and quality of service constraints (7) where time-sharing is not allowed, carrier-noncooperative transmission does not always achieve the minimal feasible sum power.

Proof of Theorem 1: We provide a proof by construction. We first choose the system parameters and rate requirements of an example system and compute the sum power needed with carrier-noncooperative (CN) transmission. Then, we derive a carrier-cooperative (CC) transmission scheme which yields a lower transmit power. Note that the statement is proven if we manage to identify a single system configuration for which we can construct a CC zero-forcing strategy that achieves lower transmit power than the optimal CN zero-forcing strategy for that configuration. Thus, we can restrict the following considerations to a system with small dimensions and a very simple channel realization without loss of generality. Similar proof techniques were, e.g., applied in [6] and [15].

Consider a system with $C = 2$ carriers, $M = 1$ transmit antenna, and $K = 2$ users with $N_k = 1$ receive antenna.[1] In this case, the noise covariance matrices $\boldsymbol{C}_k^{(c)}$ and channel matrices $\boldsymbol{H}_k^{(c)}$ are reduced

[1] Note that this is a (very simple) special case of parallel MIMO broadcast channels and, thus, a valid system to prove the theorem. Constructing a larger example with $M > 1$ and $N_k > 1$ would be possible, but not more insightful.

to scalars $C_k^{(c)}$ and $H_k^{(c)}$, which we choose to be

$$C_k^{(c)} = 1 \ \forall k, \forall c$$

$$\text{and} \quad \boldsymbol{H}_k = \begin{bmatrix} H_k^{(1)} & \\ & H_k^{(2)} \end{bmatrix} = \begin{bmatrix} 1 & \\ & 0.1 \end{bmatrix} \ \forall k. \tag{9}$$

We choose the minimum rate requirements $\varrho_k = 1$ for all k.

For CN ZFBF with zero-forcing defined either as in (4) or as in (5), only one user can be served on each carrier due to $M = 1$. On the other hand, both users have to be served due to the QoS constraints, i.e., a different user has to be scheduled on each of the carriers. As the scenario is symmetric with respect to the two users, we can choose to schedule user k on carrier $c = k$ without loss of generality. Omitting the stream index s of the only data stream of user k, we have $\gamma_k = |H_k^{(k)}|^2$, and (7) simplifies to

$$\log_2\left(1 + \left|H_k^{(k)} b_k^{(k)}\right|^2\right) \geq \varrho_k \ \Leftrightarrow \ \left|b_k^{(k)}\right|^2 \geq \frac{2^{\varrho_k} - 1}{\left|H_k^{(k)}\right|^2}. \tag{10}$$

Thus, the minimal sum transmit power fulfilling the QoS constraints of the two users with CN ZFBF is

$$P_{\text{CN}} = \sum_{k=1}^{K} \left|b_k^{(k)}\right|^2 = \sum_{k=1}^{K} \frac{2^{\varrho_k} - 1}{\left|H_k^{(k)}\right|^2} = 1 + \frac{1}{0.01} = 101. \tag{11}$$

To prove the theorem, we now have to find a CC ZFBF strategy that fulfills the QoS constraints with a lower sum transmit power. We use $S_k = 1$ data stream per user and again omit the stream index s. Given some receive filters $\boldsymbol{g}_k^{\text{H}}$, we can express any two-dimensional transmit filter vector that fulfills the zero-forcing constraints (4) and (5) for the minimal example by

$$\boldsymbol{b}_k = \tau_k \boldsymbol{P} \boldsymbol{H}_\ell^{\text{T}} \boldsymbol{g}_\ell^* \quad \text{with} \quad \boldsymbol{P} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{12}$$

where $\ell \neq k$, and τ_k is a scaling factor. Inserting these filter vectors into (8), we get

$$\gamma_k = \frac{|\boldsymbol{g}_k^{\text{H}} \boldsymbol{H}_k \boldsymbol{P} \boldsymbol{H}_\ell^{\text{T}} \boldsymbol{g}_\ell^*|^2}{\boldsymbol{g}_\ell^{\text{H}} \boldsymbol{H}_\ell \boldsymbol{H}_\ell^{\text{H}} \boldsymbol{g}_\ell \boldsymbol{g}_k^{\text{H}} \boldsymbol{g}_k} = \frac{|\boldsymbol{g}_k^{\text{H}} \boldsymbol{P} \boldsymbol{g}_\ell^*|^2}{\boldsymbol{g}_\ell^{\text{H}} \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{g}_\ell \boldsymbol{g}_k^{\text{H}} \boldsymbol{g}_k} \tag{13}$$

where we have used (9) and the identity $\boldsymbol{P}^{\text{H}} \boldsymbol{P} = \mathbf{I}_2$. The QoS constraints can be fulfilled with

$$P_{\text{CC}} = \sum_{k=1}^{K} \boldsymbol{b}_k^{\text{H}} \boldsymbol{b}_k = \sum_{k=1}^{K} \frac{2^{\varrho_k} - 1}{\gamma_k} = \frac{1}{\gamma_1} + \frac{1}{\gamma_2}. \tag{14}$$

We now have to find a choice of $\boldsymbol{g}_k^{\text{H}}$ that leads to a sum transmit power lower than P_{CN}. A possible choice are the unit-norm receive filters

$$\boldsymbol{g}_k^{\text{H}} = \left[(-1)^k \cos\left(\tfrac{\pi}{3}\right) \quad \sin\left(\tfrac{\pi}{3}\right)\right] = \left[\tfrac{(-1)^k}{2} \quad \tfrac{\sqrt{3}}{2}\right] \tag{15}$$

which achieve the subchannel gains

$$\gamma_k = \frac{\frac{3}{4}}{25 + \frac{3}{4}} \; \forall k \tag{16}$$

and the sum power

$$P_{\text{CC}} = 2\,\frac{25 + \frac{3}{4}}{\frac{3}{4}} = \frac{206}{3} < 101 = P_{\text{CN}}. \tag{17}$$

This proves the theorem. ■

The fact that CC transmission can be necessary to achieve the optimal performance in parallel MIMO broadcast channels with zero-forcing constraints is not restricted to the power minimization problem. In particular, above theorem has the following corollaries concerning throughput maximization under QoS constraints (e.g., [12], [25], [26]) and rate balancing (e.g., [10], [27]).

Corollary 1: CN transmission is not always optimal for a throughput maximization with sum power constraint $P \leq P_{\text{max}}$, zero-forcing constraints (4) or (5), and quality of service constraints (7).

Proof: Consider the system used in the proof of Theorem 1 and assume that P_{max} is chosen such that $P_{\text{CC}} < P_{\text{max}} < P_{\text{CN}}$. Then, the optimization is feasible with CC ZFBF, but infeasible with CN ZFBF. ■

Corollary 2: CN transmission is not always optimal for rate balancing with sum power constraint $P \leq P_{\text{max}}$ and zero-forcing constraints (4) or (5).

Proof: In an optimal CC rate balancing solution with per-user rates r_k, the power constraint must be active, i.e., $P_{\text{CC}} = P_{\text{max}}$ since otherwise, the rates could be increased at the cost of an increased transmit power. If $P_{\text{CN}} > P_{\text{CC}}$ is needed to achieve the same per-user rates with CN transmission, which can happen due to Theorem 1, all per-user rates have to be reduced by a common factor to obtain a feasible CN rate balancing solution with $r'_k < r_k$ and $P'_{\text{CN}} = P_{\text{max}}$. ■

IV. Power Trade-off

As in the case without zero-forcing discussed in [15], the optimization of carrier-cooperative (CC) strategies is much more involved than the carrier-noncooperative (CN) case. One reason for this is that an optimization problem in far more variables has to be solved since also the off-diagonal blocks of the filter matrices need to be optimized. Another challenge is that many existing optimization algorithms are not able to find CC solutions [2]. Therefore, it is necessary to understand under which conditions a significant gain can be expected so that performing the more complex CC optimization is worth the effort. In this section, we present a first approach towards this goal by further studying the minimal example introduced above.

We consider the subchannel gain of user 2 as a function of the subchannel gain of user 1, i.e.,

$$\gamma_2(\gamma_1) = \max_{\boldsymbol{g}_1,\boldsymbol{g}_2} \frac{\left|\boldsymbol{g}_2^{\mathrm{H}}\boldsymbol{H}_2\boldsymbol{P}\boldsymbol{H}_1^{\mathrm{T}}\boldsymbol{g}_1^*\right|^2}{\boldsymbol{g}_1^{\mathrm{H}}\boldsymbol{H}_1\boldsymbol{H}_1^{\mathrm{H}}\boldsymbol{g}_1\boldsymbol{g}_2^{\mathrm{H}}\boldsymbol{C}_2\boldsymbol{g}_2} \quad \text{s.t.} \quad \frac{\left|\boldsymbol{g}_1^{\mathrm{H}}\boldsymbol{H}_1\boldsymbol{P}\boldsymbol{H}_2^{\mathrm{T}}\boldsymbol{g}_2^*\right|^2}{\boldsymbol{g}_2^{\mathrm{H}}\boldsymbol{H}_2\boldsymbol{H}_2^{\mathrm{H}}\boldsymbol{g}_2\boldsymbol{g}_1^{\mathrm{H}}\boldsymbol{C}_1\boldsymbol{g}_1} = \gamma_1. \tag{18}$$

for the minimal example with $C = 2$ carriers, $M = 1$ transmit antenna, and $K = 2$ users with $N_k = 1$ receive antenna (cf. Section III). This optimization can be tackled by introducing the parametrization

$$\boldsymbol{g}_k^{\mathrm{H}} = \mathit{\Gamma}_k \begin{bmatrix} \mathrm{e}^{-\mathrm{j}\,\beta_k}\cos\alpha_k & \sin\alpha_k \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{C_k^{(1)}}} & \\ & \frac{1}{\sqrt{C_k^{(2)}}} \end{bmatrix} \tag{19}$$

where we can choose $\mathit{\Gamma}_k = 1$, $\beta_2 = 0$, and $\alpha_k, \beta_1 \in [0, \pi]$ without loss of generality.[2] Introducing a Lagrange multiplier $\lambda \in \mathbb{R}$ for the equality constraint, we get the Lagrange function

$$L = \frac{\left|\boldsymbol{g}_2^{\mathrm{H}}\boldsymbol{H}_2\boldsymbol{P}\boldsymbol{H}_1^{\mathrm{T}}\boldsymbol{g}_1^*\right|^2}{\boldsymbol{g}_1^{\mathrm{H}}\boldsymbol{H}_1\boldsymbol{H}_1^{\mathrm{H}}\boldsymbol{g}_1\boldsymbol{g}_2^{\mathrm{H}}\boldsymbol{C}_2\boldsymbol{g}_2} + \lambda\left(\frac{\left|\boldsymbol{g}_1^{\mathrm{H}}\boldsymbol{H}_1\boldsymbol{P}\boldsymbol{H}_2^{\mathrm{T}}\boldsymbol{g}_2^*\right|^2}{\boldsymbol{g}_2^{\mathrm{H}}\boldsymbol{H}_2\boldsymbol{H}_2^{\mathrm{H}}\boldsymbol{g}_2\boldsymbol{g}_1^{\mathrm{H}}\boldsymbol{C}_1\boldsymbol{g}_1} - \gamma_1\right) \tag{20}$$

and the KKT conditions (e.g., [28])

$$\frac{\partial L}{\partial \alpha_1} = 0, \qquad \frac{\partial L}{\partial \beta_1} = 0, \qquad \frac{\partial L}{\partial \alpha_2} = 0, \qquad \frac{\partial L}{\partial \lambda} = 0 \tag{21}$$

which can, after some algebraic manipulations, be rewritten as a system of four polynomial equations in the variables $x = \tan\alpha_1\cos\beta_1$, $y = \tan\alpha_1\sin\beta_1$, $z = \tan\alpha_2$, and λ. Solving such a polynomial system, i.e., finding all its roots, is a well investigated problem and can be done using a solver such as PHCpack [29].

To perform the change of variables described above, we have to exclude all cases where $\alpha_k = \frac{\pi}{2}$ for some k. However, such a choice of α_k corresponds to CN transmission, for which the two fractions in (18) can be easily calculated. Comparing the value of the objective function at these potential CN solutions and at all roots of the polynomial system (i.e., at all KKT points), we can find the global optimum of the maximization in (18), i.e., we can evaluate the function $\gamma_2(\gamma_1)$.

[2] Due to the special structure of $\boldsymbol{H}_k$ and $\boldsymbol{P}$, only the difference $\beta_1 - \beta_2$ plays a role. Choosing $\beta_1 \in [0, \pi]$ is sufficient since adding a value of π to β_1 is equivalent to replacing $\alpha_1 \in [0, \pi]$ by $\alpha_1' = \pi - \alpha_1 \in [0, \pi]$.

Let us now consider channel realizations where the order of the carriers according to the channel quality is the same for all users, i.e., there exists a permutation π such that

$$\frac{\left|H_k^{(\pi(1))}\right|^2}{C_k^{(\pi(1))}} \geq \frac{\left|H_k^{(\pi(2))}\right|^2}{C_k^{(\pi(2))}} \geq \cdots \geq \frac{\left|H_k^{(\pi(C))}\right|^2}{C_k^{(\pi(C))}} \; \forall k. \tag{22}$$

We call such channels *spectrally similar*. For the minimal example with $C = 2$ carriers, this definition reduces to

$$\frac{\left|H_k^{(c)}\right|^2}{C_k^{(c)}} \geq \frac{\left|H_k^{(d)}\right|^2}{C_k^{(d)}} \; \forall k \in \{1, 2\} \tag{23}$$

for some carrier c and $d \neq c$. In this case, the highest possible gain $\gamma_k = \gamma_{k,\text{max}} = \frac{|H_k^{(c)}|^2}{C_k^{(c)}}$ for user k is achieved in the minimal example by inserting $\boldsymbol{g}_k = \boldsymbol{e}_c$ and $\boldsymbol{g}_\ell = \boldsymbol{e}_d$ for $\ell \neq k$ into (13), which implies that the gain $\gamma_\ell = \gamma_{\ell,\text{min}} = \frac{|H_\ell^{(d)}|^2}{C_\ell^{(d)}}$ is achieved for user ℓ. Note that this extreme case corresponds to CN transmission. It can be easily shown that every CC strategy must fulfill $\gamma_k \leq \gamma_{k,\text{max}} \; \forall k$ and, in order to lead to a power reduction, should also fulfill $\gamma_k \geq \gamma_{k,\text{min}} \; \forall k$. Between these two boundaries, we have the following behavior.

Proposition 1: For the minimal example with $C = 2$ carriers, $M = 1$ transmit antenna, and $K = 2$ users with $N_k = 1$ receive antenna with spectrally similar channels (23), $\gamma_2(\gamma_1)$ defined in (18) is strictly decreasing for $\gamma_1 \in [\gamma_{1,\text{min}}, \gamma_{1,\text{max}}]$.

Proof of Proposition 1: Let us replace the equality constraint in (18) by an inequality constraint, and suppose that this constraint is inactive in an optimal solution, i.e., $\tilde{\gamma}_1 > \gamma_1$ with $\tilde{\gamma}_1 = \frac{|\boldsymbol{g}_1^{\text{H}} \boldsymbol{H}_1 \boldsymbol{P} \boldsymbol{H}_2^{\text{T}} \boldsymbol{g}_2^*|^2}{\boldsymbol{g}_2^{\text{H}} \boldsymbol{H}_2 \boldsymbol{H}_2^{\text{H}} \boldsymbol{g}_2 \boldsymbol{g}_1^{\text{H}} \boldsymbol{C}_1 \boldsymbol{g}_1}$. In this case, the optimal value would be equal to the optimum of the corresponding unconstrained optimization, which is $\gamma_{2,\text{max}}$. However, as $\gamma_2 = \gamma_{2,\text{max}}$ implies $\tilde{\gamma}_1 = \gamma_{1,\text{min}}$ in the case of spectrally similar channels (see above), this contradicts the assumption $\tilde{\gamma}_1 > \gamma_1 \in [\gamma_{1,\text{min}}, \gamma_{1,\text{max}}]$. Consequently, the constraint is active, and the optimization with inequality constraint is equivalent to the original optimization. Moreover, since the inequality constraint is active, increasing γ_1, i.e., making the constraint more restrictive, must lead to a decrease of the optimal value $\gamma_2(\gamma_1)$. ■

The monotonicity can also be observed in Fig. 1, which shows $\gamma_2(\gamma_1)$ for the channel realization (9). A consequence of the monotonicity is that the required sum transmit power P considered as a function of γ_1 is the sum of a decreasing function $P_1(\gamma_1)$ and an increasing function $P_2(\gamma_1)$ [cf. (14)]:

$$P(\gamma_1) = \underbrace{\frac{2^{\varrho_1} - 1}{\gamma_1}}_{P_1(\gamma_1)} + \underbrace{\frac{2^{\varrho_2} - 1}{\gamma_2(\gamma_1)}}_{P_2(\gamma_1)}. \tag{24}$$

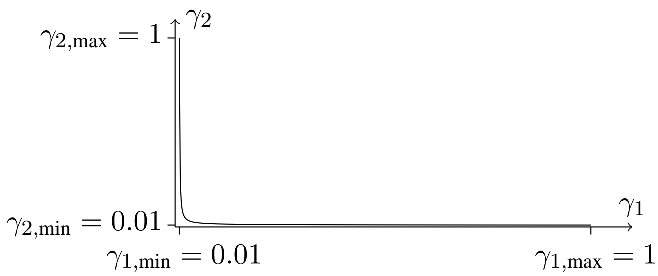

Fig. 1. $\gamma_2(\gamma_1)$ for the spectrally similar channels (9).

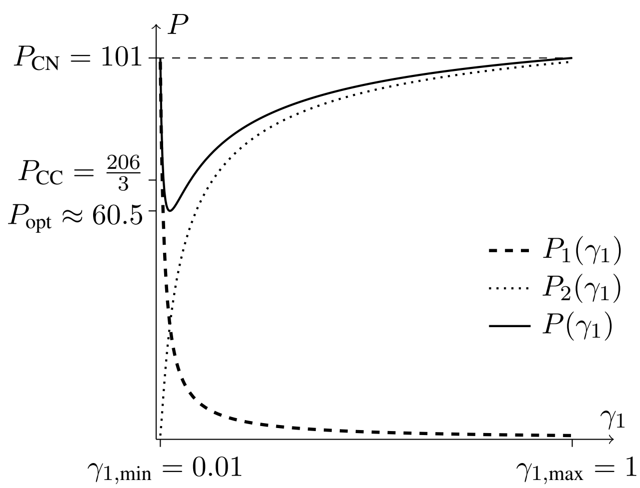

Fig. 2. $P(\gamma_1)$ for the spectrally similar channels (9) with $\varrho_k = 1 \ \forall k$.

Therefore, by modifying the value of γ_1, we can trade off the transmit power needed for user 1 against the power needed for user 2. An example can be seen in Fig. 2 for the channel realization (9) and the rate requirements $\varrho_k = 1 \ \forall k$.

In the figure, we see that even a sum transmit power P_{opt} slightly lower than P_{CC} from (17), which was used in the proof of Theorem 1, can be achieved in this scenario. To find this optimum numerically, we can apply the branch-reduce-and-bound algorithm from [30] since (24) can be written as the difference of increasing functions.

Note that we can also find scenarios with spectrally similar channel realizations for which $P(\gamma_1)$, despite being a difference of monotonic functions, is monotonic for $\gamma_1 \in [\gamma_{1,\min}, \gamma_{1,\max}]$ (e.g., for $\varrho_k \gg \varrho_\ell$). If this is the case, the sum transmit power is minimized at one of the extreme points, which correspond to CN transmission.

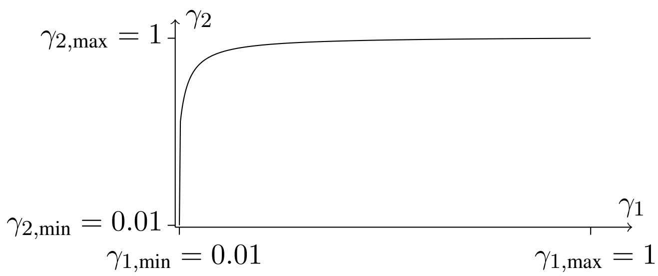

Fig. 3. $\gamma_2(\gamma_1)$ for the spectrally dissimilar channels (25).

We have seen that CC transmission might or might not be beneficial if the channels are spectrally similar. On the other hand, if the channels are not spectrally similar, CC never leads to a reduction of the sum power.

Proposition 2: If the channels are not spectrally similar according to definition (23) in the minimal example with $C = 2$ carriers, $M = 1$ transmit antenna, and $K = 2$ users with $N_k = 1$ receive antenna (cf. Section III), CN transmission achieves the optimal sum transmit power.

Proof of Proposition 2: Without loss of generality, let $\frac{|H_k^{(k)}|^2}{C_k^{(k)}} > \frac{|H_k^{(d)}|^2}{C_k^{(d)}}$ for all k and $d \neq k$. In this case, the highest possible gain $\gamma_k = \gamma_{k,\max} = \frac{|H_k^{(k)}|^2}{C_k^{(k)}}$ for user k is achieved by inserting $\boldsymbol{g}_k = \boldsymbol{e}_k$ and $\boldsymbol{g}_\ell = \boldsymbol{e}_\ell$ for $\ell \neq k$ into (13), which implies that the gain $\gamma_\ell = \gamma_{\ell,\max} = \frac{|H_\ell^{(\ell)}|^2}{C_\ell^{(\ell)}}$ is achieved for user ℓ. Since this is a CN strategy and no strategy can achieve $\gamma_k > \gamma_{k,\max}$ for any k, CN transmission achieves the minimal sum transmit power [cf. (14)] in this case. ■

An example can be seen in Fig. 3 and 4, where we have used the spectrally dissimilar channels

$$\boldsymbol{H}_1 = \begin{bmatrix} 1 & \\ & 0.1 \end{bmatrix}, \qquad \boldsymbol{H}_2 = \begin{bmatrix} 0.1 & \\ & 1 \end{bmatrix}. \tag{25}$$

Above observations have an interesting implication for practical systems. Still considering the minimal example, let us assume a scenario where the receivers encounter strong interference from another communication system on one carrier c, but not on the other carrier d. Such a situation could, for example, happen in a cellular network with inter-cell interference or in a system that operates in an unlicensed band. Treating this interference as additional noise, we obtain $C_k^{(c)} \gg C_k^{(d)}$ for both users, and it is very likely that the condition (23) for spectral similarity is fulfilled. In this case, CC transmission might lead to a significantly reduced sum transmit power.

These considerations, which are extended to larger scenarios in our companion work [24], show that

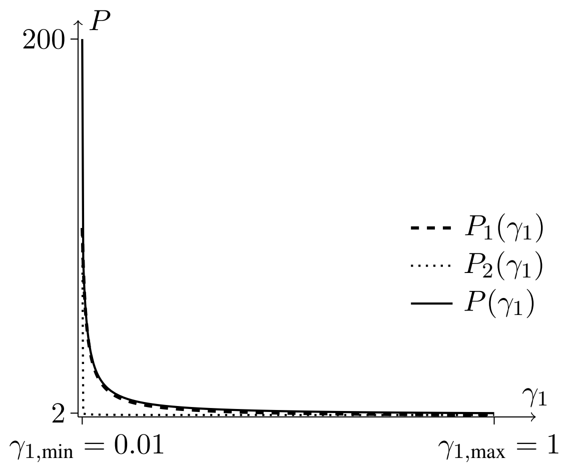

Fig. 4. $P(\gamma_1)$ for the spectrally dissimilar channels (25) with $\varrho_k = 1\ \forall k$.

spectral similarity is not only a theoretical construct, but a phenomenon that can indeed occur in a realistic scenario. Therefore, even though the propositions stated in this section only apply to a very limited special case, they provide important insights that also hold for the general case.

V. Dirty Paper Coding with Zero-Forcing

If zero-forcing is combined with dirty paper coding (e.g., [10]–[12], [16]), the zero-forcing constraints in (4) can be relaxed to

$$\boldsymbol{g}_{k,s}^{\mathrm{H}} \boldsymbol{H}_k \boldsymbol{b}_{\ell,t} = 0 \quad \forall k, s, \ell, t : \pi(\ell, t) > \pi(k, s) \tag{26}$$

where π is the encoding order of the data streams. If $\pi(\ell, t) < \pi(k, s)$, the data stream (k, s) is encoded after the stream (ℓ, t) and does not encounter interference from the stream (ℓ, t) since this interference can be precompensated by means of DPC. Therefore, zero-forcing for $\pi(\ell, t) < \pi(k, s)$ does not need to be performed by means of the transmit and receive filter matrices.

Theorem 2: In parallel MIMO broadcast channels with dirty paper coding, zero-forcing constraints (26), and quality of service constraints (7) where time-sharing is not allowed, carrier-noncooperative transmission does not always achieve the minimal feasible sum power.

Proof of Theorem 2: We provide a proof by construction using the same minimal example as in the proof of Theorem 1. The key point of the proof is to observe that also with DPC-ZF, only one user can be served on each carrier in the case of CN transmission. Therefore, $P_{\text{CN,DPC}} = P_{\text{CN}} = 101$ also holds for DPC-ZF. As optimal DPC-ZF cannot perform worse than ZFBF, we can achieve $P_{\text{CC,DPC}} \leq P_{\text{CC}} < P_{\text{CN}}$. This proves the theorem. ∎

Note that just like the proof of Theorem 1, this proof is valid for the general case, even though the provided example is only a special case of parallel MIMO broadcast channels.

Obviously, due to Theorem 2, the corollaries of Theorem 1 can be directly extended to the case of DPC-ZF, i.e., CN transmission can be suboptimal for throughput maximization with quality of service constraints and for rate balancing in parallel MIMO broadcast channels with DPC and zero-forcing.

Note that in the minimal scenario under consideration, the gap between CC and CN transmission is much more pronounced for DPC-ZF than for ZFBF: using carrier-cooperative DPC-ZF with an appropriate choice of the filter vectors, $P_{\text{CC,DPC}} \approx 20$ can be achieved.

VI. Discussion and Outlook

By constructing a minimal example, we have proven that carrier cooperation can reduce the transmit power in parallel MIMO broadcast channels with zero-forcing (ZF) constraints and minimum rate constraints if time-sharing is not allowed. This case is of high relevance for the design of practical systems since time-sharing is unattractive for practical implementation due to its high signaling overhead. The results of this paper hold for linear zero-forcing beamforming (ZFBF) as well as for zero-forcing combined with dirty paper coding (DPC-ZF), and they extend to throughput maximization with quality of service constraints and to rate balancing.

In the minimal example studied in this paper, carrier cooperation can achieve a power saving of more than 2dB for ZFBF and 7dB for DPC-ZF, and it is easy to construct examples with even higher gains. However, an important question is whether such significant gains can also be achieved in systems of larger dimensionality (more users $K > 2$, more carriers $C > 2$, and more antennas) with more realistic channel realizations (instead of constructed ones). In this case, additional questions can be taken into consideration. For instance, it could be studied whether or not carrier-cooperation across small groups of carriers instead of across all carriers suffices to obtain most of the performance gain. Another topic for future research would be to extend the study to (weighted) sum rate maximization problems or to the case where time-sharing is allowed.

In addition to trying to analytically quantify the performance gains possible with CC ZFBF in more general scenarios as a topic of future research, a quantification can also be given by means of numerical studies. Therefore, in our companion work [24], we develop a CC ZFBF algorithm, which can on average indeed achieve significant gains compared to state-of-the-art ZFBF algorithms based on CN transmission. Moreover, by extending the notion of spectrally similar channels to larger systems, we perform numerical simulations with a channel model that assumes inter-cell interference whose power varies as a function of the carrier index. In the simulations presented in [24], the proposed CC ZFBF algorithm is able to close half of the gap between the CN reference algorithm from [12] and the DPC optimum. As this improved performance comes at the cost of an iterative optimization (instead of a purely successive method such as the one used, for example, in [12]), future research should investigate whether similar gains are also achievable by optimizing CC ZFBF transmission with a purely successive algorithm.

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