Chair of Structural Analysis

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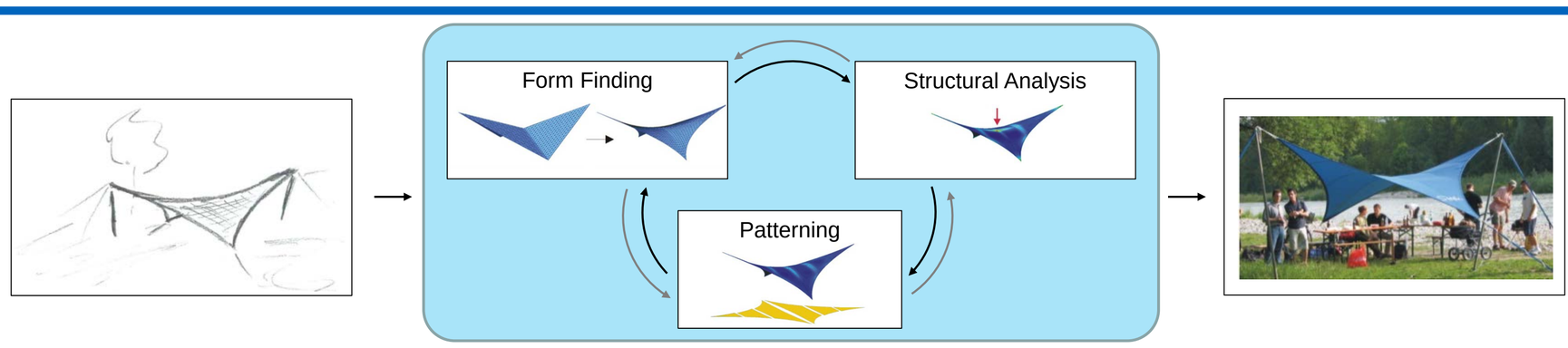
Numerical Methods for the Analysis and Design of Tensile Structures

Membrane Structures are lightweight structures, which combine optimal stress state of the material with an impressive language of shapes. The shape of membrane structures is defined by the equilibrium of surface stress and cable edge forces in tension. Throughout the whole design process of membrane structures the variation of prestress constitutes the main shaping parameter. Details of cutting pattern and compensation are affected by residual stresses from developing curved surfaces into the plane and anisotropic material properties. With the knowledge of this, numerical methods for the design and analysis of membrane structures should be able to deal with all sources of stress state in a proper way.

Design Process

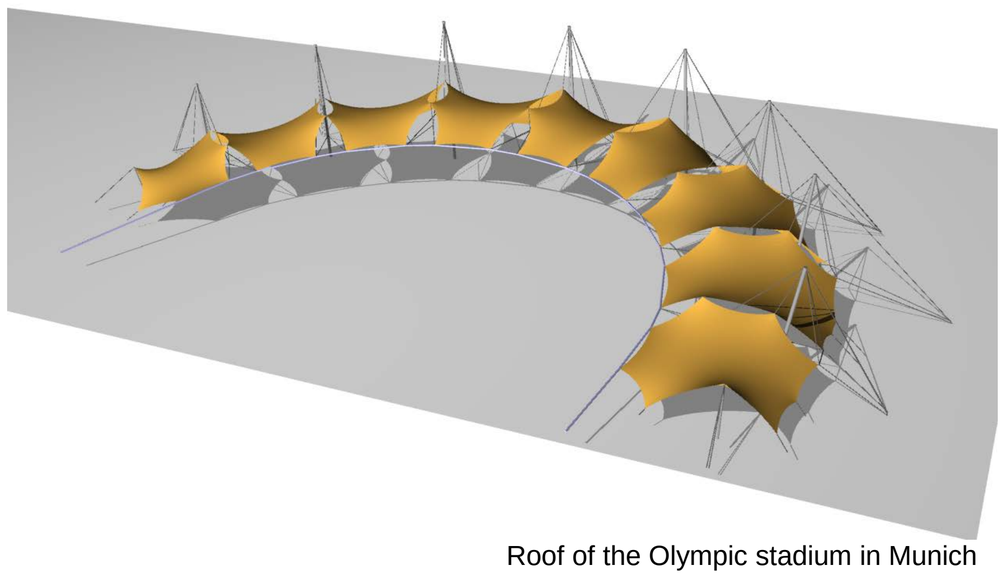
The design of a tensile structure is an integrated process were all steps influence each other. Form finding, structural analysis and cutting pattern interact to describe the final shape of the structure. None of these steps can be considered independently.



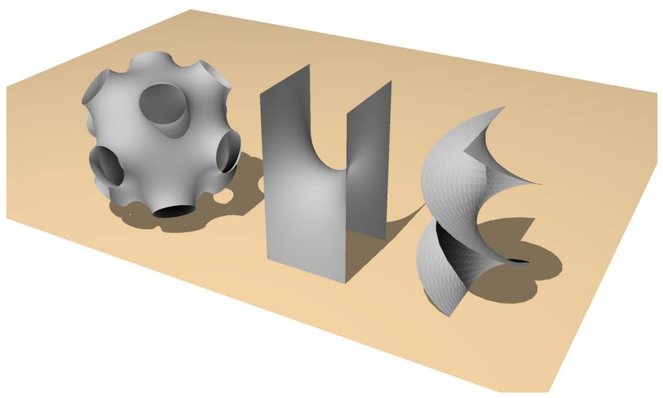
Form Finding

The first step in designing tensile structures is to find the final form. Form finding is the task to find the shape of equilibrium with respect to given surface stress state **σ** and natural (in terms of edge forces) or geometrical (e.g. clamped edges) boundary conditions. Additional loading, as e.g. internal pressure (cushions), has to be considered, too. Considering the non-linear kinematics of large deflections the equilibrium condition in the deformed, current configuration equilibrium is defined by the principle of virtual work

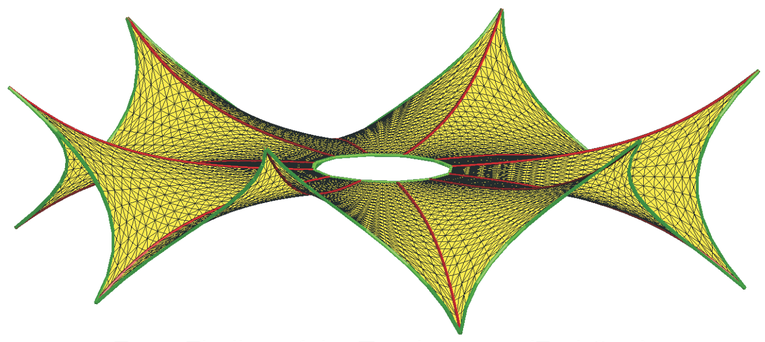
Norwegian Pavilion at the Expo 2011 in Shanghai



Roof of the Olympic stadium in Munich



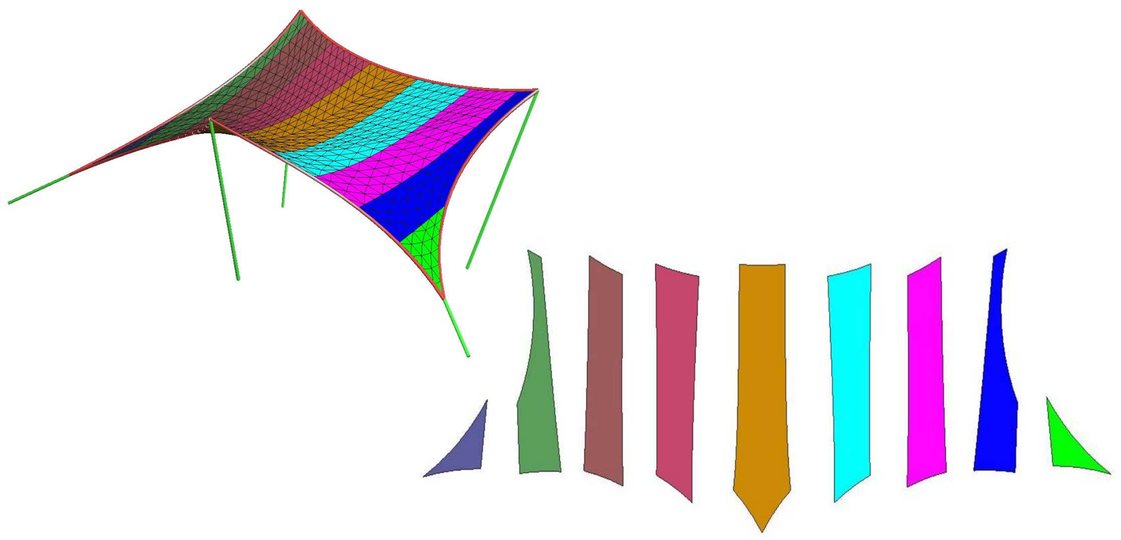
Minimal surfaces (Schön, Sherk, Helicoid)



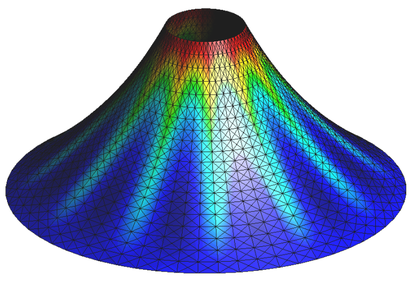
Form Finding of the Tanzbrunnen (Frei Otto)

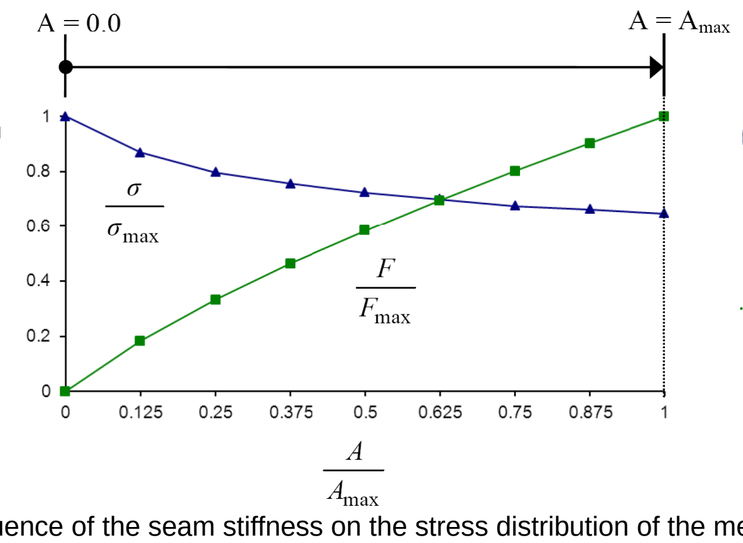
Cutting Pattern Generation

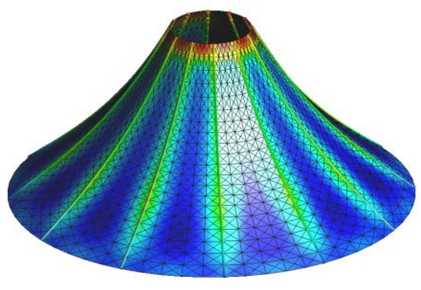
Due the nature of tensile structures the shapes are general doubly curved. Form the work of Carl Friedrich Gauß it is well known that a general doubly curved surface cannot be developed into a plane without compromises. This results in additional residual stresses when the structure will be erected. In addition, the elastic deformation due to pre-stress has to be compensated. Typically, a two stage procedure is applied consisting of (i) forced flattening of the curved surface into a plane by pure geometrical considerations and (ii) compensation of both, the intended pre-stress and the additional elastic stresses of the flattening procedure. The step which finds the best flat pattern for the tensile structure is called Cutting Pattern Generation



Realization of a 5 point membrane in a student workshop. Form finding and cutting pattern generation







Influence of the seam stiffness on the stress distribution of the membrane