Chair of Structural Analysis

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Drape Simulation and Optimized Cutting Pattern for Structures made of Carbon Fiber Reinforced Plastics (CFRP)

In various fields of civil and mechanical engineering a significant rise of lightweight design concepts can be observed, e.g. stringers and ribs (aeronautic/aerospace engineering), frame components and car body panels (automotive engineering), membrane rooftops and pneumatic structures (architectural membranes). The basic idea of lightweight design is an increasing load carrying behavior due to a combination of curvature, i.e. synclastic or anticlastic surfaces, and high tensile materials, like CFRPs or coated textiles. This leads to double-curved surfaces and geometries whose feasibility is directly linked to the shear flexibility of the material. However, the plane shape of the wrought material contradicts with the non-developable characteristics of the final structure. This contradiction leads to the crucial question a design engineer has to deal with: How should the plane structure look such that the least deviation of the spatial structure and/or fiber orientation is achieved?

Variation of Reference Strategy

The Variation of Reference Strategy (VaReS) is based on an inverse approach, i.e. the deformed configuration (3D) of the structure is given and the reference configuration (2D) is not known. Therefore, the nodal positions in the reference configuration (2D) are defined as degrees of freedom.

Considering a non-developable surface, the transformation of the plane weave into the desired shape leads to residual stresses within the membrane. MAURIN AND MOTRO came up with the idea of minimizing the sum of the residual stresses and a prescribed stress state in a global sense. The presented numerical approach solves the mentioned unconstrained optimization problem by means of the Variation of Reference Strategy.

Constitutive models

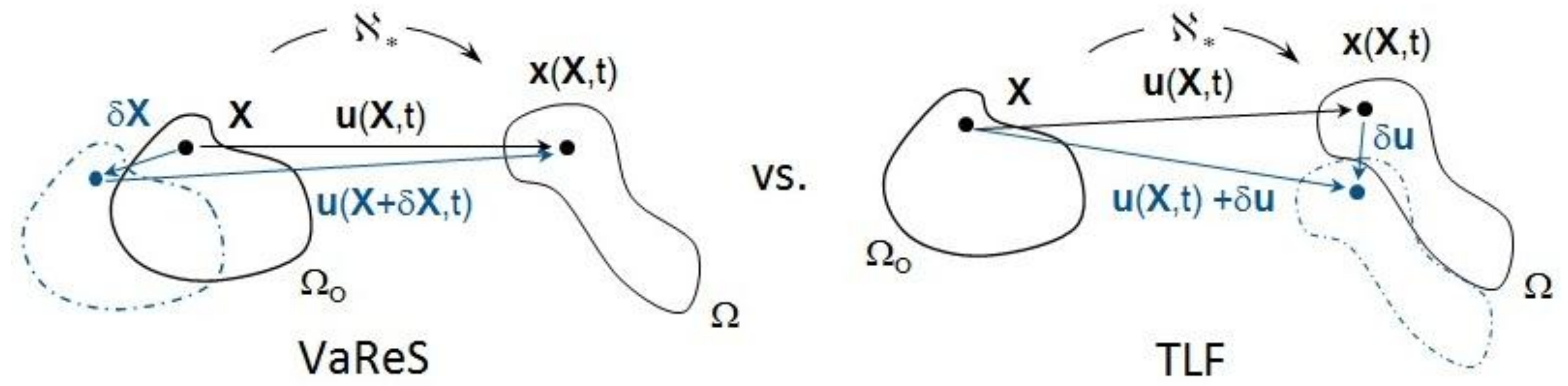
The underlying material model capturing the nonlinear and anisotropic material behavior is based on a hyperelastic approach.

In addition to the three principal invariants characterizing the isotropic behavior, two more invariants for each fiber direction measuring the fiber stretch are defined. These additional invariants depend on corresponding structural tensors extracting the fiber deformation of the right Cauchy-Green tensor.

Workflow

The outer contour of the converged reference geometry leads directly to an optimized cutting pattern.

Assuming no slip conditions between the tool and the fabric, the fiber distortions and the remaining residual stresses can be calculated by means of the deformation gradient defining the map between the optimized cutting pattern (2D) and the final structure (3D).



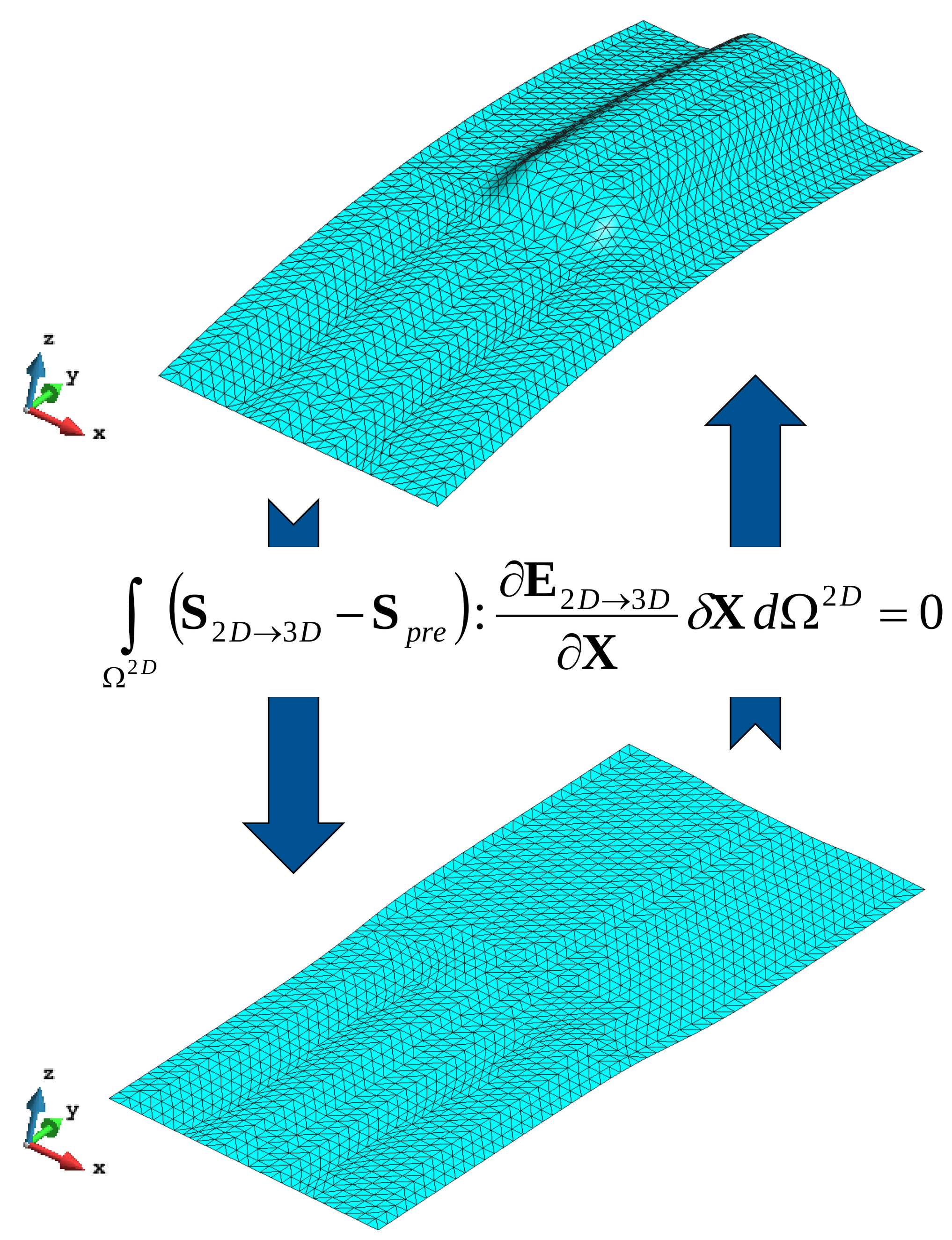
The unconstrained optimization problem: Galerkin approach

$$\min_{X_{2D}} \rightarrow \Pi = \sigma_{2D\rightarrow 3D} - \sigma_{pre} \quad \Rightarrow \quad \int_{\Omega^{3D}} \left(\sigma_{2D\rightarrow 3D} - \sigma_{pre}\right) : \delta\varepsilon_{2D\rightarrow 3D}\, d\Omega^{3D} = 0$$

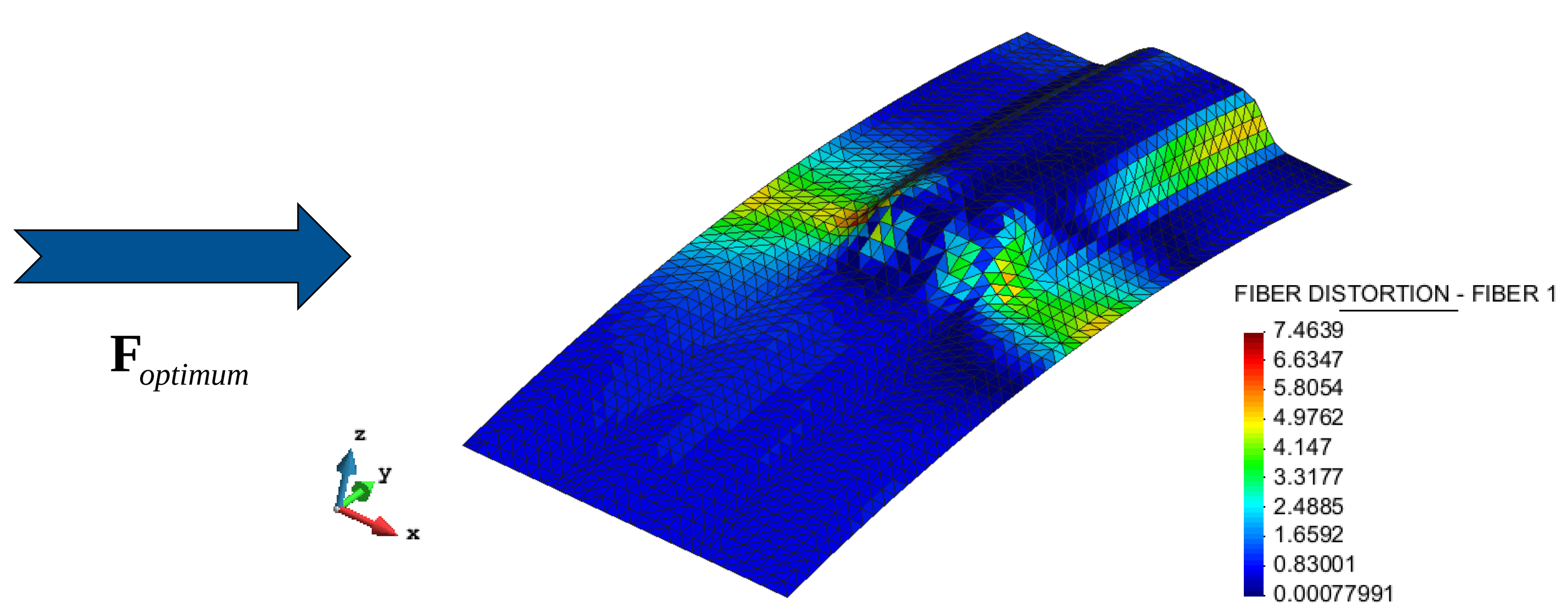
Variation of the Euler-Almansi strain tensor

$$\delta\varepsilon\left(\mathbf{U}(\mathbf{X},\delta\mathbf{X})\right) = \aleph_*\left(D_{\delta\mathbf{X}}\aleph_*^{-1}\varepsilon\left(\mathbf{U}(\mathbf{X})\right)\right) = \mathbf{F}^{-T} D_{\delta\mathbf{X}}\mathbf{E}\left(\mathbf{U}(\mathbf{X})\right)\mathbf{F}^{-1}$$

Workflow



$$\int_{\Omega^{2D}} \left(\mathbf{S}_{2D\rightarrow 3D} - \mathbf{S}_{pre}\right) : \frac{\partial \mathbf{E}_{2D\rightarrow 3D}}{\partial \mathbf{X}}\, \delta\mathbf{X}\, d\Omega^{2D} = 0$$

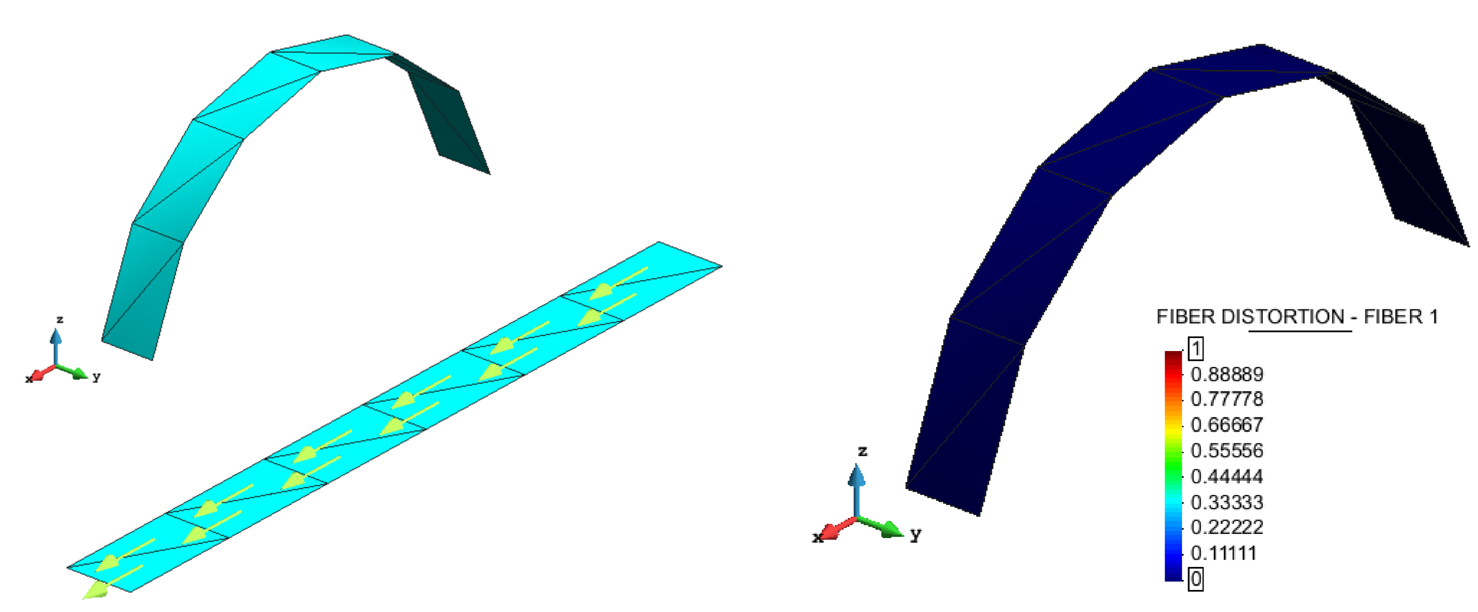


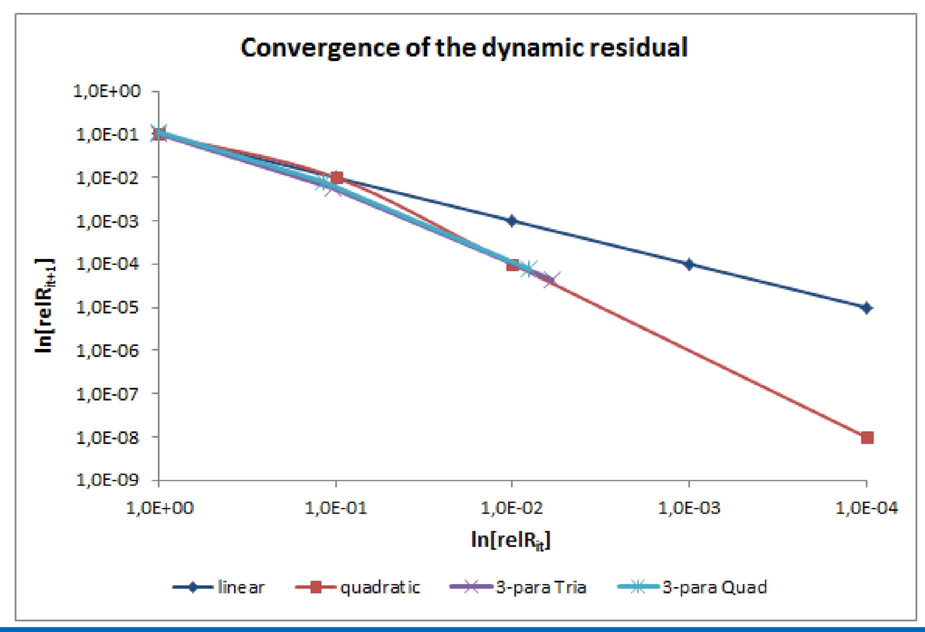
Constitutive models

$$\mathbf{S} = 2\sum_{i=1}^{n} \frac{\partial \Psi\left(I_1 \ldots I_n\right)}{\partial I_i} \frac{\partial I_i}{\partial \mathbf{C}}$$

$$\mathbf{D} = 4\sum_{i=1}^{n} \frac{\partial I_i}{\partial \mathbf{C}} \otimes \frac{\partial^2 \Psi\left(I_1 \ldots I_n\right)}{\partial I_i\, \partial \mathbf{C}} + \frac{\partial \Psi\left(I_1 \ldots I_n\right)}{\partial I_i} \frac{\partial^2 I_i}{\partial \mathbf{C}\, \partial \mathbf{C}}$$

Benchmark problem: 160 degree arc





References:

- B. Maurin and R. Motro: Cutting Pattern of Fabric Membranes with the Stress Compensation Method. International Journal of Space Structures, Vol. 14 No. 2, 1999
- A. Widhammer, R. Wüchner and K-U. Bletzinger: Drape Simulation for Non-Developable Multi-Layered CFRP Structures Focusing on Optimized Cutting Patterns, 20th ECCOMAS Conference, Vienna, 2012