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Abstract

The use of proper, i.e., circularly symmetric, complex Gaussian signals for all users is known to be optimal in broadcast channels with proper complex Gaussian noise from an information theoretic point of view, i.e., they are employed in the capacity-achieving strategy. However, such proper per-user transmit signals are not necessarily optimal for problems with quality-of-service (QoS) constraints if the transmit strategy is restricted to widely linear transceivers without time-sharing. This is shown by deriving the QoS feasibility region of the multiple-input multiple-output broadcast channel with improper Gaussian per-user transmit signals and widely linear transceivers.

Index Terms

Asymmetric complex signaling, broadcast channels, improper signals, multiuser MIMO systems, quality-of-service (QoS) region, widely linear transceivers.

EDICS Categories:

COM-MIMO MIMO communications and space-time or space-frequency coding,

COM-CODPHY Coding and Precoding design, Optimal PHY for single and multiuser systems.

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I. Introduction

The transmission of proper [1], i.e., circularly symmetric, complex Gaussian signals has been shown to be optimal in single-user multiple-input multiple-output (MIMO) channels with additive proper complex Gaussian noise [2]. According to [3] and [4], the capacity-achieving strategies in multiple-input single-output (MISO) and MIMO broadcast channels (BC) also use proper complex Gaussian transmit signals. An intuitive interpretation is as follows. In the capacity achieving transmit strategy with dirty paper coding (DPC), there is one user that does not receive any interference. Therefore, the transmit signal for this user should be proper complex Gaussian as in the single-user case. As a consequence, the next user, who receives just this one interfering signal, sees proper interference and proper noise, i.e., the effective noise is proper. Thus, for the next user, it also makes sense to use a proper transmit signal. This reasoning can be successively applied to all users. A similar reasoning was used in [5] to intuitively explain the separability of parallel MIMO BCs with DPC.

The assumption of proper complex Gaussian per-user transmit signals is generally adopted in the literature on Gaussian BCs even if DPC is not applied. However, as shown in this paper, the optimality of proper complex Gaussian per-user transmit signals does not necessarily hold if we restrict ourselves to widely linear transceivers without time-sharing.

In the context of proper signals, the term *linear transceivers* is commonly used for systems where nonlinear operations (encoding, detection, ...) are only applied to single data streams while all filtering operations that involve more than one data stream have to be linear (e.g., [6]).[1] With *widely linear transceivers*, we refer to the generalization where the originally linear filters are allowed to be widely linear [7]. The output of such filters is a linear function of the input signals and their complex conjugates or, equivalently, of their real and imaginary parts. In this paper, the latter formulation (known as composite real representation [8]) is used as it is more convenient for deriving the QoS feasibility region. Note that the real and imaginary parts of a complex baseband signal are only mathematical models for the inphase

[1] Unlike in BCs, capacity is achievable with linear transceivers in single-user MIMO channels. Thus, a restriction to (widely) linear transceivers does not have any effect there, and proper signals remain optimal.

and the quadrature components of the signal, which can be easily separated from each other. The restriction to widely linear transceivers implies that we do not allow joint encoding techniques such as DPC.

Widely linear processing is a popular concept and has been applied to communication systems that employ improper signal constellations (for instance, to comply with existing specifications and standards), e.g., [9]–[16], and to systems that encounter improper noise, e.g., [17]. Moreover, it has been studied in the context of linear-dispersion space-time codes, e.g., [18]–[21], and, recently, in interference channels [22]. The considerations in this paper differ from those in the literature: we consider a MIMO BC which does not encounter improper noise and which is studied from a Shannon rate perspective instead of for a particular signaling scheme (e.g., improper constellation, Alamouti code, …). Nevertheless, it turns out that introducing impropriety by means of widely linear processing can be beneficial also in this setting.

Time-sharing refers to applying different transmit strategies one after another to achieve some average data rate with some average transmit power. As this can lead to higher performance, but involves high signaling overhead, both cases—with and without time-sharing—are worth being considered. In this work, we study transmission without time-sharing (i.e., with one fixed transmit strategy for each given channel realization) while suboptimality of proper signals in the case with time-sharing is shown in our companion work [23].

If we consider the problem of achieving required minimal rates for all users in a BC with linear transceivers without time-sharing (e.g., [24]), the question of feasibility arises, i.e., it might happen that certain rate requirements cannot be fulfilled even if arbitrarily high transmit power is spent [24]–[26]. The same is true for BCs with improper signals transmitted by means of widely linear transceivers, but we will show that the set of feasible rates is different from the one derived in [26]. In contrast, arbitrary rate requirements are feasible in BCs that apply either DPC or time-sharing or both (cf. [27] and [28]).

By applying results from [26] to an equivalent real-valued BC, we derive the QoS feasibility region of MIMO BCs with improper per-user signals and widely linear transceivers. This region is larger than the feasibility region for the proper case. Thus, proper per-user transmit signals can be suboptimal in BCs with widely linear transceivers without time-sharing.

II. System Model

In a MIMO BC with M transmit antennas and K users, the kth user with N_k receive antennas shall recover $S_k \leq \min\{M, N_k\}$ data streams from the received signal

$$\boldsymbol{y}_k = \boldsymbol{H}_k \left(\sum_{k'=1}^{K} \boldsymbol{B}_{k'} \boldsymbol{s}_{k'} \right) + \boldsymbol{\eta}_k \tag{1}$$

(e.g., [15], [29]). Here, $\boldsymbol{s}_{k'}$ is the $S_{k'}$-dimensional vector of transmit symbols intended for user k', $\boldsymbol{B}_{k'}$ is the corresponding $M \times S_{k'}$ beamforming matrix, $\boldsymbol{H}_k$ is the $N_k \times M$ matrix describing the channel from the base station to receiver k, and $\boldsymbol{\eta}_k$ is the N_k-dimensional noise vector at receiver k. For the sake of brevity, the *per-user transmit signals* $\boldsymbol{B}_{k'}\boldsymbol{s}_{k'}$ will be simply called *transmit signals* from this point on.

We distinguish between three different cases:

a) Real-valued BC: In this case, the entries of the channel matrices $\boldsymbol{H}_k$ and the beamforming matrices $\boldsymbol{B}_k$ are real-valued, and the symbol vectors and noise vectors follow a real-valued Gaussian distribution, i.e., $\boldsymbol{s}_{k'} \sim \mathcal{N}(0, \mathbf{I}_{S_{k'}})$, and $\boldsymbol{\eta}_k \sim \mathcal{N}(0, \check{\boldsymbol{C}}_{\boldsymbol{\eta}_k})$. For given signal-to-interference-and-noise ratio (SINR) γ, the Shannon rate[2] of a stream is [30, Ch. 5]

$$r = \frac{1}{2}\log_2(1+\gamma). \tag{2}$$

b) BC with Proper Noise and Proper Transmit Signals: Now, the matrices $\boldsymbol{H}_k$ and $\boldsymbol{B}_k$ are complex, the symbol vectors and the noise vectors follow a circularly symmetric complex Gaussian distribution, i.e., $\boldsymbol{s}_{k'} \sim \mathcal{CN}(0, \mathbf{I}_{S_{k'}})$, and $\boldsymbol{\eta}_k \sim \mathcal{CN}(0, \boldsymbol{C}_{\boldsymbol{\eta}_k})$. The Shannon rate of a stream is [30, Ch. 5]

$$r = \log_2(1+\gamma). \tag{3}$$

c) BC with Proper Noise and Improper Transmit Signals: Since widely linear operations do generally not preserve propriety [8], we can generate improper transmit signals by replacing the product $\boldsymbol{B}_{k'}\boldsymbol{s}_{k'}$ in (1) by

$$\boldsymbol{x}_{k'} = \boldsymbol{B}_{\mathrm{R},k'}\,\Re\left(\boldsymbol{s}_{k'}\right) + \boldsymbol{B}_{\mathrm{I},k'}\,\Im\left(\boldsymbol{s}_{k'}\right) \tag{4}$$

with complex matrices $\boldsymbol{B}_{\mathrm{R},k'}$ and $\boldsymbol{B}_{\mathrm{I},k'}$ and a circularly symmetric input signal $\boldsymbol{s}_{k'} \sim \mathcal{CN}(0, \mathbf{I}_{S_{k'}})$. If $\boldsymbol{B}_{\mathrm{I},k'} = \mathrm{j}\,\boldsymbol{B}_{\mathrm{R},k'}$, we have $\boldsymbol{x}_{k'} = \boldsymbol{B}_{\mathrm{R},k'}\boldsymbol{s}_{k'}$ in (4), which is a linear function of $\boldsymbol{s}_{k'}$. In this case, $\boldsymbol{x}_{k'}$ is proper (cf. [8]). Otherwise, $\boldsymbol{x}_{k'}$ is a widely linear function of $\boldsymbol{s}_{k'}$ and can become improper.[3] The matrices $\boldsymbol{H}_k$ are still complex, and the noise vectors still follow a circularly symmetric complex Gaussian distribution, i.e., $\boldsymbol{\eta}_k \sim \mathcal{CN}(0, \boldsymbol{C}_{\boldsymbol{\eta}_k})$.

[2]The Shannon rates considered in this paper are upper bounds for data rates achievable with finite symbol alphabets (e.g., [30, Ch. 5]). Conversely, all powers in this paper are lower bounds to the powers required to achieve a certain rate with finite symbol alphabets.

[3]If $\boldsymbol{s}_{k'}$ is improper, $\boldsymbol{B}_{\mathrm{I},k'} \neq \mathrm{j}\,\boldsymbol{B}_{\mathrm{R},k'}$ is not needed to obtain improper $\boldsymbol{x}_{k'}$. However, in this paper, we assume proper $\boldsymbol{s}_{k'}$ without loss of generality.

Introducing a composite real representation [8], a BC with improper transmit signals can be described as real-valued BC

$$\boldsymbol{y}_{k,\text{real}} = \boldsymbol{H}_{k,\text{real}} \left(\sum_{k'=1}^{K} \tilde{\boldsymbol{B}}_{k'} \boldsymbol{s}_{k',\text{real}} \right) + \boldsymbol{\eta}_{k,\text{real}}. \tag{5}$$

The matrix $\tilde{\boldsymbol{B}}_{k'}$ is defined as

$$\tilde{\boldsymbol{B}}_{k'} = \begin{bmatrix} \Re\left(\boldsymbol{B}_{\text{R},k'}\right) & \Re\left(\boldsymbol{B}_{\text{I},k'}\right) \\ \Im\left(\boldsymbol{B}_{\text{R},k'}\right) & \Im\left(\boldsymbol{B}_{\text{I},k'}\right) \end{bmatrix} \tag{6}$$

and we use $\boldsymbol{A}_{\text{real}}$ and $\boldsymbol{a}_{\text{real}}$ to denote

$$\boldsymbol{A}_{\text{real}} = \begin{bmatrix} \Re\left(\boldsymbol{A}\right) & -\Im\left(\boldsymbol{A}\right) \\ \Im\left(\boldsymbol{A}\right) & \Re\left(\boldsymbol{A}\right) \end{bmatrix} \text{ and } \boldsymbol{a}_{\text{real}} = \begin{bmatrix} \Re\left(\boldsymbol{a}\right) \\ \Im\left(\boldsymbol{a}\right) \end{bmatrix} \tag{7}$$

for a complex matrix $\boldsymbol{A}$ and a complex vector $\boldsymbol{a}$, respectively. As each eigenvalue of $\boldsymbol{A}\boldsymbol{A}^{\text{H}}$ is a double eigenvalue of $\boldsymbol{A}_{\text{real}}\boldsymbol{A}_{\text{real}}^{\text{T}}$ (special case of [31, Theorem 1]) and since the rank of a matrix and that of its Gramian are equal, we have

$$\text{Rank}[\boldsymbol{A}_{\text{real}}] = 2\,\text{Rank}[\boldsymbol{A}]\,. \tag{8}$$

We note that $\boldsymbol{y}_{k,\text{real}} \in \mathbb{R}^{2N_k}$, $\boldsymbol{H}_{k,\text{real}} \in \mathbb{R}^{2N_k \times 2M}$, $\tilde{\boldsymbol{B}}_{k'} \in \mathbb{R}^{2M \times 2S_{k'}}$, $\boldsymbol{s}_{k',\text{real}} \in \mathbb{R}^{2S_{k'}}$, and $\boldsymbol{\eta}_{k,\text{real}} \in \mathbb{R}^{2N_k}$, i.e., the equivalent real-valued BC has $2M$ antennas at the base station and $2N_k$ antennas at user k. In the real-valued representation, the Shannon rate per data stream can be calculated from (2). However, in the calculation of the SINR γ, it has to be taken into account that the entries of $\boldsymbol{s}_{k',\text{real}}$ and $\boldsymbol{\eta}_{k,\text{real}}$ have half the variance of the entries of $\boldsymbol{s}_{k'}$ and $\boldsymbol{\eta}_k$, respectively.[4] Note that $\tilde{\boldsymbol{B}}_{k'}$ has the block structure of (7) if $\boldsymbol{B}_{\text{I},k'} = \text{j}\,\boldsymbol{B}_{\text{R},k'}$, i.e., in case of a linear transmit filter in the complex-valued BC.

III. Real-Valued MIMO BC

The QoS feasibility region is the set of all tuples of data rates (or MMSE values or SINR values or other QoS measures) that are achievable with finite transmit power using a certain type of transmit strategy (cf. e.g., [24]–[26]). We first discuss the QoS feasibility regions of real-valued MISO and MIMO BCs with linear transceivers. The results help to derive the feasibility region of the MIMO BC with improper transmit signals and widely linear transceivers.

The MMSE-based description of the QoS feasibility region of a complex-valued MISO BC derived in [25] is still valid in a real-valued MISO BC if the transmit symbols in the dual uplink [3] are Gaussian

[4]These factors of one half only influence the numerical value of the required transmit power, but not the feasibility region. Therefore, for the study below, it does not matter that the entries of $\boldsymbol{s}_{k',\text{real}}$ in the equivalent real-valued BC have variance $\frac{1}{2}$ while the real-valued BC a) was defined with variance 1.

with zero-mean and unit variance, i.e., $s_k^{\mathrm{UL}} \sim \mathcal{N}(0,1)$. Thus, the feasibility region of a real-valued MISO BC is the set of all MMSE values $\epsilon_1,\ldots,\epsilon_K$ that fulfill

$$0 \leq \epsilon_k < 1 \ \forall k \quad \text{and} \quad \sum_{k\in\mathcal{K}} \epsilon_k > |\mathcal{K}| - \mathrm{Rank}[\boldsymbol{H}_{\mathcal{K}}] \ \ \forall \mathcal{K} \subseteq \{1,\ldots,K\} \tag{9}$$

where $\boldsymbol{H}_{\mathcal{K}} \in \mathbb{R}^{M\times|\mathcal{K}|}$ is a matrix whose columns are the channel vectors $\boldsymbol{h}_k$ of all users $k \in \mathcal{K}$ in the MISO case. For a MIMO system, $\boldsymbol{H}_{\mathcal{K}} \in \mathbb{R}^{M\times\sum_{k\in\mathcal{K}} N_k}$ comprises the matrices $\boldsymbol{H}_k$ for all $k \in \mathcal{K}$ as a block row [26].

If all possible $\boldsymbol{H}_{\mathcal{K}}$ fulfill the *regularity condition* [25], [26]

$$\mathrm{Rank}[\boldsymbol{H}_{\mathcal{K}}] \geq \min\{|\mathcal{K}|, M\} \ \forall \mathcal{K} \subseteq \{1,\ldots,K\} \tag{10}$$

the feasibility criterion reduces to [25]

$$0 \leq \epsilon_k < 1 \ \forall k \quad \text{and} \quad \sum_{k=1}^{K} \epsilon_k > K - M. \tag{11}$$

Unlike in the complex-valued case as in [25], the relationship between the MMSE and the Shannon rate is given by

$$r_k = -\frac{1}{2}\log_2(\epsilon_k) \tag{12}$$

in the real-valued case. The factor of $\frac{1}{2}$ is a consequence of the real-valued setup with $s_k^{\mathrm{UL}} \in \mathbb{R}$ [consider the difference between (2) and (3)]. Therefore, in terms of Shannon rates $r_k \in [0,\infty)$, the feasibility condition (9) can be rewritten as

$$\sum_{k\in\mathcal{K}} (1 - 2^{-2r_k}) < \mathrm{Rank}[\boldsymbol{H}_{\mathcal{K}}] \ \ \forall \mathcal{K} \subseteq \{1,\ldots,K\}. \tag{13}$$

Using the same reasoning as in the complex-valued case (see [26]), we can show that any feasible rate requirement can be achieved with single-stream transmission in real-valued MIMO BCs. Thus, the feasibility condition (13) is also valid in the MIMO case as stated in the following lemma.

Lemma 1: In a real-valued MIMO BC with linear transceivers, a rate tuple $(r_1,\ldots,r_K)$ is achievable if and only if it fulfills (13).

IV. QoS Feasibility Region of MIMO BCs with Improper Transmit Signals

In a complex MIMO BC, Shannon rates that fulfill

$$\sum_{k\in\mathcal{K}} (1 - 2^{-r_k}) < \mathrm{Rank}[\boldsymbol{H}_{\mathcal{K}}] \ \ \forall \mathcal{K} \subseteq \{1,\ldots,K\} \tag{14}$$

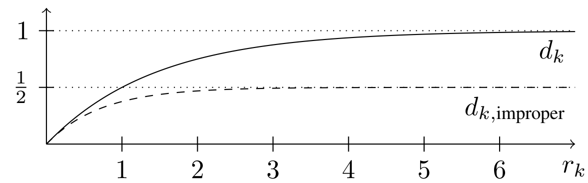

Fig. 1. EDoF for rate requirement r_k using proper/improper transmit signals.

where $\boldsymbol{H}_{\mathcal{K}} \in \mathbb{C}^{M \times \sum_{k\in\mathcal{K}} N_k}$ can be achieved with proper transmit signals and linear transceivers without time-sharing [26]. Since linear and widely linear transceivers are equivalent in the proper case [23], (14) also holds for widely linear transceivers. If the regularity condition (10) is fulfilled, (14) reduces to [28]

$$\sum_{k=1}^{K}(1-2^{-r_k}) < M. \tag{15}$$

To derive the feasibility region for improper transmit signals, we make use of the equivalent real-valued BC (5) introduced in Section II. With (5) and due to (8), the following theorem is a consequence of Lemma 1.

Theorem 1: Without restriction to proper per-user transmit signals in a MIMO BC with widely linear transceivers, a rate tuple $(r_1, \ldots, r_K)$ is achievable if and only if it fulfills

$$\sum_{k\in\mathcal{K}}(1-2^{-2r_k}) < 2\,\mathrm{Rank}[\boldsymbol{H}_{\mathcal{K}}] \ \ \forall\mathcal{K} \subseteq \{1,\ldots,K\}. \tag{16}$$

To gain intuition about QoS feasibility in MIMO BCs with improper transmit signals, we restrict ourselves to systems that fulfill the regularity condition (10) and adopt the notion of effective degrees of freedom (EDoF) [28]

$$d_k = 1 - 2^{-r_k} \in [0,1). \tag{17}$$

In the proper case, (15) can be rewritten as [28]

$$\sum_{k=1}^{K} d_k < M \tag{18}$$

which motivates the name EDoF for d_k: the system has M degrees of freedom (DoF), and the rate requirement of user k reduces the DoF available to serve the other users by d_k.

For improper transmit signals, we define the EDoF as

$$d_{k,\text{improper}} = \tfrac{1}{2}\left(1-2^{-2r_k}\right) \in [0,\tfrac{1}{2}) \tag{19}$$

so that (18) can be directly applied to the improper case. A single data stream in the equivalent real-valued system can be interpreted as half a data stream in the complex-valued system. Thus, we have

$d_{k,\text{improper}} < \frac{1}{2}$ in the improper case, where $d_{k,\text{improper}} \to \frac{1}{2}$ for $r_k \to \infty$. The upper bound $\frac{1}{2}$ equals the DoF needed for a user in case of zero-forcing with improper transmit signals since $2M$ streams can be transmitted interference-free in the equivalent real-valued BC.

On the other hand, the EDoF now converge faster to their upper bound than in the proper case (cf. Fig. 1) due to the new factor of 2 in the exponent. However, for any r_k, we have that

$$d_k - d_{k,\text{improper}} = \frac{1}{2}\left(1 - 2^{-r_k}\right)^2 > 0 \quad \forall r_k > 0 \tag{20}$$

as can be seen in Fig. 1. As the EDoF (19) for the improper case correspond to single-stream transmission in the equivalent real-valued BC, this complies with the fact that single-stream transmission is optimal for feasibility (Section III and [26]).

Due to (20), the feasibility region for proper transmit signals is a subset of the one for improper transmit signals. This does not imply a statement about the transmit power required to fulfill the rate requirements (for this question, cf. Section V), but it shows that proper signals can be suboptimal, namely if the rate requirements are feasible with improper, but not with proper transmit signals. This leads to the following theorem.

Theorem 2: In MIMO BCs using widely linear transceivers without time-sharing, proper complex Gaussian per-user transmit signals are not always optimal.

Note that the feasibility region does not depend on the channel realization as long the channels fulfill the regularity condition (10). Thus, suboptimality of proper transmit signals is not a phenomenon that only happens for a certain class of channels. This implies that it is not a measure zero event.

V. Numerical Results

In Fig. 2, we consider an example with single-antenna receivers since the globally minimal transmit power for the proper case can be computed efficiently in this scenario [24]. As achievable improper strategy, we apply the algorithm from [29] to the equivalent real-valued BC. The sum transmit power required for $r_k = \rho \; \forall k$ is normalized to the noise power and averaged (in the dB domain) over 1000 scenarios with i.i.d. circularly symmetric Gaussian channel coefficients with zero mean and unit variance. In the considered system with $M = 2$ transmit antennas, arbitrary rate requirements are achievable for $K = 2$ users, but the suboptimal improper scheme slightly outperforms the globally optimal proper scheme. For $K = 4$ users, the proper scheme is infeasible ($\sum_k d_k \geq M$) for $\rho \geq 1$ [cf. (17), (18)] while arbitrary rates are feasible with improper signals since $d_{k,\text{improper}} < \frac{1}{2} \; \forall r_k$ [cf. (19)]. For $K = 6$ users, (18) tells us that the schemes become infeasible at $\rho_{\text{proper}} \approx 0.58496$ and $\rho_{\text{improper}} \approx 0.79248$, which is again in compliance with the numerical simulation results.

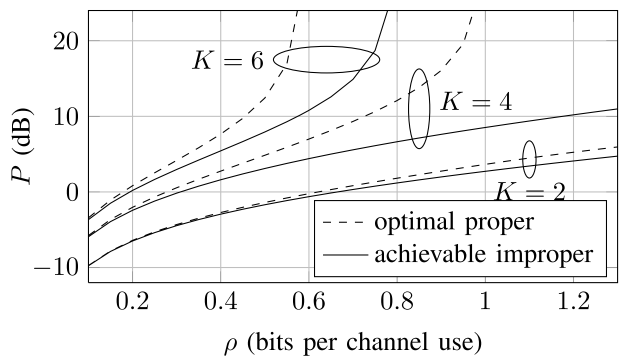

Fig. 2. Transmit power needed to serve K users with data rate $r_k = \rho$.

Moreover, the simulation results show that using improper transmit signals can decrease the required transmit power in cases where both improper and proper signals are feasible: since the solution for the proper case is globally optimal, the power gap between the curves is not caused by the applied algorithm, but inherent to the restriction to proper signals.

VI. Conclusion

For systems with improper signaling, it is well known that widely linear transceivers can outperform linear transceivers. In this paper, we have considered the converse question: in systems with widely linear transceivers, can improper signaling outperform proper signaling? In MIMO broadcast channels without time-sharing, this can indeed be the case in terms of QoS feasibility and in terms of required transmit power.

References

[1] F. D. Neeser and J. L. Massey, “Proper complex random processes with applications to information theory,” *IEEE Trans. Inf. Theory*, vol. 39, no. 4, pp. 1293–1302, Jul. 1993.

[2] I. E. Telatar, “Capacity of multi-antenna Gaussian channels,” *European Trans. Telecommun.*, vol. 10, no. 6, pp. 585–595, Nov./Dec. 1999.

[3] P. Viswanath and D. N. C. Tse, “Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality,” *IEEE Trans. Inf. Theory*, vol. 49, no. 8, pp. 1912–1921, Aug. 2003.

[4] S. Vishwanath, N. Jindal, and A. Goldsmith, “Duality, achievable rates, and sum-rate capacity of Gaussian MIMO broadcast channels,” *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2658–2668, Oct. 2003.

[5] C. Hellings and W. Utschick, “On the inseparability of parallel MIMO broadcast channels with linear transceivers,” *IEEE Trans. Signal Process.*, vol. 59, no. 12, pp. 6273–6278, Dec. 2011.

[6] D. P. Palomar and Y. Jiang, “MIMO transceiver design via majorization theory,” *Foundations and Trends® Commun. Inf. Theory*, vol. 3, no. 4–5, pp. 331–551, 2006.

[7] B. Picinbono and P. Chevalier, “Widely linear estimation with complex data,” *IEEE Trans. Signal Process.*, vol. 43, no. 8, pp. 2030–2033, Aug. 1995.

[8] T. Adalı, P. J. Schreier, and L. L. Scharf, “Complex-valued signal processing: The proper way to deal with impropriety,” *IEEE Trans. Signal Process.*, vol. 59, no. 11, pp. 5101–5125, Nov. 2011.

[9] W. Gerstacker, R. Schober, and A. Lampe, “Receivers with widely linear processing for frequency-selective channels,” *IEEE Trans. Commun.*, vol. 51, no. 9, pp. 1512–1523, Sep. 2003.

[10] D. Mattera, L. Paura, and F. Sterle, “Widely linear MMSE transceiver for real-valued sequences over MIMO channel,” presented at 14th European Signal Process. Conf. (EUSIPCO), Florence, Italy, Sep. 4–8, 2006.

[11] S. Buzzi, M. Lops, and S. Sardellitti, “Widely linear reception strategies for layered space-time wireless communications,” *IEEE Trans. Signal Process.*, vol. 54, no. 6, pp. 2252–2262, Jun. 2006.

[12] A. Mirbagheri, K. N. Plataniotis, and S. Pasupathy, “An enhanced widely linear CDMA receiver with OQPSK modulation,” *IEEE Trans. Commun.*, vol. 54, no. 2, pp. 261–272, Feb. 2006.

[13] D. Darsena, G. Gelli, and F. Verde, “Universal linear precoding for NBI-proof widely linear equalization in MC systems,” *EURASIP J. Wireless Commun. Networking*, vol. 2008, pp. 8:1–8:11, Jan. 2008.

[14] P. Xiao and M. Sellathurai, “Iterative receiver design for MIMO systems with improper signal constellations,” presented at the Int. Conf. Commun. (ICC) 2009, Dresden, Germany, Jun. 14–18, 2009.

[15] ——, “Improved linear transmit processing for single-user and multi-user MIMO communications systems,” *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1768–1779, Mar. 2010.

[16] J. Steinwandt, R. C. de Lamare, L. Wang, N. Song, and M. Haardt, “Widely linear adaptive beamforming algorithm based on the conjugate gradient method,” presented at the Int. ITG Workshop on Smart Antennas (WSA) 2011, Aachen, Germany, Feb. 24–25, 2011.

[17] Y. C. Yoon and H. Leib, “Maximizing SNR in improper complex noise and applications to CDMA,” *IEEE Commun. Lett.*, vol. 1, no. 1, pp. 5–8, Jan. 1997.

[18] B. Hassibi and B. M. Hochwald, “High-rate codes that are linear in space and time,” *IEEE Trans. Inf. Theory*, vol. 48, no. 7, pp. 1804–1824, Jul. 2002.

[19] W. Gerstacker, F. Obernosterer, R. Schober, A. Lehmann, A. Lampe, and P. Gunreben, “Equalization concepts for Alamouti’s space-time block code,” *IEEE Trans. Commun.*, vol. 52, no. 7, pp. 1178–1190, Jul. 2004.

[20] A. S. Aghaei, K. N. Plataniotis, and S. Pasupathy, “Widely linear MMSE receivers for linear dispersion space-time block-codes,” *IEEE Trans. Wireless Commun.*, vol. 9, no. 1, pp. 8–13, Jan. 2010.

[21] P. Chevalier and F. Dupuy, “Widely linear Alamouti receiver for the reception of real-valued constellations corrupted by interferences—the Alamouti-SAIC/MAIC concept,” *IEEE Trans. Signal Process.*, vol. 59, no. 7, pp. 3339–3354, Jul. 2011.

[22] Z. K. M. Ho and E. Jorswieck, “Improper Gaussian signaling on the two-user SISO interference channel,” *IEEE Trans. Wireless Commun.*, vol. 11, no. 9, pp. 3194–3203, Sep. 2012.

[23] C. Hellings and W. Utschick, “Performance gains due to improper signals in MIMO broadcast channels with widely linear transceivers,” in *Proc. Int. Conf. Acoust., Speech, Signal Process. (ICASSP) 2013*, May 2013, pp. 4379–4383.

[24] M. Schubert and H. Boche, “Solution of the multiuser downlink beamforming problem with individual SINR constraints,” *IEEE Trans. Veh. Technol.*, vol. 53, no. 1, pp. 18–28, Jan. 2004.

[25] R. Hunger and M. Joham, “A complete description of the QoS feasibility region in the vector broadcast channel,” *IEEE Trans. Signal Process.*, vol. 58, no. 7, pp. 3870–3878, Jul. 2010.

[26] M. Joham and R. Hunger, "Feasible rate region of the MIMO broadcast channel with linear transceivers," in *Proc. Int. ITG Workshop on Smart Antennas (WSA) 2010*, Feb. 2010, pp. 342–349.

[27] M. Schubert and H. Boche, "Iterative multiuser uplink and downlink beamforming under SINR constraints," *IEEE Trans. Signal Process.*, vol. 53, no. 7, pp. 2324–2334, Jul. 2005.

[28] M. Joham, C. Hellings, and R. Hunger, "QoS feasibility for the MIMO broadcast channel: Robust formulation and multi-carrier systems," in *Proc. 8th Int. Symp. Modeling Optim. Mobile, Ad Hoc, Wireless Netw. (WiOpt)*, May 2010, pp. 610–614.

[29] C. Hellings, M. Joham, and W. Utschick, "Gradient-based power minimization in MIMO broadcast channels with linear precoding," *IEEE Trans. Signal Process.*, vol. 60, no. 2, pp. 877–890, Feb. 2012.

[30] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge, UK: Cambridge University Press, 2005.

[31] B. Friedman, "Eigenvalues of composite matrices," *Math. Proc. Cambridge Philosophical Soc.*, vol. 57, no. 1, pp. 37–49, Jan. 1961.