Modeling infrastructure system performance using BN

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ABSTRACT: Aiming towards the development of a probabilistic decision-support system for near-real time emergency response and recovery following a seismic event, this paper presents methods for assessing the performance of spatially distributed infrastructure systems by use of a Bayesian Network (BN) methodology. The approach accounts for the evolving nature of available information in the post-event period. A BN model of the seismic demand that accounts for the random-field nature of ground motion intensities affecting the system components is described. The main focus of the paper is on modeling the infrastructure system through a BN. We present and compare five BN formulations for modeling performance of systems with binary component and system states. The study of a hypothetical transportation system illustrates the approaches presented in the paper.

1 INTRODUCTION

1.1 Background and Motivation

Civil infrastructure systems are logistical backbones of modern societies. Transportation networks, utility distribution systems, communication networks, as well as government, health, and public facilities are important infrastructures, vital to the welfare of society. Disruption of the service of such systems can have far-reaching economic, social, health, and safety consequences. Despite this recognized criticality, infrastructure systems remain vulnerable to a wide variety of natural and manmade hazards. In many locations around the world, earthquakes are the dominate hazard to infrastructure systems.

Decisions made by emergency management personnel immediately after a major earthquake can significantly influence the severity of the associated consequences. These crucial decisions include the deployment of emergency personnel and equipment, evacuation of people, post-event inspections, and closure (or continued operation) of critical facilities. Unfortunately, these vital decisions are often made in an ad hoc manner, under large uncertainty, with information that evolves in time. First responders are in need of a tool to aid decision-making, which properly accounts for the uncertainty prevailing in the immediate aftermath of an earthquake and which synthesizes the incoming information to reduce the uncertainty.

To address this need, we are currently working towards the development of a probabilistic decision-support system (DSS) for near-real time emergency response and recovery following a seismic event, utilizing a Bayesian Network (BN) framework. This DSS will integrate information from a wide-range of sources in near-real time to provide a comprehensive description of the state of a geographically distributed infrastructure system following an earthquake.

1.2 Objective and Scope

To date, our efforts toward the development of the probabilistic DSS have focused on modeling earthquake demand as a spatially distributed random field (Straub et al. 2008) and on investigating methods for modeling the infrastructure system as a BN. In the present paper we provide only a brief description of the seismic demand model and thereafter focus on the system modeling. Several approaches to modeling a spatially distributed network by a BN are described and compared. A case study for a transportation network demonstrates the methodology.

2 SEISMIC DEMANDS ON SPATIALLY-DISTRIBUTED INFRASTRUCTURE SYSTEM

Due to their spatially distributed nature, infrastructure systems are more likely than individual facilities to sustain damage as a result of an earthquake. Performance assessment of such a system requires: (a) characterization of the earthquake magnitude, location and other source characteristics, (b) estima-
tion of ground motion intensities at distributed points in the network with proper account of the spatial correlation structure, (c) modeling of the performance of system components, e.g., in terms of fragility functions, (d) modeling of the performance of the system in terms of the component performances and the seismic demand, and (e) system reliability assessment under different earthquake scenarios. In the context of the aforementioned near-real time DSS, we are interested in the reliability of the system and its components conditioned on any available information, e.g. known magnitude and location of the earthquake, measurements of shaking intensity at selected locations, observed performance of selected components; an application for which BNs are uniquely well-suited.

The intensity of ground motion at a site is often characterized in terms of a single measure, such as the peak ground acceleration or the spectral acceleration at a selected frequency. Let \( S_i \) denote this intensity measure at a site \( i \). Predictive models based on regression of observed data are available to relate \( S_i \) to the earthquake characteristics (Bozorgnia and Bertero, 2004). These models typically have the form

\[
\ln(S_i) = f(M, R, \theta_i) + \varepsilon_i,
\]

where \( f(M, R, \theta_i) \) is a deterministic function of the magnitude, \( M \), site-to-source distance, \( R \), and other source and site characteristics \( \theta_i \), such as type of faulting and the shear-wave velocity at the site. The zero-mean, normal random variable \( \varepsilon_i \) is the error term of the regression law. In general, the error terms \( \varepsilon_i \) and \( \varepsilon_j \) at two locations \( i \) and \( j \) are correlated due to two effects: (a) the variability in source characteristics, which is common to all sites, and (b) the random-field nature of the intensities over a spatial domain for a given set of source characteristics. Thus, the error term can be written as \( \varepsilon_i = \varepsilon_m + \varepsilon_{ir} \), where \( \varepsilon_m \) is an inter-event error term common to all sites, while \( \varepsilon_{ir} \) is an intra-event error random field. Both terms are zero-mean and Gaussian. Boore et al. (2003) and Park et al. (2007), among others, have developed autocorrelation models for \( \varepsilon_{ir} \), assuming it to be a homogeneous isotropic random field.

In a BN model of the seismic demand, the correlation between \( \varepsilon_{ir} \) for different sites implies directed links between all nodes representing these error terms. With a large number of component sites, this would render the BN computationally intractable. To overcome this difficulty, Straub et al. (2008) used the method of principle component analysis to approximate the correlation structure of \( \varepsilon_{ir} \) in terms of a few standard normal random variables \( u \), representing the most important principle components. Figure 1 shows the resulting BN model of the seismic demand for a spatially distributed infrastructure system. The node designated by “L” describes the location coordinate of the earthquake. All other nodes are according to the terminology just introduced.

3 INFRASTRUCTURE SYSTEM MODELING USING BN

3.1 Using BN to model system connectivity

Once the seismic demand model has been developed, we must consider modeling the seismic performance of the individual components of the infrastructure system as well as the performance of the system as a whole. The system components can have any number of states, but in many cases binary states, e.g., fail or survive, are sufficient to describe the function of the components. For example, a tunnel in a transportation system is either open or closed. Usually more than two states are considered when a component has a “flow” characteristic associated with its performance. For example, a bridge may be open for full capacity traffic, 50% capacity traffic, or it may be closed to traffic. Such a bridge has three states. The system performance may also be defined in terms of multiple states, e.g., the availability of certain levels of traffic flow between a set of source nodes and a set of destination nodes.

The performance of the individual components of the infrastructure system relative to a specified ground motion intensity parameter can be modeled using seismic fragility curves. Such curves are available for a wide-variety of infrastructure system components, such as the components of electrical subsystems (e.g. Straub and Der Kiureghian (2008)) or bridges in a transportation network, e.g. Mander (1999), Gardoni et al. (2003).

In this paper, we focus our attention on the issue of connectivity between the nodes of a system without consideration of flow characteristics. The components of such a system may have binary states (open or closed) and the system itself also has two states (either connectivity between the source and sink nodes exists or does not exist). The analysis for this class of systems is obviously simpler. However, the BN methodology is in not restricted to systems with binary component states and work is currently under-
A way to expand these ideas to the multi-state problem.

For systems with binary component states, we define a minimal link set (MLS) as a minimum set of components whose joint survival constitutes survival of the system, and a minimal cut set (MCS) as a minimum set of components whose joint failure constitutes failure of the system. One can show that, in general, a two-state system can be expressed as a parallel subsystem of its MLSs, or a series system of its MCSs (Der Kiureghian et al. 2007).

Friis-Hansen (2004) has proposed a BN formulation, in which system connectivity is modeled by exploiting causal relationships between the system components necessary for its survival. We have found that this approach can be effective for certain types of systems, specifically those for which it is easy to identify all MLSs. Systems with many of their components in series tend to work well with this approach.

We expand upon Friis-Hansen’s approach by considering five different formulations for modeling two-state systems using BN. These are described in conjunction with the simple example system shown in Figure 2. In this system, the five components (squares) connect points A, B, and C (circles). We assume only the square components in the system can fail. The required system performance is connectivity between the source node A and the sink node C. The MLSs of this system are \{(1,2),(1,3),(4,5)\} and its MCSs are \{(1,4),(1,5),(2,3,4),(2,3,5)\}.

### 3.2 Five BN formulations

We define a naïve BN formulation as one in which the system connectivity is modeled as a direct function of its components. Figure 3 shows the corresponding BN for the example system. As can be seen, all components are parents of one system node. For binary component states, the system node has a conditional probability table (CPT) of size \(2^n\), where \(n\) is the number of components (\(2^5 = 32\) for the example system). For systems with a large number of components, the size of the CPT will quickly cause the BN to become computationally intractable. Hence, while easy to formulate, this is not a pragmatic approach for many real-world applications, where the number of system components is often large.

We define the formulation advocated by Friis-Hansen (2004) as the explicit connectivity (EC) BN formulation. Rather than modeling the system as a child of its MLS nodes, this formulation expresses system connectivity using a causal interpretation of the connectivity paths. We can think of it as the optimist’s formulation (in contrast to a pessimist’s formulation discussed later), because one models the system by directly modeling paths that ensure survival of the system. The approach has the advantage of being intuitive and is thus useful for interaction with non-technical personnel during the modeling phase. Furthermore, due to the encoded causal relationships, the resulting BN is likely to produce CPTs that are smaller than those of the naïve or possibly the MLS formulation. Moreover, this formulation does not require identification of MLSs, though the

**Figure 3: Naïve BN formulation of example system**

We define the minimum link set BN formulation, as a BN where the system connectivity is expressed directly as a function of the MLSs. This is done by representing the system as a node whose parents are the MLSs. In turn, each MLS node has its constituent components as parents. Figure 4 shows the modeling of the example system according to this formulation. The size of the CPT for each MLS is \(2^n\) to the power of the number of its components, and the size of the system node is \(2\) to the power of the number of MLSs. For the example system, the largest CPT occurs for the system node and is of size \(2^3\).

This MLS BN formulation can take advantage of the fact that each MLS node is a series system of its components, and that the system node is a parallel system of its MLS parents. Clearly, this formulation is advantageous to the naïve formulation described above, particularly when the system has fewer MLSs than components.

**Figure 4: MLS BN formulation of example system**

We define the formulation advocated by Friis-Hansen (2004) as the explicit connectivity (EC) BN formulation. Rather than modeling the system as a child of its MLS nodes, this formulation expresses system connectivity using a causal interpretation of the connectivity paths. We can think of it as the optimist’s formulation (in contrast to a pessimist’s formulation discussed later), because one models the system by directly modeling paths that ensure survival of the system. The approach has the advantage of being intuitive and is thus useful for interaction with non-technical personnel during the modeling phase. Furthermore, due to the encoded causal relationships, the resulting BN is likely to produce CPTs that are smaller than those of the naïve or possibly the MLS formulation. Moreover, this formulation does not require identification of MLSs, though the
causal logic indirectly employs them. The formulation is always superior to the naïve formulation in terms of its computational efficiency when the number of MLSSs is smaller than the number of components in the system. This is typically the case in systems with many components in series. The disadvantage of this formulation is that constructing the BN is not as systematic as in the MLS formulation. However, it should be noted that when modeling complex systems it is of utmost importance that the modeling can be validated by third parties. Therefore, the modeling should always focus on transparency. A direct causal modeling assures this is in general possible.

Figure 5 shows the EC BN formulation for the example system. Starting at the source node A, we consider the initial steps for all paths leading to the sink node C. For the example system, these are represented by the nodes A→B and A→C. Step A→B depends on the survival of component 1, whereas step A→C depends on the survival of both components 4 and 5. The second step for the first path is B→C, the success of which depends on the success in step A→B as well as survival of either component 2 or 3. Thus, node B→C has the nodes representing step A→B and components 2 and 3 as parents. The successful arrival at the sink node C is now represented as a child of nodes A→C and B→C, survival of either of which suffices for the system to survive. In essence, the EC formulation requires the enumeration of connectivity paths (or MLSSs), but rather than representing them as nodes, the BN explicitly models each path following step-by-step causal relations. In comparison, the MLS formulation is relatively straightforward for individuals familiar with traditional system analyses, but it lacks the intuitive causal interpretation present in the EC formulation.

For the example under consideration, using the EC formulation, the size of the largest CPT is $2^3$ (which can be reduced to $2^2$ by adding additional nodes before A→C and B→C). However, a small CPT size does not necessarily translate into a reduced computational demand. In fact, in terms of the total clique table size (the sum of the sizes of all clique tables generated by the inference algorithm, where a clique is a maximally connected subgraph formed when performing computation on the BN and the clique table is the joint probability distribution over the nodes of a clique), the MLS formulation outperforms the EC formulation for this example: Using the default triangulation method in Hugin (Hugin Researcher, 2008), which for this example corresponds to the optimal triangulation, the sizes are 48 for the MLS formulation and 64 for the EC formulation.

We define the dual of the MLS formulation as the minimum cut set BN formulation, in which the system node is a child of parents representing MCS nodes, and each MCS node itself is represented as a child of nodes representing its constituent components. The system node is a series system of all the MCS nodes, whereas each MCS is a parallel system of its parent nodes. The MCS BN for the example system is shown in Figure 6. The maximum CPT size in this case is 24 and the total clique table size using Hugin’s default triangulation is 192. Thus, the MCS formulation is less advantageous than the MLS and EC formulation for this example. However, relative to the MLS formulation, the MCS formulation would be advantageous when the number of MCSs is smaller than the number of MLSSs. But, the relative advantage would also depend on the number of components within the individual MLSSs and MCSs.

We define the dual of the EC formulation as the explicit disconnectivity (EDC) BN formulation. Rather than tracing paths that ensure survival of the system, one pursues causal event paths that ensure failure of the system. This is a less intuitive approach than the EC formulation; it follows a pessimist’s perspective. Similar to the EC formulation, using the EDC formulation one can often improve upon the naïve and MCS formulations, particularly when the number of failure event paths (or MCSs) are small relative to the number of components.

Figure 7 shows the BN model of the example system according to the EDC formulation. We know disconnectivity between the source node A and the sink node C will occur if neither path A-C nor path A-B-C is open. Starting at the source node A, in or-
order to get disconnection, path A-C must be closed. This is defined by the node \( \overline{A \rightarrow C} \), which is a child of components 4 and 5 (either must fail for the path to be closed). If path A-C is closed, then we need link A-B or link B-C to be closed as well. These events are represented by the nodes \( \overline{A \rightarrow B} \) and \( \overline{B \rightarrow C} \), which are dependent not only on the states of their respective components, but also on the state of node \( \overline{A \rightarrow C} \). The largest CPT in this example is of size \( 2^2 \) and the total clique table size using Hugin’s default triangulation is 56 (smaller than the MCS and EC formulations).

Figure 7: EDC BN formulation of example system

For further refinement of the MCS and MLS formulations, it is instructive to consider the cases of ideal series and parallel systems. For a series or parallel system of \( n \) components, the naïve, MLS and MCS BN formulations appear identical and are as shown in Figure 8. Note that a series system has one MLS and as many MCSs as components, whereas a parallel system has one MCS and as many MLSs as components. Because of this, the three BN models for the two systems have identical topologies, but with different CPTs. It is easy to verify that the EC and EDC models for the series and parallel systems are as shown in Figure 9. For large number of components, this BN formulation is much more efficient than the one in Figure 8.

We can take advantage of the above formulations for series and parallel systems to improve the MLS and MCS BN formulations. Specifically, note that in the MLS formulation the system node is a parallel system of the MLS nodes. Using the idea from Figure 9, the BN in Figure 4 is reformulated in the form of Figure 10. Likewise, noting that in the MCS formulation the system node is a series system of the MCS nodes, the BN in Figure 6 is reformulated into that shown in Figure 11. Furthermore, it is noted that each MLS is a series system of its constituent components and that each MCS is a parallel system. Therefore, it is possible to use the series/parallel system representation from Figure 9 to represent the upper parts of Figure 10 and Figure 11 as well.

One can see that a significant reduction in the size of the CPTs results from these reformulations. However, this is achieved at the cost of increased number of nodes in the reformulated BNs. Definitive guidelines to achieve the most effective BN model remains an area for further study.

Figure 8: Naïve, MLS and MCS formulations of series and parallel systems

Figure 9: (a) EC and (b) EDC formulations of series and parallel systems

Figure 10: Reformulation of the MLS BN in Figure 5

Figure 11: Reformulation of the MCS BN in Figure 7
4 ILLUSTRATIVE EXAMPLE

4.1 Overview

Next, we present a case study using the hypothetical transportation system in the vicinity of an active fault, shown in Figure 12, which is a simplified version of the example in Straub et al. (2008). In this network, circles represent cities and square nodes represent bridges. System survival is defined as the event that all cities are able to reach the hospital, H, following an earthquake. Correspondingly, the system fails if any city is unable to reach the hospital (a series system of city events). We assume that the roadways remain passable following an earthquake and that only the bridges are susceptible to failure. Fragility models for bridges in terms of spectral acceleration are adapted from Gardoni et al. (2002), are used to determine the failure probability of each bridge for a given ground motion demand.

The BN used to model both the seismic demands and system performance is shown in Figure 13. An object-oriented modeling approach is used. Ovals in the figure represent the designated random variables, whereas rounded rectangles represent objects, which are themselves BNs with specified input and output nodes as described below.

The earthquake is represented by a point-source model with a uniform distribution along the fault. A finite source model with account of directivity is currently under development. The earthquake magnitude is assumed to follow a truncated exponential distribution (Araya & Der Kiureghian, 1988). The Campbell (1997) ground motion prediction equation is used to estimate the spectral acceleration at each site. The goal of this case study is primarily to demonstrate the connectivity formulations proposed herein and demonstrate the BN framework rather than to emphasize the specific numerical results. As such, empirical modeling assumptions made here should be viewed as “placeholders” rather than restrictive assumptions. In the proposed BN frame-
4.2 Modeling System Performance

4.2.1 System connectivity

As mentioned earlier, the survival of the transportation network is defined as the event that all cities can reach the hospital following an earthquake. This is a case of multiple source nodes and a single sink node. We solve this problem by creating a separate object for each city’s ability to reach the hospital, then consider the transportation network as a series system of the city objects. For each city object, any of the BN formulations described earlier can be used. One example is described below. For the representation of the series system of cities, the formulation described in Figure 9 is employed.

4.2.2 City-to-Hospital Connectivity

As an example, we consider the connectivity between City A and the hospital. The MLs are {(1), (4,2,3,6)}, and the MCSs are {(1,4), (1,2), (1,5,6), (1,5,3)}, and can be used directly to model the system using the MLS and MCS formulations. Here we focus on the EC and EDC formulations.

Figure 16 presents the EC model of the object representing the connectivity between City A and the hospital. We begin at the node “City A.” We can reach the hospital either via link A→H, whose survival depends on the survival of bridge 1, or we must travel via link A→B, which depends on the survival of bridge 4. Assuming this link survives, we must then travel link B→C to City C, which is possible only if bridge 2 survives. From City C, we can either go directly to the hospital, provided bridge 5 survives, or travel to City D on the link C→D, provided bridge 6 survives. Once at City D, we can reach the hospital, provided bridge 3 survives. Thus, we can reach the hospital from City A if any of the nodes A→H, C→H or D→H is in survival state. This represents a parallel system of the three nodes. It is noted that the input nodes of this object are the states of bridges 1-6 and the output node is the system (one or more cities not being able to reach the hospital) and of the individual bridges under the unconditional (no evidence) case as well as the following simple evidence cases:

- Case 1: The magnitude of the earthquake is measured to be between 6.75 and 7.0
- Case 2: Bridge 1 is observed to have failed
- Case 3: A sensor at the location of bridge 1 measures spectral acceleration is 0.4-0.5g

To illustrate the power of the BN framework for assessing and updating component and system reliability, we consider probabilities of component and system failure under different evidence cases. Evidence cases considered are for illustrative purposes and are selected from numerous possible cases we could include. Table 1 presents failure probabilities of the system (one or more cities not being able to reach the hospital) and of the individual bridges under the unconditional (no evidence) case as well as the following simple evidence cases:

- Case 1: The magnitude of the earthquake is measured to be between 6.75 and 7.0
- Case 2: Bridge 1 is observed to have failed
- Case 3: A sensor at the location of bridge 1 measures spectral acceleration is 0.4-0.5g
• Case 4: Bridge 1 and Bridge 2 are observed to have survived.

Table 2 presents more complex evidence cases (compound evidence from a variety of sources) as described below:

• Case 5: Earthquake magnitude is 6.75-7.0. The epicenter of the earthquake is observed to be located ~40 km from the left edge of the fault.

• Case 6: Earthquake magnitude is between 5.5 and 5.75. Spectral acceleration at bridge 4 is measured in the range 0.3-0.4g and bridge 4 is observed to have survived.

• Case 7: Earthquake magnitude is between 5.5 and 5.75. Spectral acceleration at bridge 4 is measured in the range 0.3-0.4g and bridge 4 is observed to have failed.

• Case 9: Seismological monitoring station is offline. We have received a phone call indicating that bridges 4 and 5 have failed and no ambulances are arriving from City B.

Depending on the evidence, the system failure probability ranges from less than 1% to ~5%, excluding the cases of observed system failure. Likewise, bridge failure probabilities range from less than 1% to over 40%, depending on the evidence, excluding the cases of observed bridge survival and failure.

5 CONCLUSIONS

With the eventual objective of developing a BN-based probabilistic decision support system for post-earthquake risk management, in this paper we investigated several approaches for modeling a spatially distributed infrastructure network system by a BN. These approaches included a naïve approach that directly connects component nodes to the system node, two approaches that directly utilize minimal link and cut sets, and two explicit approaches that use causal relationships to construct connectivity and disconnectivity BNs. Relative advantages of the various method from the viewpoints of modeling and computational efficiency were discussed, though definitive conclusions must await further study. Additionally, these approaches can be combined to draw upon the relative advantages of the different formulations. To illustrate the approaches outlined in this paper, we presented a hypothetical case study of a transportation network system.

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7 REFERENCES


