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***Abstract*—We focus on the design of linear beamforming based on *quality-of-service* (QoS) power minimization and rate balancing in the *downlink* (DL) of a satellite communication system with mobile and static users. Since only the rank-one covariance matrices of the channels to mobile users are known, we introduce average rate requirements for these users contrary to the perfect *channel state information* (CSI) rate requirements for the static users. Due to the structure of the ergodic rates, we cannot directly resort to the optimization techniques for the purely complete CSI vector *broadcast channel* (BC). Therefore, we propose two suitable lower and upper bounds on the ergodic rates. Incorporating these bounds in the optimization problems, instead of the ergodic expressions, the usual power minimization algorithms can be applied. This allows the computation of bounds on the optima of the considered problems. Furthermore, employing an additional power allocation on the obtained solutions, we achieve outcomes that reside close to the expected optima.**

***Index Terms*—Power minimization; rate balancing; vector BC**

I. Introduction

We consider the DL of a satellite communication system with mobile and static single-antenna receivers as described in [1]. Modeling the DL as a vector BC [2], we focus on power efficient *linear* beamformer design techniques that are able to deal with mobile and static users at the same time. To this end, we consider the following two problem formulations:

(P) *QoS power minimization*: given QoS requirements for the users, expressed as minimum rates, shall be fulfilled using minimum resources, i.e., total transmit power;

(B) *rate balancing*: given an upper bound on the total transmit power, the ratios between achieved rate and suggested target shall commonly be maximized for all users.

The difficulty with *(P)* and *(B)* in satellite communications is that only the statistics of the channels to mobile users, i.e., the rank-one covariance matrices (cf. Section II), are known for the DL phase because of the limited channel estimation capabilities due to the large round-trip time. Therefore, we resort to ergodic rate requirements for these users contrary to the perfect CSI rate targets for static users. Moreover, we design the beamformers based on the statistical CSI contrary to [3], where transmission is designed based on perfect CSI to optimize ergodic rate formulations. Unfortunately, using ergodic rate expressions, we cannot employ the efficient optimization strategies that are known for the purely complete CSI vector BC (e.g., [4], [5], and [6]) due to the lack of a general BC to *multiple access channel* (MAC) rate duality for statistical CSI [7] and missing convex reformulations.

To overcome this problem, we introduced additional zero-forcing (ZF) constraints in [1] for avoiding interference at the mobile users. For this partial ZF strategy, we were able to establish a proper uplink-downlink-SINR-duality and, moreover, solve the resulting problems in the dual MAC via the fixed-point framework in [5] and [6]. However, enforcing ZF constraints on the mobile users' channels, we strongly restrict the QoS feasible region and, therewith, the achievable rates of the remaining static users.

In this work, we follow a different approach. We propose and characterize two simple lower and upper bounds on the ergodic rates. Replacing the ergodic rate requirements in *(P)* and *(B)* with requirements on the bounds, the resulting optimizations can in turn be solved via the usual algorithms and methods. Therewith, we are able to calculate close lower and upper bounds on the optimum of the initial problems. The so obtained beamformers can then be used for an additional power allocation in the BC that keeps the spatial characteristic of the beamformers constant and only adapts the powers intended for the different users. The achieved outcomes appear to be close to the expected optima of *(P)* and *(B)*.

The remainder of this work is structured as follows. The system model and the achievable and ergodic rates are introduced in Section II and III, respectively. Next, mathematical formulations of *(P)* and *(B)* are given. In Section V, the ergodic rate lower and upper bounds are introduced. Sections VI and VII deal with the power minimization and rate balancing problems with requirements on the bounds, while the additional power allocation strategy is presented in Section VIII. Finally, the proposed methods are numerically evaluated in Section IX.

II. System and Channel Model

In the considered vector BC, an N antenna transmitter, i.e., the satellite, simultaneously conveys independent data signals $s_k \sim \mathcal{N}_{\mathbb{C}}(0,1)$ to K single antenna receivers. The data signals are linearly precoded with the beamformers $\boldsymbol{t}_k \in \mathbb{C}^N$, $k \in \{1,\ldots,K\}$. The superposition of the beamformer outcomes is then transmitted to all users. Denoting the vector channel for user k as $\tilde{\boldsymbol{h}}_k^{\mathrm{H}} \in \mathbb{C}^{1\times N}$ and the experienced additive noise as $n_k \sim \mathcal{N}_{\mathbb{C}}(0,\sigma_k^2)$, its received signal reads as

$$y_k = \tilde{\boldsymbol{h}}_k^{\mathrm{H}} \boldsymbol{t}_k + \tilde{\boldsymbol{h}}_k^{\mathrm{H}} \textstyle\sum_{i\neq k}^{K} \boldsymbol{t}_i s_i + n_k .$$

The users are differentiated into the two disjoint groups $\mathbb{P}$ and $\mathbb{S}$ with $|\mathbb{S}| = K - |\mathbb{P}|$, corresponding to *perfect CSI* users,

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e.g., fixed ground stations with line of sight to the satellite, and *statistical CSI* users, respectively, e.g., moving mobiles that suffer from local scattering and shadowing close to the ground. While we have accurate knowledge of the channel states $\tilde{\boldsymbol{h}}_k$ to perfect CSI users $k \in \mathbb{P}$, only the statistics of $\tilde{\boldsymbol{h}}_k \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \tilde{\boldsymbol{C}}_k)$, i.e., the covariance matrix $\tilde{\boldsymbol{C}}_k$, is available for the beamformer design of the statistical CSI users.

Fortunately, the channel covariance matrices $\tilde{\boldsymbol{C}}_k$ to the $|\mathbb{S}|$ statistical CSI users are essentially rank-one: the spatial signature of $\tilde{\boldsymbol{h}}_k$ remains essentially constant because of the large distance from the satellite to the earth's surface, whereas its effective gain (norm and phase) varies depending on shadowing effects and scatterers close to the user. In the remainder of this paper, we restrict to this rank-one assumption. Moreover, to simplify expositions, we only refer to the normalized channel $\boldsymbol{h}_k \triangleq \sigma_k^{-1}\tilde{\boldsymbol{h}}_k$ and covariance $\boldsymbol{C}_k = \sigma_k^{-2}\tilde{\boldsymbol{C}}_k = \boldsymbol{v}_k\boldsymbol{v}_k^{\mathrm{H}}$, with $\boldsymbol{v}_k$ being the properly scaled dominant eigenvector of $\boldsymbol{C}_k$. Note that we can write $\boldsymbol{h}_k \simeq \boldsymbol{v}_k w_k$ with $w_k \sim \mathcal{N}_{\mathbb{C}}(0,1)$.

III. Achievable and Ergodic Rates

For a given set of beamforming vectors $\{\boldsymbol{t}_i\}_{i=1}^K$, the following mutual information is achieved at the kth user

$$r_k = \log_2\left(1 + \frac{|\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{t}_k|^2}{1+\sum_{i\neq k}|\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{t}_i|^2}\right). \tag{1}$$

To exploit (1), the transmitter needs to know the current channel state $\boldsymbol{h}_k$. However, being only aware of its statistics, given by $\boldsymbol{v}_k$, we cannot use (1) for designing beamformers to statistical CSI users $k \in \mathbb{S}$. Instead, we resort to the *ergodic* mutual information which appropriately describes the statistical CSI user's average rate when assuming sufficiently long time slots for the DL phase and sufficiently fast fluctuations in w_k. The kth user's ergodic mutual information $R_k \triangleq \mathrm{E}_{\boldsymbol{h}_k}[r_k]$ reads as (cf. [1], [8])

$$R_k = \tfrac{1}{\log(2)}\varsigma\left(\tfrac{1}{\sum_{i=1}^K|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i|^2}\right) - \tfrac{1}{\log(2)}\varsigma\left(\tfrac{1}{\sum_{i\neq k}^K|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i|^2}\right), \tag{2}$$

where we defined $\varsigma : \mathbb{R}_{+,0} \mapsto \mathbb{R}_{+,0}$ as [9]

$$\varsigma(x) \triangleq \mathrm{e}^x\,\mathrm{E}_1(x) = \mathrm{e}^x\left(-\gamma - \log(x) - \sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n!n}\right) \tag{3}$$

and $\mathrm{E}_1(x) = \int_x^\infty \frac{\mathrm{e}^{-t}}{t}\,\mathrm{d}t$ is the exponential integral.

By definition, the ergodic rate R_k has great similarities to r_k in (1). Denoting the average signal and interference as

$$S_k = |\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2 \qquad I_k = \textstyle\sum_{i\neq k}|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i|^2, \tag{4}$$

respectively, we find the following monotonicity properties.

Lemma 1. *The rate r_k and the ergodic rate R_k in* (1) *and* (2)*, respectively, are non-negative and strictly …*

(a) sub-linearly increasing with S_k for fixed I_k,
(b) monotonically decreasing with I_k for fixed S_k, and
(c) sub-linearly increasing with α for the common scaling $S_k = \alpha S'_k$ and $I_k = \alpha I'_k$, with $\alpha > 0$ and fixed (S'_k, I'_k).

Proof: Noting that $|\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{t}_k|^2 = |w_k|^2 S_k$ and $\sum_{i\neq k}|\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{t}_i|^2 = |w_k|^2 I_k$, the ergodic rate is defined as

$$R_k = \mathrm{E}\left[\log_2\left(1 + \tfrac{|w_k|^2 S_k}{1+|w_k|^2 I_k}\right)\right],$$

i.e., as the average of logarithmic functions r_k. These functions r_k are sub-linearly increasing with S_k, monotonically decreasing with I_k, and sub-linearly increasing with $\alpha > 0$ for the common scaling $S_k = \alpha S'_k$ and $I_k = \alpha I'_k$. Hence, this is also valid for the average. ■

However, contrary to r_k, R_k cannot be represented in terms of the *signal-to-interference-plus-noise-ratio* (SINR). That is, the ergodic rate rather depends on the absolute values of S_k and I_k than on their ratio.

IV. Problem Formulations

Given the QoS requirements ρ_k, $k \in \{1,\ldots,K\}$, expressed as minimum rate and ergodic rate targets, the power minimization problem *(P)* reads as

$$\min_{\{\boldsymbol{t}_1,\ldots,\boldsymbol{t}_K\}} \sum_{k=1}^K \|\boldsymbol{t}_k\|_2^2 \quad \mathrm{s.t.:} \begin{cases} r_k \geq \rho_k & \forall k \in \mathbb{P}, \\ R_k \geq \rho_k & \forall k \in \mathbb{S}. \end{cases} \tag{5}$$

We remark that the constraints in (5) are active in the optimum due to the monotonicity properties of r_k and R_k (see Lemma 1). Additionally, note that (5) does not have a solution in general. In [10], a simple feasibility test was proposed for exclusively perfect CSI users (see also Section VI-A).

The closely related balancing optimization *(B)* reads as

$$\max_{\beta,\{\boldsymbol{t}_1,\ldots,\boldsymbol{t}_K\}} \beta \quad \mathrm{s.t.:} \sum_{k=1}^K \|\boldsymbol{t}_k\|_2^2 \leq P_{\mathrm{tx}}, \begin{cases} r_k \geq \beta\rho_k & \forall k \in \mathbb{P}, \\ R_k \geq \beta\rho_k & \forall k \in \mathbb{S}, \end{cases} \tag{6}$$

where the transmit power is P_{tx} in the optimum and the resulting rates are balanced, i.e., the rate of user k is $\beta_{\mathrm{opt}}\rho_k$. Note that (6) always has a solution contrary to (5).

As has been discussed in [11] and [12] for perfect CSI, (5) and (6) are inverse problems. Denoting the optimum of (6) as $\beta_{\mathrm{opt}}(P_{\mathrm{tx}})$ and the optimum of (5) as $P(\rho_1,\ldots,\rho_K)$, then

$$P(\beta_{\mathrm{opt}}(P_{\mathrm{tx}})\rho_1,\ldots,\beta_{\mathrm{opt}}(P_{\mathrm{tx}})\rho_K) = P_{\mathrm{tx}} \tag{7}$$

holds [12]. Since $\beta_{\mathrm{opt}}(P_{\mathrm{tx}})$ is monotonically increasing in P_{tx}, (6) can be solved via a bisection where β is adjusted such that $P(\beta\rho_1,\ldots,\beta\rho_K)$ is equal to P_{tx}.

Unfortunately, considering the ergodic rate constraints, neither of the problems can be solved directly. In the BC, the rates are neither convex nor concave functions of the variables $\{\boldsymbol{t}_i\}_{i=1}^K$. Therefore, (5) and (6) are nonconvex. Moreover, while the rate constraints for $k \in \mathbb{P}$ can equivalently be formulated as minimum SINR requirements $\gamma_k = 2^{\rho_k} - 1$, this is not possible for the ergodic rate constraints. Thus, we cannot reformulate problem (5) to that in [11], i.e., a power minimization with individual SINR constraints, and resort to the (convex) optimization techniques in [12] and [6] or the feasibility test in [10]. Hence, we cannot solve (6) with (5).

V. Bounds on Ergodic Mutual Information

To overcome these difficulties, we propose two pairs of lower and upper bounds for R_k next. To this end, some basic properties of the ς-function in (3) are exploited. The bounds are designed in a way, such that we can apply the usual methods and algorithms when replacing the ergodic rates with the proposed bounds in (5) and (6) (see Section VI).

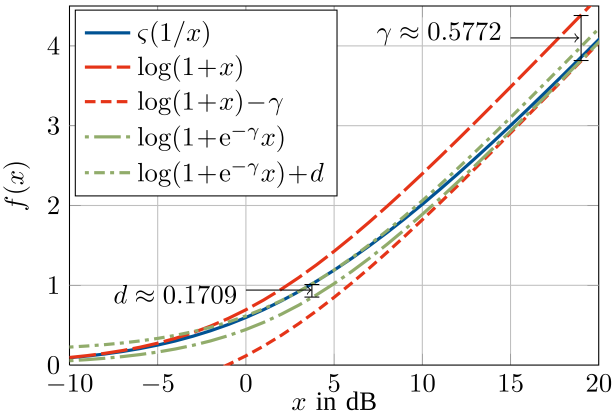

Figure 1. Plot of $\varsigma(1/x)$ and its bounds versus x in dB.

A. First Lower and Upper Bound on the Ergodic Rate

For the first upper and corresponding lower bound, we exploit the following properties of $\varsigma(1/x)$.

Lemma 2. *The difference $d_{\mathrm{U1}}(x) = \log(1+x) - \varsigma(1/x)$ is zero for $x=0$, positive for $x>0$, and strictly monotonically increasing in x. In the limit $x \to \infty$, $d_{\mathrm{U1}}(x)$ approaches the Euler-Mascheroni constant $\gamma \approx 0.5772$ [9].*

The proof is shown in Appendix A. A direct consequence of Lemma 2 is that $\varsigma(1/x)$ is bounded by

$$\log(1+x) - \gamma \le \varsigma(1/x) \le \log(1+x) \tag{8}$$

and the difference $d_{\mathrm{L1}}(x) = \varsigma(1/x) - \log(1+x) + \gamma$ is positive, approaches zero for $x \to \infty$, and is monotonically decreasing in x. This can also be seen in Figure 1, where we plotted $\varsigma(1/x)$ with its lower and upper bounds versus x in dB.

Based on (8), we propose the following conservative lower bound on the ergodic mutual information [cf. (2) and (4)]:

$$\begin{aligned} R_k \ge \underline{R}_k^{(1)} &= \log_2(1+S_k+I_k) - \log_2(1+I_k) - \tfrac{\gamma}{\log(2)} \\ &= \log_2\left(1+\tfrac{S_k}{1+I_k}\right) - \tfrac{\gamma}{\log(2)}. \end{aligned} \tag{9}$$

A drawback of this lower bound is that it becomes negative for small but reasonable S_k, what is per definition impossible for R_k (see Lemma 1). Moreover, above conservative bound is only tight if both $S_k \to \infty$ *and* $I_k = 0$, i.e., when ZF is applied and we consider the high SNR regime. Using (9), the worst case error to R_k is $\frac{\gamma}{\log(2)} \approx 0.8327$ bits per channel use.

Based on the monotonicity of $d_{\mathrm{U1}}(x)$, i.e., $d_{\mathrm{U1}}(I_k) \le d_{\mathrm{U1}}(I_k+S_k)$, we can find the following ergodic rate upper bound:

$$R_k \le \overline{R}_k^{(1)} = \log_2\left(1+\tfrac{S_k}{1+I_k}\right), \tag{10}$$

which is tight for *either* $S_k \approx 0$ *or* sufficiently large I_k. In fact, we find the following proposition proven in Appendix B.

Proposition 1. *The ergodic rate R_k (2) is upper bounded by $\overline{R}_k^{(1)}$ in (10) and differs from $\overline{R}_k^{(1)}$ by a small error, i.e., $R_k \ge \overline{R}_k^{(1)} - \delta$ with $\delta \ge 0$. Here, δ is given by*

$$\delta = \tfrac{1}{\log(2)}\varsigma(1/I_k) - \log_2(1+I_k) + \tfrac{\gamma}{\log(2)} \le \tfrac{\gamma}{\log(2)}$$

and has the following properties:

(i) *δ is monotonically decreasing with I_k,*
(ii) *$\delta = \frac{\gamma}{\log(2)}$ for $I_k = 0$, and $\delta = 0$ for $I_k \to \infty$.*

According to this proposition, we expect the upper bound to be loose when $K \le N$ and ZF beamforming strategies are applied, whereas we expect the bound to be tight in interference limited scenarios, e.g., for $N < K$ (see Section IX).

B. Second Lower and Upper Bound on the Ergodic Rate

Motivated by the observation that the bounds on $\varsigma(1/x)$ in (8) are tight for *either* $x=0$ *or* $x \to \infty$, we searched for more adequate bounds. In fact, we found that $h(x) = \log(1+e^{-\gamma}x)$ is tight for both, $x=0$ *and* $x \to \infty$.

Actually, $h(x)$ is a *tight lower bound* for $\varsigma(1/x)$. That is,

$$\varsigma(1/x) \ge \log(1+e^{-\gamma}x). \tag{11}$$

For small $x>0$, above inequality is clearly satisfied since both functions, $\varsigma(1/x)$ and $\log(1+e^{-\gamma}x)$, are zero for $x=0$, and have smoothly decreasing derivatives satisfying (cf. [9])

$$\tfrac{\mathrm{d}}{\mathrm{d}\,x}\varsigma(1/x)\big|_{x=0} = 1 > \tfrac{\mathrm{d}}{\mathrm{d}\,x}\log(1+e^{-\gamma}x)\big|_{x=0} = \tfrac{1}{e^{\gamma}}.$$

Furthermore, the difference $d_{\mathrm{L2}}(x) = \varsigma(1/x) - \log(1+e^{-\gamma}x)$ approaches zero from above for $x \to \infty$, i.e., [cf. (3)]

$$\begin{aligned} \lim_{x\to\infty} d_{\mathrm{L2}}(x) &= \lim_{x\to\infty} e^{\frac{1}{x}}\left(-\gamma+\log(x) - \sum_{n=1}^{\infty}\tfrac{(-1)^n}{n!nx^n}\right) - \log(e^{-\gamma}x) \\ &= \lim_{x\to\infty} e^{\frac{1}{x}}\left(-\gamma+\log(x)\right) - \log(x) + \gamma = 0. \end{aligned}$$

Therefore, (11) is also valid for large x. For medium x, we can see in Figure 1 that (11) holds with strict inequality.

The figure also suggest that $d_{\mathrm{L2}}(x)$ is bounded above. To find its supremum, we note that $\frac{\mathrm{d}}{\mathrm{d}\,x}d_{\mathrm{L2}}(x)$ is strictly positive for low x, negative for large x, and apparently approaches zero only for a finite x_d and in the limit $x \to \infty$. Thus, $d_{\mathrm{L2}}(x)$ appears to be pseudo-concave, why we employed a dedicated bisection search [13, Chapter 8] to accurately determine the maximal distance $d = \max_{x \ge 0} d_{\mathrm{L2}}(x)$ and x_d.

Given $d \approx 0.1709$, that is achieved at $x_d \approx 2.3665 (\triangleq 3.7411\mathrm{dB})$, we can bound $\varsigma(1/x)$ as (cf. Figure 1)

$$\log(1+e^{-\gamma}x) \le \varsigma(1/x) \le \log(1+e^{-\gamma}x) + d. \tag{12}$$

Based on these bounds for $\varsigma(1/x)$, we simply obtain a lower bound on R_k via replacing the first and second $\varsigma(\cdot)$-term in (2) with the lower and upper bound in (12), respectively. That is,

$$R_k \ge \underline{R}_k^{(2)} = \log_2\left(1+\tfrac{S_k}{e^{\gamma}+I_k}\right) - \tfrac{d}{\log(2)}. \tag{13}$$

Accordingly, an appropriate upper bound on R_k results, when we replace the first and second $\varsigma(\cdot)$-term in (2) with the upper and lower bound in (12), respectively, i.e.

$$R_k \le \overline{R}_k^{(2)} = \log_2\left(1+\tfrac{S_k}{e^{\gamma}+I_k}\right) + \tfrac{d}{\log(2)}. \tag{14}$$

Obviously, the lower bound $\underline{R}_k^{(2)}$ and the upper bound $\overline{R}_k^{(2)}$ are not tight in general. Whereas $\underline{R}_k^{(2)}$ becomes negative for sufficiently small effective SINR, i.e., $\frac{S_k}{e^{\gamma}+I_k} \approx 0$, $\overline{R}_k^{(2)}$ is always larger than $d/\log(2)$ even if the effective SINR is zero. Hence, $\overline{R}_k^{(2)}$ is clearly looser than $\overline{R}_k^{(1)}$ for low S_k or sufficiently large I_k. However, the worst case error is considerably smaller than for the bounds in the previous subsection. Given arbitrary S_k and I_k, $\overline{R}_k^{(2)}$ and $\underline{R}_k^{(2)}$ differ from R_k by maximally $\frac{2d}{\log(2)} \approx 0.4934$ bits per channel use.

VI. QoS Power Minimization with Bounds

As mentioned in Section IV, directly solving (5) is difficult. To this end, we replace the ergodic rate requirements by constraints on the proposed bounds of R_k denoted as

$$R_k^{\mathrm{B}} = \log_2\left(1+\frac{|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2}{\nu+\sum_{i\neq k}|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i|^2}\right)+\mu, \tag{15}$$

where ν and μ depend on the represented bound (see Table I).

R_k^{B}	$\underline{R}_k^{(1)}$	$\overline{R}_k^{(1)}$	$\underline{R}_k^{(2)}$	$\overline{R}_k^{(2)}$
ν	1	1	e^{γ}	e^{γ}
μ	$-\frac{\gamma}{\log(2)}$	0	$-\frac{d}{\log(2)}$	$+\frac{d}{\log(2)}$

Table I
Representation of the proposed bounds for the algorithm.

By doing so, the resulting problem can equivalently be formulated as a power minimization with SINR requirements

$$\min_{\{\boldsymbol{t}_1,\ldots,\boldsymbol{t}_K\}} \sum_{k=1}^{K}\|\boldsymbol{t}_k\|_2^2 \quad \mathrm{s.t.:}\ \ \mathrm{SINR}_k \geq \gamma_k \ \ \forall k\in\mathbb{P}, \tag{16}$$

with $\gamma_k = 2^{\rho_k}-1$ for $k\in\mathbb{P}$, $\gamma_k = 2^{\rho_k-\mu}-1$ for $k\in\mathbb{S}$, and the definition of the "signal-to-interference-and-noise ratio"

$$\mathrm{SINR}_k = \begin{cases} \frac{|\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{t}_k|^2}{1+\sum_{i\neq k}|\boldsymbol{h}_k^{\mathrm{H}}\boldsymbol{t}_i|^2} & k\in\mathbb{P},\\ \frac{|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_k|^2}{\nu+\sum_{i\neq k}|\boldsymbol{v}_k^{\mathrm{H}}\boldsymbol{t}_i|^2} & k\in\mathbb{S}. \end{cases} \tag{17}$$

For a feasibility test and for the optimization in (16), we can directly apply the usual methods and algorithms. Feasibility can for example be tested based on a reformulation to maximum *mean-square-error* (MSE) constraints [10] (see Subsection VI-A). Note that feasibility of (16) implies feasibility of (5) when R_k^{B} represents a lower bound of R_k, $k\in\mathbb{S}$. However, when we use upper bounds, feasibility of (16) is obviously only a necessary condition for the feasibility of (5).

Given feasibility, we can use efficient fixed-point methods based on the SINR-uplink-downlink-duality and the standard interference function framework (see [14], [6]) for solving (16). Depending on whether we use upper or lower bounds for R_k, $k\in\mathbb{S}$, the obtained optimum gives either a conservative or an optimistic bound for the optimum of (5).

A. QoS Feasibility

Following [10], we reformulate the SINR targets in (16) as maximum MSE requirements by noting that the maximum SINR and the *minimum MSE* (MMSE) of user k are related via $\mathrm{SINR}_k = \frac{1}{\mathrm{MMSE}_k}-1$. Additionally, since $r_k = \log_2(1+\mathrm{SINR}_k)$ and $R_k^{\mathrm{B}} = \log_2(1+\mathrm{SINR}_k)+\mu$, we can represent the MMSE requirements as

$$\mathrm{MMSE}_k \leq \varepsilon_k = \frac{1}{1+\gamma_k} = \begin{cases} 2^{-\rho_k} & \forall k\in\mathbb{P},\\ 2^{-\rho_k+\mu} & \forall k\in\mathbb{S}. \end{cases} \tag{18}$$

Note that ν does not appear in (18) since it is directly incorporated in the SINR definition of the statistical CSI users.

With (18) and under some *regularity conditions* on the effective channels $\{\boldsymbol{h}_k\}_{k\in\mathbb{P}}$ and $\{\boldsymbol{v}_k\}_{k\in\mathbb{S}}$, the MMSE targets $\{\varepsilon_k\}_{k=1}^K$ are feasible iff they lie within a polytope $\mathcal{P}$ whose bounding half-spaces are the individual box constraints $0\leq\varepsilon_k\leq 1\ \forall k\in\{1,\ldots,K\}$ and the sum MMSE constraint $\sum_{k=1}^K \varepsilon_k \geq K-N$ [10, Theorem 1]. Any point belonging to the interior of the polytope can be achieved with finite sum power. For MMSE targets $\varepsilon_k = 1$, no power is allocated to the respective user. However, note that $\varepsilon_k = 1$ results in $\rho_k = \mu$ for $k\in\mathbb{S}$, which might be negative dependent on the bound that is represented by R_k^{B} (cf. Table I).

B. Dual Downlink Model and SINR Duality

A key tool for solving (16) with the fixed-point algorithms of [6] is a reformulation into the dual uplink. Therein, K single-antenna transmitters send independent signals with power p_k, $k\in\{1,\ldots,K\}$ over the dual channels $\boldsymbol{b}_k = \boldsymbol{h}_k$ for $k\in\mathbb{P}$ and $\boldsymbol{b}_k = \frac{1}{\sqrt{\nu}}\boldsymbol{v}_k$ for $k\in\mathbb{S}$ to an N-antenna receiver. The receiver estimates the sent signals using linear equalizers $\boldsymbol{f}_k$, $k\in\{1,\ldots,K\}$ and suffers from noise $\boldsymbol{\eta}\sim\mathcal{N}_{\mathbb{C}}(\mathbf{0},\mathbf{I})$. Hence, the kth user's SINR in the dual MAC reads as

$$\mathrm{SINR}_k^{\mathrm{MAC}} = \frac{|\boldsymbol{f}_k^{\mathrm{H}}\boldsymbol{b}_k|^2 p_k}{\|\boldsymbol{f}_k\|_2^2+\sum_{i\neq k}|\boldsymbol{f}_k^{\mathrm{H}}\boldsymbol{b}_i|^2 p_i}. \tag{19}$$

Knowing that exactly the same SINRs are achievable in the vector BC and the dual vector MAC if the same total transmit power is employed [14], we can equivalently solve (16) in the dual MAC. The BC solution is then found with $\boldsymbol{t}_k = \alpha_k\boldsymbol{f}_k\ \forall k\in\{1,\ldots,K\}$ and the scalars $\{\alpha_k\}_{k=1}^K$ are obtained via solving a system of linear equations (see [14] for details).

C. Interference Function and Problem Solution

The precoder design in the BC essentially reformulates to a joint optimization of the transmit power allocation $\boldsymbol{p} = [p_1,\ldots,p_K]^{\mathrm{T}}$ and the equalizers $\boldsymbol{f}_k$, $k\in\{1,\ldots,K\}$ in the dual MAC. For this task, we suggest to apply the generic interference function framework of [5] and [6] in order to deal with the large number of users and transmit antennas in satellite communications. To this end, we define the effective interference of user k as $\mathcal{Z}_k(\boldsymbol{p},\boldsymbol{f}_k) = p_k/\mathrm{SINR}_k^{\mathrm{MAC}}$ [cf. (19)], such that the (vector) interference function is given by

$$\boldsymbol{\mathcal{Z}}(\boldsymbol{p},\boldsymbol{F}) = [\mathcal{Z}_1(\boldsymbol{p},\boldsymbol{f}_1),\ldots,\mathcal{Z}_K(\boldsymbol{p},\boldsymbol{f}_K)]^{\mathrm{T}}. \tag{20}$$

Note that $\boldsymbol{\mathcal{Z}}(\boldsymbol{p},\boldsymbol{F})$ is standard according to [5] for fixed equalizers $\boldsymbol{F} = [\boldsymbol{f}_1,\ldots,\boldsymbol{f}_K]$. Therefore, choosing the optimum equalizers, given by $\boldsymbol{f}_{\mathrm{opt},k}(\boldsymbol{p}) = \big(\mathbf{I}_N + \sum_{i\neq k}\boldsymbol{b}_i\boldsymbol{b}_i^{\mathrm{H}}p_i\big)^{-1}\boldsymbol{b}_k\sqrt{p_k}\ \ \forall k\in\{1,\ldots,K\}$, also leads to a *standard interference function* (SIF) [5] that is given by $\boldsymbol{\mathcal{I}}(\boldsymbol{p}) = \big[\min_{\boldsymbol{f}_1}\mathcal{Z}_1(\boldsymbol{p},\boldsymbol{f}_1),\ldots,\min_{\boldsymbol{f}_K}\mathcal{Z}_K(\boldsymbol{p},\boldsymbol{f}_K)\big]^{\mathrm{T}}$.

This observation and the fact that we can write $\boldsymbol{\mathcal{Z}}(\boldsymbol{p},\boldsymbol{F}) = \boldsymbol{\Psi}(\boldsymbol{F})\boldsymbol{p}+\boldsymbol{\xi}(\boldsymbol{F})$, i.e., as a linear function in $\boldsymbol{p}$, led to the two fixed-point algorithms in [6] for solving the resulting power minimization problem in the dual MAC [cf. (19)]

$$\min_{\boldsymbol{p}\geq\mathbf{0}} \mathbf{1}^{\mathrm{T}}\boldsymbol{p} \qquad \mathrm{s.t.:}\quad \boldsymbol{p}\geq\mathrm{diag}(\boldsymbol{\gamma})\boldsymbol{\mathcal{I}}(\boldsymbol{p}) \tag{21}$$

with the vector of SINR targets $\boldsymbol{\gamma} = [\gamma_1,\ldots,\gamma_K]^{\mathrm{T}}$. The details of these algorithms can also be found in [1].

VII. Rate Balancing with Bound Constraints

Replacing R_k with R_k^{B} in (6), we can write the balancing problem in a similar form as (21). Reformulating the problem in terms of SINR requirements, applying the duality of

Section VI-B, and using the interference function definition in (20), the balancing problem (6) leads to

$$\max_{\beta,\boldsymbol{p}\geq\mathbf{0}} \beta \quad \text{s.t.:} \quad \mathbf{1}^{\mathrm{T}}\boldsymbol{p} \leq P_{\text{tx}} \quad \boldsymbol{p} \geq \operatorname{diag}(\boldsymbol{\gamma}(\beta))\boldsymbol{\mathcal{I}}(\boldsymbol{p}) \qquad (22)$$

with $\boldsymbol{\gamma}(\beta) = [\gamma_1(\beta), \ldots, \gamma_K(\beta)]^{\mathrm{T}}$, $\gamma_k(\beta) = 2^{\beta\rho_k} - 1$ for $k \in \mathbb{P}$, and $\gamma_k(\beta) = 2^{\beta\rho_k - \mu} - 1$ for $k \in \mathbb{S}$.

We remark that, contrary to (6), the optimum of (22) may become negative for negative μ (cf. Table I) and sufficiently low P_{tx}. When μ is negative, some minimal transmit power $P_{\text{tx}}^{\min}$ is necessary for achieving $R_k^{\mathrm{B}} \geq 0$, $k \in \mathbb{S}$. This $P_{\text{tx}}^{\min}$ may be found via solving (16) with SINR targets $\gamma_k = 0$ for $k \in \mathbb{P}$ and $\gamma_k = 2^{-\mu} - 1$ for $k \in \mathbb{S}$.

Problem (22) can be solved with the help of (21). Based on the observation in (7), we can perform a bisection search on β where in each step the power minimization (21) is solved until the given transmit power is P_{tx} at its optimum. Here, a lower and an upper bound on β may be found via the uniform power allocation and the single user rates, respectively. Details and an implementation of the algorithm can be found in [1].

VIII. Additional Power Allocation

When calculating the BC beamformers and the actually achieved rates from the solutions of (21) or (22), we can observe: while r_k obviously meets the proposed rates for $k \in \mathbb{P}$, R_k of users $k \in \mathbb{S}$ differs from the proposed ones. This motivates an additional power allocation for reducing the necessary resources or increasing the achieved balancing factor. To this end, we represent the *given* beamformers as

$$\boldsymbol{t}_k = \boldsymbol{\tau}_k \sqrt{\theta_k}, \qquad (23)$$

with unit-norm $\boldsymbol{\tau}_k$ and $\theta_k \in \mathbb{R}_{0,+}$ denoting the portion of the transmit power that is intended for user k, and aim at solving the following problems for fixed $\{\boldsymbol{\tau}_k\}_{k=1}^K$:

1) minimize $P_{\text{tx}} = \sum_{i=1}^K \theta_i$ subject to the rate requirements $r_k \geq \rho_k \ \forall k \in \mathbb{P}$ and $R_k \geq \rho_k \ \forall k \in \mathbb{S}$;

2) maximize β subject to limited transmit power $\sum_{k=1}^K \theta_k \leq P_{\text{tx}}$ and $r_k \geq \beta\rho_k \ \forall k \in \mathbb{P}$ and $R_k \geq \beta\rho_k \ \forall k \in \mathbb{S}$.

Note that $\boldsymbol{\mathcal{T}}(\boldsymbol{\theta}) = [\mathcal{T}_1(\boldsymbol{\theta}), \ldots, \mathcal{T}_K(\boldsymbol{\theta})]^{\mathrm{T}}$ with $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_K]^{\mathrm{T}}$,

$$\mathcal{T}_k(\boldsymbol{\theta}) = \begin{cases} \theta_k / r_k & k \in \mathbb{P} \\ \theta_k / R_k & k \in \mathbb{S} \end{cases} \qquad (24)$$

is a SIF [5], i.e., it satisfies the following three axioms:

A1) $\boldsymbol{\mathcal{T}}(\boldsymbol{\theta}) > \mathbf{0}$ for $\boldsymbol{\theta} \geq \mathbf{0}$ *(Positivity)*

A2) $\boldsymbol{\mathcal{T}}(\boldsymbol{\theta}) \geq \boldsymbol{\mathcal{T}}(\boldsymbol{\theta}')$ for $\boldsymbol{\theta} \geq \boldsymbol{\theta}' \geq \mathbf{0}$ *(Monotonicity)*

A3) $\lambda\boldsymbol{\mathcal{T}}(\boldsymbol{\theta}) > \boldsymbol{\mathcal{T}}(\lambda\boldsymbol{\theta})$ for $\lambda > 1$ *(Scalability)*

Here, axioms A1)–A3) result directly from Lemma 1 and (24).

With (24) and $\boldsymbol{\rho} = [\rho_1, \ldots, \rho_K]^{\mathrm{T}}$, the power minimization problem from *1)* can equivalently be written as

$$\min_{\boldsymbol{\theta}\geq\mathbf{0}} \mathbf{1}^{\mathrm{T}}\boldsymbol{\theta} \quad \text{s.t.:} \quad \boldsymbol{\theta} \geq \operatorname{diag}(\boldsymbol{\rho})\boldsymbol{\mathcal{T}}(\boldsymbol{\theta}). \qquad (25)$$

Due to the monotonicity property of $\boldsymbol{\mathcal{T}}(\boldsymbol{\theta})$ in A2) together with the scalability property in A3), the constraint in (25) is active. This motivates the fixed-point algorithm of [5]

$$\boldsymbol{\theta}^{(n)} = \operatorname{diag}(\boldsymbol{\rho})\boldsymbol{\mathcal{T}}(\boldsymbol{\theta}^{(n-1)}), \qquad (26)$$

which produces a sequence $\{\boldsymbol{\theta}^{(n)}\}_n$ that is monotonically increasing, i.e., $\boldsymbol{\theta}^{(n+1)} \geq \boldsymbol{\theta}^{(n)}$, when starting with $\boldsymbol{\theta}^{(0)} = \mathbf{0}$ and converges to the global optimizer of (25) if it exists.

Similarly, the balancing problem *2)* can be formulated as

$$\max_{\boldsymbol{\theta}\geq\mathbf{0}} \beta \quad \text{s.t.:} \quad \mathbf{1}^{\mathrm{T}}\boldsymbol{\theta} \leq P_{\text{tx}} \quad \boldsymbol{\theta} \geq \beta \operatorname{diag}(\boldsymbol{\rho})\boldsymbol{\mathcal{T}}(\boldsymbol{\theta}). \qquad (27)$$

For its solution, we suggest again a bisection on β. In each step of this bisection we test whether there exists a power allocation $\boldsymbol{\theta}$ that satisfies the constraints. We realize this feasibility test with (26). Starting from $\boldsymbol{\theta}^{(0)} = \mathbf{0}$, if the iteration results in a $\boldsymbol{\theta}^{(n)}$ with $\mathbf{1}^{\mathrm{T}}\boldsymbol{\theta}^{(n)} > P_{\text{tx}}$, then β results in infeasible targets. Otherwise, when the iteration converges to a $\boldsymbol{\theta}$ with $\mathbf{1}^{\mathrm{T}}\boldsymbol{\theta} \leq P_{\text{tx}}$, β results in feasible targets. Initial bounds for β_{opt} are determined in the same way as in Section VII. A lower bound can be obtained with uniform power allocation $\boldsymbol{\theta} = P_{\text{tx}}\mathbf{1}$ and an upper bound with the single user power allocations $\boldsymbol{\theta} = P_{\text{tx}}\boldsymbol{e}_k$ for $k \in \{1, \ldots, K\}$.

IX. Numerical Results

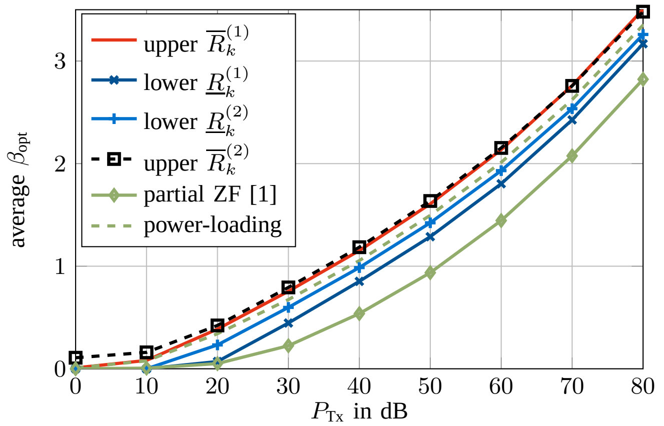

Figure 2. Achievable balancing level β in a fully loaded system with $K = 100$ users and $N = 100$ transmit antennas.

For a numerical verification of the proposed methods, we consider a GEO-stationary satellite that is directed to Munich (11°east and 48°north) and has a rectangular antenna array of $N = 100$ elements. The K perfect and statistical CSI users are randomly placed within an area that covers Europe and the channels $\{\boldsymbol{h}_k\}_{k\in\mathbb{P}}$ and $\{\boldsymbol{v}_k\}_{k\in\mathbb{S}}$ were realized with the free space path loss model (e.g., see [15]).

For the Figures 2 and 3, we considered a *fully loaded* scenario with $K = 100$ users, $|\mathbb{P}| = |\mathbb{S}| = 50$, and an *overloaded* scenario with $K = 120$ users, $|\mathbb{P}| = |\mathbb{S}| = 60$. In both scenarios, the users have targets $\rho_{2i-1} = 1$ and $\rho_{2i} = 2$ for $i \in \{1, \ldots, K/2\}$. Moreover, we generated 10 channel realizations $\boldsymbol{h}_k$, $k \in \mathbb{P}$, and $\boldsymbol{v}_k$, $k \in \mathbb{S}$ and plotted the average achieved balancing level β_{opt} versus P_{tx} in dB for the following results: *a)* the result of (22) with the bounds in Table I; *b)* the result of the partial ZF strategy from [1]; *c)* the result of the additional power loading (27) with the spatial beamformer directions $\{\boldsymbol{\tau}_k\}_{k=1}^K$ from (22) with $R_k^{\mathrm{B}} = \underline{R}_k^{(2)}$.

Figure 2 shows the results for the fully loaded setup, where β_{opt} grows unboundedly with P_{tx}, while Figure 3 considers the overloaded setup, where β_{opt} saturates for high P_{tx}. In both figures, we set the lower bound curves to zero when P_{tx} is smaller than the minimum necessary transmit power for

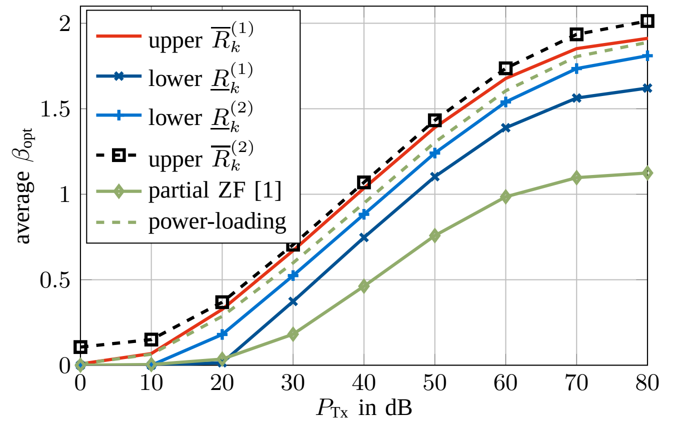

Figure 3. Achievable balancing level β in an overloaded system with $K = 120$ users and $N = 100$ transmit antennas.

satisfying $\underline{R}_k^{(i)} > 0\ \forall k \in \mathbb{S}$. Note that $\underline{R}_k^{(2)}$ exceeds $\underline{R}_k^{(1)}$ for low and medium P_{tx} since it has a considerably smaller worst case error (see Figure 2). This gap is even larger in Figure 3 since $\underline{R}_k^{(1)}$ is only tight for $I_k = 0$ and $S_k \to \infty$ which is not possible simultaneously for $N < K$. Moreover, both lower bound schemes outperform the partial ZF scheme proposed in [1] for sufficiently large P_{tx}.

Employing the additional power allocation for the simulations with $\underline{R}_k^{(2)}$ (we employed *maximum-ratio-transmission beamformers* when β_{opt} is zero) the achieved balancing level is considerably improved. In fact, this curve is close to the tight upper bound based on $\overline{R}_k^{(1)}$ and therefore also close to the optimum of (6). The second upper bound $\overline{R}_k^{(2)}$ appears to be looser than the first one $\overline{R}_k^{(1)}$ for low P_{tx}. Only for very high P_{tx} in the fully-loaded setup $\overline{R}_k^{(2)}$ is competitive to $\overline{R}_k^{(1)}$.

X. Conclusions

We presented power efficient linear DL beamformer designs with perfect CSI rates and bounds on ergodic rates. The proposed bounds have small worst-case errors and some are shown to be tight, what allowed us to compute tight bounds on the optima of the considered problems. In [16], these bounds on the optima are compared to the global optima.

Appendix

A. Proof of Lemma 2

From [9], we know that $\varsigma(z) \leq \log(1+1/z)$ and can directly infer non-negativity of $d_{\text{U1}}(x)$ via replacing z with $1/x$. Here, equality only holds for $z \to \infty$, i.e., $x = 0$. The monotonicity property follows directly from

$$\frac{\mathrm{d}}{\mathrm{d}\,x} d_{\text{U1}}(x) = \frac{1}{x^2}\varsigma(1/x) - \frac{1}{x(1+x)} \geq \frac{1}{x^2}\frac{1}{1+1/x} - \frac{1}{x(1+x)} = 0,$$

where we used $\frac{\mathrm{d}}{\mathrm{d}\,z}\varsigma(z) = \varsigma(z) - 1/z$ and $\varsigma(z) \geq \frac{1}{1+z}$ [9]. Here, equality holds if either $x = 0$ or $x \to \infty$. Finally, using $\lim_{x\to\infty} \log(1+x) = \lim_{x\to\infty} \log(x)$ and (3) we obtain

$$\lim_{x\to\infty} d_{\text{U1}}(x) = \lim_{x\to\infty} \log(x) - \mathrm{e}^{\frac{1}{x}}\left(\log(x) - \gamma - \sum_{n=1}^{\infty} \frac{(-1)^n}{n! n x^n}\right) = \gamma.$$

B. Proof of Proposition 1

The proof is based on the consequences of Lemma 2. For $x, \delta_x \geq 0$, we have that $d_{\text{U1}}(x+\delta_x) \geq d_{\text{U1}}(x)$ since $d_{\text{U1}}(\cdot)$ is a monotonically increasing function. That is,

$$\log(1+x+\delta_x) - \varsigma\left(\tfrac{1}{x+\delta_x}\right) \geq \log(1+x) - \varsigma\left(\tfrac{1}{x}\right).$$

Now, subtracting $\log(1+x)$ and adding $\varsigma\left(\frac{1}{x+\delta_x}\right)$ at both sides, multiplying them with $1/\log(2)$, and replacing x and δ_x with I_k and S_k, respectively, we see that [cf. (10)]

$$\begin{aligned}\overline{R}_k^{(1)} &= \log_2(1+I_k+S_k) - \log_2(1+I_k) \\ &\geq \tfrac{1}{\log(2)}\varsigma\left(\tfrac{1}{I_k+S_k}\right) - \tfrac{1}{\log(2)}\varsigma\left(\tfrac{1}{I_k}\right) = R_k.\end{aligned}$$

Next, starting with $d_{\text{U1}}(x+\delta_x) - d_{\text{U1}}(x) \geq 0$, we see that

$$\begin{aligned}0 &\leq \log(1+x+\delta_x) - \log(1+x) - \varsigma\left(\tfrac{1}{x+\delta_x}\right) + \varsigma\left(\tfrac{1}{x}\right) \\ &\leq \gamma - \log(1+x) + \varsigma\left(\tfrac{1}{x}\right) \leq \gamma\end{aligned}$$

because $d_{\text{U1}}(x) \leq d_{\text{U1}}(x+\delta_x) \leq \gamma$ (see Lemma 2). Doing the same replacements as above and dividing both sides by $\log(2)$, it directly follows that

$$\overline{R}_k^{(1)} - R_k \leq \tfrac{\gamma}{\log(2)} - \log_2(1+I_k) - \tfrac{1}{\log(2)}\varsigma\left(\tfrac{1}{I_k}\right) \leq \tfrac{\gamma}{\log(2)},$$

where the middle part is essentially $\frac{1}{\log(2)} d_{\text{L1}}(I_k)$ and, therefore, monotonically decreasing with I_k, equal to $\frac{\gamma}{\log(2)}$ for $I_k = 0$, and becomes zero for $I_k \to \infty$ (cf. Subsection V-A).

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