Scalar Quantizer Based Feedback of the Channel Direction Information in MU-MISO Systems

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Abstract— **The availability of the *Channel State Information* (CSI) at the transmitter is crucial for the precoder design in *Multi-user Multiple Input Single Output* (MU-MISO) systems. In *Frequency Division Duplex* (FDD) systems, CSI can be just available at the transmitter through a limited feedback channel [1], where we assume that each user estimates, normalizes, and finally quantizes its channel direction with a finite number of quantization bits relayed back error-free and quasi-instantaneous. In this paper, we consider a simple sequential uniform *scalar quantization* (SQ) scheme of the individual components (real and imaginary parts) of the *Channel Direction Information* (CDI). Although *vector quantization* (VQ) schemes [2], [3] still outperform this scalar scheme in terms of quantization error and *Bit Error Rate* (BER), the former scheme suffers from an exponential search complexity and high storage requirements at the receiver for high number of feedback bits.**

I. Introduction

The improved performance of multiple antenna systems can be realized by the availability of perfect channel state information at the transmitter. In FDD systems, however, this assumption is highly unrealistic due to the independent channels between the *downlink* (DL) and the *uplink* (UL). For such systems, a limited feedback channel is dedicated to make the CSI available at the transmitter for instance, as follows: each receiver first estimates its DL channel, normalizes its estimated channel and finally quantizes its normalized estimated channel with a finite number of quantization bits. In that case, the CSI available at the transmitter for precoding is the quantized CDI.

For the quantization of CDI, vector quantization schemes have been investigated where the transmitter and the receiver share a codebook of possible beamforming vectors indexed by a number of bits. An example of which is the *Random Vector Quantization* (RVQ) suggested in [2] such that the codebook consists of unit-norm random vectors distributed uniformly on the unit complex sphere. Unfortunately, the codebook size must increase exponentially with the number of feedback bits. Moreover, since the codebook has no structure, an exhaustive search is usually required. This applies as well to the Lloyd-type VQ scheme suggested in [3].

To avoid the exponential search complexity and the processing delay of VQ schemes, which are two limiting factors for simple receivers, we propose in this paper a simple method for quantizing the CDI by employing a uniform scalar quantizer. It is a simple scalar quantization approach that is applied to the individual real and imaginary parts of each user's normalized channel vector, which are first corrected by the phase of one of its components and then transformed to become independent and bounded. In [4] and [5] a uniform scalar quantizer for MU-MISO systems was also used. The difference with our approach is that on one hand the components (real and imaginary parts) to be quantized are unbounded, and on the other hand, more elements are needed to be fed back to the transmitter. Although VQ techniques result in smaller quantization error values as compared to our SQ scheme, SQ still does not require exponential storage requirements and accordingly has no complexity increase with the increase of feedback bits. Moreover, there is a constant quantization error gap between both schemes for high number of feedback bits. Additionally, we will show that SQ can perform as well as VQ up to a certain constant gap, which is small for moderate number of transmit antennas.

Having quantized its CDI, each receiver relays back on a dedicated feedback channel its quantized CDI with a finite number of feedback bits received erroneously and quasi-instantaneous. The quantized CDI available at the transmitter is needed for transmit processing. In this paper, the linear *Minimum Mean Squared Error* (MMSE) precoder is adopted as transmit processing whereas a simple scalar is applied at each receiver. General expressions for the MMSE precoder and receiver are given, which become specific depending on the quantization method.

The rest of the paper is organized as follows. In Section III, we explain our system model. In Section IV, the channel model is presented and the different quantization schemes are proposed. In Section V, the design of the MMSE precoder based on the quantized CDI is presented. Section VI includes the simulation results and finally conclusions and future work are drawn in Section VII.

Throughout the paper, $\mathrm{E}[\cdot]$ represents the expectation of the argument. The operators $\{\cdot\}^{\mathrm{T}}$, $\{\cdot\}^{*}$ and $\{\cdot\}^{\mathrm{H}}$ stand for the transpose, conjugate, and Hermitian of a complex number respectively. $\mathbf{1}_N$ is a column vector of identity elements of size N and $\boldsymbol{I}_N$ is the identity matrix of size $N \times N$. $\mathrm{tr}(\cdot)$ is the trace of a matrix. $\lfloor \cdot \rfloor$ denotes the floor operator which gives the largest integer smaller than or equal to the argument.

II. System Model

We consider, in a single isolated cell, the DL of MU-MISO system, i.e. a Broadcast channel (BC), where the transmitter (e.g. *Base Station* (BS) in wireless cellular systems) equipped with M transmit antennas and K decentralized receivers (e.g.

Mobile Stations (MSs)) each having a single antenna where we assume that $K \leq M$. The flat fading DL channel matrix is given by $\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2, \ldots, \boldsymbol{h}_K]^{\mathrm{T}} \in \mathbb{C}^{K \times M}$ where $\boldsymbol{h}_k \in \mathbb{C}^M$ is the channel from the BS to user k whose elements are i.i.d. zero-mean complex Gaussian random variables with variance γ_k^2.

Given the input symbols $\boldsymbol{s} \in \mathbb{C}^K$, which are drawn from a QAM modulation scheme alphabet with zero mean and covariance matrix $\boldsymbol{R}_s = \mathrm{E}[\boldsymbol{s}\boldsymbol{s}^{\mathrm{H}}] = \sigma_s^2 \boldsymbol{I}_K$, the estimated received symbols $\hat{\boldsymbol{s}} \in \mathbb{C}^K$ are expressed as:

$$\hat{\boldsymbol{s}} = \boldsymbol{G}(\boldsymbol{H}\boldsymbol{P}\boldsymbol{s} + \boldsymbol{\eta}),$$

where $\boldsymbol{P} = [\boldsymbol{p}_1, \ldots, \boldsymbol{p}_K] \in \mathbb{C}^{M \times K}$ is the linear transmit precoder such that $\boldsymbol{p}_k$ is the precoder for the k-th user, $\boldsymbol{G} \in \mathbb{C}^{K \times K}$ is the receive matrix which is restricted to be a diagonal matrix since the receivers are decentralized with diagonal elements denoted as g_k, and $\boldsymbol{\eta} \in \mathbb{C}^K$ is the additive white Gaussian noise (AWGN) that has zero mean and covariance matrix $\boldsymbol{R}_\eta = E[\boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{H}}] = \sigma_\eta^2 \boldsymbol{I}_K$.

III. Channel Quantization

In this section, we discuss the application of SQ and the VQ of the DL channel $\boldsymbol{H}$. To this end, we introduce our channel model which is similar to that presented in [6].

A. Channel Model

We adopt the same channel model as given in [6], where the entries of $\boldsymbol{h}_k$ are i.i.d complex Gaussian with variance γ_k^2. No detailed explanation is included in this section since in [6], the model is explained extensively.

As given in (48) from [6], the DL channel of user k is:

$$\begin{aligned} \boldsymbol{h}_k &\overset{(a)}{=} \hat{\boldsymbol{h}}_k + \boldsymbol{e}_k \\ &\overset{(b)}{=} b_k \boldsymbol{h}_{n,k} + \boldsymbol{e}_k \\ &\overset{(c)}{=} b_k (c_k \boldsymbol{h}_{q,k} + \boldsymbol{e}_{q,k}) + \boldsymbol{e}_k \\ &= b_k c_k \boldsymbol{h}_{q,k} + b_k \boldsymbol{e}_{q,k} + \boldsymbol{e}_k. \end{aligned} \tag{1}$$

Briefly, (a) is the estimation step such that $\hat{\boldsymbol{h}}_k$ is the estimated channel obtained using a common pilot of length $T_{\mathrm{DL}} \geq M$ emitted from the BS. The variance of the estimation error is $\mathrm{E}\left[\boldsymbol{e}_k \boldsymbol{e}_k^{\mathrm{H}}\right] = \sigma_{e_k}^2 \boldsymbol{I}_K$, where [7]

$$\sigma_{e_k}^2 = \frac{\gamma_k^2}{1 + \gamma_k^2 \frac{P_{\mathrm{DL}}}{M\sigma_\eta^2} T_{\mathrm{DL}}},$$

P_{DL} represents the total transmit power available at the BS. (b) is the normalization step where $\boldsymbol{h}_{n,k}$ is the normalized channel and $b_k = ||\hat{\boldsymbol{h}}_k||_2$ that has the following statistics available at the BS [8]:

$$\mathrm{E}\left[b_k^{-2}\right] = \frac{\gamma_k^{-2}}{(M-1)(1-\sigma_{e_k}^2)}. \tag{2}$$

(c) is the quantization step where $\boldsymbol{h}_{q,k}$ is the quantized channel and $c_k = \boldsymbol{h}_{q,k}^{\mathrm{H}} \boldsymbol{h}_{n,k} \in \mathbb{C}$ is the quantization coefficient.

Vector and scalar quantization methods to obtain $\boldsymbol{h}_{q,k}$ are explained in details in the following subsections. Moreover, the characteristics of both the quantization coefficient c_k and the quantization error $\boldsymbol{e}_{q,k}$ depend on the employed quantization scheme.

We notice here that each receiver k knows its own estimated channel $\hat{\boldsymbol{h}}_k$ while the transmitter knows all quantized channel vectors $\boldsymbol{h}_{q,k}$ which don't include information about the channel norms. In other words, the quantization coefficient c_k and the amplitude b_k are not known at the transmitter. To get rid of these unknown variables at the transmitter, it has been proposed in [6] to take the receivers g_k as follows:

$$g_k = c_k^{-1} b_k^{-1} g, \tag{3}$$

where a common scalar g for all users is introduced as an additional degree of freedom for the precoder optimization.

B. Vector Quantization

Each user k quantizes its normalized channel $\boldsymbol{h}_{n,k}$ with B quantization bits. We consider random vector quantization (RVQ) scheme [2] where each user k has a different codebook $\mathcal{C}_k$, which is also available at the BS, consisting of 2^B unit-norm random vectors $\boldsymbol{t}_{j,k} \in \mathcal{C}_k$ uniformly distributed on the complex unit sphere. For the quantization, the vector with the minimum chordal distance to $\boldsymbol{h}_{n,k}$ is chosen from $\mathcal{C}_k$ as follows:

$$\boldsymbol{h}_{q,k} = \underset{\boldsymbol{t}_{j,k} \in \mathcal{C}_k}{\operatorname{argmax}} |\boldsymbol{t}_{j,k}^{\mathrm{H}} \hat{\boldsymbol{h}}_{n,k}|. \tag{4}$$

In this case, $|c_k| = \cos\theta_k$, θ_k being the angle between $\boldsymbol{h}_{q,k}$ and $\boldsymbol{h}_{n,k}$. We have also that:

$$\mathrm{E}[\cos^{-2}\theta_k] \approx \frac{1}{\mathrm{E}[\cos^2\theta_k]} \overset{[9]}{=} \frac{1}{1 - 2^B \mathrm{Beta}(2^B, \frac{M}{M-1})}, \tag{5}$$

where the approximation becomes tight for high number of feedback bits ($B > 6$). Moreover, $\arg(c_k) \neq 0$ [6] which has to be considered in the design of the receiver. It is also worth mentioning that the quantization error $\boldsymbol{e}_{q,k}$ is orthogonal to $\boldsymbol{h}_{q,k}$, i.e., $\boldsymbol{h}_{q,k}^{\mathrm{H}} \boldsymbol{e}_{q,k} = 0$. The covariance matrix of $\boldsymbol{e}_{q,k}$ as given in [6] Eq. (62) is:

$$\mathrm{E}[\boldsymbol{e}_{q,k}^{\mathrm{H}} \boldsymbol{e}_{q,k} | \boldsymbol{h}_{q,k}] = \frac{\mathrm{E}[\sin^2\theta_k]}{M-1} (\boldsymbol{1}_M - \boldsymbol{h}_{q,k}^* \boldsymbol{h}_{q,k}^{\mathrm{T}}),$$

where

$$\mathrm{E}[\sin^2\theta_k] = 1 - \mathrm{E}[\cos^2\theta_k]. \tag{6}$$

Once user k has quantized its channel vector, it feeds back its quantized channel with B feedback bits on a dedicated feedback channel to the transmitter [1].

C. Scalar Quantization

The SQ scheme that we adopt is a simple uniform scalar quantizer. The operator $Q(.)$ represents the quantization process as follows:

$$Q(\cdot) = \mathrm{sgn}(\cdot) \left(\min\left\{ \left\lfloor \frac{1}{\delta} |(\cdot)| \right\rfloor, \frac{2^b}{2} \right\} - 0.5 \right), \tag{7}$$

where sgn represents the sign operator, δ and b are the step size and the number of quantization bits per real dimension, respectively.

A straightforward method for quantizing the channel vector is as follows. Each receiver k feeds back the non-normalized channel vector $\hat{\boldsymbol{h}}_k$, i.e. the available full-CSI. The real and the imaginary parts of $\hat{\boldsymbol{h}}_k$ are separately quantized with the uniform quantizer given in (7). Since there are $2M$ real dimensions, the per-element number of feedback bits per user denoted as b_{fullCSI} that is needed given a total budget of B bits per user is:

$$b_{\text{fullCSI}} = \frac{B}{2M}. \tag{8}$$

This method was introduced for performance comparison presented later in section V.

Now, we introduce another method that leads to significant performance improvement. Before quantization, the elements of $\boldsymbol{h}_{n,k}$ are first divided by the phase of one of its elements (say its last element) setting thus its imaginary part to zero. Then, each user k quantizes separately the real and the imaginary parts of its phase-rotated normalized channel with the uniform scalar quantizer in (7), i.e., each user aims to quantize the following real-valued vector denoted as $\boldsymbol{h}_{n,k}^{'}$:

$$\boldsymbol{h}_{n,k}^{'} = [h_1' \quad h_2' \quad h_3' \quad h_4' \quad \ldots \quad h_{2M-1}'] \in \mathbb{R}^{2M-1},$$

where the odd and the even indexes represent the phase-rotated real and imaginary elements of $\boldsymbol{h}_{n,k}$, respectively. This vector is of length $(2M-1)$ since the imaginary part of the last element after phase rotation is zero. The elements $h_m', m \in \mathbb{N}$, are bounded in the interval $[-1, +1]$ but they are still mutually dependent since the vector $\boldsymbol{h}_{n,k}^{'}$ has always unit norm.
To solve this issue, we propose that each user k quantizes its channel coefficients in the following sequential way:

$$y_m' = \frac{h_m'}{\sqrt{\sum_m^{2M-1} {h_m'}^2}} = \frac{h_m'}{\sqrt{1 - \sum_{n<m} {h_n'}^2}}, \; m \leq 2M-2, \tag{9}$$

where the second equality follows from the fact that $\sum_{m=1}^{2M-1} {h_m^{'}}^2 = 1$. Due to this fact, each receiver k feeds back only the first $2(M-1)$ elements which are computed back at the transmitter as:

$$y_m = Q\{y_m'\}\sqrt{1 - \sum_{n<m} {y_n}^2}, \; m \leq 2M-2, \tag{10}$$

whereas the $(2M-1)$-th element, i.e. the last element, is automatically recovered as:

$$y_{(2M-1)} = \sqrt{1 - \sum_{n \leq (2M-2)} {y_n}^2}. \tag{11}$$

The quantized channel vector of user k that is available at the transmitter is thus:

$$\boldsymbol{h}_{q,k} = \left[y_1 + iy_2 \quad y_3 + iy_4 \quad \ldots \quad y_{(2M-1)}\right].$$

The per-element number of feedback bits denoted as $b_{\text{Sequential}}$ per user that are needed using this sequential quantization given a total budget of B bits per user is:

$$b_{\text{Sequential}} = \frac{B}{2(M-1)}. \tag{12}$$

Comparing (12) and (8), we deduce that an increase in the quantization resolution by a factor of $M/(M-1)$ is attained. This will be the main reason for performance improvement in terms of BER.

IV. MMSE Filter for Quantized CDI

Based on the quantized CDI and the CMI statistics available at the BS, the optimum MMSE receive and precoding filters are provided in this section for the different employed quantization schemes. For the quantized full-CSI case, please refer to [5].

For the quantized CDI case, these filters were derived in [6] and read as:

$$g = \sqrt{\frac{\text{tr}\left(\left((1-\kappa)\boldsymbol{H}_q^{\text{H}}\boldsymbol{H}_q + \xi\mathbf{I}_M\right)^{-2}\boldsymbol{H}_q^{\text{H}}\boldsymbol{R}_{\boldsymbol{s}}\boldsymbol{H}_q\right)}{P_{\text{DL}}}} \tag{13}$$

$$\boldsymbol{P} = \frac{1}{g}\left((1-\kappa)\boldsymbol{H}_q^{\text{H}}\boldsymbol{H}_q + \xi\mathbf{I}_M\right)^{-1}\boldsymbol{H}_q^{\text{H}}, \tag{14}$$

$\boldsymbol{H}_q = [\boldsymbol{h}_{q,1}, \boldsymbol{h}_{q,2}, \ldots, \boldsymbol{h}_{q,K}]^{\text{T}} \in \mathbb{C}^{K \times M}$ is the quantized channel matrix available at the transmitter.
κ and ξ depend on the employed quantization scheme. These in general have the following expressions:

$$\kappa = \frac{d/(1-d)}{M-1}, \text{ and} \tag{15}$$

$$\xi = K\kappa + \sum_k \frac{(1-d)\gamma_k^{-2}}{(M-1)(1-\sigma_{e_k}^2)}\left(\sigma_{e_k}^2 + \frac{\sigma_\eta^2}{P_{\text{DL}}}\right), \tag{16}$$

where d is the quantization error given by

$$d = \text{E}[\sin^2\theta_k] = 1 - \text{E}\left[\left|\boldsymbol{h}_{q,k}^{\text{H}}\boldsymbol{h}_{n,k}\right|^2\right], \tag{17}$$

depending on the quantization scheme. For RVQ, this quantity is given in (5) and (6) in a closed form, while for the SQ case it has to be evaluated numerically by Monte-Carlo simulations.

V. Simulation Results

For a MU-MISO system with $M = 3$ and $K = 3$, we plot the uncoded BER achieved with linear MMSE precoder based on SQ schemes and the RVQ scheme versus different SNR values in dB. The results are the average of 10000 i.i.d. complex Gaussian channel realizations ($\gamma_k^2 = 1, \forall k$) where 100 QPSK symbols are sent per channel realization. Other system parameters include the DL pilots $T_{\text{DL}} = 100$ and the assumption of uncorrelated input signals and white noise with $\sigma_s^2 = 1$ and $\sigma_\eta^2 = 1$ respectively. The total budget of feedback bits is $B = 12$. In other words, we have $b_{\text{fullCSI}} = 2$ and $b_{\text{Sequential}} = 3$ for the SQs. Obviously, the RVQ scheme outperforms the suggested SQ schemes. However, there is a constant BER gap between the RVQ and the sequential SQ for fixed feedback bits at high SNR values. The large BER gap between the full CSI based SQ and the CDI based sequential SQ suggests that amplitude information needs not to be fed back but its statistics are sufficient for good precoder performance as already mentioned in subsection A of Section III. This was already concluded in [8]. Each receiver thus needs only to feed back $2(M-1)$ elements saving a pair of real dimensions. Evidently, the BER floor at high SNR values is due to the limited feedback. In Fig. 2, we compare

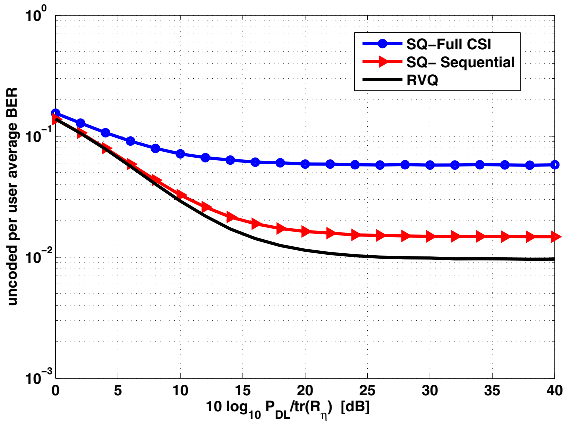

Fig. 1. QPSK: uncoded BER vrs. $10\log_{10}\frac{P_{\mathrm{DL}}}{\mathrm{tr}(\boldsymbol{R}_{\eta})}$ for Quantized CDI with different Quantization schemes.

the performance of our SQ-sequential method and the SQ method proposed in [10], which needs the same number of bits to quantize separately the phase and the amplitude of the normalized channel. This method also saves a pair of real dimensions by using an appropriate normalization. The results are in terms of BERs for $M = K = 4$ and $b = 8$ for both methods. Other system parameters are the same as before. Results show that our scheme outperforms the aforementioned method in the high SNR regime. The quantization error values

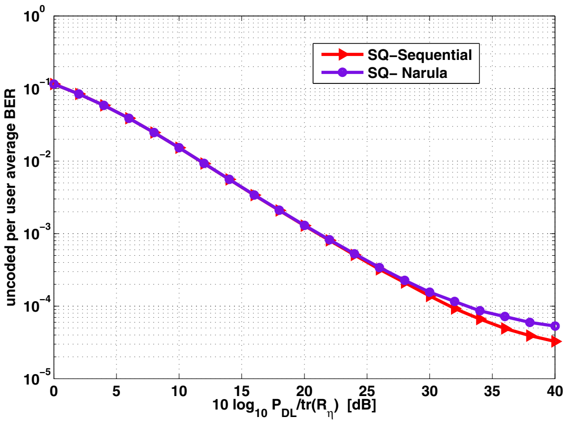

Fig. 2. QPSK: uncoded BER vrs. $10\log_{10}\frac{P_{\mathrm{DL}}}{\mathrm{tr}(\boldsymbol{R}_{\eta})}$ for Quantized CDI with different SQ schemes.

in (17) for the RVQ scheme, the SQ-Sequential method and the SQ suggested in [10] vrs. different numbers of bits per dimension are plotted in Fig. 3 for $M = 4$. As expected from the above results of the BER, the quantization error associated with the RVQ scheme is the least whereas that associated with the SQ of [10] is the worst. One can also notice the constant gap between our suggested SQ-sequential method and that for RVQ for different values of $b_{\text{Sequential}}$, which means that the performance loss as compared to VQ remains constant even in the high resolution regime.

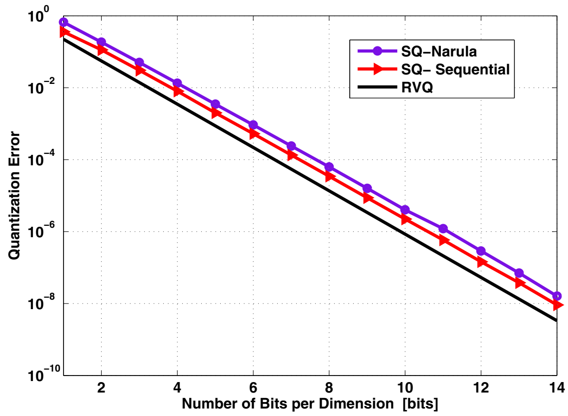

Fig. 3. Quantization Error for the different Quantization schemes.

VI. Conclusions and Future Work

For the quantization of the CDI in MU-MISO systems, we proposed a simple uniform scalar quantizer that has reduced complexity and memory requirements as compared to the RVQ scheme commonly used for these systems. These two advantages are specially essential for simple receivers such as mobile stations in cellular wireless systems. The sequential method for SQ, where only a total of $2(M-1)$ elements are needed to be fed back by each user performs quite close to VQ. Future work includes the application of scalar quantizers for the limited feedback of MU-MIMO systems.

References

[1] D. J. Love, R. W. Health Jr., W. Santipach, and M. L. Honig, "What is the Value of Limited Feedback for MIMO Channels?", in *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 54-59, Oct. 2004.

[2] C. K. Au-Yeung and D. J. Love, "On the Performance of Random Vector Quantization Limited Feedback Beamforming in a MISO System", in *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, Feb. 2007.

[3] J. C. Roh, B. D. Rao, "Transmit Beamforming in Multiple-Antenna Systems With Finite Rate Feedback: A VQ-Based Approach ", in *IEEE Trans. on Information Theory*, vol. 52, no. 3, March 2006.

[4] P. M. Castro, M. Joham, L. Castedo, and W. Utschick, "Robust Precoding for Multi-User MISO Systems with Limited-Feedback Channels", in *International ITG Workshop in Smart Antennas*, Feb. 2007.

[5] P. M. Castro, M. Joham, L. Castedo, and W. Utschick, "Optimized CSI Feedback for Robust THP Design", in *41st Asilomar Conference on Signals, Systems and Computers (ACSSC)*, Nov. 2007.

[6] M. Castañeda, I. Slim, A. Mezghani and J. A. Nossek, "Transceiver Design in Multiuser MISO Systems with Imperfect Transmit CSI", in *ITG Workshop in Smart Antennas (WSA)*, Feb. 2010.

[7] B. Hassibi and B. M. Hochwald, "How much Training is Needed in a Multiple-Antenna Wireless Link?", in *IEEE Trans. on Information Theory*, vol. 49, pp. 951-964, Apr. 2003.

[8] M. Castañeda, I. Slim and J. A. Nossek, "Transceiver Design in Multiuser MISO Systems with Limited Feedback", accepted to *IEEE Global Communications Conference (Globecom)*, Dec. 2010.

[9] N. Jindal, "MIMO Broadcast Channels with Finite Rate Feedback", in *IEEE Trans. on Information Theroy*, vol. 52, no. 11, Nov. 2006.

[10] A. Narula, M. J. Lopez, M. D. Trott, and G. W. Wornell, "Efficient Use of Side Information in Multiple-Antenna Data Transmission over Fading Channels", in *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, Oct. 1998.