Abstract—When mobile sensors are initially deployed, some sensors may be far away from the region of interest and due to the sensor’s limited sensing of range, some sensors may not be able to participate in the coverage task. This paper proposes a new algorithm on the coverage problem for mobile sensor networks which guarantees all sensors to participate in the coverage task. The algorithm is a combination of the standard gradient-based coverage algorithm and leader-following algorithm and is designed to maximize the joint detection probabilities of the events in the region of interest. First, leader sensors are selected based on the information which each sensor has gathered. The rest of the sensors will follow the leaders until they have sufficient information on the region of interest and then switch to the standard coverage algorithm. The proposed algorithm can be performed in a distributed manner. Moreover, the proposed algorithm could also improve the convergence speed of the coverage task. The results are validated through numerical simulations.

I. INTRODUCTION

Stimulated by the technological advances and the development of relatively inexpensive communication, computation, and sensing devices, the interest in the research area of coordinated networked control has majorly increased over the past years, see [1] for an excellent overview. One example is the deployment of autonomous vehicles to perform challenging tasks such as search and recovery operations, manipulation in hazardous environments, surveillance and also environmental monitoring for pollution detection and estimation. Deploying multiple agents to perform tasks is advantageous compared to the single agent case: It provides robustness to agent failure and allows to handle more complex tasks.

In this paper, we consider a mobile sensing network of vehicles equipped with sensors to sample the environment. The goal is to drive the sensors/agents to the position such that a given region is optimally covered by the sensors. In general there are three main approaches to solve the coverage control problem namely geometric, probabilistic, and potential field approach. The geometric strategy is based on the Voronoi partition and the continuous version of Lloyd algorithm, see e.g. [2]. Briefly speaking, the agents partition the given region into subregions given by Voronoi partitions and move towards the centroid of its subregion and adjust its sensing radius until all the area is covered. The advantage of the Voronoi approach is that the control law is distributed by its nature. Moreover, a lot of variations on the coverage control problem can be addressed and solved in a similar fashion by associating different Voronoi partition. Power-aware coverage algorithms for mobile networks are proposed in [3] in order to balance the energy expenditure across the network and make nodes with high power compensate for nodes with low power by incorporating partition defined by power metric. Pimenta et al. [4] used the so-called power diagram to deal with coverage control problem with heterogeneous robots, i.e., robots with different footprints.

The probabilistic based strategy is introduced in [5] where the authors consider a probabilistic network model and a density function to represent the frequency of random events taking place over the mission space. The authors develop an optimization problem that aims at maximizing coverage using sensors with limited ranges, while minimizing communication cost. A potential-field-based approach to deployment problem in an unknown environment is presented in [6]. An algorithm based on similar approach is proposed in [7] that maximizes the area coverage of a network while satisfying the constraint that every node has at least $K$ neighbors. Moreover, coverage control problem based on receding horizon control is considered in [8].

Coverage control problem considering a more realistic sensor model is considered in [9] by introducing "limited-range interactions" of the sensors, i.e., the sensing range of the sensor is restricted to a bounded region. Furthermore, the coverage algorithm for robotic visual sensor networks is proposed in [10], [11] where the sensor has a limited view angle and its sensing performance also depends on its attitude.

When the mobile sensors are initially deployed in the unknown environment, some sensors may be located far away from the region of interest. Moreover, due to the limited sensing range of the sensors, those sensors may not be able to participate in the coverage task. The main goal of this paper is to develop a new algorithm for coverage control of mobile sensor networks that guarantees the participation of all sensors in the coverage task, even if some sensors do not sense any event in the initial deployment. The idea is to combine the standard coverage algorithm with the leader-following algorithm where the leader(s), i.e., sensor(s) who has sufficient information about the environment, is selected using a voting mechanism. The sensors acting as leaders will then guide the sensors which do not have information on the environment until they gain sufficient information. Moreover, the proposed algorithm could also improve the convergence speed of the sensors. Our work in this paper
is based on the coverage algorithm developed in [5] since it can be generalized to incorporate visual sensor [10] and non-convex environment [12]. However, the proposed approach can also be applied to the other coverage algorithm, e.g., [2].

This paper is organized as follows: The problem formulation for the coverage problem is presented in Section II. The new distributed coverage algorithm is proposed in Section III. The effectiveness of the proposed algorithm is validated through numerical simulations in Section IV.

II. PROBLEM FORMULATION

A. Region of interest

Let \( Q \) be a polyhedron in \( \mathbb{R}^2 \) including its interior. The density function \( \phi(q) : Q \rightarrow \mathbb{R}_+ \) represents the probability that some event take place in \( Q \). Regions with a large value of \( \phi \) are regions of higher chances of finding a point of interest. The density function \( \phi(q) \) satisfies \( \phi(q) \geq 0 \) for all \( q \in Q \) and \( \int_{Q} \phi(q) < \infty \). In general, the region of interest \( Q \) may be very large so that there exists some areas where the value of \( \phi(q) \) approaches 0.

B. Sensor model

We consider a robotic network where each robot is equipped with limited-range omnidirectional communication and isotropic sensing capabilities. For the sake of simplicity, it is assumed that all sensors are identical, i.e., all sensors have identical capabilities for sensing, communication, computation, and mobility. Let \( s = (s_1, \ldots, s_N) \) be the location of the \( N \) identical robots/sensors moving in the region \( Q \). The kinematic model of the sensors are given by

\[
s_i(k + 1) = s_i(k) + u_i(k),
\]

where \( k \) is the iteration index, \( u_i(k) \) is the control input for the position of sensor \( i \).

The sensor is assumed to have a limited sensing range defined as follows:

Sensor Model. Each sensor has a limited sensory domain \( Q_i \) with the maximum sensing range \( R \) given by

\[
Q_i = \{q \in Q : d_i \leq R\},
\]

where \( d_i = \|q - s_i\| \).

When an event occurs at point \( q \), it emits a signal and this signal is observed by sensor \( i \) at location \( s_i \). The received signal strength, i.e., the sensing performance of the sensor is assumed to be decayed with the distance from the sensor. The degradation of the sensor performance is represented by a monotonically decreasing differentiable function \( p_i(q) \) which expresses the probability that sensor \( i \) detects the event occurring at \( q \) or indicates how poor the sensing performance is. Lower value of \( p_i(q) \) means that the point \( q \) is sensed poorly by sensor \( i \) and vice versa. Moreover, we make the following assumption on the sensing performance of the sensor.

Assumption 1.

\[
p_i(q) = 0, \quad \frac{\partial p_i(q)}{\partial d_i(q)} = 0 \quad \text{if} \ q \notin Q_i.
\]

The assumption tells us that the sensor \( i \) can only sense the point inside its region of sensing \( Q_i \). An example of the sensing performance is given for example by

\[
p_i(q) = \begin{cases} (\frac{d_i - R}{R})^2 & \text{if} \ q \in Q_i, \\ 0 & \text{otherwise} \end{cases}
\]

Next we define the information sensed by each sensor, i.e., \( I_i \) given as follows

Definition 1. The information sensed by sensor \( i \) is defined as the total received signal strength inside its sensory domain \( Q_i \). Mathematically, it is represented as follows

\[
I_i(q) = \int_{Q_i} \phi(q)p_i(q) dq.
\]

C. Optimal coverage formulation

The optimal coverage is achieved by deploying the sensors into the region of interest so that the probability of events are detected is maximized. In this paper, agents are assumed to make observations independently. When an event at \( q \) is observed by the sensors, the joint probability that this event is detected can be written as

\[
P(q, s) = 1 - \prod_{i=1}^{N} [1 - p_i(q)].
\]

Then, the optimal coverage problem can be formulated as an optimization problem which maximizes the objective function defined as

\[
F(s) = \int_{Q} \phi(q)P(q, s) dq
\]

which is the expected event detection probability by the sensors over the region of interest.

D. Distributed coverage algorithm

The control input for the kinematics of agent \( i \) is based on a gradient-based approach given as follows [5]:

\[
u_i^{\text{cov}}(k) = \beta_k \frac{\partial F}{\partial s_i}
\]

where \( \beta_k \) is a step size that should be properly chosen and \( \frac{\partial F}{\partial s_i} \) is defined as follows

\[
\frac{\partial F}{\partial s_i} = \int_{Q} \phi(q) \frac{\partial P(q, s)}{\partial s_i} dq,
\]

\[
= \int_{Q_i} \phi(q) \prod_{k \in N_i} [1 - p_k(q)] \frac{\partial p_i}{\partial d_i} \frac{\partial d_i}{\partial s_i} dq,
\]

where \( N_i \) is the neighbors of sensor \( i \) and defined as \( N_i = \{k : |s_i - s_k| \leq 2R\} \). The algorithm is distributed and drives the sensors into the region of interest. However, since it is based on a gradient-based approach and due to the limited sensing range of the sensor, there exists a condition at
the initial deployment where the information gained by some agents are close to zero, i.e., $I_i(q) \sim 0$ which results in that the control input of the sensor $u_i^\text{cov}(k) \sim 0$, i.e., the sensor could not participate in the coverage task. Formally, we call such a sensor as an isolated sensor defined as follows.

**Definition 2.** Sensor $i$ is called an isolated sensor if it collects a very small information so that it has no ability to move, i.e., $I_i \sim 0$.

Moreover, since algorithm (9) is based on a gradient-based approach, it is required some amount of time for the sensors to achieve the coverage goal.

The goal of the paper is to develop a new algorithm that guarantees the participation of all sensors in the coverage control, i.e., there will be no isolated sensors exist in the final configuration of the sensors. As an additional goal, the algorithm should also improve the convergence speed of the sensors in achieving the coverage.

**III. COVERAGE CONTROL ALGORITHM**

In this section, a new distributed algorithm is proposed combining the standard coverage algorithm and leader-following algorithm. Since the isolated sensors have relatively no information, i.e., they do not sense any events, they cannot participate in the coverage task. Thus, they need to be guided to move into the region of interest to perform the coverage task. However, no external supervisor is allowed in order to keep the algorithm to be distributed. In this paper, as a strategy, first a virtual supervisor called as leader is assigned between the sensors. The rest of the sensors then act as followers and will follow the leader(s) based on the leader-following algorithm until they gain sufficient information.

### A. Leader election

First, we consider the case where only one region of interest exists. The leader is defined as follows.

**Definition 3.** Leader sensor $l$ is a sensor which has the most information on the region of interest. Mathematically,

$$l = \arg \max_i I_i.$$  \hspace{1cm} (10)

Next, we present a simple algorithm based on a voting algorithm to assign the leader in a distributed manner. It is assumed that the sensors could receive the position of its neighbors and also information sensed by its neighbors by using the communication network. Moreover, the communication graph between the sensors is assumed as follows.

**Assumption 2.** The communication graph is static and connected.

Assumption 2 seems to be restrictive since the graph is required to be static all over the time. In the future, the case with possibly time-varying graph will be considered. The leader election algorithm is described as follows. First, each agent needs to collect its sensed information $I_i(q)$ and compute $\mu = \frac{1}{N} \sum_{i=1}^{N} I_i(q)$ which is the average of the information sensed by all sensors using, e.g., the consensus algorithm. The goal of a leader voting algorithm is to choose the sensor with the best election value ($I_i$) and broadcast the result all over the network based on a simple implementation of broadcast algorithm called flooding [13] using the multihop communication capability under assumption 2. The algorithm is initialized by agent $i$ sending a message $I_i$ to each of its neighbors. Each agent $j$ receiving one or more such messages compares the best received election value to its currently stored one. If the received value is better, its own value is deleted and the new received value is stored. The message is then forwarded to all of its neighbors except to the sender of the best value. If the received value is worse, then the agent sends its own election value to all its neighbors.

In the end, a leader or leaders will be elected. Since the sensing performance of each sensor never decreases due to the property of the gradient approach, a sensor selected as a leader will always become a leader.

### B. Leader-following algorithm

Next, we review the leader-following algorithm for a multi-leaders case [14]. In this paper we define $f$ to denote the followers and $l$ for the leaders. As the leader(s) are elected, the incidence matrix $B$ among agents can be partitioned as

$$B = \begin{bmatrix} B_f \\ B_l \end{bmatrix},$$

where $B_f \in \mathbb{R}^{n_f \times r}$, and $B_l \in \mathbb{R}^{n_l \times r}$. Here $n_f, n_l$ and $r$ are the cardinalities of the follower group, the leader group and the edge set respectively. As a result, a Graph Laplacian $L$ of the sensors is defined by

$$L = \begin{bmatrix} L_f \\ m \end{bmatrix},$$

(11)

where $L_f = B_f B_l^T$, $\eta = B_l B_l^T$, and $m = B_f B_l^T$.

The leader-following algorithm is then defined as follows

$$u_l^f(k) = L_f s_f(k) - m s_l(k),$$  \hspace{1cm} (12)

where $s_f, s_l$ are the group of followers and leaders respectively. Under assumption 2, $L_f > 0$, i.e., the followers will approach either the leader or the convex hull spanned by the leaders when the leaders move in a slow speed.

### C. Proposed distributed algorithm

In this subsection we propose a new control law to improve algorithm (9). First we introduce the following assumption.

**Assumption 3.** There exists at least one non-isolated sensor, i.e., $\exists i, I_i \neq 0$.

The proposed distributed algorithm is then given as follows.

$$u_l = u_l^\text{cov},$$  \hspace{1cm} (13)

$$u_{i,i \neq l} = u_{i,i \neq l}^\text{cov} + \alpha_i u_i^f,$$  \hspace{1cm} (14)

where $\alpha_i$ is a monotonic decreasing function w.r.t. the sensed information $I_i$. For example, $\alpha_i$ can be defined as $\alpha_i = \exp(-k_\alpha(I_i))$ where $k_\alpha$ determines how fast the
function declines. Moreover, the value of $\alpha_i$ is switched to 0, i.e., $\alpha_i(I_i) = 0$ if $I_i > \gamma_i \mu$, where $0 < \gamma_i \leq 1$ is a tuning parameter. Physically, it means that when a sensor has gathered sufficient information, then it will stop following the leader and only performs the coverage. Thus, by implementing the control law (13), (14), the participation of all sensors in the coverage task is guaranteed.

**Proposition 1.** Consider mobile sensors whose kinematic given by (1) and the control input is given by (13), (14). Thus under assumption 1-3, there will be no isolated sensors, i.e., all sensors participate in the coverage and the convergence speed is improved.

**Proof.** From assumption 3, there will always be at least a leader in the network. Moreover, since the graph is connected, then by implementing the leader-following algorithm, all the followers will approach the leader until they have a sufficient information. Furthermore, from (14), the speed of the followers are the sum of the control input of the coverage algorithm (9) and the leader-following algorithm (12), which depends on the communication topology between the sensors. Thus the convergence speed of (14) is faster than (9).

**Remark 1.** The definition of the neighbors of sensor $i$ for coverage algorithm (13), (14) is different with the one in (9). The neighbors of sensor $i$ is re-defined as $N_i = \{k : |s_i - s_k| \leq D\}$ where $D \geq 2R$ is chosen such that assumption 2 is satisfied.

Formally, the proposed algorithm can be written as algorithm 1.

**Algorithm 1 Proposed distributed algorithm**

```
loop
    for $i = 0$ to $N$ do
        Require: $I_i(q)$
        Ensure: $\mu = \sum_{i=1}^{N} I_i(q) / N$
        if $k = 0$ then
            if $I_i \neq \arg \max_k I_i$ then
                $I_i \leftarrow I_i$ (leader assignment)
                $\alpha_i \leftarrow 0$
                $u_i \leftarrow u_i^{\text{cov}}(k)$
            else
                $\alpha_i \leftarrow \exp(-k_i(I_i))$
                $u_{i,j} \leftarrow u_i^{\text{cov}} + \alpha_i u_i^{\text{g}}$
            end if
        else if $I_i > \gamma_i \mu$ then
            $I_i \leftarrow I_i$ (gain switching)
            $\alpha_i \leftarrow 0$
            $u_i \leftarrow u_i^{\text{cov}}(k)$
        else
            $\alpha_i \leftarrow \exp(-k_i(I_i))$
            $u_{i,j} \leftarrow u_i^{\text{cov}} + \alpha_i u_i^{\text{g}}$
        end if
        $k \leftarrow k + 1$
    end for
end loop
```

Next we consider a more general scenario with multiple regions of interest. The leaders are defined as follow:

**Definition 4.** Leaders $l$ are sensors with $I_l(q) > \mu$.

**Algorithm 2 Leaders voting algorithm for multi-leaders case**

- **Require:** $N$
  - $i \leftarrow 1$; $L_d \leftarrow 0$; $n \leftarrow 1$; $I_t \leftarrow 0$ (Initialization)
- **Require:** $I_i(q)$
  - Store[$i$] $\leftarrow I_i$; $I_t \leftarrow I_t$
- **Ensure:** $\mu = \mu_i$
  - for $j = 1$ to $N, j \neq i$, do
    - Sent Store[$i$] to Agent $j$
  end if
  - Store[$i$] $\leftarrow I_i$; $n \leftarrow n + 1$; $I_t \leftarrow I_t + I_j$
  - (Leader Assignment)
  - if $n = N$ then
    - if $\mu \leq \text{Store}[i]$ then
      - $L_d \leftarrow L_d + 1$; Store$L_d[i]$ $\leftarrow \text{Store}[i]$
    end if
  - if $\mu \leq \text{Store}[j]$ then
    - $L_d \leftarrow L_d + 1$; Store$L_d[j]$ $\leftarrow \text{Store}[j]$
  end if
  - Go to END
else
  - for $k = 1$ to $N, k \neq i, k \neq j$, do
    - Sent Store[$i$] and Store[$j$] to Agent $k$
    - $\text{Store}[k] \leftarrow I_k$; $n \leftarrow n + 1$; $I_t \leftarrow I_t + I_k$
  - (Leader Assignment)
  - if $n = N$ then
    - if $\mu \leq \text{Store}[i]$ then
      - $L_d \leftarrow L_d + 1$; Store$L_d[i]$ $\leftarrow \text{Store}[i]$
    end if
  - if $\mu \leq \text{Store}[j]$ then
    - $L_d \leftarrow L_d + 1$; Store$L_d[j]$ $\leftarrow \text{Store}[j]$
  end if
  - Go to END
else
  - .
  - .
  - end if
end for
end if
```

A similar method to single leader case can also be used to select the leaders, as shown in algorithm 2. Moreover, it is not only desired that each sensor participates in the coverage, but each region of interest should also be covered by at least one sensor. In order to achieve this additional goal, we introduce the following assumption.

**Assumption 4.** There exists at least one agent near each region of interest.
This assumption may not always be satisfied by the sensors at the initial deployment. In this case, the sensors could first do the exploration task using, e.g., [2] and then switch into the coverage task as discussed in [15]. Thus, by applying the control law (13), (14) and using the same argument as Proposition 1 together with assumption 4, all sensors will participate in the coverage and each region of interest will be sensed by at least one sensor.

**IV. SIMULATIONS**

In this section, the proposed algorithms will be evaluated through numerical simulations. The environment is a rectangle of size 800 x 1200 (meter). It is assumed that there are 5 sensors with sensing radius $R = 100$.

### A. Single region of interest

The density function is $\phi(q) = 55 - 0.1||q - p_i||$ with $p_1 = [400,600]$. Three sensors are isolated from the region of interest where their positions are equal to $s_1 = [130,50]$; $s_2 = [80,50]$; $s_3 = [80,100]$ while the rest are located relatively close to the target, $p_4 = [300,700]$; $p_5 = [300,400]$. As shown in Fig. 4(a), using the standard coverage algorithm (9), sensor 2 can not participate in the coverage task. Next, we apply the proposed algorithm (13), (14) where $k_o = 1 \times 10^{-6}$ and $\alpha_i = \exp(-k_o(I_i))$. Using the leader election algorithm mentioned in subsection III-A, sensor 4 is selected as the leader. As can be observed from Fig. 2, all sensors participate in the coverage task which results in a higher coverage indicated by a higher value of the objective function as shown in Fig. 1(a). Moreover, the coverage is achieved in shorter amount of time compared to the standard algorithm. The trajectories and the information gathered by each sensor are shown in Fig. 3.

### B. Multiple regions of interest

Next we evaluate the algorithm in the case of multiple region of interest which is modeled as the sum of density function $\phi_i(q) = 55 - 0.1||q - p_i||$ with $p_1 = [200,1000]$ and $p_2 = [600,800]$. Three sensors are isolated from the region of interest at the initial deployment where their positions are equal to $s_1 = [130,50]$; $s_2 = [80,300]$; $s_3 = [80,130]$, while the rest are located relatively close to targets, $p_4 = [200,700]$; $p_5 = [500,300]$. Using algorithm (9), only sensor 4 and 5 participate in the coverage task as shown in Fig. 4(b). Next we apply the proposed algorithm with $\exp(-k_o(I_i))$ and $k_o = 1 \times 10^{-6}$. Using algorithm 2, sensor 4 and sensor 5 are selected as leaders. The followers approach the convex hull spanned by the leaders as shown in Fig. 5(b) until they gather enough information and then switch into the standard coverage algorithm. As can be observed from Fig. 5(d), all sensors participate in the coverage task and each region of interest is sensed by at least one sensor. Moreover, as shown...
in Fig. 1(b), the objective function is higher than the standard algorithm. The trajectories and the information gathered by each sensor are shown in Fig. 3.

V. CONCLUSION

When mobile sensors are initially deployed, some sensors may be located far away from the region of interest and due to the sensor’s limited sensing of range, some sensors may not be able to participate in the coverage task. In this paper, a new distributed algorithm on the coverage problem for mobile sensor networks which guarantees all sensors to participate in the coverage task is proposed. In the proposed algorithm, the agents do not only exchange their position information but also the information that they have gathered. In addition, the proposed algorithm could also improve the convergence speed of the coverage task. Numerical simulations demonstrate the efficiency of the proposed algorithm. Future work will deal with time-varying communication graph and consider communication cost into the problem.

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