THE WHY AND HOW OF MULTIANTENNA SYSTEMS

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ABSTRACT

Bandwidth efficiency is of great importance for modern wireless communication systems. Increase of bandwidth efficiency normally sacrifices power efficiency, which is also important. Multiantenna systems offer the potential to improve both at the expense of utilizing multiple antennas preferably at both ends of a transmission link. We are reviewing generic performance measures for such systems in single link scenarios based on long term (LT) channel state information at the transmitter (TCSI). Extending the considerations to multiuser single cell scenarios, instantaneous TCSI will be necessary to fully exploit what multiple antennas can offer. Here we consider the problem of how to partition resources optimally to accommodate feedback of CSI from receiver to transmitter. Further extension to cellular multiuser multiantenna systems confronts us with the problem of intercell interference (ICI) which due to scheduling is a nonstationary random process. There is a huge improvement in cell sum rate to be gained, if we can make the ICI power predictable. We focus only on reviewing generic results telling us what can be achieved with multiple antennas but will not go into detailed realizations thereof.

1. INTRODUCTION

One benefit of multichannel systems is that the bandwidth efficiency can be increased without decreasing transmit power efficiency. Throughput can be increased without using more transmit power or less transmit power is needed for the same throughput compared to a single channel system. Such wireless multichannel systems can be realized with multiantenna systems. Dedicated signal processing and coding schemes can be employed to create an information channel with the desired properties to meet the challenges of future generation wireless communication systems. Antenna gain, multiplexing gain and diversity gain are introduced to measure the performance of multiantenna systems with respect to different goals: increasing the receive power, increasing the slope of information rate versus SNR and decreasing the receive power fluctuations caused by the fading channels.

In single user scenarios (multiuser scenarios making use of TDMA or FDMA as multiple access scheme can be treated as single user scenarios) linear signal processing schemes can achieve link capacity. Having only long term channel state information at the transmitter (LT-TCSI) is almost as good as instantaneous TCSI. Therefore, the performance measures are derived based on LT-TCSI. The fundamental trade-off among the three figures of merit is demonstrated.

In multiuser scenarios with spatial multiple access nonlinear techniques are necessary to approach capacity. These nonlinear techniques (dirty paper coding or the suboptimal Tomlinson-Harashima-Precoding) need instantaneous TCSI. This is much more demanding in terms of feedback capacity needed. Therefore, optimum resource partitioning for payload data and feedback is an important problem. A first step towards the solution of this partitioning problem is shown for a bidirectional link with multiple antennas on one end of the link.

Multicell scenarios add another layer of complexity: we are faced with multiuser interference, which is not under control of the base station, which is serving multiple users, but comes from the neighboring cells. This interference power is subject to scheduling decisions of the neighboring base stations, which cannot be coordinated by *e.g.* successive encoding. Preliminary results show that there are huge improvements possible if we can make the intercell interference predictable, which is an attractive challenge for future research.

2. LIMITATIONS OF SINGLE ANTENNA SYSTEMS

The capacity of single antenna systems in bits per channel use is given by

$$C = \log_2\left(1 + \frac{|h|^2 P_{\mathrm{T}}}{B N_0}\right) = \log_2(1 + \mathrm{SNR}),$$

where h is the complex channel gain, N_0 the noise power density and B the noise bandwidth. In the case of Nyquist pulse shaping split equally between transmitter and receiver the spacing of channel uses is $T = 1/B$. Only in the very low SNR-regime, the capacity is growing linearly with the transmit power P_{T}.

The bandwidth efficiency η_{B} of such a system is given as the rate, at which information in bits per second can be

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transferred in each Hz of bandwidth $B_{\mathrm{S}} = (1+\rho)B$ occupied by the signal with ρ being the roll-off factor:

$$\eta_{\mathrm{B}} = \frac{B \log_2(1 + \mathrm{SNR})}{(1+\rho)B} \approx \Big|_{\rho \ll 1} \log_2(1 + \mathrm{SNR}) = C. \tag{1}$$

Obviously, the larger η_{B}, the better the bandwidth is being utilized. The transmit power efficiency η_{P} measures the relative utilization of the SNR for achieving the maximum possible capacity:

$$\eta_{\mathrm{P}} = \frac{C}{\mathrm{SNR}} = \frac{\log_2(1 + \mathrm{SNR})}{\mathrm{SNR}} = \frac{\eta_{\mathrm{B}}}{2^{\eta_{\mathrm{B}}} - 1}. \tag{2}$$

The larger η_{P} is, the better the transmit power is utilized. Power efficiency and bandwidth efficiency are conflicting goals as is visualized in Fig. 1(a). Power efficiency is optimized in the low SNR-regime ($\lim_{\mathrm{SNR} \to 0} \eta_{\mathrm{P}} = \eta_{\mathrm{P,max}} = 1/\ln 2$) while the bandwidth efficiency is optimized in the high SNR-regime ($\lim_{\mathrm{SNR} \to 0} \eta_{\mathrm{B}} = \eta_{\mathrm{B,max}} = \infty$). An economic compromise is achieved maximizing the product of both:

$$\mathrm{SNR}_{\mathrm{eco}} = \underset{\mathrm{SNR}}{\mathrm{argmax}}\, \eta_{\mathrm{B}} \eta_{\mathrm{P}}.$$

This problem has a unique solution and tells that a power economic single antenna system provides an information rate of no more than $\eta_{\mathrm{B}}^{\mathrm{eco}} = 2.3$ bits per second for each Hz of bandwidth operating at an $\mathrm{SNR}_{\mathrm{eco}} \approx 4$ ($\overset{\triangle}{=}$ 6 dB) as shown in Fig. 1(b).

Although this concept of economic operation is not a strict rule, it nevertheless sets the stage, where the operating point of a single antenna system should approximately be. If the required throughput of a system significantly exceeds $\eta_{\mathrm{B}}^{\mathrm{eco}}$, then the use of multiantenna systems should be considered.
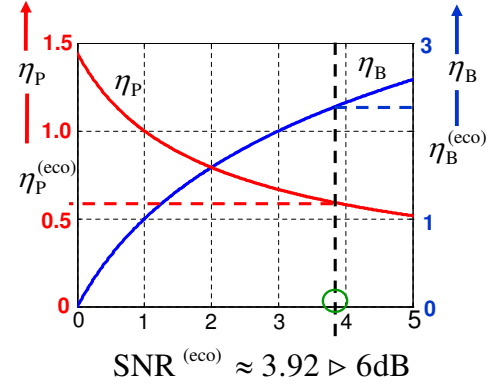
3. MULTICHANNEL SYSTEMS

The limitation of single channel systems in increasing bandwidth efficiency and power efficiency simultaneously can be overcome by the use of several parallel channels. By maintaining a given transmit power efficiency, the bandwidth efficiency can be increased by the number of channels.

By comparing a single channel system and an L-channel system on the basis of achieving the same capacity C, we see that the total transmit power can be greatly reduced through the use of multiple channels
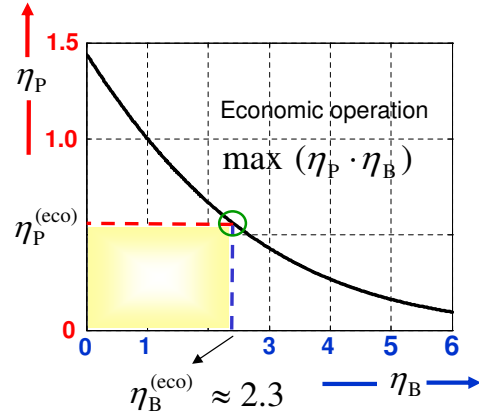
$$\eta = \frac{P_{\mathrm{T},L}}{P_{\mathrm{T},1}} = L \cdot \frac{2^{C/L} - 1}{2^C - 1}.$$

This transmit power reduction η is visualized in Fig. 2. As expected, as soon as the number of channels is large enough that the required capacity does not exceed the economic bandwidth efficiency

$$C \leq \eta_{\mathrm{B},L}^{\mathrm{eco}} = L \cdot 2.3 \quad \text{bits/sec/Hz,}$$

$$\mathrm{SNR}^{(\mathrm{eco})} \approx 3.92 \rhd 6\mathrm{dB}$$

(a)

$$\eta_{\mathrm{B}}^{(\mathrm{eco})} \approx 2.3$$

(b)

Fig. 1. Economic operation: (a) Bandwidth efficiency η_{B} and transmit power efficiency η_{P} versus SNR, (b) transmit power efficiency η_{P} versus bandwidth efficiency η_{B}

the main reduction in total transmit power has been achieved. Any further increase in the number of channels does not improve transmit power efficiency substantially.
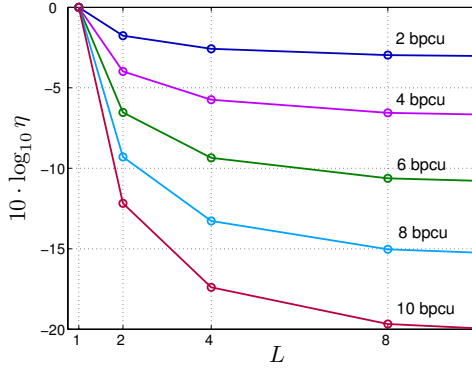

Fig. 2. Possible total transmit power reduction versus number of utilized channels L

It has to be noted, that it has been assumed that all L channels have the same channel gain. Multichannel systems which are realized on the basis of multiantenna systems with appropriate cooperative transmit and receive processing will not really meet that assumption. But the trend depicted in Fig. 2 still holds.

4. SINGLE USER MULTIANTENNA SYSTEMS

Systems with multiple antennas on both ends of the link, *i.e.* multiple-input multiple-output (MIMO) systems offer the potential for a substantial increase of both bandwidth and power efficiency.

It is well known [1] that the capacity of a MIMO system can grow linearly with the number of transmit or receive antennas, whichever is smaller. The capacity of such a MIMO system can be achieved with linear signal processing. Assuming perfect instantaneous channel knowledge at both ends of the link, both the receiver and the transmitter can apply the left and the right singular vectors respectively to diagonalize and decompose the MIMO link into L decoupled, parallel SISO links. The estimate $\hat{s}$ of a signal vector s transmitted over a flat fading MIMO channel can be described by

$$\hat{s} = BHTP^{1/2}s + Bn = y + n', \qquad (3)$$

where $P^{1/2} = \mathrm{diag}\{\sqrt{P_i}\}_{i=1}^{L}$ with $\mathrm{tr}(P) = P_{\mathrm{T}}$ describes the distribution of the total transmit power to the L individual streams, T is the linear mapping from the L streams to the N transmit antennas, H is the MIMO channel mapping the N-dimensional transmit vector x onto the M-dimensional receive vector y, which is corrupted by additive noise n and then mapped to $\hat{s}$, the estimate for the L-dimensional data signal s, with the aid of the linear transform B. The number of

streams has to be smaller or equal to the rank of the channel matrix $L \leq \mathrm{rank}(H) \leq \min(M, N)$.

With the SVD of $H = U\Sigma V^{\mathrm{H}}$ and using $T = V$ and $B = U^{\mathrm{H}}$, the MIMO channel matrix is diagonalized and water filling can be applied to optimally distribute the available transmit power to the L data streams thereby maximizing the mutual information $I(s, \hat{s})$ and achieve capacity.

$$
\begin{aligned}
\hat{s} &= U^{\mathrm{H}} \cdot U\Sigma V^{\mathrm{H}} \cdot V \cdot P^{1/2}s + U^{\mathrm{H}}n \\
&= \mathrm{diag}\{\sigma_i \cdot \sqrt{P_i}\}_{i=1}^{L} \cdot s + n'. \qquad (4)
\end{aligned}
$$

It has been shown [2], that especially in scenarios with spatial correlation, the capacity is not significantly sacrificed, if the transmitter has only long term channel state information (LT-CSI), which is much easier to provide by either a low rate feedback channel or via reciprocity even in a frequency division duplex (FDD) system. Therefore, it will be assumed for the remaining part of this section, that the transmitter knows only $\mathrm{E}[H^{\mathrm{H}}H] = Q\Lambda Q^{\mathrm{H}}$ and utilizes the eigenvectors Q for the linear mapping from streams to transmit antennas. Based on this assumption of eigenbeamforming, three performance measures for MIMO systems will be derived: antenna gain, multiplexing gain and diversity gain.

4.1. Antenna Gain

We define antenna gain G_{A} as the ratio between the average receive power of the MIMO system under consideration $P_{\mathrm{R}}^{\mathrm{MIMO}}$ and the average receive power $P_{\mathrm{R}}^{\mathrm{SISO}}$ a single-input single-output (SISO) system would get having an average channel coefficient taken from the MIMO channel:

$$
\begin{aligned}
P_{\mathrm{R}}^{\mathrm{MIMO}} &= \mathrm{E}_{H}[P_{\mathrm{R}}(H)] = \mathrm{E}_{H}\left[\mathrm{E}\left[\|y\|_2^2 \,\middle|\, H\right]\right] \\
&= \mathrm{E}_{H}\left[\mathrm{E}\left[\mathrm{tr}\left(yy^{\mathrm{H}}\right) \middle| H\right]\right] \\
&= \mathrm{E}_{H}\left[\mathrm{tr}\left(HQPQ^{\mathrm{H}}H^{\mathrm{H}}\right)\right] \\
&= \mathrm{tr}(\Lambda P), \qquad (5)
\end{aligned}
$$

$$
\begin{aligned}
P_{\mathrm{R}}^{\mathrm{SISO}} &= P_{\mathrm{T}}\,\mathrm{E}\left[|h_{ij}|^2\right] = \frac{P_{\mathrm{T}}}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}\mathrm{E}\left[|h_{ij}|^2\right] \\
&= \frac{P_{\mathrm{T}}}{MN}\,\mathrm{E}\left[\|H\|_{\mathrm{F}}^2\right] = \frac{P_{\mathrm{T}}}{MN}\,\mathrm{E}\left[\mathrm{tr}(H^{\mathrm{H}}H)\right] \\
&= \frac{P_{\mathrm{T}}\,\mathrm{tr}(\Lambda)}{MN}. \qquad (6)
\end{aligned}
$$

Therefore we get

$$
G_{\mathrm{A}} = \frac{P_{\mathrm{R}}^{\mathrm{MIMO}}}{P_{\mathrm{R}}^{\mathrm{SISO}}} = \left(N\frac{\mathrm{tr}(\Lambda P)}{\mathrm{tr}(\Lambda)\,\mathrm{tr}(P)}\right)M = G_{\mathrm{A}}^{\mathrm{Tx}} \cdot G_{\mathrm{A}}^{\mathrm{Rx}}, \quad (7)
$$

which is maximized by powering up only the strongest eigenmode

$$
G_{\mathrm{A}}^{\max} = MN\frac{\lambda_{\max}}{\sum_{i=1}^{L}\lambda_i},
$$

i.e. by restricting the system to only one stream $L = 1$.

This antenna gain can be split into a transmit and receive antenna gain (7) where the power loading $\boldsymbol{P}$ has an influence on G_A^{Tx} while $G_A^{\mathrm{Rx}} = M$ is independent of $\boldsymbol{P}$. If the channel is rank deficient, especially if $\mathrm{rank}\left(\mathrm{E}[\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}]\right) = 1$, then the largest antenna gain can be achieved

$$G_A^{\max}\left(\mathrm{rank}\left(\mathrm{E}[\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}]\right) = 1\right) = MN.$$

If all eigenmodes are equally strong ($\lambda_1 = \lambda_2 = \cdots = \lambda_L$) and $L = N \leq M$, then we have

$$G_A^{\max}\left(\lambda_{\max}/\lambda_{\min} = 1\right) = M$$

completely independent of the power loading $\boldsymbol{P}$. Note, that the antenna gain in this case is due to the M receive antennas only, the transmit antennas do not contribute. For a SISO system the antenna gain is obviously one.

4.2. Multiplexing Gain

An important aspect of MIMO systems is the possibility to transmit $L > 1$ parallel streams simultaneously in the same bandwidth. We define the multiplexing gain G_{M} as the slope of the ergodic capacity as a function of the logarithmic transmit power P_{T} divided by the variance of the noise σ_n^2:

$$G_{\mathrm{M}}^{\mathrm{erg}} = \frac{\mathrm{d}\,\mathrm{E}[C]}{\mathrm{d}\log_2(P_{\mathrm{T}}/\sigma_n^2)}. \tag{8}$$

However, the evaluation of the expectation is rather involved for MIMO systems with correlated fading. We therefore replace the ergodic capacity, which is the average capacity of the fading MIMO channel

$$C_{\mathrm{erg}} = \mathrm{E}\left[\sum_{i=1}^{L}\log_2\left(1 + \frac{P_i'}{\sigma_n^2}\lambda_i'\right)\right],$$

where λ_i' are the eigenvalues of $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}$ and the P_i' are determined according to water filling by the capacity of the average channel, where water filling is performed in accordance with λ_i, the eigenvalues of $\mathrm{E}[\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}]$

$$\bar{C} = \sum_{i=1}^{L}\log_2\left(1 + \frac{P_i}{\sigma_n^2}\lambda_i\right). \tag{9}$$

Now we get for the multiplexing gain

$$\begin{aligned} G_{\mathrm{M}} &= \frac{\mathrm{d}\bar{C}}{\mathrm{d}\log_2(P_{\mathrm{T}}/\sigma_n^2)} = \sum_{i=1}^{L}\frac{\lambda_i P_i}{\sigma_n^2 + \lambda_i P_i} \\ &= \mathrm{tr}\left(\boldsymbol{\Lambda}\boldsymbol{P}(\boldsymbol{1}\sigma_n^2 + \boldsymbol{\Lambda}\boldsymbol{P})^{-1}\right). \end{aligned} \tag{10}$$

Due to Jensen's inequality this multiplexing gain G_{M} is an upper bound for the multiplexing gain based on the derivation

of C_{erg}, *i.e.* $G_{\mathrm{M}} \geq G_{\mathrm{M}}^{\mathrm{erg}}$. This upper bound is usually fairly tight as is illustrated in Fig. 3(b). In the high SNR regime, this multiplexing gain becomes

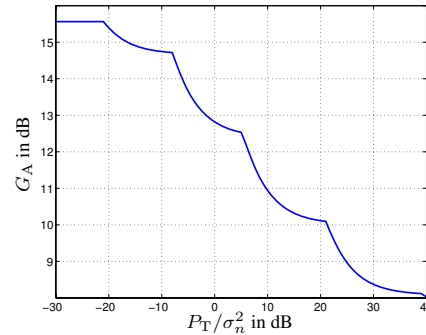$$\lim_{\sigma_n^2 \to 0} G_{\mathrm{M}} = \mathrm{tr}(\boldsymbol{1}_L) = L,$$

where L is the number of eigenmodes which actually are powered up. For a SISO system, the multiplexing gain obviously is equal to one. G_{M} is upper bounded by $\min(M, N)$.
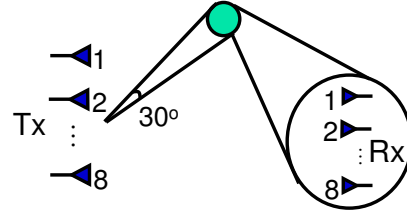
(a) Scenario of an 8×8 MIMO system with one DoD with $30°$ angle spread and diffuse scattering on the receive side
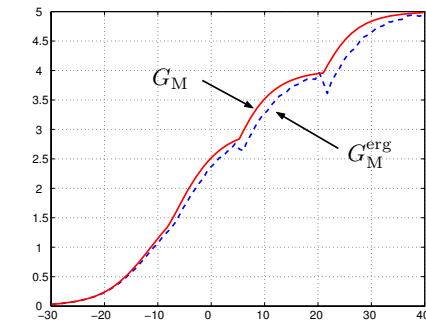
(b) Comparison of G_{M} and $G_{\mathrm{M}}^{\mathrm{erg}}$ for the scenario (a)

(c) G_{A} for the scenario (a)

Fig. 3.

In Fig. 3(a) a scenario with 8 transmit and 8 receive antennas is shown. There is one main direction of departure with an angle spread of $30°$ while at the receive side rich diffuse

scattering has been assumed ($\mathrm{E}[\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}] = \alpha\mathbf{1}_M$). The power allocation $\boldsymbol{P}$ used for generating Fig. 3(b) and Fig. 3(c) was water filling based on the eigenvalues of $\mathrm{E}[\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H}]$. It can be seen that with increasing transmit power, the number of eigenmodes, which are powered up, is increasing leading to a decrease in antenna gain and to an increase in multiplexing gain.

4.3. Diversity Gain

We define diversity gain as the ratio between the squared average receive signal power and the variance of that receive signal power

$$G_{\mathrm{D}} = \frac{(\mathrm{E}_{\boldsymbol{H}}[P_{\mathrm{R}}(\boldsymbol{H})])^2}{\mathrm{var}\,(P_{\mathrm{R}}(\boldsymbol{H}))}, \tag{11}$$

where $P_{\mathrm{R}}(\boldsymbol{H}) = \mathrm{E}\left[\|\boldsymbol{y}\|_2^2\,\big|\,\boldsymbol{H}\right]$ (see Eq. (5)).

This diversity gain can be expressed with the covariance matrix of the effective channel including eigenbeamforming

$$\begin{aligned}\boldsymbol{R} &= \mathrm{E}\left[\mathrm{vec}(\boldsymbol{H}\boldsymbol{Q}\boldsymbol{P}^{1/2})\cdot\left(\mathrm{vec}(\boldsymbol{H}\boldsymbol{Q}\boldsymbol{P}^{1/2})\right)^{\mathrm{H}}\right]\\ &\overset{\mathrm{EVD}}{=} \boldsymbol{T}\boldsymbol{D}\boldsymbol{T}^{\mathrm{H}}.\end{aligned}$$

The numerator of Eq. (11) can be simplified to

$$\begin{aligned}\left(\mathrm{E}_{\boldsymbol{H}}\left[\mathrm{E}\left[\|\boldsymbol{y}\|_2^2\,\big|\,\boldsymbol{H}\right]\right]\right)^2 &= (\mathrm{tr}(\boldsymbol{\Lambda}\boldsymbol{P}))^2\\ &= (\mathrm{tr}(\boldsymbol{R}))^2 = (\mathrm{tr}(\boldsymbol{D}))^2,\end{aligned}$$

while the denominator is more elaborate [3] and leads to

$$\mathrm{E}_{\boldsymbol{H}}\left[\left(\mathrm{E}\left[\|\boldsymbol{y}\|_2^2\,\big|\,\boldsymbol{H}\right]\right)^2\right] = \|\boldsymbol{R}\|_{\mathrm{F}}^2 = \mathrm{tr}(\boldsymbol{R}^2) = \mathrm{tr}(\boldsymbol{D}^2).$$

Therefore, we have for the diversity gain

$$G_{\mathrm{D}} = \frac{(\mathrm{tr}(\boldsymbol{R}))^2}{\mathrm{tr}(\boldsymbol{R}^2)} = \frac{(\mathrm{tr}(\boldsymbol{D}))^2}{\mathrm{tr}(\boldsymbol{D}^2)} = G_{\mathrm{D}}^{\mathrm{Tx}}\cdot G_{\mathrm{D}}^{\mathrm{Rx}},$$

which can be split into a transmit and receive diversity gain if the transmit side and receive side fading processes are independent. In this case, the covariance matrix $\boldsymbol{R}$ of the effective channel $\boldsymbol{Z} = \boldsymbol{H}\boldsymbol{Q}\boldsymbol{P}^{1/2}$ can be decomposed into the Kronecker product of a transmit covariance and a receive covariance

$$\boldsymbol{R} = \frac{1}{\mathrm{tr}\left(\mathrm{E}[\boldsymbol{Z}^{\mathrm{H}}\boldsymbol{Z}]\right)}\left(\mathrm{E}[\boldsymbol{Z}^{\mathrm{H}}\boldsymbol{Z}]\right)^{\mathrm{T}}\otimes\mathrm{E}[\boldsymbol{Z}\boldsymbol{Z}^{\mathrm{H}}],$$

where the transmit diversity gain and the receive diversity gain can be written as

$$\begin{aligned}G_{\mathrm{D}}^{\mathrm{Tx}} &= \frac{\left(\mathrm{tr}(\mathrm{E}[\boldsymbol{Z}^{\mathrm{H}}\boldsymbol{Z}])\right)^2}{\mathrm{tr}\left((\mathrm{E}[\boldsymbol{Z}^{\mathrm{H}}\boldsymbol{Z}])^2\right)}\\ G_{\mathrm{D}}^{\mathrm{Rx}} &= \frac{\left(\mathrm{tr}(\mathrm{E}[\boldsymbol{Z}\boldsymbol{Z}^{\mathrm{H}}])\right)^2}{\mathrm{tr}\left((\mathrm{E}[\boldsymbol{Z}\boldsymbol{Z}^{\mathrm{H}}])^2\right)} = \frac{\left(\mathrm{tr}(\mathrm{E}[\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}])\right)^2}{\mathrm{tr}\left((\mathrm{E}[\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}])^2\right)}.\end{aligned}$$

Table 1. Maximum performance gains

$G_{\mathrm{A}}^{\mathrm{Tx}}$	$G_{\mathrm{A}}^{\mathrm{Rx}}$	G_{A}	G_{M}	$G_{\mathrm{D}}^{\mathrm{Tx}}$	$G_{\mathrm{D}}^{\mathrm{Rx}}$	G_{D}
1.93	3	5.79	1.58	3	3	9
2.85dB	4.77dB	7.62dB	1.99dB	4.77dB	4.77dB	9.54dB

The last expression for $G_{\mathrm{D}}^{\mathrm{Rx}}$ shows that the unitary eigenbeamforming $\boldsymbol{Q}$ and power loading $\boldsymbol{P}$ on the transmit side does not affect the receive diversity gain at all. Therefore, power loading does influence transmit diversity gain only and the overall diversity gain can be optimized by appropriate power loading optimizing $G_{\mathrm{D}}^{\mathrm{Tx}}$ only. Since the transmit diversity gain can be computed as

$$G_{\mathrm{D}}^{\mathrm{Tx}} = \frac{(\mathrm{tr}(\boldsymbol{\Lambda}\boldsymbol{P}))^2}{\mathrm{tr}(\boldsymbol{\Lambda}^2\boldsymbol{P}^2)},$$

which achieves its maximum N, if $\mathrm{rank}(\boldsymbol{H}) = N$ and the transmit power is distributed such that all entries of the diagonal matrix $\boldsymbol{\Lambda}\boldsymbol{P}$ are equal, $i.e.$, $\lambda_1 P_1 = \lambda_2 P_2 = \cdots = \lambda_N P_N$ and therefore, all N eigenmodes are powered up.

The overall G_{D} is upper limited by MN, a value which can be achieved if $G_{\mathrm{D}}^{\mathrm{Tx}}$ is optimized by power loading and achieves a value of N and if the receive side fading is i.i.d. leading to $G_{\mathrm{D}}^{\mathrm{Rx}} = M$.

4.4. Trade-off between Antenna Gain, Multiplexing Gain and Diversity Gain

The three performance gains are maximized by three different power distributions, which are summarized here:

$$\begin{aligned}G_{\mathrm{A}}^{\mathrm{Tx}} &= N\cdot\frac{\mathrm{tr}(\boldsymbol{\Lambda}\boldsymbol{P})}{\mathrm{tr}(\boldsymbol{\Lambda})\,\mathrm{tr}(\boldsymbol{P})} \quad P_1 = P_{\mathrm{T}}, P_2 = \cdots = P_N = 0,\\ G_{\mathrm{M}} &= \mathrm{tr}\left(\boldsymbol{\Lambda}\boldsymbol{P}(\mathbf{1}\sigma_n^2 + \boldsymbol{\Lambda}\boldsymbol{P})^{-1}\right)\\ &\qquad P_i = \sigma_n^2\max\left(0, \frac{\xi}{\sqrt{\lambda_i}} - \frac{1}{\lambda_i}\right), i = 1, \ldots, L\\ &\qquad \text{with } \xi > 0 \text{ such that } \sum_{i=1}^{L} P_i = P_{\mathrm{T}},\\ G_{\mathrm{D}}^{\mathrm{Tx}} &= \frac{(\mathrm{tr}(\boldsymbol{\Lambda}\boldsymbol{P}))^2}{\mathrm{tr}(\boldsymbol{\Lambda}^2\boldsymbol{P}^2)} \quad P_i = \frac{P_{\mathrm{T}}}{\lambda_i\sum_{i=1}^{N}\frac{1}{\lambda_i}}.\end{aligned}$$

Obviously, there is no way to maximize all three gains or any pair of gains simultaneously. Therefore, there is a trade-off among these three performance measures, which will be illustrated with a simple example with $N = M = 3$ and with $\lambda_1 = 9, \lambda_2 = 4$ and $\lambda_3 = 1$ and with $P_{\mathrm{T}}/\sigma_n^2 = 1$. The individually achievable maximum gains are given in Table 1.

Fig. 4 shows the values of our three performance gains, which are jointly achievable. Note that not all combinations
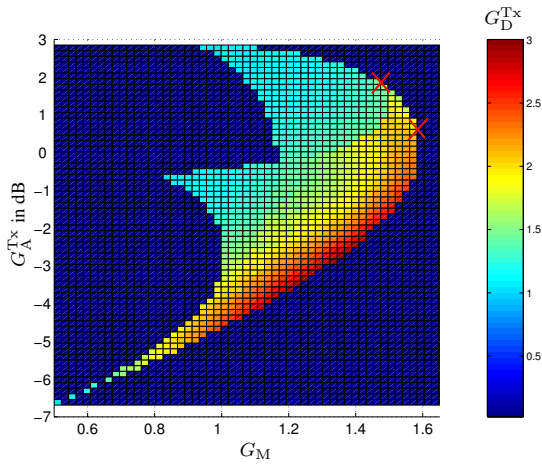
Fig. 4. Fundamental gain trade-off

of G_A^{Tx} and G_M permit a solution. If a solution exists, then the corresponding G_D^{Tx} is computed and coded with a color according to the right hand scale in Fig. 4. The dark areas correspond to situations where no solution exists.

Now we are confronted with the basic question: which triple (G_A, G_M, G_D) is the best? This question can only be answered taking into account the application and the associated quality of service requirement. In Fig. 4 two triples are highlighted: one maximizes ergodic mutual information and, therefore, would fit an application where long term average throughput is the figure of merit while the second one maximizes peak signal to noise ratio of images and therefore would be suitable for video streaming applications.

Of course we find the solutions for different application on the right hand front of the permissible area of (G_A, G_M) pairs, which is kind of a Pareto-front comprising efficient combinations of G_A and G_M.

5. MULTIUSER SINGLE CELL SYSTEMS

Having K users to be served by one base station in the downlink (broadcast channel, BC) of a multiple antenna communication system, we have to take into account multiuser interference in addition to noise, and therefore, have to deal with SINR values instead of SNR values only. The capacity region of MIMO broadcast channels has been studied [4], [5], [6] and can be achieved by dirty paper coding [7]. The principle of successive encoding assigns a code to user k such that this user receives interference only from the signals not already encoded. The application of this concept affords instantaneous TCSI in contrast to single link consideration of the previous section. Therefore, feedback becomes much more important, because it will occupy a much larger share of the available resources.
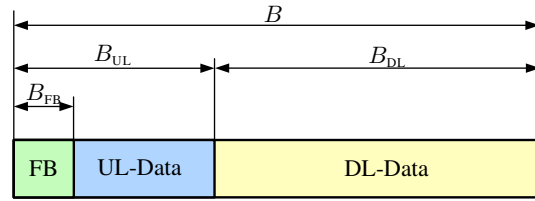

Fig. 5. The available bandwidth is split into a band for downlink and a band for uplink, which accommodates the feedback band. The usual frequency gap between uplink and downlink is not shown.

5.1. Feedback

The more accurate instantaneous TCSI is available, the closer the capacity region can be achieved. But the provision of that TCSI takes away from the resources available for payload data transmission. Therefore, an optimum partition of resources for feedback and for payload data has to be found.

We will consider an FDD system with N transmit antennas and a single receive antenna in the downlink and a single transmit antenna and N receive antennas in the uplink. Therefore, feedback is needed only to provide the downlink multi-antenna transmitter with CSI. The split of the overall bandwidth B as shown in Fig. 5 should be carried out according to

$$(B_{\mathrm{UL}}^{\mathrm{opt}}, B_{\mathrm{DL}}^{\mathrm{opt}}, B_{\mathrm{FB}}^{\mathrm{opt}}) =$$
$$\underset{(B_{\mathrm{UL}}, B_{\mathrm{DL}}, B_{\mathrm{FB}})}{\mathrm{argmax}} \quad (\bar{R}_{\mathrm{DL}}(B_{\mathrm{DL}}, B_{\mathrm{FB}}) + \bar{R}_{\mathrm{UL}}(B_{\mathrm{UL}}, B_{\mathrm{FB}}))$$
$$\text{s.t.} \quad B_{\mathrm{DL}} > 0, \quad B_{\mathrm{UL}} > 0, \quad 0 < B_{\mathrm{FB}} \le B_{\mathrm{UL}},$$
$$B_{\mathrm{UL}} + B_{\mathrm{DL}} = B, \quad \bar{R}_{\mathrm{UL}} = \mu \bar{R}_{\mathrm{DL}}. \tag{12}$$

Thereby $\bar{R}_{\mathrm{UL}}$ and $\bar{R}_{\mathrm{DL}}$ are the average payload data rates with $\mu \ge 0$ as symmetry factor accounting for different payload requirement in the two different directions. In order to solve the above optimization problem, we rely on the following assumptions:

- the channel is i.i.d. frequency flat block fading with block length T and feedback is issued per block

- the channel gains between any receive and transmit antenna are uncorrelated

- channel estimation errors at the receivers are negligible, the necessary resources for pilots are neglected

- random vector quantization of normalized channel vectors using b bits per antenna

- feedback bits are protected by capacity approaching error control coding and can be decoded correctly with negligible outage

- payload data in up- and downlink is also protected by capacity approaching error control coding.

Based on this set of assumptions a closed-form solution of the above optimization problem (12) can be derived [8].

The results obtained from this optimization are depicted in Fig. 6 for different numbers of transmit antennas N and shows a considerable part of the overall bandwidth for feedback.
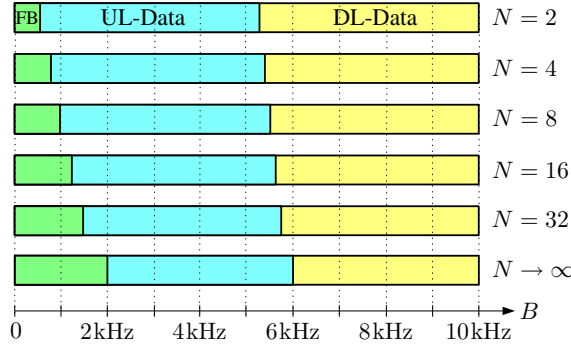
Fig. 6. Optimum partitioning of the available bandwidth B for different number of antennas

Further work is required to incorporate space division multiple access and channel aware scheduling in the optimum feedback bandwidth allocation.

6. MULTIUSER MULTICELL SYSTEMS

The substantial difference between a multicell and a single cell system is the intercell interference (ICI). Utilizing the whole available bandwidth in all cells of a network (frequency re-use one), most users will suffer more from ICI than from thermal noise. The ICI power levels experienced by each user have to be known to the serving base station to enable the application of capacity approaching transmit processing. However, the ICI power levels depend on the transmit processing of the other base stations in the surrounding cells. If all base stations work independently, the ICI power becomes unpredictable, leading to a mismatch between the assumed ICI power used to compute the transmit processing and the true ICI power which leads to a link adaptation which either is too optimistic and therefore leads to packet losses and, therefore, necessitate retransmissions, or is too pessimistic and therefore does not exploit the resources fully.

There are different ways to deal with this mismatch problem:

1. Genie Assistance: the mismatch could be avoided completely if all base stations would know the true ICI powers generated by their neighbors. This could be achieved either with a centralized approach at the expense of a huge signalling overhead in a so called backhaul network or by a decentralized iterative approach with a huge signalling overhead over the up- and downlink channels within the cells.

2. Conservative Gambling: one can also accept the ICI power mismatch and use conservative link adaptation. This will lead to the aforementioned underloading of the system and, depending on the back-off, occasional packet losses.

3. Isolation: a way of not completely avoiding but reducing the mismatch is by reducing ICI power level by increasing the frequency re-use factor. Of course, the bandwidth available per cell is thereby greatly reduced.

4. Stabilization: by refraining from user specific transmit processing the mismatch can be avoided. Of course this approach gives away the benefits of spacial signal processing.

In this section, we investigate the influence of these four approaches on the cell sum-capacity in the cellular Gaussian broadcast channel. We restrict the discussion to the case, where only the base stations are equipped with multiple antennas and the mobile user equipment has only a single antenna.

The details of the simulation scenario can be found in [9]. In addition, the cell sum-rate for a single cell scenario is also given as an upper bound. The results are shown with the cumulative distribution function (cdf) of the sum-rate in Fig. 7.
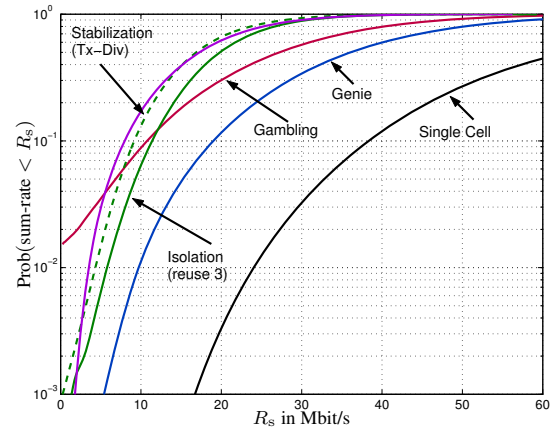
Fig. 7. CDF of cell sum-rate

We can observe from Fig. 7 that the genie-approach performs for virtually all outage probabilities at a rate of approximately 50% with respect to the single cell scenario, while conservative gambling is not that bad on average but very poor at low outage probabilities, say less than 10%. Isolation

with a re-use factor of three makes the problem less severe, but note that the relatively simple and robust transmit diversity scheme (stabilization) is performing similarly.

Comparing the genie approach with stabilization in low outage probability region ($\leq 5\%$) we see a big gap: the cell sum-rate can be increased by $\sim 160\%$ by the genie approach. Therefore, an important task and attractive challenge for future research is to find a practical, implementable approach performing close to or even exceeding the genie approach.

7. CONCLUSIONS

Multiple antenna systems have come a long way already. Many important results have been derived and it is obvious that multiple antennas are an important ingredient for many future wireless system generations and applications. But this does not mean that a mature state, where we know the answer to all important questions, has been reached. There are lots of important open problems necessitating intensive future research and only a few have been touched upon here.

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