ON THE GEOMETRICAL STRUCTURE OF NETWORK EQUATIONS

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ABSTRACT

In the past, there have been several attempts to relate *Hamiltonian equations*, which describe conservative systems, to electrical networks. In control theory the ideas of Hamiltonian systems were expanded to *affine Hamiltonian systems* with *feedback* that are input/output systems with Hamiltonian vectorfields.

In this paper the principles of affine Hamiltonian systems to a class of electrical networks are applied. It is shown that voltage and current coordinates admit a Poisson structure and how input/output Poisson systems with feedback are related to electrical networks.

1. INTRODUCTION

In the past, there has been a great interest in a general description of electrical networks. Every now and then, there were attempts to apply the theory of *Hamiltonian systems* to electrical networks for several reasons [6], [2], [9], to mention just a few. Another approach was the formulation of *Brayton-Moser's equations* [4], which is a formulation of network equations as a gradient system. However, the development in linear network theory exploited concepts of energy conservation of subsystems rather than the gradient nature of the whole system, though, each linear lossless multiport terminated with resistors is in fact a gradient system. Hence, it seems worthwhile to tackle nonlinear networks from the same point of view or more specifically, to separate the lossless part of the system from the terminal resistors.

In recent years, mathematical control theorists investigated *affine Hamiltonian systems* as a natural extension of Hamiltonian systems. Brockett was the first to discuss this point [5]. See [11] for a recent summary. *Noncanonical* Hamiltonian systems are associated with *Poisson manifolds*, where the trajectories evolve on [1]. Only recently, there has been a revision of Hamiltonian formulations for electrical networks from the point of view of *Poisson manifolds* [9], [10]. So far, the networks under consideration do not admit sources and resistive elements in general.

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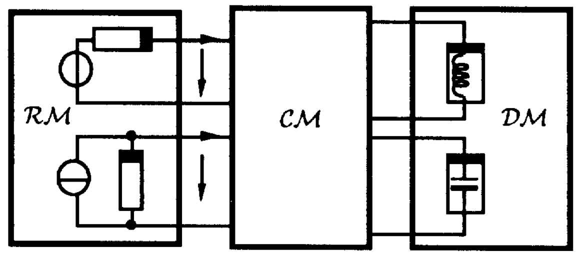

Figure 1: The network model

In this contribution, it is proved that a quite general class of nonlinear LC–networks is equivalent to dynamical systems on Poisson manifolds. The issue of the proper choice of the state variables is touched upon in this context. Furthermore, a relation between affine Hamiltonian systems and RLC-networks is established.

2. NETWORK MODEL

A suitable network model for our purpose is depicted in Fig. 1. The model consists of three circuit blocks denoted by $\mathcal{CM}$, $\mathcal{RM}$, $\mathcal{DM}$. They contain the connecting multiport, resistive elements and reactive elements. The subcircuit $\mathcal{DM}$ consists of uncoupled nonlinear/linear reactive elements. These elements have to admit an algebraic description by a function $\delta : \mathcal{R} \to \mathcal{R}$. Take, for example, the inductors which are either flux or current controlled. The resistive multiport contains nonlinear resistors in series or in parallel with an ideal source. We assume minimal state space realization. This is equivalent to the existence of a tree containing *all* capacitors and *no* inductors. The current-controlled resistors with voltage source are located in the tree (index T) and the co-tree contains the voltage-controlled resistors with current sources (index C). For more general resistors see [8].

The connecting n-port ($\mathcal{CM}$) is governed by *Kirchhoff's equations*:

$$\begin{bmatrix} 0 & \boldsymbol{A} \\ \boldsymbol{B}^{\mathrm{T}} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{i} \end{bmatrix} = \boldsymbol{0}. \qquad (1)$$

The matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ describe the connection of the circuit elements. Because each port is specified by voltage and current we have $2n$ variables for $\mathcal{CM}$. Moreover, (1) gives rise to a n-dimensional linear subspace of the $2n$ dimensional signal space $\left(\left[\boldsymbol{u}^{\mathrm{T}} \quad \boldsymbol{i}^{\mathrm{T}}\right]^{\mathrm{T}}\right)$. A parametrization of this linear subspace is given in terms of n port variables s_k (voltages and currents) [3], i.e.,

$$\begin{aligned} i_k &= i_k(s_1, s_2, \ldots, s_n) & (2)\\ u_k &= u_k(s_1, s_2, \ldots, s_n) \qquad k = 1(1)n. & (3) \end{aligned}$$

This parametrization is not unique but in any case global (linear subspace). We apply symmetric sign convention for the ports of $\mathcal{CM}$.

To specify $\boldsymbol{A}$ and $\boldsymbol{B}$ further suppose that $\mathcal{DM}$ contains n_C capacitors and n_L inductors as well as $\mathcal{RM}$ contains $n_{RT} + n_{RC}$ resistors. It holds $n = n_{RT} + n_{RC} + n_L + n_C$. The capacitor voltages (charge) and the inductor currents (flux) determine the state of dynamical system. The capacitor voltage and the inductor current are taken as parameters. Finally, the ports of the resistors are parametrized either by the port voltage for the tree elements and current for the co-tree elements. In this way the above parametrization becomes unique. This defines the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ as ($\boldsymbol{I}$ identity matrix)

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{I}_{n_C} & & -\boldsymbol{N}_1^{\mathrm{T}} & -\boldsymbol{N}_2^{\mathrm{T}} \\ & \boldsymbol{I}_{n_{RT}} & -\boldsymbol{N}_3^{\mathrm{T}} & -\boldsymbol{N}_4^{\mathrm{T}} \end{bmatrix} \quad (4)$$

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{N}_1 & \boldsymbol{N}_3 & \boldsymbol{I}_{n_L} & \\ \boldsymbol{N}_2 & \boldsymbol{N}_4 & & \boldsymbol{I}_{n_{RC}} \end{bmatrix}^{\mathrm{T}}. \quad (5)$$

so that the connecting multiport is governed by

$$\begin{bmatrix} \boldsymbol{u}_L \\ \boldsymbol{u}_{RC} \\ \boldsymbol{i}_C \\ \boldsymbol{i}_{RT} \end{bmatrix} = \underbrace{\begin{bmatrix} & & -\boldsymbol{N}_1 & -\boldsymbol{N}_3 \\ & & -\boldsymbol{N}_2 & -\boldsymbol{N}_4 \\ \boldsymbol{N}_1^{\mathrm{T}} & \boldsymbol{N}_2^{\mathrm{T}} & & \\ \boldsymbol{N}_3^{\mathrm{T}} & \boldsymbol{N}_4^{\mathrm{T}} & & \end{bmatrix}}_{\boldsymbol{\Gamma}} \begin{bmatrix} \boldsymbol{i}_L \\ \boldsymbol{i}_{RC} \\ \boldsymbol{u}_C \\ \boldsymbol{u}_{RT} \end{bmatrix} \quad (6)$$

Proposition 1 ([10]) *Equation (6) describes a linear manifold $\mathcal{M}$ endowed with a Poisson structure and the structure matrix $\boldsymbol{\Gamma}$.*

It is, roughly speaking, a smooth manifold with a bilinear map which is skew-symmetric and suffices the Jacobi identity and the Leibniz rule. For a definition of Poisson manifolds see, for example, [1]. The matrix $\boldsymbol{\Gamma}$ is skew-symmetric and the Jacobi identity is fulfilled because the structure matrix is constant. It is the Poisson structure we are focussing on in this paper.

3. DYNAMICAL SYSTEMS DESCRIPTION

Nonlinear dynamical systems are usually separated into two classes, namely *gradient systems* and *Hamiltonian systems* with characteristic types of differential equations.

3.1. HAMILTONIAN EQUATIONS

With the class of Hamiltonian systems one usually associates equations of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \boldsymbol{q} \\ \boldsymbol{p} \end{bmatrix} = \begin{bmatrix} & \boldsymbol{I} \\ -\boldsymbol{I} & \end{bmatrix} \nabla H(\boldsymbol{q}, \boldsymbol{p}) = \boldsymbol{J} \nabla H(\boldsymbol{q}, \boldsymbol{p}). \quad (7)$$

Here $\boldsymbol{q}$, $\boldsymbol{p}$ denote the state of the system and H the Hamiltonian function. Those systems are called canonical Hamiltonian systems due to $\boldsymbol{J}$. They can be generalized to non-canonical Hamiltonian systems by replacing the structure matrix $\boldsymbol{J}$ by a matrix $\boldsymbol{K}$ such that all properties of a Poisson manifold still hold. $\boldsymbol{K}$ can also depend on the states.

This class of systems is related to Fig. 1, if there are no resistors and sources in the circuit. The energy stored in the reactive elements is given by the Hamiltonian function. Two essentially different choices for the state space coordinates exist.

3.1.1. CHARGE-FLUX COORDINATES

A possible coordinate choice for the state would be flux and charge as used in [2] and [9]. Hence, the nonlinear reactive elements are required to be either charge or flux controlled. This choice leads straightforward to an explicit state space differential equation. This is due to the following relations for $\mathcal{DM}$

$$\begin{aligned} \boldsymbol{u}_L &= \tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\varphi} & \boldsymbol{i}_L &= \tfrac{\mathrm{d}}{\mathrm{d}\boldsymbol{\varphi}} E_L(\boldsymbol{\varphi}) \\ \boldsymbol{i}_C &= \tfrac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{q} & \boldsymbol{u}_C &= \tfrac{\mathrm{d}}{\mathrm{d}\boldsymbol{q}} E_C(\boldsymbol{q}). \end{aligned} \quad (8)$$

E_L and E_C denote the total stored energy in a inductive and capacitive elements respectively. With this choice one obtains a dynamical system on a Poisson manifold given by the connecting multiport.

Proposition 2 ([10]) *The dynamical system $\mathcal{N}_{H,\varphi,q}$: $\{\mathcal{CM}, \mathcal{DM}\}$*

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \boldsymbol{\varphi} \\ \boldsymbol{q} \end{bmatrix} = \begin{bmatrix} & \boldsymbol{N} \\ -\boldsymbol{N}^{\mathrm{T}} & \end{bmatrix} \nabla H(\boldsymbol{\varphi}, \boldsymbol{q}) = \boldsymbol{\Gamma} \nabla H(\boldsymbol{\varphi}, \boldsymbol{q}) \quad (9)$$

with $H = E_L + E_C$ and $\boldsymbol{\Gamma} = \mathrm{const.}$ is called a Hamiltonian system on a Poisson manifold or Poisson system.

The Poisson tensor $\boldsymbol{K} = \boldsymbol{\Gamma}$ is constant and skew-symmetric so that it is easy to verify the Jacobi identity. Rank deficiency of $\boldsymbol{\Gamma}$ is excluded by the equality of the state space dimension and the number of reactive elements.

3.1.2. VOLTAGE-CURRENT COORDINATES

In case of voltage or current controlled elements, the description as a dynamical system on Poisson manifolds becomes much more involved. In this specific case the differential equations are not explicit. One can introduce a lossless transformation mapping nonlinear reactive elements onto linear reactive elements. The idea is to define a new inductance L_m by [7]

$$\frac{\mathrm{d}}{\mathrm{dt}}\varphi(i_L) = L(i_L)\frac{\mathrm{d}}{\mathrm{dt}}i_L = \sqrt{L_m(i_L)}\frac{\mathrm{d}}{\mathrm{dt}}\left(\sqrt{L_m(i_L)}i_L\right) = u_L. \tag{10}$$

For a given $L(i_L)$ can computes $L_m(i_L)$. The introduction of new coordinates by a mapping

$$\tilde{i}_L = \sqrt{L_m(i_L)}i_L \qquad \tilde{u}_L = \frac{u_L}{\sqrt{L_m(i_L)}} \tag{11}$$

denote a nonlinear transformer with "turns ratio" $n(i_L) = \sqrt{L_m(i_L)}$. Note that $\tilde{i}$, $\tilde{u}$ still have the physical meaning of currents and voltages. The linear reactive element in the new coordinates is governed by $\frac{\mathrm{d}}{\mathrm{dt}}\tilde{i}_L = \tilde{u}_L$. If L_m is negative, then we use a linear inductor with element value -1 ($\frac{\mathrm{d}}{\mathrm{dt}}\tilde{i}_L = -\tilde{u}_L$) to obtain a reasonable transformation (11). A similar derivation holds for the capacitors. The turns ratio of the whole multiport transformer $\mathcal{TM}$ reads

$$\boldsymbol{M} = \begin{bmatrix} \sqrt{L_m(i_L)} & \\ & \sqrt{C_m(u_C)} \end{bmatrix}. \tag{12}$$

The matrix is diagonal as a consequence of the assumption of uncoupled reactive elements (see above). The transformation (11) depends on the old coordinates. It is favourable to find a form such that the turns ratio of the transformer depends on the new coordinates. Needless to say, $L_m(i_L)$ and $\tilde{L}_m(\tilde{i}_L)$ are different in general. These details are omitted in the sequel. On the one hand, the Hamiltonian function for the now linear energy storing elements is given by the positive definite quadratic form (pos. elements) $\tilde{H} = \frac{1}{2}\left(\tilde{\boldsymbol{i}}_L^{\mathrm{T}}\tilde{\boldsymbol{i}}_L + \tilde{\boldsymbol{u}}_C^{\mathrm{T}}\tilde{\boldsymbol{u}}_C\right)$. On the other hand, the energy $H(i_L, u_C)$ in the old coordinates is given by

$$H = \int \begin{bmatrix} i_L \\ u_C \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} i_L \\ u_C \end{bmatrix} \mathrm{d}t = \frac{1}{2}\begin{bmatrix} i_L \\ u_C \end{bmatrix}^{\mathrm{T}} \boldsymbol{M}^2 \begin{bmatrix} i_L \\ u_C \end{bmatrix}. \tag{13}$$

The energy content in the circuit is not changed by the coordinate transformation $H(\boldsymbol{i}_L, \boldsymbol{u}_C) = \tilde{H}\left(\tilde{\boldsymbol{i}}_L, \tilde{\boldsymbol{u}}_C\right)$. The interconnection matrix $\boldsymbol{\Gamma}$ turns to

$$\boldsymbol{K} = \boldsymbol{M}^{-1}\boldsymbol{\Gamma}\boldsymbol{M}^{-1} \tag{14}$$

with entries $K_{ij} = M_{ii}^{-1}\Gamma_{ij}M_{jj}^{-1}$. The matrix $\boldsymbol{K}$ substitutes the description of the connecting multiport $\boldsymbol{\Gamma}$.

Proposition 3 *The LC-network $\mathcal{N}_{\tilde{H},\tilde{u},\tilde{i}} : \{\mathcal{CM}, \mathcal{DM}\}$*

$$\frac{\mathrm{d}}{\mathrm{dt}}\tilde{\boldsymbol{x}} = \boldsymbol{K}(\tilde{\boldsymbol{x}})\nabla\tilde{H}(\tilde{\boldsymbol{x}}) = \boldsymbol{K}(\tilde{\boldsymbol{x}})\tilde{\boldsymbol{x}} \tag{15}$$

defines a dynamical system on a Poisson manifold with Poisson tensor $\boldsymbol{K}$ ($K_{ij} = M_{ii}^{-1}\Gamma_{ij}M_{jj}^{-1}$).

Proof: Skew-symmetry and bilinearity are not altered by the transformation (14). To show the Jacobi identity one has to check

$$0 = \sum_{l=1}^{n_L+n_C} K_{lj}\frac{\partial K_{ik}}{\partial \tilde{x}_l} + K_{li}\frac{\partial K_{kj}}{\partial \tilde{x}_l} + K_{lk}\frac{\partial K_{ji}}{\partial \tilde{x}_l} \tag{16}$$

for all tupel i, j, k, $1 \le i, j, k \le (n_L + n_C)$. One obtains as nonvanishing terms

$$\begin{aligned} 0 \;=\; & -K_{ij}K_{ik}\frac{\partial M_{ii}/\partial \tilde{x}_k}{M_{ii}} - K_{ik}K_{ji}\frac{\partial M_{ii}/\partial \tilde{x}_k}{M_{ii}} \\ & -K_{ji}K_{kj}\frac{\partial M_{jj}/\partial \tilde{x}_k}{M_{jj}} - K_{jk}K_{ji}\frac{\partial M_{jj}/\partial \tilde{x}_k}{M_{jj}} \\ & -K_{kj}K_{ik}\frac{\partial M_{kk}/\partial \tilde{x}_k}{M_{kk}} - K_{ki}K_{kj}\frac{\partial M_{kk}/\partial \tilde{x}_k}{M_{kk}}. \end{aligned} \tag{17}$$

Due to the skew-symmetry of $\boldsymbol{K}$, (16) follows. ◊

A combination of coordinates (q, φ), $(\tilde{u}, \tilde{i})$ admits to cope with different kinds of dependencies in the reactive elements. Either way, the system is Poisson.

Note that Hamiltonian systems are governed by homogeneous differential equations. This seems sufficient for classical mechanics but not for applications in engineering. Therefore, in mathematical control theory the concept of Hamiltonian systems was extended to *affine Hamiltonian systems*.

3.2. AFFINE HAMILTONIAN EQUATIONS

The dynamical system (7) becomes an input/output system with m inputs v_i and m outputs y_i by the following extensions [5], [11]:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{dt}}\begin{bmatrix} \boldsymbol{q} \\ \boldsymbol{p} \end{bmatrix} &= \boldsymbol{K}\nabla H_0(\boldsymbol{q}, \boldsymbol{p}) - \sum_{i=1}^{m} \boldsymbol{K}\nabla H_i(\boldsymbol{q}, \boldsymbol{p})\, v_i \\ \boldsymbol{y} &= [H_1, H_2, \ldots, H_m]^{\mathrm{T}}. \end{aligned} \tag{18}$$

The functions H_i denote the interaction Hamiltonian and H_0 the internal energy. The number of inputs and outputs is always equal in this definition. This is justified by the natural pairing between input and output coordinates for energy balance. The change of the internal energy H_0 is given by $\frac{\mathrm{d}}{\mathrm{dt}}H_0 = \sum_{i=1}^{m} v_i \frac{\mathrm{d}}{\mathrm{dt}}H_i$.

Example: An example of an affine Hamiltonian system is shown in Fig. 2. It is governed by the equations

$$\frac{\mathrm{d}}{\mathrm{dt}}\begin{bmatrix} \varphi \\ q \end{bmatrix} = \begin{bmatrix} & 1 \\ -1 & \end{bmatrix}\nabla H_0 - \begin{bmatrix} & 1 \\ -1 & \end{bmatrix}\nabla H_1 i_0$$

with Hamiltonian $H_0 = \frac{q^2}{2C} + \frac{\varphi^2}{2L}$, $H_1 = \varphi$ and $\nabla H_1 = [1 \quad 0]^{\mathrm{T}}$. As the natural output of this circuit one obtains $y = H_1 = \varphi = \int u_L \mathrm{d}t = \int u_C \mathrm{d}t$. The source power is given by $\frac{\mathrm{d}}{\mathrm{dt}}H_0 = u_L i_0$. ◊

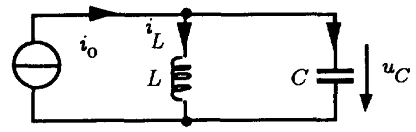

Figure 2: Affine Hamiltonian system

Definition 1 *Let $\mathcal{M}$ be a manifold with Poisson tensors $\boldsymbol{K}_0, \boldsymbol{K}_1, \ldots, \boldsymbol{K}_n$ and $H_0, H_1, \ldots, H_n \in C^\infty(\mathcal{M})$. Then we call*

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \boldsymbol{i} \\ \boldsymbol{u} \end{bmatrix} &= \boldsymbol{K}_0 \nabla H_0(\boldsymbol{i}, \boldsymbol{u}) - \sum_{i=1}^{m} \boldsymbol{K}_i \nabla H_i(\boldsymbol{i}, \boldsymbol{u})\, v_i \\ \boldsymbol{y} &= \sum_{i=1}^{m} \nabla H_i^{\mathrm{T}} \boldsymbol{K}_i \nabla H_0 + \\ & \quad [H_{m+1}(\boldsymbol{v}), H_{m+2}(\boldsymbol{v}), \ldots, H_{2m}(\boldsymbol{v})]^{\mathrm{T}} . \end{aligned} \tag{19}$$

an affine Poisson system. For each of these tensors $\boldsymbol{K}_i$ hold the properties of a Poisson brackets.

The definition varies from (18) by multiple tensors $\boldsymbol{K}_i$ and a direct functional dependence between input and output. The latter is often omitted in control.

Proposition 4 *The LC network $\mathcal{N}_{P,\varphi,q} : \{\mathcal{CM}, \mathcal{DM}\}$ with resistor-free sources is an affine Poisson system (19).*

Proof: The Poisson tensors $\boldsymbol{K}_i$ are obtained from $\boldsymbol{\Gamma}$ as $\boldsymbol{K}_0 = \begin{bmatrix} & \boldsymbol{N}_1 \\ -\boldsymbol{N}_1^{\mathrm{T}} & \end{bmatrix}$ and the $\boldsymbol{K}_i$ are constructed from the columns of $\boldsymbol{N}_2$ and rows of $\boldsymbol{N}_3^{\mathrm{T}}$ by embedding a column i of $\boldsymbol{N}_2(:, i)$ into a $n_L + n_C$ square zero matrix in row and column i

$$\boldsymbol{K}_i = \begin{bmatrix} & \boldsymbol{O} \quad \boldsymbol{N}_2(:, i) \quad \boldsymbol{O} \\ \boldsymbol{O} & \\ -\boldsymbol{N}_2^{\mathrm{T}}(:, i) & \\ \boldsymbol{O} & \end{bmatrix}.$$

The construction for $\boldsymbol{N}_3^{\mathrm{T}}$ goes along the same line. The functions H_i are taken such that ∇H_i is a canonical vector with ± 1 in position i and zeros otherwise. The first term for $\boldsymbol{y}$ follows from comparison with (6) and the last reads $\begin{bmatrix} & \boldsymbol{N}_4 \\ -\boldsymbol{N}_4^{\mathrm{T}} & \end{bmatrix} \boldsymbol{v}$. ◊

For voltage-current state coordinates, one is able to incorporate the resistors as a feedback between output y_i and input u_i [11]. For example a linear feedback that is a linear resistor with source V_i yields the additional relation $v_i = -k_i y_i + V_i$. It is easy to identify the feedback parameter k_i as the element value of the resistor. The negative sign is due to the symmetric sign convention of $\mathcal{CM}$. Such systems can be called *affine Hamiltonian system with feedback.* Taking feedback into account one runs into serious trouble with charge and flux as state coordinates because the output equations have to be expressed in these coordinates.

4. CONCLUSION

Electrical networks of reactive and resistive elements belong to the class of gradient systems. However, a more refined network model by extraction of resistors reveals a Poisson structure of the connecting multiport. The terminal resistors provide a feedback between some output and some input ports. Such a geometrical structure could be exploited for network synthesis or numerical simulation.

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