Data Processing based on Geometric Feature Detection

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Abstract— **We report on novel ideas to support the equipment of technical systems with cognitive capabilities on the software side. We represent a given data set by points in some Euclidean space and search for certain properties of associated geometric objects, that are characteristic for the given data set concerning various aspects.**

In this paper, our focus is on symmetry detection in the context of identifying traffic signs from visual data. Also, we present an algorithm which can be used to determine the direction of a sound source from head related impulse response (HRIR) data.

I. Introduction

Recently, it has become a key issue in engineering, to equip technical systems with cognitive capabilities, such as perception, recognition and even learning features. Here, we report on a class of algorithms developed in order to support this kind of achievements on the software side. For example, it might be used to effiently store and analyse large sets of audio as well as video data.

The algorithms are based on the idea, to identify specific geometric features of a point set embedded in an Euclidean n-space and describing a given data set. Possible geometric features to identify are symmetries, topological and metric properties up through manifold structures.

In this paper, we propose a class of this kind of algorithms, that are rather robust with respect to noise in the data set and efficient concerning the complexity. This is achieved by smoothing and compressing the given data set through a convexification procedure applied to the corresponding point set.

Here we present versions of the algorithm for symmetry detection of point sets in 2- and 3-space (Sections 2 and 3) as well as for determining metric properties, such as the volume of the convex hull of a point set (Section 4). For illustration, we consider visual data representing geometric objects in 2- and 3-space as well as head related impulse response audio data.

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II. Symmetry Detection in 2D

The main idea of our algorithms is to study not the whole point set $A = \{A_1, .., A_m\}$ $A_i \in \mathbb{R}^2$, but just a small subset $\tilde{A} \subset A$, which contains important information about the structure of A.

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A. Characteristic Subset

On the one hand it is essential that the subset $\tilde{A}$, which will be analyzed by our algorithm, is as small as possible to reduce the complexity. On the other hand $\tilde{A}$ should contain all important information about the point set A, or at least about the convex hull of the point set.

We define a subset $\tilde{A}$ of A which is called the set of characteristic vertices of A to achieve both goals. $\tilde{A}$ is supposed to represent the set of real vertices of the point set A. So $\tilde{A}$ is a subset of the set of extreme points A_K of A.

Definition 2.1: A point set A is called convex, if for every pair of points $x, y \in A$, every point on the line segment connecting x and y is in A.

Definition 2.2: The convex hull of a point set A is the smallest convex point set A_{con}, such that A is a subset of A_{con}.

Definition 2.3: An extreme point of a convex point set A is a point of A_{con} which does not lie in any open line segment connecting two points of A_{con}. The set of all extreme points of a point set A is called A_K.

The Quickhull-Algorithm (which is described in [6]) determines the convex hull and the set of extreme points A_K of every 2-dimensional, as well as 3-dimensional, point set.

If the data, which are described by the point set A, are exact, it is obvious to choose $\tilde{A} = A_K$. But in case of inaccuracies, there are some extreme points, which do not correspond to a characteristic vertex of the point set. The point set which desribes, for example, a traffic sign, which has (nearly) the shape of a regular 8-gon, will have in most cases more than 8 extreme points (see Figure 3).

To search for a subset $\tilde{A} \subset A_K$ which represent the real vertices we solve the following optimization problem, where $dcon \in \mathbb{R}$ is a positive integer.

$OP(A_K, dcon)$**:**

$$\min_{\tilde{A} \subseteq A_K} \sum_{w \in (A_K \setminus \tilde{A})} d(w, \tilde{A}_{con}).$$

with the constraints:

$$C1 : d(w, \tilde{A}_{con}) \leq dcon \qquad \forall w \in (A_K \setminus \tilde{A})$$

$$C2 : |\tilde{A}| = \min_{\tilde{A}_i \subseteq A_K \wedge \tilde{A}_i | C1} |\tilde{A}_i|.$$

where:

$$d(w, \tilde{A}_{con}) = \min_i |w - a_i| \quad a_i \in A_{con}.$$

An algorithm, which solves $OP(A_K, dcon)$, in an efficient way, can be found in [10]. This thesis also contain a detailed discussion of the parameter $dcon$.

Definition 2.4: A subset $\tilde{A} \subset A_K$ is called the set of characteristic ($dcon$-) vertices of A if $\tilde{A}$ solves the optimization problem $OP(A_K, dcon)$.

B. Symmetry Value

To decide whether the set of characteristic vertices $\tilde{A}$ of a point set A has the symmetry of a regular n-gon (n-symmetry), we define a symmetry value for every n:

Definition 2.5: The symmetry value $d_n(A)$ of a point set $A \subset \mathbb{R}^2$ concerning the n-symmetry ($n \in \mathbb{N}, n > 1$) is the distance between A and B_n, where B_n is generated by rotating A about a fixed rotation center M by an angle of $\frac{2\pi}{n}$ degrees.

Even if the point set A is not exactly n-symmetric, we define:

Definition 2.6: The rotation center M of $A = \{A_1, .., A_m\} \subset \mathbb{R}^2$ is defined as follows:

$$M := \frac{\sum_{i=1:m} A_i}{m}.$$

The distance between two point sets A and B could be measured in different ways. One method, which calculates the distance in consideration of all entries of A and B, is to determine a Minimal Perfect Matching (MPM) in a bigraph $G(A,B)$. It is only possible to use this method, if $|A| = |B|$ holds, which is no problem in our case, since B is generated by rotating A.

The complete weighted bigraph $G(A,B)$ is constructed in the following way. One of the corresponding partitions consists of the points $w_1, .., w_m \in A$ and the other one consists of the points $v_1, .., v_m \in B$.
The weight of an edge is the distance between the two vertices, assosiated to the edge.

The problem of finding a MPM in $G(A,B)$ can be formulated as a linear program $LP(U)$, where the entries of the mxm matrix U are defined to be:

$$u_{ij} = d(w_i, v_j).$$

$LP(U)$**:**

$$\min_x \sum_{i,j} u_{ij} x_{ij}$$

with the constraints:

$$\sum_j x_{ij} = 1 \quad \forall i$$

$$\sum_i x_{ij} = 1 \quad \forall j$$

$$x_{ij} \in \{0, 1\}.$$

Fig. 1. Traffic Sign

$LP(U)$ can be solved by the Hungarian Method [5], for instance.

To compare different symmetry values and to decide whether A is approximatively n-symmetric, we define a comparative value for every symmetry value:

Definition 2.7: The comparative value $\Delta^n_{i,...,j}(A)$ of a symmetry value $d_n(A)$ is defined as:

$$\Delta^n_{i,...,j}(A) = \max_{k \in \{i,...,j\} \setminus \{l \mid n \mod l = 0\}} \frac{d_n(A)}{d_k(A)} * 100.$$

Depending on the comparative value of the different symmetry values, our algorithm assignes an approximate n-symmetry to a point set A.

Definition 2.8: We assign an approximate n-symmetry to a point set A if and only if the comparative value $\Delta^n_{i,...,j}(A)$ is less than 25.

C. Example

We will illustrate the symmetry detection algorithm by the following example. The picture in figure 1 shows the traffic sign "STOP", which is (approximative) 8-symmetric (as well as 2- and 4-symmetric, but we are just interested in the symmetry of the highest order). A is a set of 2-dimensional points which consists of all red pixel in the picture (see Figure 2).

With the help of the Quickhull-Algorithm we determined the point set A_K, which is shown in figure 3. As one can see $|A_K| = 31 > 8$ includes more points than the number of vertices of the traffic sign. Hence it makes sense to choose $\tilde{A} \neq A_K$. The set of characteristic vertices ($dcon = 3$) is illustrated in figure 4.

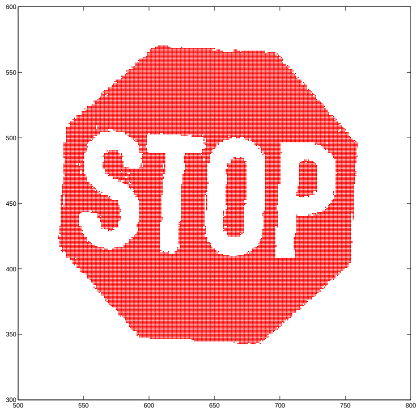

Fig. 2. Point set A

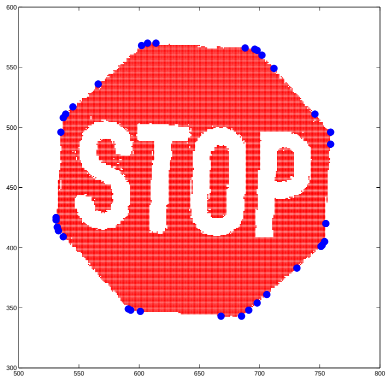

Fig. 3. The blue points are the set of extreme points A_K

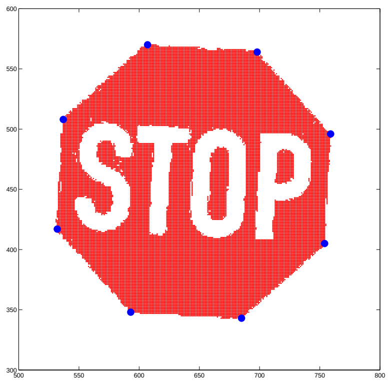

Fig. 4. The blue points are the set of characteristic vertices $\tilde{A}$

n	$d_n(\tilde{A})$
2	4.6
3	250.8
4	7.1
5	300.6
6	250.8
7	107.8
8	**6.5**
$\Delta^8_{2,..,8}$	**6.0**

Fig. 5. Symmetry Values, Comparative Value (n=8)

The symmetry values $d_1(\tilde{A}),..,d_8(\tilde{A})$ and the comparative value of $d_8(\tilde{A})$ are listed in figure 5. All other comparative values are not less than 25. So our algorithm assignes an 8-symmetry to the set of characteristic vertices of A.

III. Symmetry Detection in 3D

The algorithm, which is introduced in the chapter above, can be extended to find also rotation symmetries of the convex hull of a given point set $A = \{A_1,..,A_m\}$ $A_i \in \mathbb{R}^3$. In contrast to the 2-dimensional case, a point set $A \subset \mathbb{R}^3$ is called n-symmetric, if it is invariant under a rotation by $\frac{2\pi}{n}$ degrees about some rotation axis $g : x + \lambda v$ $x, v \in \mathbb{R}^3, \lambda \in \mathbb{R}$.

A. Characteristic Subset

The approach to find a subset $\tilde{A}$ of A, which is at the one hand as small as possible and has at the other hand similar characteristic symmetry properties as A is analog to the approach in the 2-dimensional case.

The first step is to determine the convex hull A_{con} and the set of extreme points A_K of A. This can also be done by the Quickhull-Algorithm [6].

Once we have the set of extreme points A_K, we want to find a (small) subset $\tilde{A}$, which represents the real vertices of the point set. This means we are searching for a point set $\tilde{A}$, the convex hull $\tilde{A}_{con}$ of which is similar to A_{con}. We define:

Definition 3.1: A subset $\tilde{A}$ of a 3-dimensional point set A is called the set of characteristic ($dcon-$)vertices of A, if $\tilde{A} := \{P_1,..,P_m\}$ fulfills the following conditions (where $dcon \in \mathbb{R}^+$ is a positive integer):

1)
$$\tilde{A} \subseteq A_K.$$

2)
$$d(w, \tilde{A}_{con}) \leq dcon \quad \forall w \in (A_K \setminus \tilde{A}).$$

3)
$$\max_{w \in (A_K \setminus (\tilde{A} \setminus P_i))} d(w, (\tilde{A} \setminus P_i)_{con}) > dcon \quad \forall i \in \{1,..,m\}.$$

The modified definition in comparison to the 2-dimensional case is neccessary due to the fact, that the algorithm which solves **OP** (see chapter 1) is too time consuming in the 3-dimensional case, since every extreme point has more than 2 neighbours (for details see [10]).

B. Rotation Axes

As mentioned above, we need to find possible rotation axes to decide whether a point set is (approximate) n-symmetric or not. As opposed to the 2-dimensional case it is a non-trivial problem to find such possible rotation axes. The following theorem gives an idea of how to determine them:

Theorem 3.1: Let be $A \subset \mathbb{R}^3$ a n-symmetric point set ($n \in \mathbb{N} \,\wedge\, n \geq 2$) with respect to the rotation axis g, A_K the set of extreme points of A, A_M the set of geometric centers of all faces of A_{con} and A_S the set of all midpoints of the edges of the convex hull. Furthermore let M be defined as in Definition 2.6. Then it holds that:

$$M \in g \quad \wedge \quad |g \cap (A_K \cup A_M \cup A_S)| = 2.$$

Proof:
See [10]

Let us keep in mind that we are not only interested in exact but also in approximative symmetries. Hence an axis g which satisfies the equation in theorem 3.1 within a noise-tolerance ε is also accepted to be a rotation axis.

So, we determine all possible rotation axes by considering every 2-tuple (P_1, P_2) of the points in $(A_K \cup A_M \cup A_S)$ and determining the distance $d(g_{P_1P_2}, M)$ of M to the line $g_{P_1P_2}$, connecting P_1 and P_2. If

$$d(g_{P_1P_2}, M) < \varepsilon \;,\;\; \varepsilon \in \mathbb{R}^+$$

holds, then

$$g := P_1 + \lambda(P_2 - P_1) \quad (\lambda \in \mathbb{R})$$

is a possible rotation axis.

C. Symmetry Value

Since a point set $A \subset \mathbb{R}^3$ is exactly n-symmetric, if A is invariant under a rotation by $\frac{2\pi}{n}$ degrees about an axis g we define the symmetry value analogously to Definition 2.5

Definition 3.2: The symmetry value $d_n(A)$ of a point set $A \subset \mathbb{R}^3$ concerning the n-symmetry ($n \in \mathbb{N}, n > 1$) is the distance between A and B_n, where B_n is generated by rotating A about a fixed rotation axis g by an angle of $\frac{2\pi}{n}$ degrees.

The comparative value of a symmetry value (Definition 2.7) as well as the assignment of symmetries (Definition 2.8) are defined in the same way as in the 2-dimensional case. Moreover, the distance between two point sets A and B is the value of the Minimal Perfect Matching in the bigraph $G(A,B)$, which is constructed identically to the bigraph in the 2-dimensional case.

D. Example

We want to analyse a point set $A \subset \mathbb{R}^3$ ($|A| = 10338$) which represents the surface of a polyhedra, which is 5-symmetric with respect to a certain rotation axis (see Figure 6). We determined the convex hull of A and computed the set of extreme points A_K (see Figure 7; $|A_K| = 197$) of A.

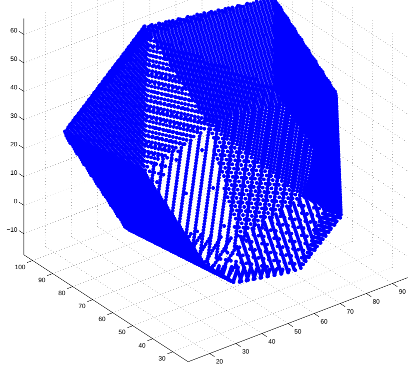

Fig. 6. Point Set A

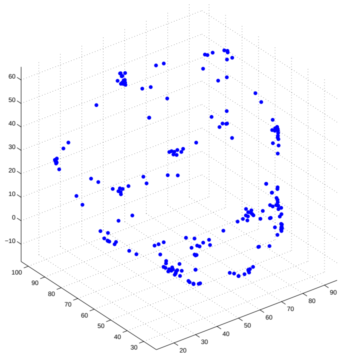

Fig. 7. Extreme Points A_K

Our algorithm found a subset $\tilde{A}$ of characteristic vertices and searched for possible rotation axes. The result is the point set $\tilde{A}$ in Figure 8 ($|\tilde{A}| = 16$) and the rotation axis g, which is also shown in Figure 8. The symmetry values, as well as the comparative value of d_5 , concerning rotation about g are listed in Figure 9. Accordingly, a 5-symmetry is assigned to A.

IV. Analysis of the Structure of Audio-Data

The main idea of both algorithms introduced in the sections above, is to analyse the structure of a point set A by studying not the whole point set, but just a characteristic subset $\tilde{A} \subset A$. This procedure can be used not only to identify approximate symmetries. As another application we want to illustrate how it can be used to analyse head related impulse response data (HRIR data) [3].

We use HRIR data from the CIPIC HRTF DATABASE [2], which is a public-domain database of high-spatial-resolution HRTF (head related transfer functions) measurments for 45

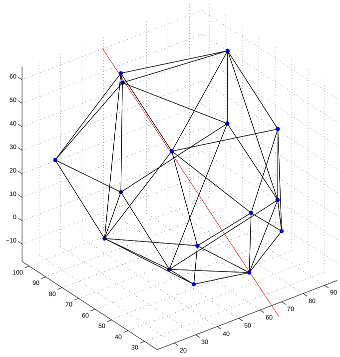

Fig. 8. $\tilde{A}$, g

n	$d_n(\tilde{A})$
2	13.09
3	12.47
4	9.05
5	**0.79**
6	6.35
7	10.67
8	13.52
$\Delta^5_{2,..,8}$	**12.44**

Fig. 9. Symmetry Values

different test persons. The database includes for every test person the HRIR at 25 different interaural-polar azimuths α and 50 different interaural-polar elevations β for both ears. The HRIR data could be used to determine the direction of a sound source, for instance. The HRIR data of different test persons at one fixed angle are not identical, because they depend on the personal anthropometry.

Hence the problem is to extract significant information from HRIR data [1], which is common for all test persons.

To this end, we first analysed the HRIR data corresponding to 10 test persons. For each ear and for any of the 25 available values for the azimuth $\alpha \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ we constructed the point set $A \subset \mathbb{R}^3$ as follows:

$$A = \{(\beta_9, t_{20}, HRIR(\alpha, \beta_9, t_{20})), .., (\beta_{41}, t_{80}, HRIR(\alpha, \beta_{41}, t_{80}))\},$$

where $\beta_j = -\frac{\pi}{4} + (j-1) * \frac{\pi}{32}$ $(j = 9, .., 41)$ is a selection of 33 out of the 50 available elevations and $t_i = i * 0.01$ $(i = 20, .., 80)$ denote a sequence of times. Obviously the cardinality of A is $|A| = 33 * 61 = 2013$.

Moreover, we determined the convex hull A_{con} of A and identified a set of characteristic vertices $\tilde{A}$. In Figures 10 and 11, A and $\tilde{A}_{con}$ are displayed for one test person, one ear and one azimuth.

Next we introduced functions $V^L(\alpha)$ and $V^R(\alpha)$ which assign, to any available value of α, separately for the left and the right ear, respectively, the volume of $\tilde{A}_{con}(\alpha)$ averaged over the 10 test persons. Finally, we chose polynomicals $p^L(\alpha)$ and $p^R(\alpha)$ (Figure 12) of degree 5 which interpolate

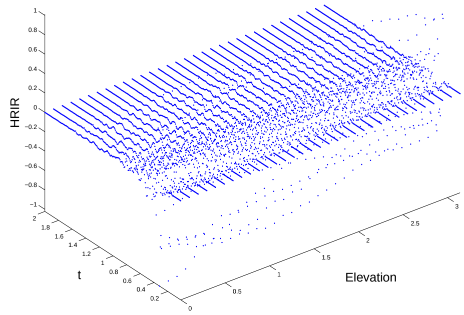

Fig. 10. Point Set B

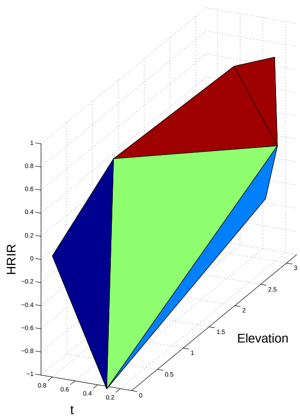

Fig. 11. $\tilde{A}_{con}$

(in the sense of a least square fit) the discrete functions $V^L(\alpha)$ and $V^R(\alpha)$, respectively, in the range $\alpha \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

It turns out that the polynomicals $p^L(\alpha)$ and $p^R(\alpha)$ represent quite well all the test persons included in the CIPIC HRTF DATABASE. So in particular, they can be used to solve the inverse problem: Given HRIR data for some person (or a robot having ears) at sequences of elevation β_j and time values t_i, as above, one can read off the corresponding azimuth α^* from the diagram in Figure 12. Of course one looks for an α^* that best fits the left- and right-ear curves in a certain sense.

The discrete points in Figure 13 represent results obtained in this way for a single test person. The vertical distance to the solid line describes the error of the computed α-values. It is within a range of 8 degrees. So the actual and the computed azimuths agree quite well. The correlation coefficient is:

$$\rho > 0.99.$$

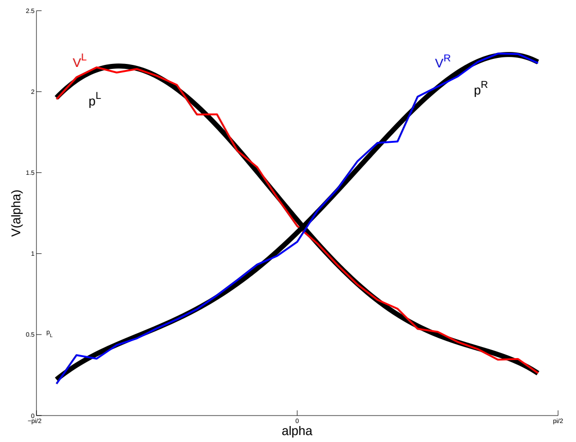

Fig. 12. Average functions V^L, V^R, Polynominals p^L, p^R

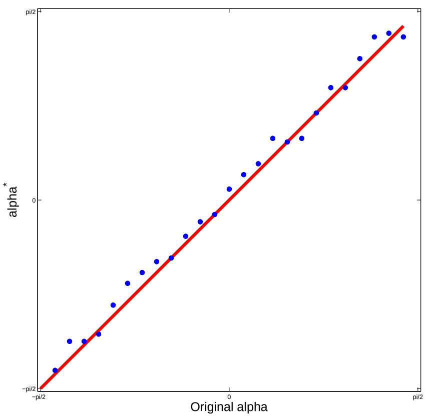

Fig. 13. Determination of azimuths

V. Conclusion

Compared to other symmetry detection algorithms ([4], [7], [8]) our algorithms have various advantages:

- Since usually $|\tilde{A}| \ll |A|$ holds, the complexity of our algorithms is significantly less than that of other algorithms.
- The sensitivity to noise is comparatively low and therefore the reliability of our algorithms is relatively high, since the point set A is regularized by convexification.
- In various cases, the symmetry of the convex hull of a point set A is a characteristic attribute of the data, which are represented by A, even if the whole point set A is not symmetric.

Our analysis of the HRIR data shows, that there are various applications where it is possible to find associated point sets A, which contain characteristic information about the given data set

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