ON MAXIMIZING THE SUM NETWORK MISO BROADCAST CAPACITY

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ABSTRACT

Recently, it has been shown that *dirty paper coding* (DPC) achieves the sum rate capacity of the Gaussian *multi-user multiple-input single-output* (MU-MISO) broadcast channel of a *single isolated* cell. However, when considering a multi-cell scenario, i.e., a cellular network, the optimal strategy to maximize the sum rate capacity in each of the cells is still unknown. Nevertheless, based on a game-theoretic framework, DPC can be applied at each cell as a decentralized strategy in a cellular network, in order to maximize the *sum broadcast capacity of the network*. By treating the cells in the network as players in a strategic cooperative game, *simultaneous* iterative waterfilling can be performed, i.e., every cell computes its optimal beamforming vectors according to DPC and by considering the intercell interference generated in the previous iteration. At each iteration the beamforming vectors for each user in each cell are updated with the gradient projection algorithm in order to maximize the sum network broadcast capacity. The algorithm is repeated until it converges, i.e., a local maximum is achieved. This theoretic result approaches the maximum rate that can be transmitted in the downlink of a network. Additionally, in order to introduce some fairness into the network, we consider in a similar way as the previous problem, the task of minimizing the sum of the mean square errors of all the users in the network.

1. INTRODUCTION

Consider a Gaussian *multi-user multiple-input single-output* (MU-MISO) broadcast channel of a *single isolated* cell with M transmit antennas with K users. Lately, it has been shown that *dirty paper coding* (DPC) achieves the sum rate capacity of this broadcast channel [1] and moreover, also its capacity region [2]. Now let us consider a cellular network composed of C cells sharing the same physical resources, i.e., time and bandwidth. The main difference between such a cellular system and a non-cellular system is the *intercell interference* [3, 4]. Thus, in a cellular downlink scenario the performance of a cell does not depend solely on its own transmit strategy but also on the transmit strategies of the other cells in the network. The sum cell capacity achieved by a given cell depends on its own decision, i.e., its set of selected beamforming vectors, and on the decisions of the other cells in the network, i.e., the set of selected beamforming vectors in the other cells, which produce the intercell interference.

However, based on a game-theoretic framework and allowing no cooperation between the cells, DPC can be applied at each cell of the network, such that each cell tries to maximizes its sum cell capacity with a transmit power constraint and based on the previous intercell interference. This *multi-objective optimization problem* (MOP) can be seen as a competitive maximization, since the performance and the decisions of the cells are mutually coupled. Due to this competitive nature of the multicell context we need to adopt an iterative algorithm [5]. By treating the C cells as players in a *non-cooperative* game, *simultaneous* iterative waterfilling can be performed, i.e., every cell computes its optimal beamforming vectors according to DPC and by considering the intercell interference generated in the previous iteration [3]. The algorithm is repeated until it converges, i.e., a *Nash equlibrium* is found. Nevertheless, the competitive nature of the cellular system does not guarantee in general the convergence of such an iterative scheme [5]. In this non-cooperative game, the players, i.e., the base stations, must know the channels from the base station to the users in its own cell and the intercell interference experienced in the previous iteration at each of the users in its own cell.

In this work, we take a different and rather theoretical approach by not focusing on the multi-objective problem discussed above but actually on a single objective problem: the maximization of the *sum broadcast capacity of the network* with a power constraint per base station. To this end, we allow a *partial* cooperation between the cells. We consider the cells in the network as players in a *strategic cooperative* game instead of the non-cooperative game described above. In the proposed approach, the beamforming vectors in each cell are first computed with DPC and based on the intercell interference generated with the beamforming vectors from the other cells in the previous iteration. Contrary to the approach described above, the beamforming vectors are afterwards updated such that we maximize the sum network broadcast capacity. We still need to adopt an iterative algorithm, for which we employ the gradient projection algorithm. The algorithm is repeated until a local optimum is achieved. In this cooperative game, we assume that each player (base station) must know the channel from its base station to the users in its own cell and to the users in the other cells. Additionally, we assume that the base station knows the intercell interference experienced in the previous iteration at each of the users in its own cell. This can be achieved by having a *remote central processor* (RCP) [6], which knows all the channels from every base station to all the users in the network. In the RCP, the beamforming vectors of all the cells in the network are computed and then distributed to the base stations, instead of performing the algorithm iteratively over the whole network. We do not assume that the symbols, to be transmitted to the users, are known by the RCP and therefore, in this sense we refer that the proposed scheme has *partial* cooperation between the players.

With this approach and cost function we are interested in computing the maximum rate that can be transmitted in the downlink of all the cells of a cellular network. As an upper bound of the sum network broadcast capacity, we take the sum of the maximum sum capacity of each cell in the network achieved without intercell interference, i.e. as if each cell were *isolated*. As a lower bound, we have the strategic cooperative game described above, where the sum network broadcast capacity is maximized. The proposed scheme serves as a lower bound since it has not been proved that DPC is the optimum strategy in this context. However, we show that the lower bound comes close to the *unachievable* upper bound described above, and hence, comes also close to the maximum sum network capacity that can be achieved in the downlink of the network.

Additionally, in order to introduce some fairness into the system we also consider a scheme which minimizes the sum *mean square error* (MSE) of all the users in the network, i.e., the *sum network MSE* in the downlink of the network.

To this end, this paper is organized as follows. In section 2, the cellular network model is discussed. In Section 3 we discuss the problem of optimizing a cellular network. We treat the maximization of the sum network broadcast capacity in Section 4, while in Section 5 we undertake the task of minimizing the sum MSE of all the users in the network. Afterwards, we present simulation results and a comparison in Section 6. We conclude the paper with a summary and comments about the discussed topic in Section 7.

2. CELLULAR NETWORK MODEL

We consider a Gaussian flat-fading broadcast channel of a cellular network consisting of C cells (and base stations). For convenience, we model the shape of each cell as an hexagon [7]. We assume sectorization throughout the network, such that three base stations are co-located at the one position in order to form a *site* as shown in Fig. 1, where each dot represents three co-located base stations. The distance between adjacent sites is denoted by *intersite distance* (ISD).

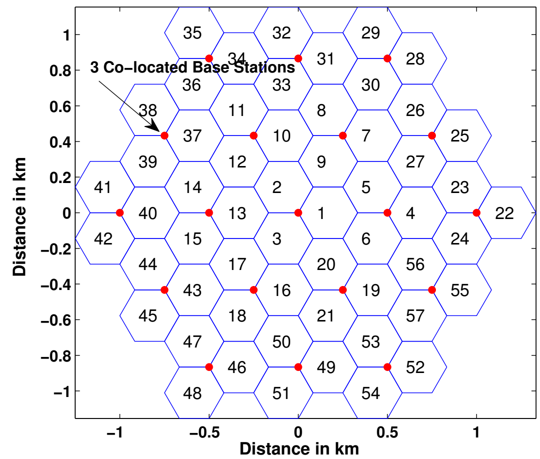

Fig. 1. Cellular Network

The base station at each cell has M_c transmit antennas and there are K_c single-antenna users per cell. The received signal at a given time t[1] of user $k \in \{1, 2, \cdots, K_c\} = \mathcal{K}_c$ in cell $c \in \{1, 2, \cdots, C\} = \mathcal{C}$ is given by

$$y_{c,k} = \mathbf{h}_{c,c,k}^{\mathrm{T}} \mathbf{p}_{c,k} x_{c,k} + \sum_{(c',k') \neq (c,k)} \mathbf{h}_{c',c,k}^{\mathrm{T}} \mathbf{p}_{c',k'} x_{c',k'} + n_{c,k}, \quad (1)$$

where $\mathbf{h}_{c',c,k} \in \mathbb{C}^{M_{c'}}$ is the channel vector from the base station at cell c' to the user k at cell c, $\mathbf{p}_{c',k'} \in \mathbb{C}^{M_{c'}}$ is the beamforming vector for user k' in cell c', $x_{c',k'}$ is the transmitted signal for user k' in cell c', and $n_{c,k}$ is the additive white Gaussian noise at the receiver of user k in cell c. Additionally, we have that $n_{c,k}$ are zero-mean complex Gaussian random variables with variance $\mathrm{E}[|n_{c,k}|^2] = \sigma_n^2$ and that $\mathrm{E}[|x_{c,k}|^2] = 1 \; \forall c, k$.

The channel vector $\mathbf{h}_{c',c,k}$ is given by

$$\mathbf{h}_{c',c,k} = \sqrt{\rho_{c',c,k}} \cdot \mathbf{g}_{c',c,k}, \quad (2)$$

where $\mathbf{g}_{c',c,k} \in \mathbb{C}^{M_{c'}} \; \forall \, k, c$ are zero-mean unit-variance complex Gaussian random variables and $\rho_{c',c,k}$ represents the average gain of the channel from base station c' to user k in cell c, due to the antenna gain, path-gain and the log-normal shadowing. In detail $\rho_{c',c,k}$ is expressed as

$$\rho_{c',c,k} = G_{c',c,k} \cdot \left(\frac{\lambda/1\mathrm{m}}{4\pi}\right)^2 \cdot \left(\frac{d_{c',c,k}}{1\mathrm{m}}\right)^{-\kappa} \cdot 10^{\frac{\xi_{c',c,k}}{10}}, \quad (3)$$

[1] For ease of notation, we have omitted the time index t in the following.

where κ is the pathloss exponent [7] and $G_{c',c,k}$ is the antenna gain which is a function of the angle of departure of the signal from base station c' to user k in cell c [4]. Let us recall that we assume sectorization throughout the network and hence, we need to have a correctly shaped beam pattern in order to enable approximately the same average receiver power on the whole cell edge. Therefore, the antenna gain depends on the position (angle) of the user k at cell c with respect to the boresight of the antenna array of the base station c' [4]. Additionally, $d_{c',c,k}$ is the distance between the base station c' and user k in cell c and, the $\xi_{c',c,k}$ models the effects of shadowing and we assume it is a zero-mean Gaussian random variable with variance σ_{s}^2. Moreover, we have that λ represents the wavelength of the carrier, and here we will consider $\lambda = 0.15$ m. Note that when $c' = c$, we are considering the channel from the base station c which is serving user k in the cell c.

To simulate one network realization we randomly place K_c users, $c \in \mathcal{C}$, in each of the C cells of the network. The users in each cell are assumed to uniformly distributed over the hexagonal cell [4].

3. OPTIMIZING THE CELLULAR NETWORK

In this section we consider as figure of merit the *sum network broadcast capacity*. Although it might seem as an unconventional performance measure, we consider this metric in order to gain insight into the maximum capacity that can be achieved in the downlink of all the cells in a given network. The sum network broadcast capacity C_{N} is given by

$$C_{\text{N}} = \sum_{c\in\mathcal{C}} \sum_{k\in\mathcal{K}_c} I(x_{c,k}, y_{c,k}), \tag{4}$$

where $I(x_{c,k}, y_{c,k})$ is the mutual information between $x_{c,k}$ and $y_{c,k}$.

If the channels and the intercell interference power experienced by the all the users in a cell are known at their respective base station, we can perform DPC as a transmit strategy to the users. With DPC, the mutual information $I(x_{c,k}, y_{c,k})$ between $x_{c,k}$ and $y_{c,k}$ is given by (5)-(6) at the bottom of this page. Note that $A_{c,k} = B_{c,k} + |\mathbf{h}^T_{c,c,k} \cdot \mathbf{p}_{c,k}|^2$.

3.1. Precoding Order of the Users

Note that in (5), the *intracell interference* for user k in cell c is

$$\sum_{k'>k} \left|\mathbf{h}^{\text{T}}_{c,c,k} \cdot \mathbf{p}_{c,k'}\right|^2, \tag{7}$$

while the *intercell interference* experienced by user k in cell c is

$$\begin{aligned}\sum_{c'\neq c, k'\in\mathcal{K}_c} \left|\mathbf{h}^{\text{T}}_{c',c,k}\mathbf{p}_{c',k'}\right|^2 &= \sum_{c'\in\{\mathcal{C}\backslash c\}} \mathbf{h}^{\text{T}}_{c',c,k} \left(\sum_{k'\in\mathcal{K}'_c} \mathbf{p}_{c',k'}\mathbf{p}^{\text{H}}_{c',k'}\right) \mathbf{h}^*_{c',c,k} \\ &= \sum_{c'\in\{\mathcal{C}\backslash c\}} \mathbf{h}^{\text{T}}_{c',c,k} \cdot \mathbf{Q}_{c'} \cdot \mathbf{h}^*_{c',c,k},\end{aligned} \tag{8}$$

where $\mathbf{Q}_{c'}$ is the sum of the transmit covariance matrices of the users $k' \in \mathcal{K}'_c$ in cell c', i.e.,

$$\mathbf{Q}_{c'} = \sum_{k'\in\mathcal{K}'_c} \mathbf{p}_{c',k'}\mathbf{p}^{\text{H}}_{c',k'}. \tag{9}$$

From (8), we can see that for the users in cell c, the precoding order of the users employed by the DPC in the interfering cells $c' \in \{\mathcal{C}\backslash c\}$, is *irrelevant* in terms of sum capacity. Furthermore, for the intercell interference the specific beamforming vectors of the users in the interfering cells are also irrelevant. Given a set of selected beamforming vectors $\mathbf{p}_{c',k'}$ for $k' \in \mathcal{K}_{c'}$ decided upon by cell c', the intercell interference generated by base station c' on a user k in cell c is *independent* of the precoding order employed by the interfering cell c', since given the channel vector $\mathbf{h}_{c',c,k}$ what determines the intercell interference is actually $\mathbf{Q}_{c'}$. Additionally, it is also known that for achieving the sum cell capacity of a given cell of interest c, the precoding order of the users $k \in \mathcal{K}_c$ of the cell of interest is also *irrelevant*. The precoding order does not matter, since the sum capacity can be achieved with any arbitrary ordering, which can be shown by means of the uplink-downlink duality [8]. Thus, when performing DPC in each cell of the network, the precoding order of the users in each cell is *irrelevant* for maximing either the sum cell capacity in each cell or the sum network broadcast capacity.

$$\begin{aligned} I(x_{c,k}, y_{c,k}) &= \log\left(1 + \frac{\left|\mathbf{h}^{\text{T}}_{c,c,k} \cdot \mathbf{p}_{c,k}\right|^2}{\sigma_n^2 + \sum_{k'>k} \left|\mathbf{h}^{\text{T}}_{c,c,k} \cdot \mathbf{p}_{c,k'}\right|^2 + \sum_{c'\neq c, k'\in\mathcal{K}_c} \left|\mathbf{h}^{\text{T}}_{c',c,k} \cdot \mathbf{p}_{c',k'}\right|^2}\right) \qquad (5) \\ &= \log\left(\frac{\sigma_n^2 + \sum_{k'\geq k} \left|\mathbf{h}^{\text{T}}_{c,c,k} \cdot \mathbf{p}_{c,k'}\right|^2 + \sum_{c'\neq c, k'\in\mathcal{K}_c} \left|\mathbf{h}^{\text{T}}_{c',c,k} \cdot \mathbf{p}_{c',k'}\right|^2}{\sigma_n^2 + \sum_{k'>k} \left|\mathbf{h}^{\text{T}}_{c,c,k} \cdot \mathbf{p}_{c,k'}\right|^2 + \sum_{c'\neq c, k'\in\mathcal{K}'_c} \left|\mathbf{h}^{\text{T}}_{c',c,k} \cdot \mathbf{p}_{c',k'}\right|^2}\right) = \log\left(\frac{A_{c,k}}{B_{c,k}}\right). \qquad (6) \end{aligned}$$

3.2. Non-cooperative Game

Before we discuss the proposed approaches, let us briefly review the non-cooperative game commented in the introduction. Given a power constraint per base station, we consider the multi-objective problem of maximizing the sum cell capacity of each cell in the network without any cooperation between the cells, i.e., we have the following non-cooperative game:

$$(\mathcal{G}_0): \quad \begin{array}{ll} \underset{\mathbf{p}_c}{\text{maximize}} & \sum_{k\in\mathcal{K}_c} I(x_{c,k}, y_{c,k}) \\ \text{subject to} & \sum_{k\in\mathcal{K}_c} \|\mathbf{p}_{c,k}\|_2^2 = \text{tr}\ \mathbf{Q}_c \leq P_\text{T} \end{array} \quad \forall c \in \mathcal{C}, \tag{10}$$

where $\mathbf{p}_c \triangleq (\mathbf{p}_{c,k})_{k\in\mathcal{K}_c}$, i.e., the beamforming vectors of all the users in cell c. For this game, we assume that we perform DPC at each cell as the transmit strategy. Due to the competitive nature of the multicell scenario an iterative algorithm must be employed in order to solve the problem. To this end we employ the gradient projection algorithm. At each iteration the beamforming vectors are computed based on the intercell interference generated with the beamforming vectors from the interfering cells in the previous iteration. Afterwards, the beamforming vectors in each cell are updated such that the sum cell capacity of each cell is maximized. The algorithm is repeated until a Nash equilibrium is obtained. Note then that every cell makes the best decision for selecting its set of beamforming vectors based on the decision of the interfering cells.

3.3. Cooperative Games

The existence of several users, each with its own channel quality in a cell, makes the definition of a global measure of the system quality very difficult. Furthermore, in a cellular network, not only do we have the multi-user scenario in each cell but also a multi-cell scenario, making it then, even more difficult to define a global performance metric of the network quality. As a consequence, a variety of design criteria have to be discussed. In this work we consider two cooperative games with different performance measures for optimizing a cellular network:

Game 1: Maximization of the sum network broadcast capacity with a power constraint per base station, i.e.,

$$(\mathcal{G}_1): \quad \begin{array}{ll} \underset{\mathbf{p}}{\text{maximize}} & C_\text{N} = \sum_{c\in\mathcal{C}} \sum_{k\in\mathcal{K}_c} I(x_{c,k}, y_{c,k}) \\ \text{subject to} & \sum_{k\in\mathcal{K}_c} \|\mathbf{p}_{c,k}\|_2^2 = \text{tr}\ \mathbf{Q}_c \leq P_\text{T} \end{array} \quad \forall c \in \mathcal{C}, \tag{11}$$

where $\mathbf{p} \triangleq (\mathbf{p}_{c,k})_{(c,k)\in\mathcal{C}\times\mathcal{K}_c}$, i.e., the beamforming vectors of all the users in the network. For this game we will assume that we will also perform DPC in each cell but with partial cooperation among the cells. As discussed in Section 3.1, when maximizing the sum network broadcast capacity, the precoding of the users of the DPC is irrelevant.

Game 2: Minimization of the sum of the mean square errors of the users in the network with a power constraint per base station.

$$(\mathcal{G}_2): \quad \begin{array}{ll} \underset{\mathbf{p}}{\text{minimize}} & \epsilon_\text{N} = \sum_{(c,k)\in\mathcal{C}\times\mathcal{K}_c} \epsilon_{c,k} \\ \text{subject to} & \sum_{k\in\mathcal{K}_c} \|\mathbf{p}_{c,k}\|_2^2 = \text{tr}\ \mathbf{Q}_c \leq P_\text{T} \end{array} \quad \forall c \in \mathcal{C}, \tag{12}$$

where $\epsilon_{c,k}$ is the MSE of user k in cell c and ϵ_N denotes the sum of the MSE of all the users in the network. Here we assume that we employ as before DPC as the transmit strategy, which can be implemented in practice with *Tomlison-Harashima Precoding* (THP). Contrary to game $\mathcal{G}_1$, the order of the users does play a role for minimizing the sum MSE. In this case, we assume a heuristic order of the users, where the user with the weakest channel would be precoded first and the strongest user in the cell last. This is to avoid the high intracell interference that would be caused by the weakest users who require more power. By precoding the weakest user first, we are able to remove its interference on the other users precoded afterwards through DPC.

Since the capactiy region or the MSE region are not convex, we aim to finding local maximums and local minimas for the cooperative games $\mathcal{G}_1$ and $\mathcal{G}_2$, respectively. Due to competitive nature of the multicell context we still need to adopt an iterative algorithm, for which we will use the gradient projection algorithm.

4. MAXIMIZING THE SUM NETWORK BROADCAST CAPACITY

As stated before, we assume the existence of a remote central processor (RCP) which knows all the channel vectors $\mathbf{h}_{c',c,k}$ $\forall\ c', c, k$ but the symbols to be transmitted to the users of a given base station are only known at each respective base station. If the symbols were known at the RCP, then we would have a *macrocell*, where a centralized DPC achieves the maximum sum network capacity [8, 9].

We are interested in computing the beamforming vectors in order to maximize the sum network broadcast capacity with P_T available power per base station. We iteratively solve $\mathcal{G}_1$ with the projection gradient algorithm. Let us now compute the gradient of C_N with respect to each beamforming vector $\mathbf{p}_{c,k}$, i.e.,

$$\nabla_{\mathbf{p}_{c,k}} C_\text{N} = \sum_{(c',k')\in\mathcal{C}\times\mathcal{K}_c} \nabla_{\mathbf{p}_{c,k}} I(x_{c',k'}, y_{c',k'}), \tag{13}$$

To this end we need the following gradients:

1. $\nabla_{\mathbf{p}_{c,k}} I(x_{c,k}, y_{c,k})$, where $I(x_{c,k}, y_{c,k})$ corresponds to its own tranmission of user k at cell c, i.e.,

$$\nabla_{\mathbf{p}_{c,k}} I(x_{c,k}, y_{c,k}) = \mathbf{h}^*_{c,c,k} \cdot \mathbf{h}^{\mathrm{T}}_{c,c,k} \cdot \mathbf{p}_{c,k} \cdot \left(\frac{1}{A_{c,k}}\right).$$

2. $\nabla_{\mathbf{p}_{c,k}} I(x_{c,k'}, y_{c,k'})$ with $k' \neq k$, where $I(x_{c,k'}, y_{c,k'})$ corresponds to the transmissions of the other users in the same cell c where user k lies, i.e., $\nabla_{\mathbf{p}_{c,k}} I(x_{c,k'}, y_{c,k'}) =$

$$\begin{cases} \mathbf{h}^*_{c,c,k'} \mathbf{h}^{\mathrm{T}}_{c,c,k'} \mathbf{p}_{c,k} \cdot \left(\frac{1}{A_{c,k'}} - \frac{1}{B_{c,k'}}\right) & \text{for} \quad k > k', \\ 0 & \text{for} \quad k < k'. \end{cases}$$

3. $\nabla_{\mathbf{p}_{c,k}} I(x_{c',k'}, y_{c',k'})$, with $c' \neq c$, where $I(x_{c',k'}, y_{c',k'})$ corresponds to the transmissions of the other users in the other cells, i.e., $\nabla_{\mathbf{p}_{c,k}} I(x_{c',k'}, y_{c',k'}) =$

$$\mathbf{h}^*_{c,c',k'} \cdot \mathbf{h}^{\mathrm{T}}_{c,c',k'} \cdot \mathbf{p}_{c,k} \cdot \left(\frac{1}{A_{c',k'}} - \frac{1}{B_{c',k'}}\right).$$

With an initial set of beamforming vectors, we iteratively perform a gradient update at step $l+1$ as

$$\tilde{\mathbf{p}}^{(l+1)}_{c,k} = \mathbf{p}^{(l)}_{c,k} + \gamma \cdot \nabla_{\mathbf{p}_{c,k}} C_{\mathrm{N}} \ \ \forall\, k, c. \tag{14}$$

The employed step size γ is

$$\gamma = \sqrt{\frac{C \cdot P_{\mathrm{T}}}{\sum\limits_{(c,k)\in\mathcal{C}\times\mathcal{K}_c} \|\nabla_{\mathbf{p}_{c,k}} C_{\mathrm{N}}\|_2^2}} \cdot \frac{1}{d+1}, \tag{15}$$

where d may be increased, in order to reduce the step size adaptively and force convergence. Now, the new updates for the beamforming vectors $\mathbf{p}^{(l+1)}_{c,k}$ are calculated by projecting $\tilde{\mathbf{p}}^{(l+1)}_{c,k}$ on the feasible set $\mathcal{P}$ given by the constraints

$$\sum_{k\in\mathcal{K}_c} \|\mathbf{p}_{c,k}\|_2^2 \ \leq\ P_{\mathrm{T}}\ \forall c, \tag{16}$$

i.e.,

$$\mathbf{p}^{(l+1)}_{c,k} = \left[\tilde{\mathbf{p}}^{(l+1)}_{c,k}\right]_{\mathcal{P}}. \tag{17}$$

To this end we need to solve the following optimization problem

$$\begin{aligned} \mathbf{p}^{(l+1)}_{c,k} \quad &= \quad \underset{\mathbf{p}'_{c,k}}{\operatorname{argmin}} \, \|\mathbf{p}'_{c,k} - \tilde{\mathbf{p}}^{(l+1)}_{c,k}\|_2^2 \\ &\quad\ \text{s.t.} \sum_{k\in\mathcal{K}_c} \|\mathbf{p}'_{c,k}\|_2^2 \leq P_{\mathrm{T}} \quad \forall c. \end{aligned} \tag{18}$$

The solution of this problem is obvious: if $\sum \|\tilde{\mathbf{p}}^{(l+1)}_{c,k}\|_2^2 > P_{\mathrm{T}}$ for a given c, then we normalize the $\tilde{\mathbf{p}}^{(l+1)}_{c,k}$ for $k \in \mathcal{K}_c$ in cell c, to satisfy the power constraint:

$$\mathbf{p}^{(l+1)}_{c,k} \longleftarrow \sqrt{\frac{P_{\mathrm{T}}}{\sum_{k\in\mathcal{K}_c} \|\tilde{\mathbf{p}}_{c,k}\|_2^2}} \cdot \tilde{\mathbf{p}}^{(l+1)}_{c,k}, \tag{19}$$

otherwise,

$$\mathbf{p}^{(l+1)}_{c,k} \longleftarrow \tilde{\mathbf{p}}^{(l+1)}_{c,k}. \tag{20}$$

5. MINIMIZING THE SUM NETWORK MSE

In the literature, many researchers considered the maximization of the sum capacity as metric of the overall efficiency of the system. However, due to the very different SINR's of the substreams transmitted from a given base station, this transmission policy requires careful bit allocation to match each subchannel's capacity and achieve a prescribed *bit error rate* (BER). Bit allocation not only increases the coding/decoding complexity but it is also inherently capacity reducing because of the finite constellation granularity and the *shaping loss*. Alternative design criteria like the minimization of the sum *mean square error* (MSE) or the minimization of the maximum MSE, seem to be more efficient in practical systems, since they allocate more power to the channel for high SNR, and additionally are independent of the constellation size.

Hence, in this section we focus on the minimization of the sum of the MSE of all the users in the networks, i.e., game $\mathcal{G}_2$ given in (12). With such a performance metric we take into account fairness, which is in contrast to game $\mathcal{G}_1$ which does not consider at all any fairness. Note that (6) can also be written as

$$I(x_{c,k}, y_{c,k}) = -\log\left(\epsilon_{c,k}\right), \tag{21}$$

where

$$\epsilon_{c,k} = \frac{B_{c,k}}{A_{c,k}}, \tag{22}$$

is the *mean square error* (MSE) of the transmission of user k in cell c. Thus, the problem of maximizing the network capacity is equivalent to the minimization of the geometric mean of the MSE's, i.e. game $\mathcal{G}_1$ minimizes $\prod_{(c,k)}^{\mathcal{C}\times\mathcal{K}_c} \epsilon_{c,k}$. This strategy clearly could lead to unfair rate allocations, since the users with the strongest channels will be favorized. The MSE criteria can be therofore considered as a kind of compromise between *overall efficiency* and *total fairness* between users, as we will observe in the simulations. So now let us consider $\mathcal{G}_2$, the minimization of the sum MSE of the users in the network:

$$\min \epsilon_{\mathrm{N}} \quad \text{s.t.} \sum_{k\in\mathcal{K}_c} \|\mathbf{p}_{c,k}\|_2^2 \leq P_{\mathrm{T}}\ \forall c, \tag{23}$$

with $\epsilon_{\mathrm{N}} = \sum\limits_{(c,k)\in\mathcal{C}\times\mathcal{K}_c} \epsilon_{c,k}$ as the sum network MSE.

To this end, the beamforming vectors $\mathbf{p}_{c,k}\ \forall c, k$ will now be computed with the gradient projection algorithm, but the updates of the beamforming vectors at each iteration are computed such that the sum network MSE is minimized. We iteratively perform a gradient update at step $l+1$ as

$$\tilde{\mathbf{p}}^{(l+1)}_{c,k} = \mathbf{p}^{(l)}_{c,k} - \gamma \cdot \nabla_{\mathbf{p}_{c,k}} \epsilon_{\mathrm{N}} \ \ \forall\, k, c. \tag{24}$$

Similarly as it was done for game $\mathcal{G}_1$ in Section 4, to compute the gradient $\nabla_{\mathbf{p}_{c,k}} \epsilon_{\mathrm{N}}$ we need the following gradients:

1. $\nabla_{\mathbf{p}_{c,k}} \epsilon_{c,k}$, where $\epsilon_{c,k}$ corresponds to the MSE of user k's tranmission at its own cell c, i.e.,

$$\nabla_{\mathbf{p}_{c,k}} \epsilon_{c,k} = \mathbf{h}^*_{c,c,k} \cdot \mathbf{h}^{\mathrm{T}}_{c,c,k} \cdot \mathbf{p}_{c,k} \left(-\frac{B_{c,k}}{A^2_{c,k}} \right).$$

2. $\nabla_{\mathbf{p}_{c,k}} \epsilon_{c,k'}$ with $k' \neq k$, where $\epsilon_{c,k'}$ corresponds to the MSE's of the transmissions of the other users in the same cell c where user k lies, i.e., $\nabla_{\mathbf{p}_{c,k}} \epsilon_{c,k'} =$

$$\begin{cases} \mathbf{h}^*_{c,c,k'} \mathbf{h}^{\mathrm{T}}_{c,c,k'} \mathbf{p}_{c,k} \cdot \left(\frac{1-B_{c,k'}}{A^2_{c,k'}} \right) & \text{for} \quad k > k', \\ 0 & \text{for} \quad k < k'. \end{cases}$$

3. $\nabla_{\mathbf{p}_{c,k}} \epsilon_{c',k'}$, with $c' \neq c$, where $\epsilon_{c',k'}$ corresponds to the MSE's of the transmissions of the other users in the other cells, i.e.,

$$\nabla_{\mathbf{p}_{c,k}} \epsilon_{c',k'} = \mathbf{h}^*_{c,c',k'} \cdot \mathbf{h}^{\mathrm{T}}_{c,c',k'} \cdot \mathbf{p}_{c,k} \cdot \left(\frac{1 - B_{c',k'}}{A^2_{c',k'}} \right).$$

The beamforming vectors are then updated using (24) and (19) or (20) with γ given by (15).

6. SIMULATION RESULTS AND COMPARISON

Consider now an interference limited network with 57 sectorized cells, with an intersite distance of 0.500 km as shown in Fig. 1 with $P_{\mathrm{T}} = 20$ W for every cell in the network. We assume that $K_c = 4$ users $\forall c$ cells in the network, and so, we have 4 users uniformly distributed in each cell. Additionally, we assume that $M_c = 4$ antennas $\forall c$ base stations in the network and that the users experience flat independent Rayleigh fading over the antennas. The variance of the shadowing is given by $\sigma_{\mathrm{s}} = 8$ dB and the beam pattern employed by the antennas is the one described in [4].

For the following, let us consider three cases:

1. **Lower Bound Non-cooperative Game:** corresponds to the case where each cell tries to maximize its own sum capacity with DPC based on the intercell interference generated in the previous iteration, i.e., game $\mathcal{G}_0$ given in (10).

2. **Lower Bound Partial Cooperative Game:** corresponds to the proposed approach where all the cells *partially cooperate* in order to maximize the sum network broadcast capacity. In each cell we perform DPC based on the intercell interference generated in the previous iteration based on the partial cooperation among the players, i.e., game $\mathcal{G}_1$ given in (11).

3. **Upper Bound Cooperative Game:** corresponds to the case where each cell maximizes its own sum capacity with DPC without any intercell interference, i.e., assuming that each cell is isolated.

The first case represents a lower bound of the non-cooperative game $\mathcal{G}_0$, since it has not been shown that applying DPC in a decentralized way over the cells in the network is the optimal strategy for game $\mathcal{G}_0$. As for the cooperative game $\mathcal{G}_1$, applying DPC over all the cells with the partial cooperation[2] among the cells is not the optimal strategy for this game and hence, it represents a lower bound [10]. As for the last case, it is clear that this represents an upper bound to the partial cooperative game of $\mathcal{G}_1$. Note that this upper bound for a cellular network, where the cells inherently suffer intercell interference, is of course unachievable!

Based on these definitions, in Fig. 2 we show the average sum capacity per cell for 170 network realizations for cells 1-28 for the three bound mentioned above. As expected the smallest average sum cell capacity is achieved by the lower bound of the non-cooperative game, while the largest corresponds to the upper bound of the cooperative game. Nevertheless, notice that the lower bound of the cooperative game $\mathcal{G}_1$ is not far away from the *unachiavable* upper bound of $\mathcal{G}_1$. Although that the DPC is the optimal strategy for a single cell, it also seems that DPC is a viable strategy in the multi-cell context. Also notice that the performance of the DPC in game $\mathcal{G}_0$ is also not far away from the lower bound of game $\mathcal{G}_1$.

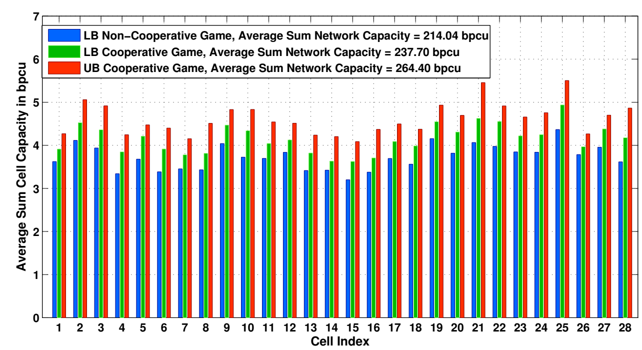

Fig. 2. Average Sum Capacity per Cell for 170 realizations

In order to observe the performance of the sum network broadcast capacity over several network realizations, Fig. 3 depicts the abovementioned bounds for 50 network realizations. Again it can be seen that at each realization and on average the proposed scheme, i.e. applying DPC in game $\mathcal{G}_1$, comes close to the unachievable upper bound of the cooperative game $\mathcal{G}_1$.

Now consider the results obtained with game $\mathcal{G}_2$, i.e., the minimization of the sum MSE of all the users in the network. To this end, let us consider the same simulation scenario described before, except that we know assume that there are 10

[2] i.e., we assume that a RCP knows the channels from every base station to every user in the network, but the symbols to be transmitted to the users are not known by the RCP.

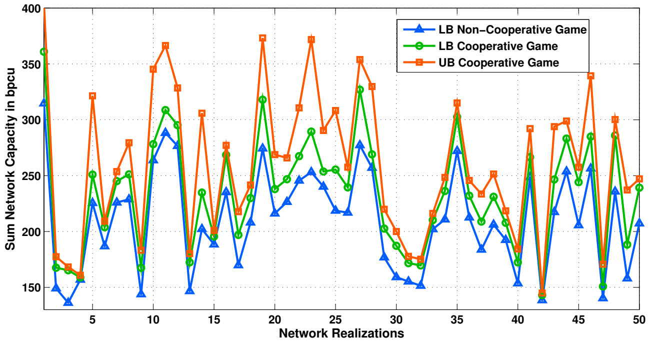

Fig. 3. Sum Network Capacity for Different Network Realizations

randomly uniformly distributed users in each cell of the network, i.e., $K_c = 10 \; \forall c$. In Fig. 4, we depict the *cumulative distribution function* (cdf) of the sum cell capacity achieved over several network realizations for the three bounds discussed previously and the sum cell capacity achieved with game $\mathcal{G}_2$. If is interesting to notice that the distribution with the smallest variance is the one corresponding to game $\mathcal{G}_2$. This is as expected since when minimizing the sum MSE, we introduce fairness and there is a less variation of the sum cell capacities.

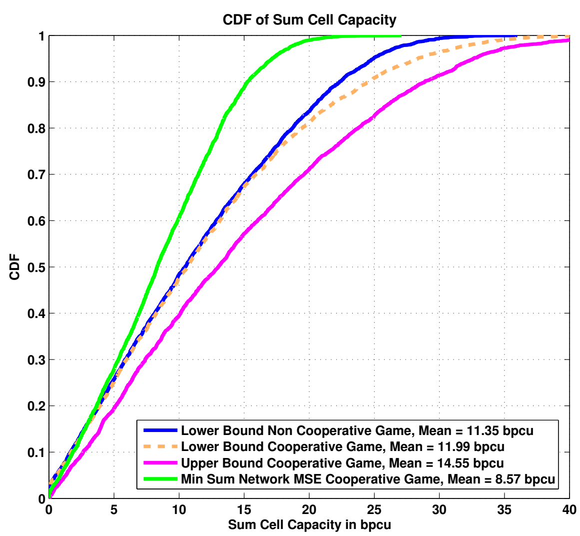

Fig. 4. CDF of the Sum Cell Broadcast Capacity

However, when looking into the region of the 5% value of the cdf as shown in Fig. 5 with the abcissa in the logarithmic scale, we can see that the approach of game $\mathcal{G}_2$ is better than the lower bound of the cooperative game $\mathcal{G}_1$ aiming at maximizing the sum network broadcast capacity. At 5%, the lower bound of game $\mathcal{G}_0$ and the lower bound of game $\mathcal{G}_1$ achieve both a sum cell capacity of about 0.54 bpcu, while when minimizing the sum MSE we achieve a sum cell capacity of 0.88 bpcu, which represents a gain of more than 60%. Furthermore, we have that the upper bound of the cooperative game $\mathcal{G}_1$ achieves a sum cell capacity of 1.31.

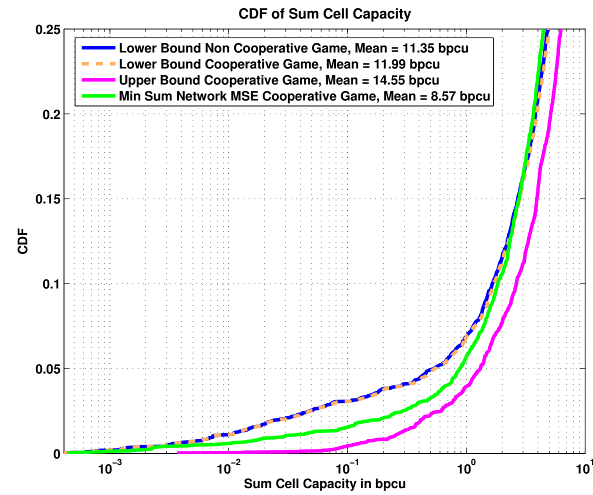

Fig. 5. 5% Region of the CDF of the Sum Cell Broadcast Capacity

Now let us focus now on the CDF of the sum MSE of the users in a cell for the three bounds discussed previously and game $\mathcal{G}_2$, as shown in Fig. 6. Taking as performance metric the sum MSE of the users in a cell, we have that the performance of game $\mathcal{G}_2$ is the best. This is the same simulation scenario described above with $K_c = 10$ user for each cell $c \in \mathcal{C}$ in the network with 4 transmit antennas at each base station. Note that performance with respect to this figure of merit of the lower bound of games $\mathcal{G}_0$ and $\mathcal{G}_1$ are basically the same.

7. SUMMARY AND FURTHER REMARKS

In this work, we have undertaken the task of optimizing a cellular network. As stated before, in a *multi-user* context and even more in a *multi-cell* context it is very difficult to define a meaningful performance measure of the system. On the one hand, we have considered the maximization of the sum network broadcast capacity, i.e. the maximum rate that can be achieved in the downlink in all of the cells of the network. On the other hand, we have also consider the minimization of the sum of the MSE's of all the users in the network to provide fairness. Due to the competitive nature of the multi-cell context, an iterative algorithm, i.e., the gradient projection algorithm, was employed to find a solution to the discussed games. However, due to the non-convexity of the capacity

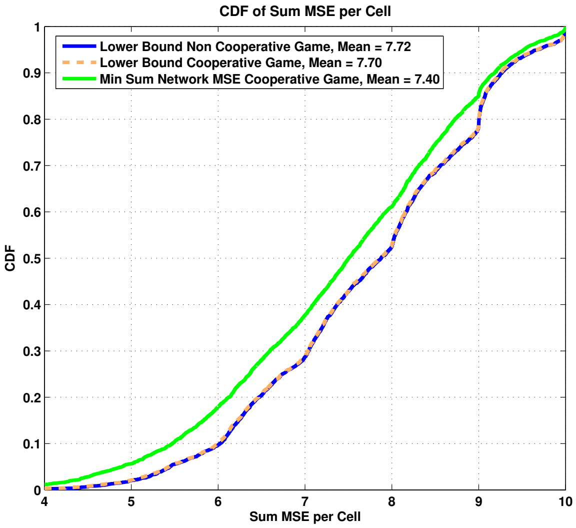

Fig. 6. CDF of the Sum MSE of the users per Cell

and the MSE region, the solutions we can achieve are just local optima.

We have considered the use of dirty paper coding as the transmit strategy employed by each of the cells of the network based on the intercell interference generated in the previous iteration. It was shown that the order of the users is irrelevant for the maximization of the sum cell capacity or the maximization of the sum network capacity, since the intercell interference depends on the transmit covariance matrices of the interfering cells, but not on the specific order or beamforming vectors of the users in the interfering cells.

Moreover, we could observe that the suboptimal solution to the partially cooperative game $\mathcal{G}_1$, i.e., the lower bound of the cooperative game $\mathcal{G}_1$, comes close to the unachievable upper bound of the isolated cell capacity. We recall that this suboptimal strategy is based on the employment of DPC in each of the cells of the network. Hence, considering DPC in the multi-cell context seems heuristically to be a viable approach. Let us recall that the performance of the investigated lower bound of the non-cooperative game $\mathcal{G}_0$ was also obtained with DPC.

We have also analyzed the minimization of the sum MSE's of all the users in the network, not only to introduce fairness but also as a more practical approach than applying dirty paper coding to achieve the sum capacity. As we have also discussed here, each performance metric leads to different conclusions and hence, an appropiate definition of a figure of merit is still an open question in the cellular scenario.

As for future work, it would be interesting to consider decentralized schemes which can achieve a performance close to that of the partially cooperative game described in this paper. In this way one can ease the assumptions of a RCP which knows all the channels from every base station to all the users in the netwok. An interesting to the tradeoff issue between fairness and overall efficiency consists in combining both solutions of the maximum sum network broadcast capacity and the minimium sum network MSE by means of *time-sharing*, which may lead to an improvement on the entire region of the cdf of the sum cell capacity.

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