POINT-TO-POINT MIMO MMSE VECTOR PRECODING AND THP ACHIEVING CAPACITY

Michael Joham, Hans Brunner, Raphael Hunger, David A. Schmidt, Wolfgang Utschick

Associate Institute for Signal Processing
Technische Universität München, 80290 Munich, Germany
e-mail: {joham,brunner,hunger,dschmidt,utschick}@tum.de

ABSTRACT

Non-linear precoding for *point-to-point* (P2P) *multiple-input multiple-output* (MIMO) systems is considered. First, the *minimum mean square error* (MMSE) optimal *vector precoding* (VP) is presented for different receiver structures, viz., weighted identity matrix, diagonal matrix, weighted unitary matrix, and matrix without particular structure. Whereas the former two structures can also be applied to the vector broadcast channel, the latter two are only realizable for cooperative receivers. Second, VP is derived that minimizes the MSE but is restricted to maximize the mutual information of the MIMO channel. Third, the corresponding *Tomlinson-Harashima precoding* (THP) is found by applying the nearest-plane approximation to the computation of the perturbation signal. The resulting maximum mutual information THP clearly outperforms the state-of-the-art P2P-MIMO THP based on the *generalized triangular decomposition* (GTD) with respect to MSE and BER.

Index Terms— MIMO systems, non-linear transceivers, information rates, MMSE design, vector precoding.

1. INTRODUCTION

The maximum mutual information of P2P-MIMO channels can be achieved by linear transceivers, where the transmit covariance matrix is the waterfilling solution (e.g., [1]). When restricting to linear transceivers, the transmit covariance matrix resulting from the MMSE optimization takes a different form and is therefore not maximizing the mutual information (e.g., [2]). However, it was shown in [3, 4], that the MMSE optimal *decision feedback equalization* (DFE) and THP reach maximum mutual information and lead to equal MSEs for all scalar data streams. Thus, no bit-loading is necessary.

With finite-length codes, DFE suffers from error propagation contrary to THP. However, THP has the disadvantage of power & shaping and modulo loss due to the modulo operators at the transmitter and the receiver, respectively. The P2P-MIMO THP design presented in [3, 5] is based on the standard assumption that the output of the modulo operator at the transmitter is uncorrelated (e.g., [6]). This assumption is, however, only an approximation and the resulting transmit covariance matrix is not maximizing the mutual information.

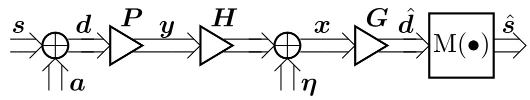

Fig. 1. P2P-MIMO System with Modulo Receiver

We propose VP [7, 8] for P2P-MIMO systems, since VP overcomes the problem of power loss and has no error propagation. Besides MMSE designs that are not necessarily maximizing the mutual information, we also optimize VP for P2P-MIMO systems by minimizing the MSE under the constraint that the transmit covariance matrix is equal to the waterfilling solution. By applying the nearest-plane approximation [9] to the closest point search necessary for the computation of the perturbation signal, we find new THP schemes for P2P-MIMO systems that outperform the GTD based designs of [3, 5].

2. SYSTEM MODEL

The data symbols $\boldsymbol{s} \in \mathbb{A}^B$ are perturbed by $\boldsymbol{a} \in \mathbb{M}^B$ to get the virtual desired signal $\boldsymbol{d} \in \mathbb{C}^B$, where $\mathbb{A}$ denotes the modulation alphabet and $\mathbb{M} = \tau\mathbb{Z}+\mathrm{j}\tau\mathbb{Z}$. Note that the transmitter has the freedom to add the perturbation signal $\boldsymbol{a}$ due to the modulo operator $\mathrm{M}(\bullet)$ with the modulo constant τ at the receiver (e.g., [10]). From the linear transformation of $\boldsymbol{d}$ by the precoding filter $\boldsymbol{P} \in \mathbb{C}^{N\times B}$, the transmit signal $\boldsymbol{y} = \boldsymbol{P}\boldsymbol{d} \in \mathbb{C}^N$ results which propagates over the channel $\boldsymbol{H} \in \mathbb{C}^{M\times N}$, is perturbed by the noise $\boldsymbol{\eta} \in \mathbb{C}^M \sim \mathcal{N}_\mathbb{C}(\boldsymbol{0}, \boldsymbol{C}_{\boldsymbol{\eta}})$, and filtered by the equalizer $\boldsymbol{G} \in \mathbb{C}^{B\times M}$ to get the estimate (see Fig. 1)

$$\hat{\boldsymbol{d}} = \boldsymbol{G}\boldsymbol{H}\boldsymbol{P}\boldsymbol{d} + \boldsymbol{G}\boldsymbol{\eta} \in \mathbb{C}^B \tag{1}$$

for $\boldsymbol{d}$. For notional brevity, we assume that $M = B \leq N$ and that $\boldsymbol{H}$ has full rank.

3. LINEAR DESIGN

The transmit covariance matrix maximizing the mutual information is the waterfilling solution resulting from

$$\boldsymbol{C}_{\mathrm{wf}} = \underset{\boldsymbol{C}_{\boldsymbol{y}}}{\operatorname{argmax}}\, \mathrm{I}(\boldsymbol{y};\boldsymbol{x}) \quad \text{s.t.: } \mathrm{E}\left[\|\boldsymbol{y}\|_2^2\right] \leq E_{\mathrm{tx}} \tag{2}$$

where $\boldsymbol{C_y} = \mathrm{E}[\boldsymbol{y}\boldsymbol{y}^{\mathrm{H}}]$ is the transmit covariance matrix and $\mathrm{I}(\boldsymbol{y};\boldsymbol{x}) = \log_2\det(\mathbf{I}+\boldsymbol{C_y}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{C_\eta}^{-1}\boldsymbol{H})$. With the reduced *eigenvalue decomposition* (EVD) of $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{C_\eta}^{-1}\boldsymbol{H} = \boldsymbol{U}\boldsymbol{\Phi}\boldsymbol{U}^{\mathrm{H}}$, where $\boldsymbol{U}\in\mathbb{C}^{N\times B}$ and the diagonal elements of $\boldsymbol{\Phi}\in\mathbb{R}_{0,+}^{B\times B}$ are sorted in non-increasing order, we have (e.g., [1])

$$\boldsymbol{C}_{\mathrm{wf}} = \boldsymbol{U}\left(\mu\mathbf{I}-\boldsymbol{\Phi}^{-1}\right)_+\boldsymbol{U}^{\mathrm{H}}. \tag{3}$$

Here, μ is chosen to fulfill $\mathrm{tr}(\boldsymbol{C}_{\mathrm{wf}}) = E_{\mathrm{tx}}$ and $(\bullet)_+$ applies $\max(0,\bullet)$ element-wise. The corresponding precoder is

$$\boldsymbol{P}_{\mathrm{wf}} = \boldsymbol{U}\left(\mu\mathbf{I}-\boldsymbol{\Phi}^{-1}\right)_+^{1/2}\boldsymbol{\Xi}^{-1/2}\boldsymbol{V}^{\mathrm{H}}. \tag{4}$$

The B eigenvalues of $\boldsymbol{C_d} = \mathrm{E}[\boldsymbol{d}\boldsymbol{d}^{\mathrm{H}}] = \boldsymbol{V}\boldsymbol{\Xi}\boldsymbol{V}^{\mathrm{H}}$ are on the diagonal of $\boldsymbol{\Xi}\in\mathbb{R}_{0,+}^{B\times B}$ in non-increasing order and the respective unitary modal matrix is $\boldsymbol{V}\in\mathbb{C}^{B\times B}$. Using this precoder, the capacity can be achieved by employing bit-loading, infinite block length, and an optimal receiver.

The MMSE optimal linear precoder and equalizer obey

$$\{\boldsymbol{P}_{\mathrm{lin}},\boldsymbol{G}_{\mathrm{lin}}\} = \operatorname*{argmin}_{\{\boldsymbol{P},\boldsymbol{G}\}}\mathrm{E}\left[\|\boldsymbol{d}-\hat{\boldsymbol{d}}\|_2^2\right]\quad\text{s.t.: }\mathrm{E}\left[\|\boldsymbol{y}\|_2^2\right]\le E_{\mathrm{tx}} \tag{5}$$

can be found with similar steps as in [2], and are respectively

$$\boldsymbol{P}_{\mathrm{lin}} = \boldsymbol{U}\left(\mu\boldsymbol{\Phi}^{-1/2}\boldsymbol{\Xi}^{-1/2}-\boldsymbol{\Phi}^{-1}\boldsymbol{\Xi}^{-1}\right)_+\boldsymbol{V}^{\mathrm{H}} \tag{6}$$

and $\boldsymbol{G}_{\mathrm{lin}} = (\boldsymbol{C_d}^{-1}+\boldsymbol{P}_{\mathrm{lin}}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{C_\eta}^{-1}\boldsymbol{H}\boldsymbol{P}_{\mathrm{lin}})^{-1}\boldsymbol{P}_{\mathrm{lin}}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{C_\eta}^{-1}$. The scalar μ is chosen to fulfill $\mathrm{tr}(\boldsymbol{C}_{\mathrm{lin}}) = E_{\mathrm{tx}}$ and the MMSE transmit covariance matrix $\boldsymbol{C}_{\mathrm{lin}} = \boldsymbol{P}_{\mathrm{lin}}\boldsymbol{C_d}\boldsymbol{P}_{\mathrm{lin}}^{\mathrm{H}}$ is clearly different from $\boldsymbol{C}_{\mathrm{wf}}$.

4. MMSE VP FOR P2P MIMO SYSTEMS

The statistics of $\boldsymbol{d} = \boldsymbol{s}+\boldsymbol{a}$ are unknown, since it is hard to find the statistics of $\boldsymbol{a}$. Therefore, we must use the time average instead of the expectation, i.e.,

$$\boldsymbol{C_d} = \frac{1}{Q}\sum_{q=1}^{Q}\boldsymbol{d}[q]\boldsymbol{d}^{\mathrm{H}}[q] \tag{7}$$

with the block length $Q\ge B$ and the time index q (cf. [8]).

4.1. Weighted Identity Equalizer

In [8], the receiver was restricted to be a weighted identity matrix, i.e., $\boldsymbol{G} = g\mathbf{I}$. This restriction led to a closed-form solution for the precoder $\boldsymbol{P}_{\mathrm{BC}}$ whose structure is independent of the perturbation signal. Moreover, a computation rule for the perturbation signal $\boldsymbol{a}[q]$ was found in [8] that can be solved by standard algorithms for a closest-point search in a lattice.

4.2. Diagonal Equalizer

For the case of a vector broadcast channel, a diagonal equalizer, i.e., $\boldsymbol{G} = \mathrm{diag}(g_1,\ldots,g_B)$, is the most general setup. Since the scalar receivers can be found in closed form for a given transmitter and also the transmitter depends on the receivers in closed form, an *alternating optimization* (AO) was proposed for the MMSE design of linear precoders for such a setup in [11]. This approach to linear precoding can be easily extended to solve the VP problem. Not only the transmitter and the receivers are computed alternately, but also the perturbation signal is recomputed in every iteration. Since every single step reduces the MSE, the iteration converges (cf. [11]).

4.3. Weighted Unitary Equalizer

An interesting generalization of a weighted identity matrix receiver is a weighted unitary matrix equalizer, i.e., $\boldsymbol{G} = g\boldsymbol{\Gamma}^{\mathrm{H}}$ with $\boldsymbol{\Gamma}\boldsymbol{\Gamma}^{\mathrm{H}} = \mathbf{I}$. Again, the MSE can be minimized by an AO. For some normalized equalizer $\boldsymbol{\Gamma}^{\mathrm{H}}$ and perturbation $\boldsymbol{a}[q]$ (i.e., $\boldsymbol{C_d}$ is given), the precoder minimizing the MSE can be expressed as ($\boldsymbol{H}$ is replaced by $\boldsymbol{\Gamma}^{\mathrm{H}}\boldsymbol{H}$ in the solution of [8])

$$\boldsymbol{P}_{\mathrm{un}} = g_{\mathrm{un}}^{-1}\boldsymbol{H}^{\mathrm{H}}\left(\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}+\xi\mathbf{I}\right)^{-1}\boldsymbol{\Gamma} \tag{8}$$

with $\xi = \mathrm{tr}(\boldsymbol{C_\eta})/E_{\mathrm{tx}}$. The scalar at the receiver g_{un} follows from $\mathrm{tr}(\boldsymbol{P}_{\mathrm{un}}\boldsymbol{C_d}\boldsymbol{P}_{\mathrm{un}}^{\mathrm{H}}) = E_{\mathrm{tx}}$. Substituting $\boldsymbol{P}_{\mathrm{un}}$ and g_{un} into the MSE $\varepsilon = \sum_{q=1}^{Q}\mathrm{E}[\|\boldsymbol{d}[q]-\hat{\boldsymbol{d}}[q]\|_2^2|\boldsymbol{s}[q]]/Q$ yields

$$\varepsilon_{\mathrm{un}} = \xi\,\mathrm{tr}\left(\boldsymbol{\Gamma}^{\mathrm{H}}\left(\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}+\xi\mathbf{I}\right)^{-1}\boldsymbol{\Gamma}\boldsymbol{C_d}\right). \tag{9}$$

With Lagrangian multipliers, the solution to the optimization

$$\boldsymbol{\Gamma}_{\mathrm{un}} = \operatorname*{argmin}_{\boldsymbol{\Gamma}}\varepsilon_{\mathrm{un}}\quad\text{s.t.: }\boldsymbol{\Gamma}\boldsymbol{\Gamma}^{\mathrm{H}} = \mathbf{I}$$

can easily be obtained and reads as

$$\boldsymbol{\Gamma}_{\mathrm{un}} = \boldsymbol{Q}\boldsymbol{V}^{\mathrm{H}} \tag{10}$$

with the unitary $\boldsymbol{Q}$ of the EVD $\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}} = \boldsymbol{Q}\boldsymbol{\Upsilon}\boldsymbol{Q}^{\mathrm{H}}$, where the diagonal elements of $\boldsymbol{\Upsilon}$ are sorted in non-increasing order.

Minimizing the MSE $\varepsilon_{\mathrm{un}}$ with respect to the perturbation for given $\boldsymbol{\Gamma}^{\mathrm{H}}$ can be split into optimizations for each time index $q = 1,\ldots,Q$ and due to (7) and $\boldsymbol{d}[q] = \boldsymbol{s}[q]+\boldsymbol{a}[q]$,

$$\boldsymbol{a}_{\mathrm{un}}[q] = \operatorname*{argmin}_{\boldsymbol{a}\in\mathbb{M}^B}\left\|(\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}+\xi\mathbf{I})^{-1/2}\boldsymbol{\Gamma}\left(\boldsymbol{s}[q]+\boldsymbol{a}\right)\right\|_2^2. \tag{11}$$

Combining the last results, we propose following AO. In every step, we assume that $\boldsymbol{P}_{\mathrm{un}}$ and g_{un} are used. Then, the MSE $\varepsilon_{\mathrm{un}}$ is minimized alternately with respect to $\boldsymbol{\Gamma}$ and $\boldsymbol{a}[q]$ for $q = 1,\ldots,Q$, where the other quantity is kept fixed. Since the MSE is reduced in every step, the iteration converges.

4.4. Unconstrained Equalizer

If the MSE $\varepsilon = \sum_{q=1}^{Q}\mathrm{E}[\|\boldsymbol{d}[q]-\hat{\boldsymbol{d}}[q]\|_2^2|\boldsymbol{s}[q]]/Q$ is minimized with respect to $\boldsymbol{P}$ and $\boldsymbol{G}$ for given $\boldsymbol{a}[q]$, we have to solve an optimization as in (5). With the linear MMSE equalizer $\boldsymbol{G}_{\mathrm{unc}} = (\boldsymbol{C_d}^{-1}+\boldsymbol{P}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{C_\eta}^{-1}\boldsymbol{H}\boldsymbol{P})^{-1}\boldsymbol{P}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{C_\eta}^{-1}$, we get

$$\varepsilon_{\mathrm{unc}} = \mathrm{tr}\left(\left(\boldsymbol{C_d}^{-1}+\boldsymbol{P}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{C_\eta}^{-1}\boldsymbol{H}\boldsymbol{P}\right)^{-1}\right).$$

From (6), we can conclude that

$$\boldsymbol{P}_{\text{unc}} = \boldsymbol{U}\boldsymbol{\Phi}^{-1/2}\boldsymbol{\Psi}\boldsymbol{V}^{\mathrm{H}}\boldsymbol{C}_{\boldsymbol{d}}^{-1/2} \tag{12}$$

with $\boldsymbol{C}_{\boldsymbol{d}}^{1/2} = \boldsymbol{V}\boldsymbol{\Xi}^{1/2}\boldsymbol{V}^{\mathrm{H}}$ and $\boldsymbol{\Psi} = (\mu\mathbf{I} - \boldsymbol{\Phi}^{-1/2}\boldsymbol{\Xi}^{-1/2})_{+}$. Substituting $\boldsymbol{P}_{\text{unc}}$ into ε_{unc} leads to

$$\varepsilon_{\text{unc}} = \operatorname{tr}\left(\boldsymbol{V}\left(\mathbf{I}+\boldsymbol{\Psi}^2\right)^{-1}\boldsymbol{V}^{\mathrm{H}}\boldsymbol{C}_{\boldsymbol{d}}\right). \tag{13}$$

Due to (7), the q-th perturbation can be obtained with

$$\boldsymbol{a}_{\text{unc}}[q] = \underset{\boldsymbol{a}\in\mathbb{M}^B}{\operatorname{argmin}} \left\|(\mathbf{I}+\boldsymbol{\Psi}^2)^{-1/2}\boldsymbol{V}^{\mathrm{H}}\left(\boldsymbol{s}[q]+\boldsymbol{a}\right)\right\|_2^2. \tag{14}$$

We propose following AO to solve the MSE minimization for an equalizer $\boldsymbol{G}$ with arbitrary structure. We assume in every step that $\boldsymbol{G}_{\text{unc}}$ is used. Then, ε_{unc} is minimized alternately with respect to the perturbation $\boldsymbol{a}[q]$ for $q = 1,\ldots,Q$ and $\boldsymbol{P}$, i.e., we alternate between the evaluation of (14) and the update of $\boldsymbol{V}$ (modal matrix of $\boldsymbol{C}_{\boldsymbol{d}}$) and $\boldsymbol{\Psi}$ [see (12)]. Again, every step reduces the MSE and the iteration converges.

5. MAXIMUM MUTUAL INFORMATION MMSE VP

In this Section, we investigate the optimization, where the MSE is minimized under the constraint that the transmit covariance matrix is the waterfilling solution:

$$\{\boldsymbol{P}_{\text{cap}}, \boldsymbol{G}_{\text{cap}}, \boldsymbol{a}_{\text{cap}}[q]\} = \underset{\{\boldsymbol{P},\boldsymbol{G},\boldsymbol{a}[q]\}}{\operatorname{argmin}}\ \varepsilon \quad \text{s.t.: } \boldsymbol{P}\boldsymbol{C}_{\boldsymbol{d}}\boldsymbol{P}^{\mathrm{H}} = \boldsymbol{C}_{\text{wf}} \tag{15}$$

with the MSE $\varepsilon = \sum_{q=1}^{Q}\mathrm{E}[\|\boldsymbol{d}[q]-\hat{\boldsymbol{d}}[q]\|_2^2|\boldsymbol{s}[q]]/Q$. Under the heuristical assumption that $\boldsymbol{y}$ is Gaussian, this maximizes the mutual information between $\boldsymbol{x}$ and $\boldsymbol{y}$. We will show that this significantly improves the BER and MSE. Note that no explicit transmit power constraint is necessary, since $\operatorname{tr}(\boldsymbol{C}_{\text{wf}}) = E_{\text{tx}}$. The optimal receiver is the linear MMSE equalizer $\boldsymbol{G}_{\text{cap}} = (\boldsymbol{C}_{\boldsymbol{d}}^{-1} + \boldsymbol{P}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{C}_{\boldsymbol{\eta}}^{-1}\boldsymbol{H}\boldsymbol{P})^{-1}\boldsymbol{P}^{\mathrm{H}}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{C}_{\boldsymbol{\eta}}^{-1}$ and from (4), we deduce that

$$\boldsymbol{P}_{\text{cap}} = \boldsymbol{U}\left(\mu\mathbf{I}-\boldsymbol{\Phi}^{-1}\right)_{+}^{1/2}\boldsymbol{V}^{\mathrm{H}}\boldsymbol{C}_{\boldsymbol{d}}^{-1/2}. \tag{16}$$

With above equalizer and precoder, the MSE reads as

$$\varepsilon_{\text{cap}} = \operatorname{tr}\left(\boldsymbol{V}\boldsymbol{\Theta}\boldsymbol{V}^{\mathrm{H}}\boldsymbol{C}_{\boldsymbol{d}}\right). \tag{17}$$

where $\boldsymbol{\Theta} = (\mathbf{I} + (\mu\boldsymbol{\Phi}-\mathbf{I})_{+})^{-1}$. Thus, the q-th perturbation minimizing above MSE ε_{cap} can be found with [see also (7)]

$$\boldsymbol{a}_{\text{cap}}[q] = \underset{\boldsymbol{a}\in\mathbb{M}^B}{\operatorname{argmin}} \left\|\boldsymbol{\Theta}^{1/2}\boldsymbol{V}^{\mathrm{H}}\left(\boldsymbol{s}[q]+\boldsymbol{a}\right)\right\|_2^2. \tag{18}$$

The AO minimizing the MSE ε_{cap} in (17), i.e., the equalizer $\boldsymbol{G}_{\text{cap}}$ and the precoder $\boldsymbol{P}_{\text{cap}}$ are used, alternately computes the perturbation $\boldsymbol{a}[q]$ with (18) for $q = 1,\ldots,Q$ and the modal matrix $\boldsymbol{V}$ of $\boldsymbol{C}_{\boldsymbol{d}} = \sum_{q=1}^{Q}\boldsymbol{d}[q]\boldsymbol{d}^{\mathrm{H}}[q]/Q = \boldsymbol{V}\boldsymbol{\Xi}\boldsymbol{V}^{\mathrm{H}}$. Every step reduces the MSE. Thus, the iteration converges.

6. MMSE THP FOR P2P MIMO SYSTEMS

THP minimizing the MSE for P2P MIMO systems with an unconstrained equalizer was proposed in [3, 5], where the standard assumption was used that the scalar outputs of the modulo operator at the transmitter are mutually uncorrelated (e.g., [6]). Under this assumption, it can be shown that the MMSE design leads to a mutual information maximizing waterfilling transmit covariance matrix and the MSE for all data streams is the same [3, 5]. Unfortunately, the assumption of uncorrelatedness is only an approximation and the maximum mutual information is not reached with the resulting transmit covariance matrix.

We employ the result that THP is a restricted VP, where the perturbation signal is not found via the full closest point search in a lattice but Babai's nearest plane approximation [9] is used (e.g., [12]).

The rules for perturbation signal computation of the proposed schemes can be written as [cf. (11), (14), and (18)]

$$\boldsymbol{a}_{\text{opt}}[q] = \underset{\boldsymbol{a}\in\mathbb{M}^B}{\operatorname{argmin}}\ \|\boldsymbol{A}(\boldsymbol{s}[q]+\boldsymbol{a})\|_2^2$$

where $\boldsymbol{A}$ depends on the employed VP design. Clearly, $\boldsymbol{A}$ can be replaced by $\boldsymbol{D}^{1/2}\boldsymbol{L}\boldsymbol{\Pi}$ from the symmetrically permuted Cholesky factorization $\boldsymbol{\Pi}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A}\boldsymbol{\Pi}^{\mathrm{T}} = \boldsymbol{L}^{\mathrm{H}}\boldsymbol{D}\boldsymbol{L}$ with diagonal $\boldsymbol{D} \in \mathbb{R}_{0,+}^{B\times B}$, unit lower triangular $\boldsymbol{L} \in \mathbb{C}^{B\times B}$, and the unitary permutation matrix $\boldsymbol{\Pi} \in \{0,1\}^{B\times B}$. We use the algorithm of [13] to compute above factorization, since it successively minimizes the MSE. The nearest plane approximation finds the entries of $\boldsymbol{a}[q]$ successively, i.e., the k-th entry $a_{b_k}[q]$ of $\boldsymbol{\Pi}\boldsymbol{a}[q]$ is computed with the already found values for the first $k-1$ entries kept fixed. Mathematically,

$$a_{\text{np},b_k}[q] = \underset{a\in\mathbb{M}}{\operatorname{argmin}}\left|s_{b_k}[q] + a + \sum_{i=1}^{k-1}\ell_{k,i}(s_{b_i}[q] + a_{\text{np},b_i}[q])\right|^2.$$

Here, $\ell_{k,i}$ is the k-th element in the i-th column of $\boldsymbol{L}$. Fortunately, above optimization can be solved by a simple rounding operation and the operation of the transmitter can be reformulated as a feedback loop with a modulo operation (see e.g., [12]), i.e., we end up with a THP transmitter.

Clearly, the successive computation of the perturbation signal as for THP is suboptimal compared to the full search employed for VP. Thus, it is not ensured that the update of the perturbation signal in any of the proposed AOs leads to a reduction of the MSE compared to the previous step. We see that a convergence cannot be proven. However, we observed that the iterations in fact converge.

7. SIMULATION RESULTS

We employed a channel model with i. i. d. unit variance Rayleigh fading coefficents and 10000 channel realizations. The AOs where initialized with $\boldsymbol{V} = \mathbf{I}$ and the iteration was stopped, when the relative change in MSE was below 10^{-4}.

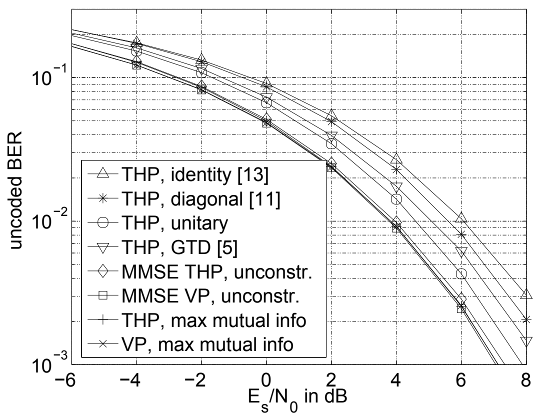

Fig. 2. BER vs. SNR for $M = B = 4$ Antennas, QPSK

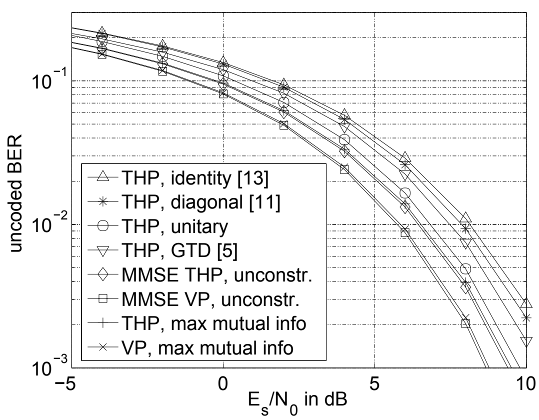

Fig. 3. BER vs. SNR for $M = B = 10$ Antennas, 16QAM

We omit the mutual information results, because the curves lie very close to each other. We observed that the MMSE optimal VP and THP offer a higher mutual information than the GTD based THP of [3, 5]. As expected, the maximum mutual information design of Section 5 delivers the highest mutual information. The difference to MMSE VP and THP is small and for high SNR, the gain over GTD THP is about 0.8 dB and 0.25 dB for the 4×4 and 10×10 system, respectively.

In Fig. 2, we see that the maximum mutual information and the MMSE designs have nearly the same BER performance for a 4×4 MIMO system with QPSK, if VP or THP is used. Interestingly, the state-of-the-art GTD THP is even outperformed by the MMSE design with weighted unitary equalizer.

For a 10×10 system with 16QAM as in Fig. 3, a difference between VP and THP for the maximum mutual information and the MMSE designs can be observed. This behavior can be explained by the bad performance of the nearest plane approximation for ill-behaved generator matrices that are more likely for larger systems. Again, the GTD THP design is worse than the unitary equalizer design, whose results are closer to that of MMSE THP than for the 4×4 system.

8. REFERENCES

[1] I. E. Telatar, “Capacity of Multi-antenna Gaussian Channels,” *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585–595, November/December 1999.

[2] H. Sampath, P. Stoica, and A. Paulraj, “Generalized Linear Precoder and Decoder Design for MIMO Channels Using the Weighted MMSE Criterion,” *IEEE Transactions on Communications*, vol. 49, no. 12, pp. 2198–2206, December 2001.

[3] Y. Jiang, J. Li, and W. W. Hager, “Joint Transceiver Design for MIMO Communications Using Geometric Mean Decomposition,” *IEEE Transactions on Signal Processing*, vol. 53, no. 10, pp. 3791–3803, October 2005.

[4] F. Xu, T. N. Davidson, J.-K. Zhang, and K. M. Wong, “Design of Block Transceivers With Decision Feedback Detection,” *IEEE Transactions on Signal Processing*, vol. 54, no. 3, pp. 965–978, March 2006.

[5] R. Hunger, M. Riemensberger, and W. Utschick, “Near Capacity Tomlinson-Harashima Precoding with Uniform Stream Properties,” in *Proc. IEEE/ITG WSA 2007*, February 2007.

[6] R. F. H. Fischer, *Precoding and Signal Shaping for Digital Transmission*, John Wiley & Sons, 2002.

[7] B. M. Hochwald, C. B. Peel, and A. L. Swindlehurst, “A Vector-Perturbation Technique for Near-Capacity Multi-Antenna Multi-User Communication—Part II: Perturbation,” *IEEE Transactions on Communications*, vol. 53, no. 3, pp. 195–202, March 2005.

[8] D. A. Schmidt, M. Joham, and W. Utschick, “Minimum Mean Square Error Vector Precoding,” *Accepted for publication in European Transactions on Telecommunications*, 2006.

[9] L. Babai, “On Lovász’ Lattice Reduction and the Nearest Lattice Point Problem,” *Combinatorica*, vol. 6, no. 1, pp. 1–13, 1986.

[10] M. Joham, D. A. Schmidt, J. Brehmer, and W. Utschick, “Finite-Length MMSE Tomlinson-Harashima Precoding for Frequency Selective Vector Channels,” *IEEE Transactions on Signal Processing*, vol. 55, no. 6, pp. 3073–3088, June 2007.

[11] R. Hunger, W. Utschick, D. Schmidt, and M. Joham, “Alternating Optimization for MMSE Broadcast Precoding,” in *Proc. ICASSP 2006*, May 2006, vol. IV, pp. IV–757–IV–760.

[12] M. Joham, D. A. Schmidt, H. Brunner, and W. Utschick, “Symbol-Wise Order Optimization for THP,” in *Proc. ITG/IEEE WSA 2007*, February 2007.

[13] K. Kusume, M. Joham, W. Utschick, and G. Bauch, “Cholesky Factorization with Symmetric Permutation Applied to Detecting and Precoding Spatially Multiplexed Data Streams,” *IEEE Transactions on Signal Processing*, vol. 55, no. 6, pp. 3089–3103, June 2007.