Field Modelling of a Multiconductor Digital Bus

Hristomir Yordanov [1], Michel Ivrlac [2], Josef Nossek [2], and Peter Russer [1]

[1] *Institute for High Frequency Engineering, Munich University of Technology*
Arcisstrasse 21, 80333 Munich, Germany, Tel.: + 49 89 289 23373
yordanov@tum.de

[2] *Institute for Circuit Theory and Signal Processing*
Munich University of Technology, Arcisstrasse 21, 80333 Munich, Germany

***Abstract*— A study of the transmission line parameters of a multiconductor transmission line, used as an on-chip digital interconnect is presented in this work. As a first step a quasi-static approach in connection with Schwarz-Christoffel mapping has been used to determine the capacitance per unit length values. Then the obtained results have been used to solve the multiconductor transmission line equations in frequency domain. The resulting frequency response was used to compute the pulse distortion and the crosstalk effect in a real-case on-chip digital bus.**

I. Introduction

With the increase of the on-chip data transfer rate to several 10 Gbit/sec the spatio-temporal intersymbol interference (auto-interference) within the multiconductor interconnect systems starts to play a role on the device performance. In order to minimize the effects of such parasitic phenomena different solutions are proposed, like space-coding techniques [1]. The proper design of such tools requires a thorough knowledge of the electrical parameters of the digital bus. Therefore a precise electromagnetic modeling of the multiwired interconnect systems is required.

In this work an on-chip digital bus with equidistant conductors of equal cross section is considered. The quasi-static parameters are computed under the assumption for symmetry using even-odd mode analysis [2] and Schwarz-Cristoffel transformation [3]. The obtained results are used to solve the multiconductor transmission line equations in frequency domain, thus obtaining the frequency response of the digital interconnect [4]. Then the pulse distortion is computed using simple Fourier transformation.

The earliest attempts for analytical computation of rectangular coupled lines between parallel grounded plates were based on the assumption of zero conductor thicknesses and adding a correction term for the finite thickness case. Cohn, for example, has calculated the transmission line parameters for the zero-thickness case using conformal mapping methods [5]. Getsinger has extended Cohn's work to computing the fringing field capacitances at the conductor edges, but he has imposed restrictions on the conductor dimensions [6].

In this work a numerical inversion of the Schwarz-Christoffel conformal mapping is used to produce exact results for the even and odd mode capacitances. The only assumption that is made is for symmetry. No restrictions on the width to height ratio of the conductor cross section are made. The multiconductor transmission line equations are also solved numerically.

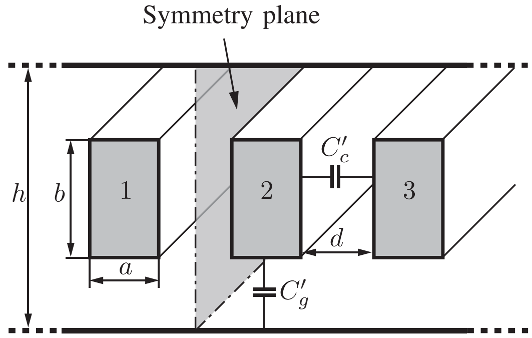

Fig. 1. A cross section of three-wire digital interconnect embedded between ground plates with a coupling and a ground capacitance

Section II gives a short theoretical introduction to multiconductor transmission lines and to Schwarz-Christoffel mapping. Section III describes the application and section IV presents the parameters, computed with the described technique.

II. Theory

A. Multiconductor Transmission Lines

In this section the TEM modes of a multiconductor transmission line filled with homogenous isotropic dielectric material used for digital interconnect bus are treated. A cross section of such line with three signal conductors between two ground planes is presented in Fig. 1. In order to fully characterize a line, consisting of n conductors and reference ground plane, the voltage $v_k(z,t)$ and the current $i_k(z,t)$ of each line are needed. These variables can be summarized in the following vectors

$$\boldsymbol{v}(z,t) = [v_1(z,t), v_2(z,t), ..., v_n(z,t)]^T, \quad (1)$$
$$\boldsymbol{i}(z,t) = [i_1(z,t), i_2(z,t), ..., i_n(z,t)]^T. \quad (2)$$

The relation between these two vectors can be expressed via the frequency-domain telegrapher's equation in matrix form

$$\frac{\mathrm{d}^2\boldsymbol{V}(z)}{\mathrm{d}z^2} = \boldsymbol{Z'Y'V}(z), \quad (3)$$
$$\frac{\mathrm{d}^2\boldsymbol{I}(z)}{\mathrm{d}z^2} = \boldsymbol{Y'Z'I}(z), \quad (4)$$

where

$$\boldsymbol{Z'} = \boldsymbol{R'} + j\omega\boldsymbol{L'}, \tag{5}$$
$$\boldsymbol{Y'} = \boldsymbol{G'} + j\omega\boldsymbol{C'}. \tag{6}$$

are respectively the complex impedance and complex admittance per unit length. $\boldsymbol{R'}$ is the resistance per unit length, which models the ohmic losses in the conductors, $\boldsymbol{G'}$ is the conductivity per unit length, which accounts for the dielectric losses in the material, in which the line is embedded, and $\boldsymbol{C'}$ and $\boldsymbol{L'}$ are respectively the capacitance and inductance per unit length. In the case of quasi-TEM modes the following relation between $\boldsymbol{C'}$ and $\boldsymbol{L'}$ holds

$$\boldsymbol{L'C'} = \boldsymbol{C'L'} = \frac{1}{c^2}\mathbf{1}, \tag{7}$$

where c is the phase velocity of the TEM mode and $\mathbf{1}$ is the unity matrix.

In order to find a close form solution of the transmission line equations the uncoupled modal voltages and currents are introduced

$$\tilde{\boldsymbol{V}} = \boldsymbol{M}_V \boldsymbol{V}, \tag{8}$$
$$\tilde{\boldsymbol{I}} = \boldsymbol{M}_I \boldsymbol{I}. \tag{9}$$

The transformation matrices $\boldsymbol{M}_V$ and $\boldsymbol{M}_I$ can be found by diagonalizing the product $\boldsymbol{Z'Y'}$ as follows

$$\tilde{\gamma}^2 = \tilde{\boldsymbol{Z}}'\tilde{\boldsymbol{Y}}' = \boldsymbol{M}^{-1}\boldsymbol{Z'Y'M}, \tag{10}$$

where $\tilde{\gamma}$ is the modal propagation coefficient diagonal matrix, and $\tilde{\boldsymbol{Z}}'$ and $\tilde{\boldsymbol{Y}}'$ are the diagonalized $\boldsymbol{Z'}$ and $\boldsymbol{Y'}$ matrices

$$\tilde{\boldsymbol{Z}}' = \boldsymbol{M}_V^{-1}\boldsymbol{Z'}\boldsymbol{M}_I = \mathrm{diag}[\tilde{Z}_1, \tilde{Z}_2, ..., \tilde{Z}_n], \tag{11}$$
$$\tilde{\boldsymbol{Y}}' = \boldsymbol{M}_I^{-1}\boldsymbol{Y'}\boldsymbol{M}_V = \mathrm{diag}[\tilde{Y}_1, \tilde{Y}_2, ..., \tilde{Y}_n]. \tag{12}$$

The relation between $\boldsymbol{M}_V$, $\boldsymbol{M}_I$, and $\boldsymbol{M}$ is given by

$$\boldsymbol{M} = \boldsymbol{M}_V = (\boldsymbol{M}_I^T)^{-1}. \tag{13}$$

The characteristic impedance matrix is defined as

$$\boldsymbol{Z}_0 = \boldsymbol{M}\tilde{\gamma}\boldsymbol{M}^{-1}\tilde{\boldsymbol{Z}}. \tag{14}$$

The boundary conditions for solving equations (3) and (4) are applied by introducing matrices $\boldsymbol{Z}_S$, accounting for the source impedances, and $\boldsymbol{Z}_L$, accounting for the load impedances, as well as vectors $\boldsymbol{V}_S$ and $\boldsymbol{V}_L$, accounting for the impressed source and load voltages.

Finally we need to solve for the modal currents $\tilde{\boldsymbol{I}}$ the following matrix equation

$$\begin{bmatrix} (\boldsymbol{Z}_0 + \boldsymbol{Z}_{\boldsymbol{S}})\boldsymbol{M} & (\boldsymbol{Z}_0 - \boldsymbol{Z}_S)\boldsymbol{M} \\ (\boldsymbol{Z}_0 - \boldsymbol{Z}_L)\boldsymbol{M}e^{-\tilde{\gamma}l} & (\boldsymbol{Z}_0 - \boldsymbol{Z}_L)\boldsymbol{M}e^{\tilde{\gamma}l} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{I}}^{(+)} \\ \tilde{\boldsymbol{I}}^{(-)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{V}_S \\ \boldsymbol{V}_L \end{bmatrix}, \tag{15}$$

where $\tilde{\boldsymbol{I}}^{(+)}$ and $\tilde{\boldsymbol{I}}^{(-)}$ are the amplitudes of the solution of the diagonalized equation (4)

$$\tilde{\boldsymbol{I}}(z) = e^{-\tilde{\gamma}z}\tilde{\boldsymbol{I}}^{(+)} + e^{\tilde{\gamma}z}\tilde{\boldsymbol{I}}^{(-)}. \tag{16}$$

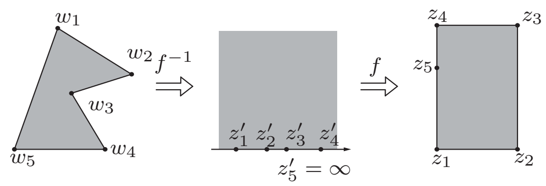

Fig. 2. Mapping an arbitrary simple bounded polygon to a rectangle

B. Schwarz-Christoffel Mapping to Simply Connected Polygonal Domains

The Schwarz-Christoffel mapping is a conformal mapping technique, which maps the upper half of the complex plane Ω_z into a polygonal domain by preserving the local angles [3], [7]. This transformation is described by the Schwarz-Christoffel formula

$$f(z) = A + C\int^z \prod_{k=1}^{n-1}(\zeta - z_k)^{\alpha_k - 1}\mathrm{d}\zeta. \tag{17}$$

In this equation z_k are the points on the real axis of the Ω_z domain, which are mapped into the polygon vertices, α_k are the internal angles of the polygon in counterclockwise direction, normalized to π, and A and C are complex constants. Under this transformation a capacitance, which characterizes the original structure characterizes the image too.

A simply connected planar domain can be defined as the interior of a planar closed line, that does not contain any holes. Using the inverse of the SC transformation any simply connected polygon can be mapped into the upper half of the image plane. Using one more SC transformation we can map the upper half-plane into a canonical shape like a rectangle (see Fig. 2). This is the mapping that will be used for symmetric transmission lines, as shown in the next section.

III. Application

The following procedure is used for computing the transmission line parameters. First the capacitance per unit of length $\boldsymbol{C'}$ and resistance per unit of length $\boldsymbol{R'}$ matrices are calculated. The losses in the dielectric are neglected, therefore the conductance per unit of length matrix $\boldsymbol{G'}$ is considered $\mathbf{0}$. From the $\boldsymbol{C'}$ matrix the inductance per unit of length matrix $\boldsymbol{L'}$ is computed using (7). The multiconductor transmission line equations are solved by numerically inverting (15).

A. Computation of the Capacitance Matrix

In order to compute the static capacitance matrix of the transmission line even-odd mode analysis is utilized. If an electric wall inserted at the plane of symmetry between the signal conductors (see Fig. 1) only one signal conductor can be considered. The capacitance between this conductor and ground is the odd-mode capacitance C'_o. If a magnetic wall is inserted at the plane of symmetry the capacitance of the resulting structure is the even-mode capacitance C'_e (see Fig. 3). The ground and the coupling capacitances are

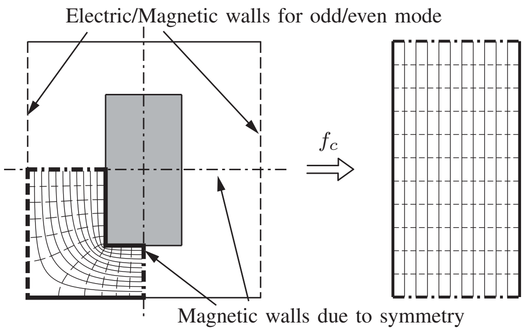

Fig. 3. Mapping a quoter of an even/odd mode equivalent structure to a rectangle

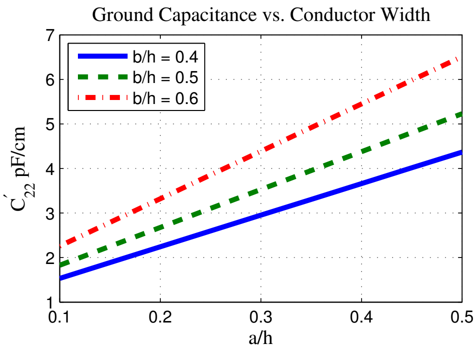

Fig. 5. Ground capacitance vs. geometry for digital transmission line, filled in with silicon. Here $d/h = 0.125$

connected with the even and odd mode capacitance with the following equations

$$C'_g = C'_o, \tag{18}$$

$$C'_c = \frac{1}{4}(C'_o - C'_e). \tag{19}$$

The even and odd mode capacitances have been computed by making use of the symmetry of the structure, as depicted in Fig. 3. Schwarz-Christoffel mapping is applied to the quoter of the structure, shown in bold lines. The capacitance per unit of length of the resulting rectangle is computed as the ratio of the electric wall length to the magnetic wall length times the dielectric permitivity of the medium. The Schwaz-Christoffel mapping is performed numerically using an integrated MATLAB toolbox [8].

Up to now it was considered that the ground capacitances of all signal conductors are equal. This does not hold for the end bus lines in the case of small distance between conductors d and long extended ground planes. In order to find the ground capacitance of the end conductors we consider the SC mapping, shown in Fig. 4, where the ground planes have been extended to infinity. Here we can use only one symmetry plane. The ground capacitance of the end conductors is equal to twice the capacitance of the equivalent rectangle.

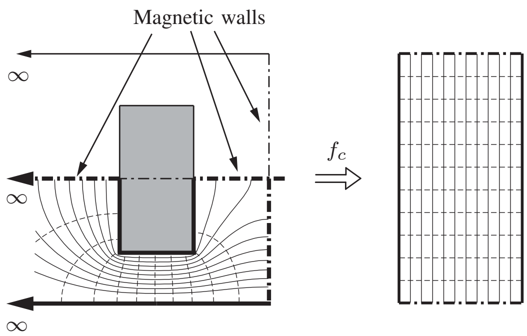

Fig. 4. Mapping half of an end-conductor equivalent structure to a rectangle

Finally the capacitance per unit of length matrix is constructed. Its diagonal elements are the sum of all capacitances, connected to the corresponding conductor, and its off-diagonal elements are the coupling capacitances between the corresponding conductors with a negative sign, as shown below

$$\boldsymbol{C'} = \begin{bmatrix} C'_g + C'_c & -C'_c & \dots & 0 \\ -C'_c & C'_g + 2C'_c & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & C'_g + C'_c \end{bmatrix}. \tag{20}$$

B. Computation of the Inductance and Resistance Matrices

The inductance per unit of length matrix $\boldsymbol{L'}$ is computed from the capacitance per unit of length matrix $\boldsymbol{C'}$ from the following equation

$$\boldsymbol{L'} = \frac{1}{c^2}\boldsymbol{C'}^{-1}. \tag{21}$$

This equation is only approximative for lossy lines, but due to the low resistivity of the used conductor this approximation can be used.

The clock frequency of the digital bus under consideration is several hundred megahertz and the bus dimensions are in the range of several hundred nanometers, therefore we can assume an uniform current distribution in the conductors. Therefore the resistance per unit length matrix $\boldsymbol{R'}$ can be computed from the metal conductivity σ and the bus dimensions as follows

$$R'_{ij} = \begin{cases} \frac{1}{\sigma ab} & \text{for } i = j, \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

IV. RESULTS

The above described computational procedure was implemented on a MATLAB code. The results for the capacitances per unit length for a transmission line, filled in with silicon, are presented in Fig. 5 and Fig. 6. Since the capacitance is only a function of the ratio of the line dimensions, all geometry is normalized to the distance between the ground planes h.

As a real case study a 4-line digital bus with the dimensions, shown in table I is investigated. In order to solve (15) the following values were chosen. The load and source impedance matrices $\boldsymbol{Z}_L$ and $\boldsymbol{Z}_S$ were chosen as diagonal matrices with a value of 50 Ω for every diagonal element. This was done in order to compare the results with the data, obtained from the

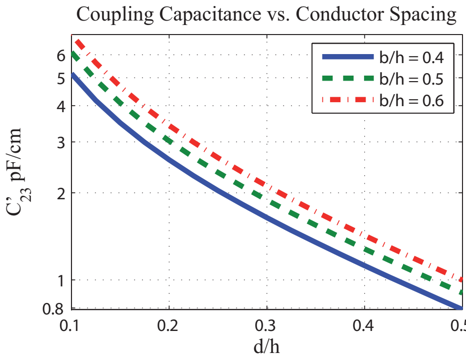

Fig. 6. Coupling capacitance vs. geometry for digital transmission line, filled in with silicon. Here $a/h = 0.25$

TABLE I

REAL DIGITAL BUS DIMENSIONS

Description	Notation	Dimension
Conductor width	a	200 nm
Conductor height	b	400 nm
Distance between conductors	d	100 nm
Distance between ground plates	h	800 nm
Line length	l	1mm

commercial MoM-based simulator MOMENTUM from ADS. The load voltages vector $\boldsymbol{V}_L$ was set to zero, and the source voltage vector $\boldsymbol{V}_S$ was set to zero except for one element, which was set to one, providing an excitation, as follows

$$\boldsymbol{V}_S = [1\ 0\ 0\ 0]^T. \tag{23}$$

The frequency response for the crosstalk voltage at the far end of the line is presented in Fig. 7. The simulation time required by MOMENTUM on a Pentium 4 based computer with 2.4 GHz clock frequency and 1 GB of RAM was in the order of 60 minutes, while the equation-solving routine requires less than a second on the same machine.

In order to compute the pulse distortion more realistic values for the load and source impedance matrices $\boldsymbol{Z}_L$ and $\boldsymbol{Z}_S$ were chosen. The source impedance matrix $\boldsymbol{Z}_S$ has been set to zero, and the diagonal elements of the load impedance matrix $\boldsymbol{Z}_L$ were set to 10 MΩ. After performing a Fourier transform the shape of the distorted pulse at the far end of the line, shown in Fig. 8, was obtained. The results are compared with the data, obtained from SPICE simulation of the equivalent lumped-element circuit of the bus, using the previously computed capacitance and resistance values.

V. CONCLUSIONS

A precise electromagnetic model of a multiconductor transmission line, used as an on-chip digital bus was presented. The structure was investigated under static conditions using numerical inversion of the Schwarz-Christoffel transformation. Using the even-odd mode analysis techniques the capacitance, inductance and resistance per unit length matrices were calculated. The dependance of the electrostatic parameters on the transmission line dimensions was explored. Using the obtained results the frequency response and the pulse distortion of a real-case on-chip digital bus was obtained. The advantages of the proposed method are its accuracy, the lack of geometrical limitations and the algorithm efficiency.

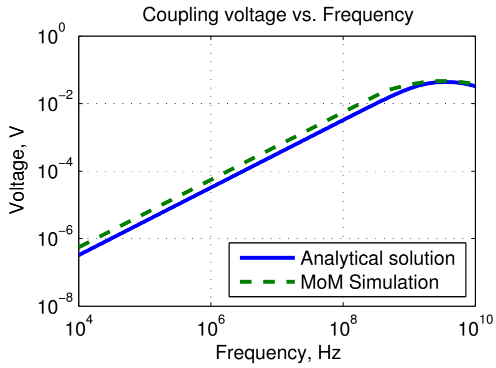

Fig. 7. Frequency response of the crosstalk voltage at the far end of the line. Comparison of the results, obtained by solving the transmission line equations, and the data, obtained by full-wave analysis

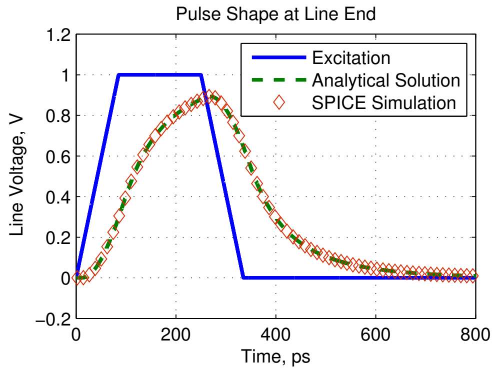

Fig. 8. Pulse distortion at the far end of the line. Comparison of the data, obtained with the proposed method, and the results from a SPICE simulation of the equivalent lumped-element circuit of the bus

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