ANALYSIS OF THE IMPACT OF CHANNEL ESTIMATION ERRORS ON THE DECOMPOSITION OF MULTIUSER MIMO CHANNELS

Pedro Tejera, Wolfgang Utschick*, Gerhard Bauch†, Josef A. Nossek**

* Munich University of Technology
Institute for Circuit Theory and Signal Processing
Arcisstrasse 21, 80290 Munich, Germany

† DoCoMo Communications Laboratories Europe GmbH
Landsbergerstr. 308-312, 80687 Munich, Germany

ABSTRACT

In the work at hand a general procedure to analyze the impact of channel estimation errors on the performance of decomposition techniques for multiuser MIMO channels is presented. In particular, this procedure is applied to a decomposition technique called cooperative zero-forcing with successive encoding and successive allocation method (CZF-SESAM). Based on the resulting analytical expressions the transmitter is able to adjust bit and power loading so that in spite of estimation errors transmission quality requirements can still be met.

1. INTRODUCTION

CZF-SESAM is a decomposition technique for multiuser MIMO channels first introduced in [1]. The algorithm proceeds successively assigning a new spatial dimension to a certain user at each step in such a way that no interference is caused on previously assigned dimensions. If at each step the dimension associated with the largest singular value is chosen, the algorithm is nearly optimum in terms of sum capacity [1]. Also, in terms of capacity region this algorithm shows a nearly optimum performance for a wide range of scenarios [2].

Here, the focus is on the downlink of a communication system. CZF-SESAM is applied in order to decompose the MIMO broadcast channel and known interference is eliminated at the transmitter by employing subtraction and modulo arithmetics [3]. A transmission quality constraint is imposed to the system in terms of a maximum symbol error rate (SER) that must not be exceeded over any of the subchannels and the objective is to maximize the sum rate under this constraint. To this end, on the set of decoupled subchannels, the rate maximizing greedy bit and power loading algorithm presented in [4] is employed assuming square QAM modulation alphabets.

In this paper, we present an analytical framework for evaluating performance degradation of such a system due to estimation errors. The derived analytical results make it possible to perform a readjustment of the loading so that in spite of estimation errors compliance with the target SER is guaranteed. Even though the analysis refers to the system mentioned above, the way of proceeding is very general and can be applied to any other decomposition technique with or without successive encoding (see references in [5]). The structure of this paper is as follows. The system model is introduced in Section 2. In Section 3, some features of CZF-SESAM are recalled which are basic for the understanding of the analysis. Section 4 presents the impact analysis of estimation errors on system performance. Section 5 sketches a simple approach to adapt loading to estimation errors in order to fulfil quality contraints. Finally, some numerical results are presented in Section 6.

2. SYSTEM MODEL

A downlink system is considered with K users and t transmit antennas. User k has r_k antennas and its receive signal is given by the usual MIMO flat fading model, i.e. $\boldsymbol{y}_k = \boldsymbol{H}_k\boldsymbol{x} + \boldsymbol{n}_k$, where $\boldsymbol{H}_k \in \mathbb{C}^{r_k \times t}$ is the corresponding channel matrix, $\boldsymbol{n}_k$ is a noise vector with independently zero-mean Gaussian distributed entries of variance $\sigma^2 = 1$ and $\boldsymbol{x} = \boldsymbol{V}\boldsymbol{P}^{1/2}\boldsymbol{s}$ is the transmit signal. Here, $\boldsymbol{V}$ is a beamforming matrix of unit norm column vectors, $\boldsymbol{P}$ is a diagonal power loading matrix and $\boldsymbol{s}$ is the vector of unit variance transmit symbols. A power constraint applies to the transmit signal, $\mathrm{E}\{\boldsymbol{x}^{\mathrm{H}}\boldsymbol{x}\} = \mathrm{Tr}\{\boldsymbol{P}\} \leq P_{\mathrm{Tx}}$. Considering all channel matrices in the system, the composite channel matrix $\boldsymbol{H} = [\ \boldsymbol{H}_1^{\mathrm{T}} \ \cdots \ \boldsymbol{H}_K^{\mathrm{T}}\ ]^{\mathrm{T}}$ can be defined.

3. DECOMPOSITION WITH PERFECT ESTIMATES

3.1. CZF-SESAM

The output of CZF-SESAM is a set of pairs of unit-norm receive and transmit weighting vectors,

1-4244-0469-X/06/$20.00 ©2006 IEEE

each of them characterizing a spatial dimension, i.e. $\mathcal{D} = \{(\boldsymbol{u}_{\pi(d)}, \boldsymbol{v}_{\pi(d)}) | d \in \{1, \ldots, D\}\}$.

The function $\pi : \{1, \ldots, D\} \to \{1, \ldots, K\} \times \mathbb{N}$ is used to index the vectors and maps dimensions to pairs $(\pi_1(d), \pi_2(d))$, formed by the user the dimension has been assigned to and a natural number that references a dimension among all assigned to a specific user.

The channel gain of any dimension or subchannel d is given by $\lambda_d = \boldsymbol{u}_{\pi(d)}^{\mathrm{H}} \boldsymbol{H}_{\pi_1(d)} \boldsymbol{v}_{\pi(d)}$, and, since $\boldsymbol{u}_{\pi(d)}$ and $\boldsymbol{v}_{\pi(d)}$ are the left and right singular vectors of a projection of the matrix $\boldsymbol{H}_{\pi_1(d)}$, $\lambda_d \boldsymbol{u}_{\pi(d)} = \boldsymbol{H}_{\pi_1(d)} \boldsymbol{v}_{\pi(d)}$ [1].

If $\ell < d$,

$$\boldsymbol{u}_{\pi(\ell)}^{\mathrm{H}} \boldsymbol{H}_{\pi_1(\ell)} \boldsymbol{v}_{\pi(d)} = 0, \tag{1}$$

i.e. no interference is caused on previously assigned dimensions. On the contrary, if $\ell > d$ and $\pi_1(\ell) \neq \pi_1(d)$, in general, $\boldsymbol{u}_{\pi(\ell)}^{\mathrm{H}} \boldsymbol{H}_{\pi_1(\ell)} \boldsymbol{v}_{\pi(d)} \neq 0$. However, if information is encoded in the same order in which the respective subchannels were allocated, this remaining interference can be efficiently neutralized [1].

3.2. Cancellation of known interference

Let

$$\boldsymbol{V} = [\ \boldsymbol{v}_{\pi(1)} \quad \boldsymbol{v}_{\pi(2)} \quad \cdots \quad \boldsymbol{v}_{\pi(D)}\ ], \tag{2}$$

$\boldsymbol{G}_k = \boldsymbol{H}_k \boldsymbol{V} \boldsymbol{P}^{1/2}$, and $\boldsymbol{g}_k^d$ the dth column of $\boldsymbol{G}_k$. The vector of signals received by user k is given by

$$\boldsymbol{y}_k = \boldsymbol{G}_k \boldsymbol{s} + \boldsymbol{n}_k. \tag{3}$$

Provided that dimension d has been assigned to this user, i.e. $\pi_1(d) = k$, optimum detection over dimension d can be performed by first applying the corresponding receive weighting vector $\boldsymbol{u}_{\pi(d)} = \boldsymbol{g}_k^d / \|\boldsymbol{g}_k^d\|$ to $\boldsymbol{y}_k$ and then scaling the resulting signal with $\|\boldsymbol{g}_k^d\|^{-1}$. From (3) and considering (1) and (2) we obtain

$$y_d = s_d + \sum_{\ell=1}^{d-1} \frac{\boldsymbol{g}_k^{d,\mathrm{H}} \boldsymbol{g}_k^\ell}{\|\boldsymbol{g}_k^d\|^2} s_\ell + \frac{\boldsymbol{g}_k^{d,\mathrm{H}} \boldsymbol{n}_k}{\|\boldsymbol{g}_k^d\|^2}. \tag{4}$$

The second term of the above expression represents the remaining interference due to previously encoded signals and, thus, it is known at the time at which signal s_d is generated. This knowledge can be used to design signal s_d so as to cancel this interference. A practical approach, to which we adhere in this paper, consists of subtracting the known interference from the intended symbol and applying a modulo operation in order to limit the power of the resulting signal [3]. Correspondingly, assuming symbol $\tilde{s}_d$ is intended for transmission over subchannel d, signal s_d can be written as,

$$s_d = \underbrace{\tilde{s}_d - \tau_1 n - j\tau_2 m}_{\tilde{s}_d'} - \sum_{\ell=1}^{d-1} \frac{\boldsymbol{g}_k^{d,\mathrm{H}} \boldsymbol{g}_k^\ell}{\|\boldsymbol{g}_k^d\|^2} s_\ell. \tag{5}$$

Here, τ_1 and τ_2 are the moduli in the real and imaginary axes respectively and $n, m \in \mathbb{Z}$ are chosen so that real and imaginary parts of s_d lie within the modulo intervals $[-\tau_1/2,\ \tau_1/2]$ and $[-\tau_2/2,\ \tau_2/2]$, respectively. Plugging (5) into (4), we obtain

$$y_d = \tilde{s}_d' + \frac{\boldsymbol{g}_k^{d,\mathrm{H}} \boldsymbol{n}_k}{\|\boldsymbol{g}_k^d\|^2}.$$

If $\tilde{s}_d'$ is correctly detected, application of the modulo operator on this signal returns $\tilde{s}_d$.

4. PERFORMANCE ANALYSIS WITH ESTIMATION ERRORS

4.1. Estimation errors at the base station

In the following a time division duplex (TDD) system is considered in which channel matrices in the uplink are simply the transposes of channel matrices in the downlink. In such a system, in order to provide the base station with channel knowledge, each user k may simultaneously send pilot sequences of length B over each of its r_k antennas. Let $\boldsymbol{Q}_k \in \mathbb{C}^{r_k \times B}$ denote the matrix formed by r_k pilot sequences transmitted by user k. The signal received by the base station is given by

$$\boldsymbol{Y} = \boldsymbol{H}^{\mathrm{T}} \boldsymbol{Q} + \boldsymbol{N} \in \mathbb{C}^{t \times B},$$

where $\boldsymbol{Q} = [\ \boldsymbol{Q}_1^{\mathrm{T}} \ \cdots \ \boldsymbol{Q}_K^{\mathrm{T}}\ ]^{\mathrm{T}}$ and $\boldsymbol{N}$ is a matrix of additive Gaussian noise with zero-mean, unit variance, uncorrelated entries. This expression can be rearranged as $\boldsymbol{y} = \tilde{\boldsymbol{Q}} \boldsymbol{h} + \boldsymbol{n}$, where $\boldsymbol{y}$, $\boldsymbol{h}$ and $\boldsymbol{n}$ are vectors obtained by stacking the columns of matrices $\boldsymbol{Y}$, $\boldsymbol{H}^{\mathrm{T}}$ and $\boldsymbol{N}$, respectively, and $\tilde{\boldsymbol{Q}} = (\boldsymbol{Q}^{\mathrm{T}} \otimes \boldsymbol{I}_t)$. The maximum likelihood estimate of $\boldsymbol{h}$ is given by $\hat{\boldsymbol{h}} = (\tilde{\boldsymbol{Q}}^{\mathrm{H}} \tilde{\boldsymbol{Q}})^{-1} \tilde{\boldsymbol{Q}}^{\mathrm{H}} \boldsymbol{y}$, the estimation error by $\boldsymbol{e} = (\tilde{\boldsymbol{Q}}^{\mathrm{H}} \tilde{\boldsymbol{Q}})^{-1} \tilde{\boldsymbol{Q}}^{\mathrm{H}} \boldsymbol{n}$ and, if the sequences are chosen to be mutually orthogonal and with the same average power $P_{\mathrm{p}}^{\mathrm{up}}$, its covariance matrix reads $\mathrm{E}\{\boldsymbol{e}\boldsymbol{e}^{\mathrm{H}}\} = \sigma_{\mathrm{Tx}}^2 \boldsymbol{I}_{(r_1 + \ldots + r_k)t}$ with $\sigma_{\mathrm{Tx}}^2 = 1/(P_{\mathrm{p}}^{\mathrm{up}} B)$, i.e. the error variance is the same for all channel coefficients and estimation errors of different coefficients are uncorrelated.

4.2. Estimation errors at the mobile stations

In the downlink, if $\pi_1(d) = k$, all user k must know to perform optimum detection is vector $\boldsymbol{g}_k^d$ [6]. To provide the mobile stations with this knowledge the base station may simultaneously transmit orthogonal pilot sequences of length B and average power ρ over each allocated dimension. Defining matrix $\boldsymbol{Q} \in \mathbb{C}^{D \times B}$ formed by D training sequences corresponding to D assigned dimensions, user k receives

$$\boldsymbol{Y} = \boldsymbol{G}_k \boldsymbol{Q} + \boldsymbol{N}_k.$$

Let $\boldsymbol{g}_k = [\ \boldsymbol{g}_k^{1,\mathrm{T}} \ \cdots \ \boldsymbol{g}_k^{D,\mathrm{T}}\ ]^{\mathrm{T}}$. A maximum likelihood estimate $\hat{\boldsymbol{g}}_k$ of this vector can be computed following the steps described in the previous section. The covariance matrix of the corresponding estimation error $\boldsymbol{f}_k$ is now given by $\mathrm{E}\{\boldsymbol{f}_k\boldsymbol{f}_k^{\mathrm{H}}\} = \sigma^2_{\mathrm{Rx}}\boldsymbol{I}_{Dr_k}$ with $\sigma^2_{\mathrm{Rx}} = 1/(\rho B)$.

4.3. Performance analysis

The base station first computes matrix $\boldsymbol{V}$ and matrix $\boldsymbol{P}$ based on the uplink estimates $\hat{\boldsymbol{H}}_k$. Accordingly, for user k,

$$\boldsymbol{G}_k = \hat{\boldsymbol{G}}_k - \boldsymbol{F}_{\mathrm{Tx},k} = (\hat{\boldsymbol{H}}_k - \boldsymbol{E}_k)\boldsymbol{V}\boldsymbol{P}^{1/2},$$

where $\boldsymbol{E}_k$ is the matrix of estimation errors with covariance σ^2_{Tx}. The coefficients of matrix $\boldsymbol{G}_k$ are estimated at the receiver as it was described in the previous section. As a result we obtain

$$\hat{\hat{\boldsymbol{G}}}_k = \boldsymbol{G}_k + \boldsymbol{F}_{\mathrm{Rx},k} = \hat{\boldsymbol{G}}_k - \boldsymbol{F}_{\mathrm{Tx},k} + \boldsymbol{F}_{\mathrm{Rx},k},$$

where $\boldsymbol{F}_{\mathrm{Rx},k}$ is the matrix of estimation errors with covariance σ^2_{Rx}.

If $\pi(d) = k$, user k will try to detect symbol $\tilde{s}_d$ as explained in Section 3.2 but now rather than (4) the following expression is obtained,

$$y_d = \frac{\hat{\hat{\boldsymbol{g}}}_k^{d,\mathrm{H}}\boldsymbol{g}_k^d}{\|\hat{\hat{\boldsymbol{g}}}_k^d\|^2}s_d + \sum_{\ell\neq d}\frac{\hat{\hat{\boldsymbol{g}}}_k^{d,\mathrm{H}}\boldsymbol{g}_k^\ell}{\|\hat{\hat{\boldsymbol{g}}}_k^d\|^2}s_\ell + \frac{\hat{\hat{\boldsymbol{g}}}_k^{d,\mathrm{H}}\boldsymbol{n}_k}{\|\hat{\hat{\boldsymbol{g}}}_k^d\|^2}. \quad (6)$$

The base station, in turn, generates s_d also based on the estimate $\hat{\boldsymbol{G}}_k$, which yields

$$s_d = \tilde{s}'_d - \sum_{\ell=1}^{d-1}\frac{\hat{\boldsymbol{g}}_k^{d,\mathrm{H}}\hat{\boldsymbol{g}}_k^\ell}{\|\hat{\boldsymbol{g}}_k^d\|^2}s_\ell. \quad (7)$$

Finally, plugging (7) into (6) and observing $\hat{\boldsymbol{g}}_k^{d,\mathrm{H}}\hat{\boldsymbol{g}}_k^\ell = 0\ \forall\ell > d$ we get

$$y_d(\boldsymbol{f}^d_{\mathrm{Tx},k}, \boldsymbol{f}^d_{\mathrm{Rx},k}) = \frac{\hat{\hat{\boldsymbol{g}}}_k^{d,\mathrm{H}}\boldsymbol{g}_k^d}{\|\hat{\hat{\boldsymbol{g}}}_k^d\|^2}\tilde{s}'_d + \\ + \sum_{\ell\neq d}\left(\frac{\hat{\hat{\boldsymbol{g}}}_k^{d,\mathrm{H}}\boldsymbol{g}_k^\ell}{\|\hat{\hat{\boldsymbol{g}}}_k^d\|^2} - \frac{\hat{\hat{\boldsymbol{g}}}_k^{d,\mathrm{H}}\boldsymbol{g}_k^d}{\|\hat{\hat{\boldsymbol{g}}}_k^d\|^2}\frac{\hat{\boldsymbol{g}}_k^{d,\mathrm{H}}\hat{\boldsymbol{g}}_k^\ell}{\|\hat{\boldsymbol{g}}_k^d\|^2}\right)s_\ell + \frac{\hat{\hat{\boldsymbol{g}}}_k^{d,\mathrm{H}}\boldsymbol{n}_k}{\|\hat{\hat{\boldsymbol{g}}}_k^d\|^2}. \quad (8)$$

Noting $\boldsymbol{g}_k^d = \hat{\boldsymbol{g}}_k^d - \boldsymbol{f}^d_{\mathrm{Tx},k}$ and $\hat{\hat{\boldsymbol{g}}}_k^d = \hat{\boldsymbol{g}}_k^d - \boldsymbol{f}^d_{\mathrm{Tx},k} + \boldsymbol{f}^d_{\mathrm{Rx},k}$, (8) can be linearized with respect to $\boldsymbol{f}^d_{\mathrm{Tx},k}$ and $\boldsymbol{f}^d_{\mathrm{Rx},k}$ using a Taylor expansion about zero,

$$y_d \approx y_d(0,0) + \boldsymbol{f}^{d,\mathrm{T}}_{\mathrm{Tx},k}\left.\frac{\partial y_d}{\partial\boldsymbol{f}^d_{\mathrm{Tx},k}}\right|_0 + \boldsymbol{f}^{d,\mathrm{H}}_{\mathrm{Tx},k}\left.\frac{\partial y_d}{\partial\boldsymbol{f}^{d,*}_{\mathrm{Tx},k}}\right|_0 + \\ + \boldsymbol{f}^{d,\mathrm{T}}_{\mathrm{Rx},k}\left.\frac{\partial y_d}{\partial\boldsymbol{f}^d_{\mathrm{Rx},k}}\right|_0 + \boldsymbol{f}^{d,\mathrm{H}}_{\mathrm{Rx},k}\left.\frac{\partial y_d}{\partial\boldsymbol{f}^{d,*}_{\mathrm{Rx},k}}\right|_0 =$$

$$= \tilde{s}'_d - \frac{\hat{\boldsymbol{g}}_k^{d,\mathrm{H}}\boldsymbol{f}^d_{\mathrm{Rx},k}}{\|\hat{\boldsymbol{g}}_k^d\|^2}\tilde{s}'_d + \sum_{\ell\neq d}2\mathrm{Re}\left\{\hat{\boldsymbol{g}}_k^{d,\mathrm{H}}\boldsymbol{f}^d_{\mathrm{Tx},k}\right\}\frac{\hat{\boldsymbol{g}}_k^{d,\mathrm{H}}\boldsymbol{g}_k^\ell}{\|\hat{\boldsymbol{g}}_k^d\|^4}s_\ell \\ + \sum_{\ell\neq d}\left(2\mathrm{Re}\left\{\hat{\boldsymbol{g}}_k^{d,\mathrm{H}}\boldsymbol{f}^d_{\mathrm{Rx},k}\right\}\frac{\hat{\boldsymbol{g}}_k^{d,\mathrm{H}}\boldsymbol{f}^\ell_{\mathrm{Tx},k}}{\|\hat{\boldsymbol{g}}_k^d\|^4} - \frac{\boldsymbol{f}^{d,\mathrm{H}}_{\mathrm{Tx},k}\boldsymbol{g}_k^\ell}{\|\hat{\boldsymbol{g}}_k^d\|^2}\right)s_\ell \\ + \sum_{\ell\neq d}\left(\frac{\boldsymbol{f}^{d,\mathrm{H}}_{\mathrm{Rx},k}\boldsymbol{g}_k^\ell}{\|\hat{\boldsymbol{g}}_k^d\|^2} - \frac{\hat{\boldsymbol{g}}_k^{d,\mathrm{H}}\boldsymbol{f}^\ell_{\mathrm{Tx},k}}{\|\hat{\boldsymbol{g}}_k^d\|^2} - \frac{\boldsymbol{f}^{d,\mathrm{H}}_{\mathrm{Rx},k}\hat{\boldsymbol{g}}_k^d}{\|\hat{\boldsymbol{g}}_k^d\|^2}\frac{\hat{\boldsymbol{g}}_k^{d,\mathrm{H}}\hat{\boldsymbol{g}}_k^\ell}{\|\hat{\boldsymbol{g}}_k^d\|^2}\right)s_\ell \\ + \left(1 - \frac{2\mathrm{Re}\left\{\boldsymbol{f}^{d,\mathrm{H}}_{\mathrm{Rx},k}\hat{\boldsymbol{g}}_k^d\right\}}{\|\hat{\boldsymbol{g}}_k^d\|^2} + \frac{2\mathrm{Re}\left\{\boldsymbol{f}^{d,\mathrm{H}}_{\mathrm{Tx},k}\hat{\boldsymbol{g}}_k^d\right\}}{\|\hat{\boldsymbol{g}}_k^d\|^2}\right)\frac{\hat{\boldsymbol{g}}_k^{d,\mathrm{H}}\boldsymbol{n}_k}{\|\hat{\boldsymbol{g}}_k^d\|^2} \\ + \left(\boldsymbol{f}^{d,\mathrm{H}}_{\mathrm{Rx},k} - \boldsymbol{f}^{d,\mathrm{H}}_{\mathrm{Tx},k}\right)\boldsymbol{n}_k = \tilde{s}'_d + w_d.$$

In this expression the first term corresponds to the desired signal and all other terms constitute disturbing signals due to noise and estimation errors. Now, computing the variance of the resulting equivalent noise term w_d we obtain

$$\sigma^2_{w_d} = \frac{1}{\|\hat{\boldsymbol{g}}_k^d\|^2} + \frac{\sigma^2_{\mathrm{Rx}}}{\|\hat{\boldsymbol{g}}_k^d\|^4}\left(\sum_{\ell\neq d}\hat{\boldsymbol{g}}_k^{\ell,\mathrm{H}}\left(\boldsymbol{I}_{r_k} - \frac{\hat{\boldsymbol{g}}_k^d\hat{\boldsymbol{g}}_k^{d,\mathrm{H}}}{\|\hat{\boldsymbol{g}}_k^d\|^2}\right)\hat{\boldsymbol{g}}_k^\ell\right) + \\ \frac{\sigma^2_{\mathrm{Tx}}\sum_{\ell\neq d}p_\ell}{\|\hat{\boldsymbol{g}}_k^d\|^2} + \frac{r_k\sigma^2_{\mathrm{Tx}}\sigma^2_{\mathrm{Rx}}\sum_{\ell\neq d}p_\ell}{\|\hat{\boldsymbol{g}}_k^d\|^4} + \frac{\sigma^2_{\mathrm{Tx}}p_d\sum_{\ell\neq d}\|\hat{\boldsymbol{g}}_k^\ell\|^2}{\|\hat{\boldsymbol{g}}_k^d\|^4} \\ + \frac{r_k\sigma^4_{\mathrm{Tx}}p_d\sum_{\ell\neq d}p_\ell}{\|\hat{\boldsymbol{g}}_k^d\|^4} + \frac{r_k\sigma^2_{\mathrm{Tx}}p_d}{\|\hat{\boldsymbol{g}}_k^d\|^4} + \frac{r_k\sigma^2_{\mathrm{Rx}}}{\|\hat{\boldsymbol{g}}_k^d\|^4} + \frac{\sigma^2_{\mathrm{Rx}}|\tilde{s}'_d|^2}{\|\hat{\boldsymbol{g}}_k^d\|^2},$$

where p_ℓ denotes the ℓth entry in the diagonal of matrix $\boldsymbol{P}$. In order to estimate transmission quality, w_d is considered to be circularly symmetric complex Gaussian distributed, which is not true but enormously simplifies matters and still provides a good approximation to system perfomance. Based on this assumption a good estimate of the SER on dimension d is given by

$$\mathrm{SER}_d \approx 4Q\left(\sqrt{\frac{\mu_d^2}{2\sigma^2_{w_d}}}\right), \quad (9)$$

where $Q(\cdot)$ is the Gaussian tail function and μ_d^2 is the minimum distance between neighbouring points of the chosen constellation with power 1.

5. ERROR ADAPTIVE LOADING

As explained in Section 1 the transmitter performs bit and power loading based on a target SER. If this loading is done taking the channel estimate as the actual channel, estimation errors will lead to a violation of the target SER. However, using the analytical results derived in the previous section, the transmitter can readjust the bit and power load obtained on each subchannel so that in spite of estimation errors the quality constraint is met. The simplest way of doing this is by removing as many bits as needed in order to get below the target SER. This is equivalent to choosing a modulation alphabet with larger μ_d (see (9)). Note that as long as the power loading remains constant, the variance of the equivalent noise does not change.

6. NUMERICAL RESULTS

Fig. 1 shows performance degradation of CZF-SESAM in a setting with $t = 4$ transmit antennas, $K = 2$ users and $r_k = 2$ receive antennas per user. The horizontal axis indicates the mean square error (MSE) of the channel estimation, which we define as $\mathrm{MSE} = \sigma^2_{\mathrm{Rx}} t / P_{\mathrm{Tx}} = \sigma^2_{\mathrm{Tx}}$. The vertical axis indicates average SER, where the average is taken over a large number of realizations of matrix $\boldsymbol{H}$, whose entries are independent and identically distributed according to a zero-mean complex Gaussian distribution with unity covariance. Solid and dashed lines represent simulated performance for targets $\mathrm{SER} = 0.1$ and $\mathrm{SER} = 0.01$, respectively. Dotted lines are obtained analytically estimating SER for each channel realization. Lines are plotted for four different values of signal-to-noise ratio $\mathrm{SNR} = P_{\mathrm{Tx}}/\sigma^2$.

As the error variance increases performance degrades. Sensitivity grows with increasing transmit power. Indeed, low SNR values produce very conservative bit loadings with large distances between neighboring points of the resulting signal constellations, whereas high SNR values yield heavily loaded subchannels where neighboring points are very close. This makes detection very sensitive to any unexpected source of noise. Analytical results match the simulated results very well especially for low to moderate estimation errors, which is the range for which the linear approximation is good enough, and for low and moderate SER values, which is the range in which (9) is good enough.

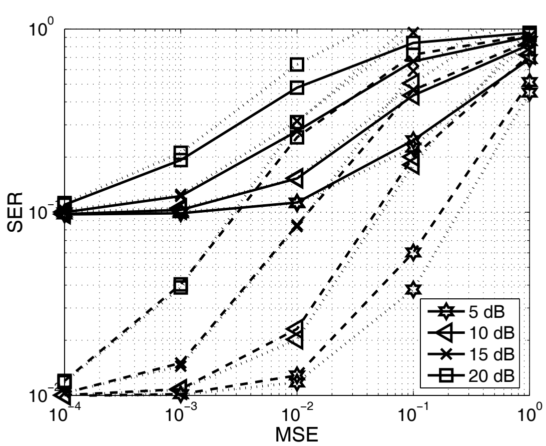

Figure 1. Performance impact of estimation errors. $t = 4$, $K = 2$, $r_k = 2$.

For the same settings, Fig. 2 shows performance after bit loading readjustment. It can be observed that for MSE values up to 0.1 analytical results allow to perform a readjustment of the bit load on each subchannel such that compliance with the target SER is still possible.

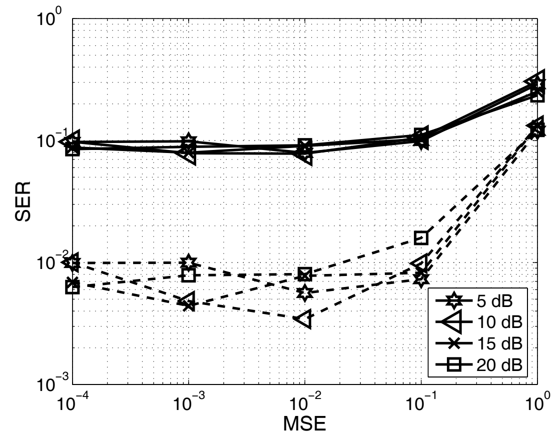

Figure 2. Performance of CZF-SESAM with estimation errors and readjusted loading. $t = 4$, $K = 2$, $r_k = 2$.

7. REFERENCES

[1] P. Tejera, W. Utschick, G. Bauch, and J. A. Nossek, "A Novel Decomposition Technique for Multiuser MIMO," in *IEEE/ITG Workshop on Smart Antennas*, Duisburg, Apr. 2005.

[2] J. Brehmer, A. Molin, P. Tejera, and W. Utschick, "Low Complexity Approximation of the MIMO Broadcast Channel Capacity Region," in *ICC'06*, Istanbul, 2006.

[3] G. Ginis and J. Cioffi, "A Multi-user Precoding Scheme achieving Crosstalk Cancellation with Application to DSL Systems," in *Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, Oct. 2000, pp. 1627–1631.

[4] J. A. C. Bingham, "Multicarrier Modulation for Data Transmission: an Idea whose Time Has Come," *IEEE Commun. Mag.*, vol. Vol. 28, pp. 5–14, May 1990.

[5] P. Tejera, W. Utschick, G. Bauch, and J. A. Nossek, "Sum-Rate Maximizing Decomposition Approaches for Multiuser MIMO-OFDM," in *PIMRC 05*, Berlin, Sep. 2005.

[6] P. Tejera, W. Utschick, G. Bauch, and J. A. Nossek, "Efficient Implementation of Successive Encoding Schemes for the MIMO OFDM Broadcast Channel," in *ICC 06*, Istanbul, 2006.