Extension of Linear and Nonlinear Transmit Filters for Decentralized Receivers

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***Abstract*: For the downlink of a multi-user system with decentralized receivers, we extend both linear and nonlinear transmit filters. As a special case of the joint optimization of transmitter and receiver, transmit processing emerges from restricting the receivers to be scalar. Such a setup then corresponds to the downlink of a multi-user system with decentralized receivers that cannot cooperate and cannot combine their received signals in order to obtain the symbol estimates of the data streams associated to them. Instead of designing the transmitter based on the common assumption that all receivers use the same scalar weight, i. e. the factor of the *automatic gain control* (AGC), we allow different weights and end up with modified transmitter structures. Thus, performance improvements can be obtained especially for scenarios, where the receivers have different average channel powers due to path loss or shadowing. Consequently, smaller bit error rates can be allocated to users with strong channels compared to the conventional case, whereas users with weak channel powers show almost no deterioration.**

1. Introduction

Transmit processing at the base station has gained a lot of attraction in wireless communications, see [1, 2, 3], [4], or [5]. In the downlink of a time division duplex system, signal processing can be transferred from the receiver side to the transmitter side (base station), which has less stringent constraints in terms of available computing power and power supply. Due to this shift of the computational load, simple and cheap receivers are feasible offering long standby durations. Complex operations like channel estimation and FIR filtering for equalization become obsolete. As highlighted in [6], transmit processing can be regarded as a special case of the joint optimization of transmitter and receiver. An additional constraint in the joint optimization ensures, that the receivers are kept very simple by performing only a scalar weighting to their received signals even in frequency selective scenarios. Note that when completely omitting *any* kind of signal processing at the receivers, many optimization criteria like unbiasedness for example could not be maintained due to the limited available transmit power. The main computational load thus is located at the transmitter side, being an attractive scheme for the downlink in wireless multi-user systems. Due to the decentralized receivers (e.g. [7, 8, 9, 10, 11]), non-cooperative receive processing has to be applied as the individual users have different locations. This broadcast channel scenario leads to a further restriction of the "receiver side" compared to the general single-user *multiple-input multiple-output* (MIMO) case [12], [13],

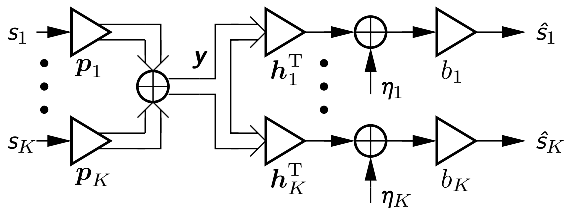

Figure 1: Downlink of multi-user *multiple-input single-output* (MISO) system.

where the signals of all receiving antennas are combined in order to estimate the data symbols of the allocated data streams. Instead, each data stream is associated to a different user implying that the receiving antennas are located somewhere in the coverage area of the serving base station, making cooperation impossible. The diagonalization of the channel at both receiver and transmitter thus does not work as in [12], [13], and [14].

A popular approach for transmit processing is to allocate identical scalar weights to all receivers, following from the initial approach of shifting the equalization from the receiver to the transmitter, e.g. see [6, 15]. We drop this constraint and allow for different user weights leading to modified transmitter structures. Note that also in [7, 8, 9], different scalar weights are applied, leading to a suboptimum solution there since the heuristic approach is not based on an optimization criterion. Whether the receiver units apply scalar weights or not does not change the individual instantaneous *signal-to-noise ratios* (SNRs) as both noise and signals of each user are amplified by the same factor. However, the assumption of the different receiver weighting surprisingly leads to *new transmit filter structures*. Note that the user weights can be interpreted as an AGC making sure that the signal level is correct. For QAM modulation alphabets where symbol information is also stored in the amplitude not only in the phase, the application of the correct weight is indispensable.

This paper is organized as follows: In Section 2, we discuss the system model underlying all derivations and simulation results. The linear transmit matched filter, the linear transmit zero-forcing filter, and the linear transmit Wiener filter are extended in Section 3, whereas the modified nonlinear zero-forcing filter with *Tomlinson-Harashima-Precoding* (THP) is derived in Section 4. Having presented simulation results in Section 5, this paper concludes in Section 6.

2. System model and notation

Fig. 1 shows the downlink of the broadcast channel [10]. For the sake of simplicity, we restrict ourselves to non-dispersive channels with multiple transmit antennas, as the main principles of the extension towards different scaling factors hold in this case, too. For frequency selective channels, the modifications can be implemented in a similar way, and surprisingly, latency time optimization is mostly decoupled from the determination of all other parameters. The complex-valued data symbols of the K users are denoted by s_k, $k \in \{1,\ldots,K\}$ and are stacked in the column vector

$$\boldsymbol{\mathsf{s}} = [\mathsf{s}_1,\ldots,\mathsf{s}_K]^{\mathrm{T}} \in \mathbb{C}^K.$$

As we focus on frequency flat channels, no FIR filtering has to be applied at the transmitter. Hence, the transmit filter consists of K spatial filters $\boldsymbol{p}_k \in \mathbb{C}^{N_\mathrm{a}}$ corresponding to the K columns of the precoding filter matrix

$$\boldsymbol{P} = [\boldsymbol{p}_1,\ldots,\boldsymbol{p}_K] \in \mathbb{C}^{N_\mathrm{a}\times K}.$$

Here, N_a denotes the number of antenna elements deployed at the transmitter. The k-th row $\boldsymbol{h}_k^{\mathrm{T}}$ of the channel matrix

$$\boldsymbol{H} = [\boldsymbol{h}_1,\ldots,\boldsymbol{h}_K]^{\mathrm{T}} \in \mathbb{C}^{K\times N_\mathrm{a}}$$

reflects the channel coefficients comprising the transmission coefficients from all N_a transmit antennas to the k-th receiver. The entries of $\boldsymbol{h}_k$ are complex-valued realizations of N_a zero-mean Gaussian random variables each having the same variance $\sigma^2_{\mathsf{h}_k}$. Different path losses of the users become manifest in different variances $\sigma^2_{\mathsf{h}_k}$ of the random variables generating the entries of the K channel vectors $\boldsymbol{h}_k$. For the derivations in Sections 3 and 4, we assume perfect *channel state information* (CSI) at the transmitter side. Inaccuracies in CSI due to channel estimation or time-variant scenarios can be overcome by channel prediction and a robust paradigm, see [16] or [17]. The noise vector $\boldsymbol{\eta}$ contains the perturbations at the K receivers. Finally, the user weight of user k is represented by $b_k \in \mathbb{R}_{+,0}$, again $k \in \{1,\ldots,K\}$. For notational simplicity, we introduce the *diagonal* matrix

$$\boldsymbol{B} = \mathbf{diag}\{b_k\}_{k=1}^K \in \mathbb{R}_{+,0}^{K\times K},$$

which now no longer has to be a scaled identity matrix $\beta^{-1}\boldsymbol{I}_K$ as in [6] and [18] where all users had to apply the same weight β^{-1}, i.e. $b_k = \beta^{-1}\forall k$. With these definitions, the estimate $\hat{\boldsymbol{\mathsf{s}}}$ for the user symbols in $\boldsymbol{\mathsf{s}}$ reads as

$$\hat{\boldsymbol{\mathsf{s}}} = \boldsymbol{B}\boldsymbol{H}\boldsymbol{P}\boldsymbol{\mathsf{s}} + \boldsymbol{B}\boldsymbol{\eta} \ \in \mathbb{C}^K. \tag{1}$$

Notation: Deterministic vectors and matrices are denoted by lower and upper case bold letters. The respective random variables are written in sans serif font. The operators $\mathrm{E}[\cdot]$, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, $(\cdot)^+$, $(\cdot)^*$, and $\mathrm{tr}(\cdot)$ stand for expectation with respect to symbols and noise, transposition, Hermitian transposition, pseudo inverse, complex conjugate, and trace of a matrix, respectively. The scalar element in row b and column c of the matrix $\boldsymbol{A}$ is denoted by $[\boldsymbol{A}]_{b,c}$, and $\boldsymbol{I}_K$ stands for the $K\times K$ identity matrix, whose k-th column is $\boldsymbol{e}_k$, and $\|\cdot\|_2$ is the Euclidean norm. The zero-matrix of dimension $m\times n$ reads as $\mathbf{0}_{m\times n}$.

3. Extension of linear transmit filters

In this section, we extend three well-known *conventional* linear transmit filters to their respective *diagonal* versions with different user weights, namely the *transmit matched filter* (TxMF), the *transmit zero-forcing filter* (TxZF), and the *transmit Wiener filter* (TxWF).

3.1 Diagonal Transmit Matched Filter

The *diagonal transmit matched filter* (DTxMF) maximizes

$$\begin{aligned}\gamma(\boldsymbol{P}, b_1,\ldots,b_K) &= \frac{\left|\mathrm{E}\left[\hat{\boldsymbol{\mathsf{s}}}^{\mathrm{H}}\boldsymbol{\mathsf{s}}\right]\right|^2}{\mathrm{E}\left[\|\boldsymbol{\mathsf{s}}\|_2^2\right]\mathrm{E}\left[\|\boldsymbol{B}\boldsymbol{\eta}\|_2^2\right]} \\ &= \frac{|\mathrm{tr}(\boldsymbol{B}\boldsymbol{H}\boldsymbol{P}\boldsymbol{R}_{\mathsf{s}})|^2}{\mathrm{tr}(\boldsymbol{R}_{\mathsf{s}})\,\mathrm{tr}(\boldsymbol{B}\boldsymbol{R}_{\boldsymbol{\eta}}\boldsymbol{B})} \in \mathbb{R},\end{aligned} \tag{2}$$

where (2) rather describes the total SNR of a MIMO system than that of a multi-user MISO system (MIMO with decentralized receivers), but otherwise a weighted sum of the individual SNRs would have to be minimized which we do not consider here. The desired transmit filter $\boldsymbol{P}_{\mathrm{MF}}$ and the user weights $\boldsymbol{B}_{\mathrm{MF}}$ follow from

$$\begin{aligned}\{\boldsymbol{P}_{\mathrm{MF}}, \boldsymbol{B}_{\mathrm{MF}}\} &= \underset{\{\boldsymbol{P},\boldsymbol{B}\}}{\mathrm{argmax}}\,\gamma(\boldsymbol{P}, b_1,\ldots,b_K) \\ \text{subject to: } &\mathrm{E}\left[\|\boldsymbol{P}\boldsymbol{\mathsf{s}}\|_2^2\right] \le E_{\mathrm{tr}}, \text{ and} \\ &\boldsymbol{B} = \textstyle\sum_{k=1}^K b_k \boldsymbol{e}_k\boldsymbol{e}_k^{\mathrm{T}}.\end{aligned} \tag{3}$$

The first constraint in (3) ensures that the transmit power does not exceed E_{tr}, whereas the second constraint forces the receivers not to cooperate, so the estimate $\hat{\mathsf{s}}_k$ of the data symbol s_k from user k has to be computed from the received signal $\mathsf{r}_k = \boldsymbol{h}_k^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\mathsf{s}} + \eta_k$, other signals r_l with $l \neq k$ must not be utilized. For uncorrelated data symbols, i.e. diagonal $\boldsymbol{R}_{\mathsf{s}} = \mathrm{E}[\boldsymbol{\mathsf{s}}\boldsymbol{\mathsf{s}}^{\mathrm{H}}]$, solving (3) returns a transmit filter

$$\boldsymbol{P} = \sqrt{\frac{E_{\mathrm{tr}}}{\mathrm{tr}(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{B}\boldsymbol{R}_{\mathsf{s}}\boldsymbol{B}\boldsymbol{H})}}\,\mathrm{e}^{\mathrm{j}\varphi}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{B}, \tag{4}$$

which still depends on the choice of $\boldsymbol{B}$, and $\varphi \in [0; 2\pi)$ may be chosen freely as it does not change the total SNR. By means of (4), (2) reduces to

$$\begin{aligned}\gamma(b_1,\ldots,b_K) &= \frac{E_{\mathrm{tr}}}{\mathrm{tr}(\boldsymbol{R}_{\mathsf{s}})}\frac{\mathrm{tr}(\boldsymbol{B}\boldsymbol{R}_{\mathsf{s}}\boldsymbol{B}\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}})}{\mathrm{tr}(\boldsymbol{B}\boldsymbol{R}_{\boldsymbol{\eta}}\boldsymbol{B})} \\ &= \frac{E_{\mathrm{tr}}}{\sum_{k=1}^K \sigma^2_{\mathsf{s}_k}}\frac{\sum_{k=1}^K b_k^2\sigma^2_{\mathsf{s}_k}\|\boldsymbol{h}_k\|_2^2}{\sum_{k=1}^K b_k^2\sigma^2_{\boldsymbol{\eta}_k}},\end{aligned} \tag{5}$$

with $\sigma^2_{\mathsf{s}_k} = [\boldsymbol{R}_{\mathsf{s}}]_{k,k}$, and $\sigma^2_{\boldsymbol{\eta}_k} = \mathrm{E}[|\eta_k|^2]$. The total SNR $\gamma(b_1,\ldots,b_K)$ in (5) is maximized by choosing

$$b_k = 0 \quad \forall k \neq k_{\mathrm{MF}} \ \text{ and } \ b_{k_{\mathrm{MF}}} \neq 0, \tag{6}$$

k_{MF} denoting the only served user following from the maximum metric

$$k_{\mathrm{MF}} = \underset{k\in\{1,\ldots,K\}}{\mathrm{argmax}}\,\frac{\sigma^2_{\mathsf{s}_k}}{\sigma^2_{\boldsymbol{\eta}_k}}\|\boldsymbol{h}_k\|_2^2. \tag{7}$$

Eqn. (7) results from the partial derivative of (5) with respect to all b_k, leading to an eigenvalue problem. The magnitude of $b_{k_{\mathrm{MF}}}$ is not of relevancy, since it is revoked in (4), the power constraint consequently is fulfilled. Altogether, the transmit filter $\boldsymbol{P}_{\mathrm{MF}}$ and the user weights $b_{\mathrm{MF},1}, \ldots, b_{\mathrm{MF},K}$ read as

$$\begin{aligned} \boldsymbol{P}_{\mathrm{MF}} &= \sqrt{\frac{E_{\mathrm{tr}}}{\sigma^2_{\boldsymbol{s}_{k_{\mathrm{MF}}}}}} \mathrm{e}^{\mathrm{j}\varphi} \frac{\boldsymbol{h}^*_{k_{\mathrm{MF}}}}{\|\boldsymbol{h}_{k_{\mathrm{MF}}}\|_2} \boldsymbol{e}^{\mathrm{T}}_{k_{\mathrm{MF}}}, \\ \boldsymbol{b}_{\mathrm{MF},k_{\mathrm{MF}}} &= 1, \quad \boldsymbol{b}_{\mathrm{MF},k} = 0\ \forall k \neq k_{\mathrm{MF}}. \end{aligned} \tag{8}$$

From (8) we can conclude, that the *diagonal* version of the *transmit matched filter* only accounts for the data stream of user k_{MF}, whereas all other data streams are discarded ($b_k = 0\ \forall k \neq k_{\mathrm{MF}}$). In contrast, the *conventional* TxMF serves *all* users according to their channel powers.

3.2 Diagonal Transmit Zero-Forcing Filter

The *diagonal transmit zero-forcing filter* (DTxZF) is characterized by complete interference cancellation and unbiasedness of the desired signals. In the noise free case, i.e. $\boldsymbol{\eta} = \boldsymbol{0}_{K\times 1}$, the estimate $\hat{\boldsymbol{s}}$ is therefore identical to the original symbol vector $\boldsymbol{s}$. The cost function to optimize is the mean square error

$$\begin{aligned} \varepsilon(\boldsymbol{P}, b_1, \ldots, b_K) &= \mathrm{E}\left[\|\hat{\boldsymbol{s}} - \boldsymbol{s}\|_2^2\right] \\ &= \sum_{k=1}^{K} b_k^2 \sigma^2_{\boldsymbol{\eta}_k} \ \in \mathbb{R}_{+,0} \end{aligned} \tag{9}$$

of the symbols $\boldsymbol{s}$ and their estimates $\hat{\boldsymbol{s}}$, which boils down to the simple sum of the weighted noise variances $\sigma^2_{\boldsymbol{\eta}_k}$ because of the interference removal constraint $\boldsymbol{BHP} = \boldsymbol{I}_K$. The optimization underlying the *transmit zero-forcing filter* reads as

$$\begin{aligned} &\{\boldsymbol{P}_{\mathrm{ZF}}, \boldsymbol{B}_{\mathrm{ZF}}\} = \underset{\{\boldsymbol{P},\boldsymbol{B}\}}{\mathrm{argmin}}\, \varepsilon(\boldsymbol{P}, b_1, \ldots, b_K) \\ &\text{subject to: } \boldsymbol{BHP} = \boldsymbol{I}_K,\ \mathrm{E}\left[\|\boldsymbol{Ps}\|_2^2\right] \leq E_{\mathrm{tr}}, \text{ and} \\ &\qquad \boldsymbol{B} = \textstyle\sum_{k=1}^{K} b_k \boldsymbol{e}_k \boldsymbol{e}_k^{\mathrm{T}}. \end{aligned} \tag{10}$$

The first constraint in (10) corresponds to unbiasedness and complete interference suppression. Using the method of Lagrangian multipliers, we find

$$\boldsymbol{P}_{\mathrm{ZF}} = \boldsymbol{H}^{\mathrm{H}} \left(\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}\right)^{-1} \boldsymbol{B}_{\mathrm{ZF}}^{-1}, \tag{11}$$

implying a dissipated transmit power $\mathrm{E}\left[\|\boldsymbol{Ps}\|_2^2\right] = \sum_{k=1}^{K} b_k^{-2} \sigma^2_{\boldsymbol{s}_k} [(\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}})^{-1}]_{k,k}$. Limiting this value by E_{tr} and minimizing (9) finally leads to the scalar weights

$$b_{\mathrm{ZF},k} = \sqrt[4]{\frac{\alpha_k}{\sigma^2_{\boldsymbol{\eta}_k}}} \sqrt{\frac{\sum_{m=1}^{K} \sqrt{\alpha_m \sigma^2_{\boldsymbol{\eta}_m}}}{E_{\mathrm{tr}}}}, \tag{12}$$

and the substitution $\alpha_k = \sigma^2_{\boldsymbol{s}_k} [(\boldsymbol{H}\boldsymbol{H}^{\mathrm{H}})^{-1}]_{k,k}$. Inserting (11) and (12) into (9), we find the mean square error of the *diagonal transmit zero-forcing filter*

$$\varepsilon_{\mathrm{ZF}} = \tfrac{1}{E_{\mathrm{tr}}} \left(\textstyle\sum_{k=1}^{K} \sqrt{\alpha_k \sigma^2_{\boldsymbol{\eta}_k}}\right)^2. \tag{13}$$

Eqns. (11) and (12) reveal, that different scalar MSEs $\varepsilon_k = \mathrm{E}[|s_k - \hat{s}_k|]^2]$ are allocated to the different users in case of the *diagonal* version even for $\sigma^2_{\boldsymbol{\eta}_k} = \sigma^2_{\boldsymbol{\eta}}\ \forall k \in \{1, \ldots, K\}$, whereas the *conventional* counterpart with equal user weights assigns identical MSEs. If the noise variances $\sigma^2_{\boldsymbol{\eta}_k}$ and the symbol variances $\sigma^2_{\boldsymbol{s}_k}$ are invariant w.r.t. k, the MSE quotient in a two user scenario can be expressed as

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{b_1^2}{b_2^2} = \frac{\|\boldsymbol{h}_2\|_2}{\|\boldsymbol{h}_1\|_2}, \tag{14}$$

i.e. the user with the stronger channel is favored and will exhibit a smaller MSE. For more than two users, the MSE quotient expressions become more complicated but the bottom line stays the same — "stronger" users will be preferred and will feature a smaller bit error rate.

3.3 Diagonal Transmit Wiener Filter

Based on the same cost function as the DTxZF, the *diagonal transmit Wiener filter* (DTxWF) minimizes the mean square error

$$\varepsilon(\boldsymbol{P}, b_1, \ldots, b_K) = \mathrm{E}\left[\|\boldsymbol{s} - \hat{\boldsymbol{s}}\|_2^2\right], \tag{15}$$

but *without* an unbiasedness and interference cancellation constraint, similar to the *conventional* TxWF [6], [18]:

$$\begin{aligned} &\{\boldsymbol{P}_{\mathrm{WF}}, \boldsymbol{B}_{\mathrm{WF}}\} = \underset{\{\boldsymbol{P},\boldsymbol{B}\}}{\mathrm{argmin}}\, \varepsilon(\boldsymbol{P}, b_1, \ldots, b_K) \\ &\text{subject to: } \mathrm{E}\left[\|\boldsymbol{Ps}\|_2^2\right] \leq E_{\mathrm{tr}} \text{ and} \\ &\qquad \boldsymbol{B} = \textstyle\sum_{k=1}^{K} b_k \boldsymbol{e}_k \boldsymbol{e}_k^{\mathrm{T}}. \end{aligned} \tag{16}$$

The transmit filter minimizing (16) can be expressed as

$$\boldsymbol{P}_{\mathrm{WF}} = \left(\boldsymbol{H}^{\mathrm{H}} \boldsymbol{B}_{\mathrm{WF}}^2 \boldsymbol{H} + \frac{\mathrm{tr}\left(\boldsymbol{B}_{\mathrm{WF}}^2 \boldsymbol{R}_{\boldsymbol{\eta}}\right)}{E_{\mathrm{tr}}} \boldsymbol{I}_{N_{\mathrm{a}}}\right)^{-1} \boldsymbol{H}^{\mathrm{H}} \boldsymbol{B}_{\mathrm{WF}}, \tag{17}$$

whereupon it turns out to be advantageous to split up all user weights via $b_k = \beta'^{-1} b'_k$ in order to easily fulfill the power constraint in (16), since $\boldsymbol{P}_{\mathrm{WF}} = \beta' \boldsymbol{P}'$ and $\boldsymbol{P}'$ is independent of β'. In addition, evaluating (15) with (17) reveals, that only the quotients b_k/b_l are of interest, so splitting $b_k = \beta'^{-1} b'_k$ solves the first constraint in (16). For $K = 2$ users, the optimum receiver factors in $\boldsymbol{B}_{\mathrm{WF}}$ can be computed analytically by maximizing the MSE

$$\varepsilon(b_1, b_2) = \frac{c_1 b_1^4 + c_2 b_2^4 + c_{12} b_1^2 b_2^2}{d_1 b_1^4 + d_2 b_2^4 + d_{12} b_1^2 b_2^2} \tag{18}$$

from (15) having applied (17), with the constraint of real-valued *non-negative* b_k. The real-valued constants c_1, c_2, c_{12}, d_1, d_2, d_{12} depend on the channel matrix $\boldsymbol{H}$, the available maximum transmit power E_{tr}, and on the noise variances $\sigma^2_{\boldsymbol{\eta}_1}$, $\sigma^2_{\boldsymbol{\eta}_2}$. For $K \geq 3$, numerical solutions have to be applied. In any case, the *diagonal* version features the possibility — according to the given SNR E_{S}/N_0 — to switch off individual data streams. This property can also be observed in the single user MIMO case with *fully* cooperating receiving antennas, where less data streams than $\min(K, N_{\mathrm{a}})$ are transmitted over the dominant eigenmodes when the SNR is too small, cf. [12], [13], and [14]. Thereby, we observe

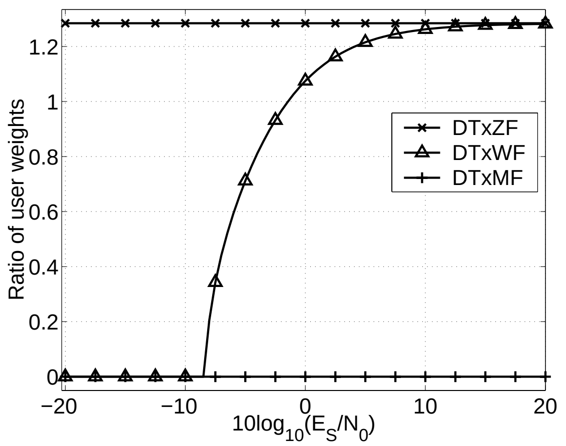

Figure 2: Ratio b_1/b_2 of the diagonal zero-forcing (DTxZF), Wiener (DTxWF), and matched filter (DTxMF) for different SNR values and an instantaneous channel realization with $\|\boldsymbol{h}_2\|_2^2/\|\boldsymbol{h}_1\|_2^2 \approx 2.9$.

the familiar convergence properties of the *conventional* TxWF, i.e. the DTxWF converges to the DTxZF for a large SNR, and to the DTxMF for the SNR reaching zero. The latter implies, that only a single user will be served, cf. (8). This behavior is shown in Fig. 2, where the quotient b_1/b_2 of the presented linear diagonal filters is plotted versus the average SNR. In this special realization of the channel, $\|\boldsymbol{h}_2\|_2^2/\|\boldsymbol{h}_1\|_2^2 \approx 2.9$, $\sigma_{\boldsymbol{\eta}_1}^2 = \sigma_{\boldsymbol{\eta}_2}^2$ and $\sigma_{\boldsymbol{s}_1}^2 = \sigma_{\boldsymbol{s}_2}^2$ was assumed. From (14), we obtain $b_{\text{ZF},1}/b_{\text{ZF},2} \approx \sqrt[4]{2.9} \approx 1.3$ independently from the SNR for the diagonal zero-forcing filter (cross marker). In contrast, the diagonal matched filter (plus marker) sets $b_{\text{MF},1} = 0$, cf. (6) and (7), since user two features a stronger channel. For the diagonal Wiener filter (triangle up marker), the first user is discarded if the SNR is below approx. -9 dB. Increasing the SNR, the quotient $b_{\text{WF},1}/b_{\text{WF},2}$ continuously increases, and asymptotically, the ratio $b_{\text{ZF},1}/b_{\text{ZF},2}$ will be reached.

4. Extension of the nonlinear transmit Zero-Forcing Filter with Tomlinson-Harashima-Precoding

By adding nonlinear modulo operators

$$\begin{aligned}\mathrm{M}(x) &= x - \left\lfloor \frac{\mathrm{Re}(x)}{\tau} + \frac{1}{2} \right\rfloor \tau - \mathrm{j} \left\lfloor \frac{\mathrm{Im}(x)}{\tau} + \frac{1}{2} \right\rfloor \tau \\ &= x + a\end{aligned} \tag{19}$$

both at transmitter and receivers, the *diagonal transmit zero-forcing filter* is equipped with THP [6], [7], [8], [9], and [15]. The constant τ depends on the used modulation alphabet (e. g. [8], [19], [20]) and $\lfloor\cdot\rfloor$ denotes the floor operator. Different from the originally intended purpose of the THP in [21] and [22], where intersymbol-interference in a single-user single-input single-output system is eliminated by a non-linear recursive structure at the transmitter, we apply this approach to the broadcast scenario in the frequency flat case, yielding a spatial precoding instead of a temporal one. The idea of allowing for different user weights b_k can also be found in [7, 8, 9], indeed it is not based upon an optimization there and thus leads to a suboptimum solution. Despite the nonlinearity of the transmit filter with the transmit signal $\boldsymbol{y} = \boldsymbol{P}\mathrm{M}(\boldsymbol{\Pi}\boldsymbol{s} + \boldsymbol{F}\boldsymbol{v})$, the framework from Subsection 3.2 can again be applied to obtain the desired zero-forcing filter, since $\mathrm{M}(x) = x + a$ with the auxiliary signal a (cf. Eqn. 19). All filter computations now rest upon the vector $\boldsymbol{d} = \boldsymbol{\Pi}^{\mathrm{T}}(\boldsymbol{I}_K - \boldsymbol{F})\boldsymbol{v} \in \mathbb{C}^K$ denoting the input of the unitary permutation matrix

$$\boldsymbol{\Pi} = \sum_{k=1}^{K} \boldsymbol{e}_k \boldsymbol{e}_{i_k}^{\mathrm{T}} \in \{0;1\}^{K\times K} \tag{20}$$

describing the precoding order (cf. Fig. 3 and see [15]). i_k denotes the index of the user precoded at the kth step. Note that the precoding order has significant impact on the bit error rate. The feedback filter $\boldsymbol{F} \in \mathbb{C}^{K\times K}$ has to be lower triangular, as only already precoded symbols can be fed back. The input of the linear filter $\boldsymbol{P}$ is denoted by $\boldsymbol{v} \in \mathbb{C}^K$, having a diagonal covariance matrix

$$\mathrm{E}[\boldsymbol{v}\boldsymbol{v}^{\mathrm{H}}] = \mathbf{diag}\{\sigma_{v_k}^2\}_{k=1}^K \in \mathbb{R}_+^{K\times K},$$

since we make the assumption, that symbol which ran through the modulo operation are almost uncorrelated among each other. Due to the modified transmitter structure, the MSE now reads as

$$\begin{aligned}&\varepsilon(\boldsymbol{P}, \boldsymbol{F}, \boldsymbol{\Pi}, b_1, \ldots, b_K) = \\ &\mathrm{E}\left[\|\boldsymbol{d} - \boldsymbol{B}(\boldsymbol{H}\boldsymbol{P}\boldsymbol{v} + \boldsymbol{\eta})\|_2^2\right] = \sum_{k=1}^{K} b_k^2 \sigma_{\boldsymbol{\eta}_k}^2,\end{aligned} \tag{21}$$

and the zero-forcing constraint looks slightly different compared to (10), cf. [15]:

$$\begin{aligned}\{\boldsymbol{P}, \boldsymbol{F}, \boldsymbol{B}, \boldsymbol{\Pi}\}_{\mathrm{ZF}} = &\operatorname*{argmin}_{\{\boldsymbol{P},\boldsymbol{F},\boldsymbol{B},\boldsymbol{\Pi}\}} \varepsilon(\boldsymbol{P}, \boldsymbol{F}, \boldsymbol{\Pi}, \boldsymbol{B}) \\ \text{subject to: } &\boldsymbol{B}\boldsymbol{H}\boldsymbol{P} = \boldsymbol{\Pi}^{\mathrm{T}}(\mathbf{1}_K - \boldsymbol{F}), \\ &\mathrm{E}\left[\|\boldsymbol{P}\boldsymbol{v}\|_2^2\right] \le E_{\mathrm{tr}}, \\ &\boldsymbol{F} \text{ lower triangular, and} \\ &\boldsymbol{B} = \textstyle\sum_{k=1}^{K} b_k \boldsymbol{e}_k \boldsymbol{e}_k^{\mathrm{T}}.\end{aligned} \tag{22}$$

Forcing $\boldsymbol{F}$ to be lower triangular with zero main-diagonal can be transferred into K constraints $\boldsymbol{S}_k \boldsymbol{F} \boldsymbol{e}_k = \mathbf{0}_{k\times 1}$, with the selection matrix $\boldsymbol{S}_k = [\boldsymbol{I}_k, \mathbf{0}_{k\times(K-k)}] \in \{0,1\}^{k\times K}$. The closed form solution of (22) can be found for an arbitrary number K of users and reads as

$$\begin{aligned}\boldsymbol{P}_{\mathrm{ZF}} &= \sum_{k=1}^{K} \boldsymbol{H}^{\mathrm{H}} \boldsymbol{B}_{\mathrm{ZF}} \boldsymbol{\Pi}_{\mathrm{ZF}}^{\mathrm{T}} \boldsymbol{S}_k^{\mathrm{T}} \\ &\quad \left(\boldsymbol{S}_k \boldsymbol{\Pi}_{\mathrm{ZF}} \boldsymbol{B}_{\mathrm{ZF}} \boldsymbol{H}\boldsymbol{H}^{\mathrm{H}} \boldsymbol{B}_{\mathrm{ZF}} \boldsymbol{\Pi}_{\mathrm{ZF}}^{\mathrm{T}} \boldsymbol{S}_k^{\mathrm{T}}\right)^{-1} \boldsymbol{S}_k \boldsymbol{e}_k \boldsymbol{e}_k^{\mathrm{T}} \\ &= \sum_{k=1}^{K} b_{\mathrm{ZF},i_k}^{-1} \left(\boldsymbol{\Pi}_k^{(\mathcal{O})} \boldsymbol{H}\right)^{+} \boldsymbol{e}_{i_k} \boldsymbol{e}_k^{\mathrm{T}}, \\ \boldsymbol{F}_{\mathrm{ZF}} &= \boldsymbol{I}_K - \boldsymbol{\Pi}_{\mathrm{ZF}} \boldsymbol{B}_{\mathrm{ZF}} \boldsymbol{H} \boldsymbol{P}_{\mathrm{ZF}}, \text{ and} \\ b_{\mathrm{ZF},i_k} &= \sqrt[4]{\frac{\alpha_k}{\sigma_{\boldsymbol{\eta}_{i_k}}^2}} \sqrt{\frac{\sum_{m=1}^{K} \sqrt{\alpha_m \sigma_{\boldsymbol{\eta}_{i_m}}^2}}{E_{\mathrm{tr}}}},\end{aligned} \tag{23}$$

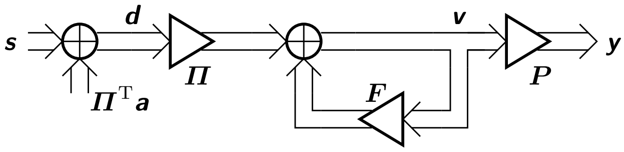

Figure 3: Quasi-linear representation of the THP without modulo operation $\mathrm{M}(\bullet)$ but auxiliary signal $\boldsymbol{\Pi}^{\mathrm{T}}\boldsymbol{a}$ instead.

with $\alpha_k = \sigma_{v_k}^2 \|(\boldsymbol{\Pi}_k^{(\mathcal{O})}\boldsymbol{H})^+ \boldsymbol{e}_{i_k}\|_2^2$, and $\boldsymbol{\Pi}_k^{(\mathcal{O})} = \boldsymbol{I}_K - \sum_{j=k+1}^{K} \boldsymbol{e}_{i_j}\boldsymbol{e}_{i_j}^{\mathrm{T}} = \boldsymbol{\Pi}_{\mathrm{ZF}}^{\mathrm{T}}\boldsymbol{S}_k^{\mathrm{T}}\boldsymbol{S}_k\boldsymbol{\Pi}_{\mathrm{ZF}}$. A suboptimum sorting order for $\mathcal{O} = (i_1, \ldots, i_K)$ with $\boldsymbol{\Pi}_{\mathrm{ZF}} = \sum_{k=1}^{K} \boldsymbol{e}_k\boldsymbol{e}_{i_k}^{\mathrm{T}}$, which does not need to check all $K!$ permutations, reads as

$$i_k = \underset{\ell \in \{1,\ldots,K\}\setminus\{i_{k+1},\ldots,i_K\}}{\operatorname{argmin}} \left\| \left(\boldsymbol{\Pi}_k^{(\mathcal{O})}\boldsymbol{H}\right)^+ \boldsymbol{e}_\ell \right\|_2^2 \quad (24)$$
$$\forall k \in \{K, K-1, \ldots, 1\},$$

and can be found in [15] as well as a more detailed derivation of the similar *conventional* TxZF THP.

5. Simulation Results

Averaged over 80000 channel realizations, uncoded *bit-error-rates* (BERs) are presented depending on the mean E_{S}/N_0 ratio. Note that the average channel power of user 1 is ten times the mean channel power of user 2, i.e. $\sigma_{h_1}^2 = 10\sigma_{h_2}^2$, as for identical average powers, only small gains can be achieved by extending the filters towards their diagonal counterparts. Fig. 4 shows the zero-forcing variants and highlights the behavior already stated in Subsection 3.2, cf. (14): The stronger user (square marker) dramatically benefits, whereas the weaker user (star marker) has to face only a slight degradation. Note that the BERs of user 1 and user 2 (circle marker) are identical for the conventional zero-forcing filter (TxZF), since we assumed identical noise variances. The user averaged BER (triangle down marker) of the DTxZF is always smaller than the one of the conventional TxZF (circle marker), but since it is mainly governed by the weaker user, the user-averaged gain is small.

Similar results are obtained by the Wiener filters, cf. Fig. 5. The *conventional* version already allocates different MSEs to the users according to their average channel powers, and the *diagonal* version allows for a further reduction of the BER of the stronger user (square marker vs. circle marker). For an SNR value of approx. -2 dB, there is almost a boundary point between the curves of the diagonal version and the conventional version. For this SNR, $b_1 \approx b_2$ holds on average, hence both variants almost coincide. If the number of transmit antennas is reduced to $N_{\mathrm{a}} = 2$, Figs. 6 and 7 are obtained for the zero-forcing and Wiener filter types, respectively. The *diagonal* zero-forcing version still allows for a large reduction of the BER of user 1, whereas the extension of the conventional Wiener filter towards its diagonal counterpart brings about smaller gains compared to the $N_{\mathrm{a}} = 4$ antennas case.

The nonlinear zero-forcing filters with THP are shown in Fig. 8, where all $K = 4$ users have the same average channel power. The *unitary* version [7] is outperformed by the *diagonal* version and behaves similar to the *conventional* one, especially in the high-SNR region, although the *conventional* TxZF THP in [15] makes use of identical user weights, whereas the *unitary* variant permits different scalars.

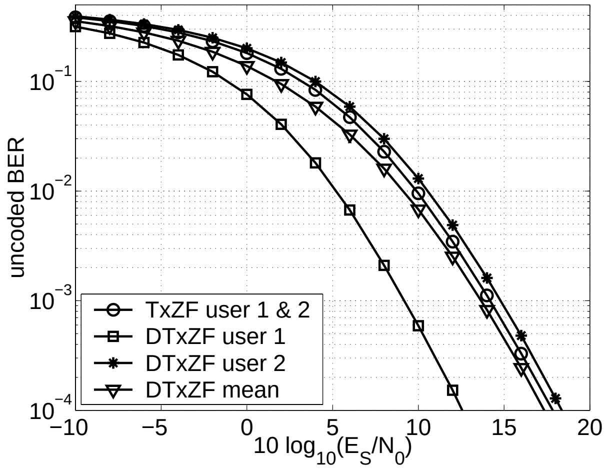

Figure 4: *Conventional* (TxZF) vs. *diagonal* (DTxZF) transmit zero-forcing filter for $K = 2$ users, $N_{\mathrm{a}} = 4$ antennas.

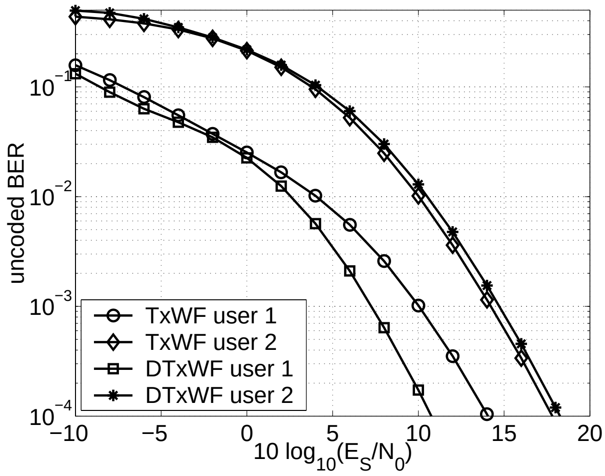

Figure 5: *Conventional* (TxWF) vs. *diagonal* (DTxWF) transmit Wiener filter for $K = 2$ users, $N_{\mathrm{a}} = 4$ antennas.

6. Conclusion

We addressed the extension of transmit filters for decentralized receivers. The modified filters allow for performance improvements especially when individual users exhibit different mean channel powers. Both linear and nonlinear transmitter structures can be extended by the presented paradigm.

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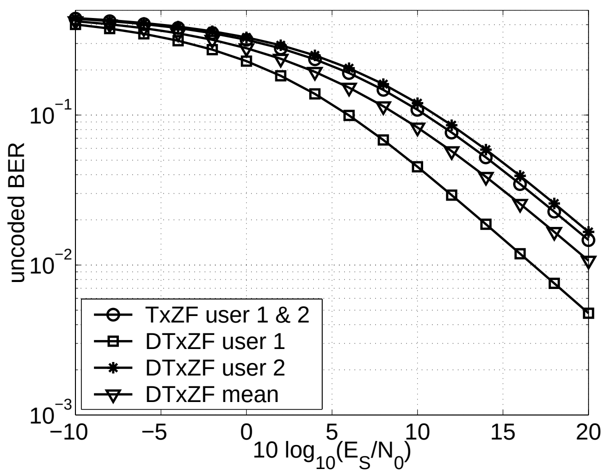

Figure 6: *Conventional* (TxZF) vs. *diagonal* (DTxZF) transmit zero-forcing filter for $K = 2$ users, $N_{\mathrm{a}} = 2$ antennas.

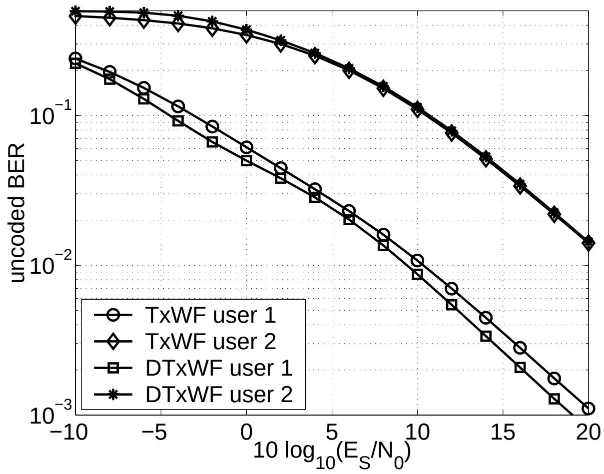

Figure 7: *Conventional* (TxWF) vs. *diagonal* (DTxWF) transmit Wiener filter for $K = 2$ users, $N_{\mathrm{a}} = 2$ antennas.

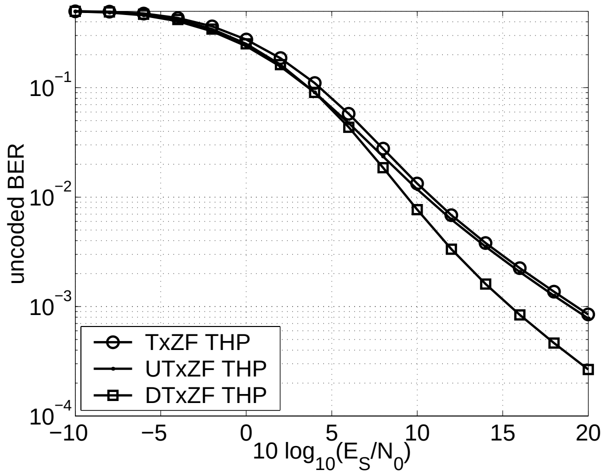

Figure 8: *Conventional* (TxZF) vs. *unitary* (UTxZF, [7]) vs. *diagonal* (DTxZF) transmit zero-forcing filter with THP for $K = 4$ users, $N_{\mathrm{a}} = 4$ antennas (user averaged).

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