On Channel Estimation and Equalization of OFDM Systems with Insufficient Cyclic Prefix

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***Abstract*— Inherent inter-symbol and inter-carrier interference elimination ability of cyclic prefixed OFDM transmission fails for the case of multipath fading channels when the Channel Impulse Response (CIR) length exceeds the duration of Cyclic Prefix (CP). Conventional channel estimation and equalization schemes, if applied to this case of insufficient CP, suffer significant performance degradation. We propose, in this paper, a channel estimation scheme that enables estimation of the complete CIR even beyond the CP length. We then design an optimal MMSE based equalizer for the suppression of insufficient CP generated interference. A robust and low complexity version of this equalizer is also derived. Simulation results for the proposed schemes show significant performance gain at low SNRs and drastic reduction of the error floors at high SNRs and more importantly, as opposed to earlier schemes, without any loss in transmission efficiency.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has proven to be an efficient underlying technology for wireless communication. The major motivation for OFDM comes from its relatively simple way of handling the frequency selective channels encountered in wireless mobile environments. With the use of a *Cyclic Prefix* (CP) — a set of last few data samples prepended at the start of a block — a frequency selective channel is effectively transformed into parallel, interference free, narrowband sub-channels [1]. Moreover, the scheme is computationally efficient due to simple realization via IDFT and DFT operations at the transmitter and receiver respectively. Owing to its superior performance and simpler implementation, OFDM has been adopted in major wireless communication standards like DVB, HIPERLAN and E-UTRA [2].

The length of cyclic prefix plays a vital role in determining the performance of an OFDM system. Too long CP leads to a considerable reduction of system efficiency $N/(N+\nu)$, N being the original data block length and ν the CP length. The length of CP, on the other hand, needs to be greater than or equal to the CIR length because otherwise we encounter *Inter-Carrier* and *Inter-Symbol Interferences* abbreviated henceforth as ICI and ISI respectively. The presence of ISI and ICI complicates the otherwise simple receiver structure in many aspects. The system model for channel estimation and equalization gets much more complex and, if ignored, these interferences lead to significant performance loss.

II. System Model

We consider an OFDM system with N sub-carriers, of which N_u sub-carriers are being used for actual transmission. The remaining $2N_o$, so called null sub-carriers, at the two extremes of the spectrum are left un-used to provide frequency guard bands and thereby avoid interference between different systems. The user symbol vector at the k_{th} time instant $\boldsymbol{S}_k = [S_k(0)\ S_k(1)\dots S_k(N_u-1)]^{\mathrm{T}} \in \mathbb{C}^{N_u}$ is appended by these null sub-carriers to yield a N-dimensional frequency domain symbol vector $\boldsymbol{X}_k \in \mathbb{C}^N$. A $N-$pt IDFT is then applied to yield time domain signal

$$\boldsymbol{x}_k = \boldsymbol{F}^{\mathrm{H}}\boldsymbol{X}_k = \boldsymbol{F}^{\mathrm{H}}\begin{bmatrix}\boldsymbol{0}_{N_o}\\ \boldsymbol{S}_k\\ \boldsymbol{0}_{N_o}\end{bmatrix}, \tag{1}$$

where $\boldsymbol{F} \in \mathbb{C}^{N\times N}$ is the unitary Fourier matrix i.e. $\boldsymbol{F}\boldsymbol{F}^{\mathrm{H}} = \boldsymbol{I}$. A Cyclic prefix of length ν is added to this time signal and with the notation $x_k(-i) = x_k(N-i)$ for $i = 1, 2, \dots, \nu$ for the CP, the cyclic prefixed signal

$$\boldsymbol{x}_k^{\mathrm{CP}} = [x_k(-\nu), x_k(-\nu+1)\dots x_k(0)\dots x_k(N-1)]^{\mathrm{T}} \tag{2}$$

is then transmitted over a channel with CIR length L. The CIR denoted by the vector $\boldsymbol{h} = [h_0, h_1 \dots h_{L-1}]^{\mathrm{T}} \in \mathbb{C}^L$ is initially assumed to be time-invariant and its length L is considered to be larger than ν, the duration of Cyclic Prefix, throughout the paper unless otherwise stated.

III. Problem Formulation

In case of insufficient CP, i.e. $\nu < L-1$, the received signal even after discarding the CP is contaminated by inter-symbol and inter-carrier interferences. The origin of these interference components can be seen in the following convolution equation. The vector $\boldsymbol{y}_k = [y(0), y(1)\dots y(N-1)]^{\mathrm{T}} \in \mathbb{C}^N$ denotes the received signal after removal of CP.

$$\boldsymbol{y}_k = \begin{bmatrix} h_{L-1} & \dots & h_0 & 0 & \dots & 0\\ 0 & \ddots & & \ddots & \ddots & \vdots\\ \vdots & & \ddots & & \ddots & 0\\ 0 & \dots & \dots & h_{L-1} & \dots & h_0 \end{bmatrix} \begin{bmatrix} x_{k-1}(N-E)\\ \vdots\\ x_{k-1}(N-1)\\ x_k(-\nu)\\ \vdots\\ x_k(N-1)\end{bmatrix} + \tilde{\boldsymbol{\eta}}_k \tag{3}$$

where $E = L - \nu - 1$ is the exceeding channel length which, if greater than zero, leads to interferences and $\tilde{\boldsymbol{\eta}}_k \in \mathbb{C}^N$ is the gaussian noise. The channel matrix $\in \mathbb{C}^{N \times (N+L-1)}$ in the above equation can be extended and partitioned into two $\mathbb{C}^{N \times N}$ matrices, the first one $\boldsymbol{H}_{\text{ISI}}$ corresponds to the previous transmitted block and as such introduces ISI while the second one $\boldsymbol{H}_{\text{CURR}}$ corresponds to the current block only. This $\boldsymbol{H}_{\text{CURR}}$ matrix can be further decomposed for analytical convenience and interpretation into a circulant matrix with all tap coefficients in each of its circularly shifted row and the residual matrix that leads to ICI. The channel output after these decompositions can be given as

$$\boldsymbol{y}_k = \boldsymbol{H}_{\text{CIRC}}\boldsymbol{x}_k - \boldsymbol{H}_{\text{ICI}}\boldsymbol{x}_k + \boldsymbol{H}_{\text{ISI}}\boldsymbol{x}_{k-1} + \tilde{\boldsymbol{\eta}}_k, \tag{4}$$

where the matrices $\boldsymbol{H}_{\text{CIRC}}, \boldsymbol{H}_{\text{ICI}}, \boldsymbol{H}_{\text{ISI}} \in \mathbb{C}^{N \times N}$ are given as under,

$$\boldsymbol{H}_{\text{CIRC}} = \begin{bmatrix} h_0 & 0 & \dots & 0 & h_{L-1} & \dots & h_1 \\ \vdots & \ddots & \ddots & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & & \ddots & h_{L-1} \\ h_{L-1} & & & \ddots & \ddots & & 0 \\ 0 & \ddots & & & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \dots & 0 & h_{L-1} & \dots & \dots & h_0 \end{bmatrix} \tag{5}$$

$$\boldsymbol{H}_{\text{ICI}} = \begin{bmatrix} \mathbf{0}_{E \times (N-E-\nu)} & \boldsymbol{H}_{\text{I}} & \mathbf{0}_{E \times \nu} \\ \mathbf{0}_{(N-E) \times (N-E-\nu)} & \mathbf{0}_{(N-E) \times E} & \mathbf{0}_{(N-E) \times \nu} \end{bmatrix} \tag{6}$$

$$\boldsymbol{H}_{\text{ISI}} = \begin{bmatrix} \mathbf{0}_{E \times (N-E)} & \boldsymbol{H}_{\text{I}} \\ \mathbf{0}_{(N-E) \times (N-E)} & \mathbf{0}_{(N-E) \times E} \end{bmatrix} \tag{7}$$

with the upper triangular (interference originating) matrix $\boldsymbol{H}_{\text{I}} \in \mathbb{C}^{E \times E}$ given below,

$$\boldsymbol{H}_{\text{I}} = \begin{bmatrix} h_{L-1} & \dots & \dots & h_{\nu+1} \\ 0 & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & h_{L-1} \end{bmatrix}. \tag{8}$$

The conventional OFDM receiver proceeds by taking a N-point DFT of the received symbol, leading to

$$\begin{aligned} \boldsymbol{Y}_k &= \boldsymbol{F}\boldsymbol{H}_{\text{CIRC}}\boldsymbol{F}^{\text{H}}\boldsymbol{X}_k - \boldsymbol{F}\boldsymbol{H}_{\text{ICI}}\boldsymbol{x}_k + \boldsymbol{F}\boldsymbol{H}_{\text{ISI}}\boldsymbol{x}_{k-1} + \boldsymbol{F}\tilde{\boldsymbol{\eta}}_k \\ &= \boldsymbol{H}\boldsymbol{X}_k - \boldsymbol{F}\boldsymbol{H}_{\text{ICI}}\boldsymbol{x}_k + \boldsymbol{F}\boldsymbol{H}_{\text{ISI}}\boldsymbol{x}_{k-1} + \boldsymbol{\eta}_k \end{aligned} \tag{9}$$

where $\boldsymbol{H} \in \mathbb{C}^{N \times N}$ is a diagonal matrix obtained by EVD of the circulant matrix [3] i.e. $\boldsymbol{H}_{\text{CIRC}} = \boldsymbol{F}^{\text{H}}\boldsymbol{H}\boldsymbol{F}$ and as such can also be expressed as $\boldsymbol{H} = \text{diag}[\boldsymbol{F}_{N \times L}\boldsymbol{h}]$ implying that it contains the *Channel Frequency Response* (CFR) along its diagonal elements.

Note that for $\nu \geq L - 1$ the matrix $\boldsymbol{H}_{\text{I}}$ is a null matrix and both the interference terms vanish. Numerous methods exist that deal with the task of channel estimation and equalization for this simple case [4], [5], [6], [7], but very few schemes have been proposed for the case of insufficient CP [8], [9]. Also proposed in the literature is the concept of channel shortening in time domain. Among these [10], [11], [12] require channel knowledge and as such are not applicable here. The blind channel shorteners [13], [14], [15] on the other hand require quite a large number of symbols to converge and so are not suitable for high speed networks like HIPERLAN and E-UTRA where the transmission consists of discontinuous short bursts called TTIs (Transmission Time Intervals).

IV. Proposed Pilot Signal Configuration and Channel Estimation

For systems with length of CP exceeding the CIR length, it is well established [16] that the pilot symbols should be equispaced along time as well as along frequency. It has been recently shown in [17], however, that instead of equispaced individual pilot symbols, equispaced groups of pilot symbols, along the frequency axis (at the expense of larger spacing) show superior performance in the case of rapid channel variations where ICI causes a major degradation. A similar result, as we will show in the sequel, holds true for channels longer than the CP length. Consecutive and suitably designed pilot symbols along time axis enable complete elimination of interference terms and greatly simplify the task of channel estimation for systems with insufficient CP.

The underlying idea behind the proposed pilot signal configuration is that for any arbitrary unknown channel profile mutual cancellation of ISI and ICI can be achieved by having the pilot symbols consecutive and then by forcing

$$\boldsymbol{F}\boldsymbol{H}_{\text{ICI}}\boldsymbol{x}_k \doteq \boldsymbol{F}\boldsymbol{H}_{\text{ISI}}\boldsymbol{x}_{k-1} \tag{10}$$

to eliminate interference components in (9). More insight can be gained by a closer examination of the matrices $\boldsymbol{H}_{\text{ICI}}$ and $\boldsymbol{H}_{\text{ISI}}$ which are similar except for the circular shift of their columns implying that the two consecutive time domain symbols should be chosen to be circular shifted versions of each other. The requirement can even be relaxed because of the null matrices in $\boldsymbol{H}_{\text{ICI}}$ and $\boldsymbol{H}_{\text{ISI}}$, so that given the estimate of CIR length, only $E = L - \nu - 1$ samples at appropriate positions need to be identical in consecutive blocks. Based on this the so called adaptive bit loading was proposed in [9] requiring respective portions of *all consecutive pilot and data blocks* to be circular shifted versions. The scheme does achieve interference suppression but at the cost of significant reduction in transmission efficiency especially for longer channels. Moreover arriving at the same time domain signal at specific locations in consecutive symbols is difficult at least in practice for OFDM systems although it can easily be achieved for single carrier based transmission with CP.

The scheme proposed here requires *no change in data blocks* so it achieves the same transmission efficiency and is rather based on pilot symbol re-arrangement followed by better, interference suppressing, equalization scheme. Instead of equispaced pilot symbol transmission, the transmitter is supposed to transmit two pilot symbols in succession accompanied by a larger spacing in time. The successive pilot blocks can easily be chosen (even for OFDM) to satisfy the requirement of mutual interference cancellation in (10) *without making any assumptions on CIR length*, so that the reception of second pilot block is free of both ICI and ISI and the conventional system model,

$$\boldsymbol{Y}_k = \boldsymbol{H}\boldsymbol{X}_k + \boldsymbol{\eta}_k, \tag{11}$$

holds instead of (9). Since only the central N_u sub-carriers are of interest the system model reduces further to

$$\boldsymbol{Y}_{N_u} = \boldsymbol{P}\boldsymbol{F}_{N_u}\boldsymbol{h} + \boldsymbol{\eta}_{N_u} \tag{12}$$

where the matrix $\boldsymbol{P} \in \mathbb{C}^{N_u \times N_u}$ containing the transmitted pilot symbols on its diagonal is same as $\mathrm{diag}[\boldsymbol{S}_k]$. Note that we omit the subscript k because in absence of interference blocks can be treated individually. The matrix $\boldsymbol{F}_{N_u} \in \mathbb{C}^{N_u \times L}$ is the respective portion of the DFT matrix. ML or MMSE based channel estimation for the CIR, $\boldsymbol{h}$, can now be pursued through one of the following equations [4]

$$\hat{\boldsymbol{h}}_{\mathrm{ML}} = \left(\boldsymbol{F}_{N_u}^{\mathrm{H}}\boldsymbol{P}^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{\eta}}^{-1}\boldsymbol{P}\boldsymbol{F}_{N_u}\right)^{-1}\boldsymbol{F}_{N_u}^{\mathrm{H}}\boldsymbol{P}^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{\eta}}^{-1}\boldsymbol{Y}_{N_u} \tag{13}$$

$$\begin{aligned}\hat{\boldsymbol{h}}_{\mathrm{MMSE}} = &\boldsymbol{R}_{\boldsymbol{hh}}\boldsymbol{F}_{N_u}^{\mathrm{H}}\left(\boldsymbol{F}_{N_u}\boldsymbol{R}_{\boldsymbol{hh}}\boldsymbol{F}_{N_u}^{\mathrm{H}} + \left(\boldsymbol{P}^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{\eta}}^{-1}\boldsymbol{P}\right)^{-1}\right)^{-1} \\ &\left(\boldsymbol{P}^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{\eta}}^{-1}\boldsymbol{P}\right)^{-1}\boldsymbol{P}^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{\eta}}^{-1}\boldsymbol{Y}_{N_u}.\end{aligned} \tag{14}$$

In this way the proposed pilot signal configuration enables the estimation of CIR even beyond the CP length, which is otherwise impossible. The CFR can now be obtained via pre-multiplication with the DFT matrix i.e.

$$\hat{\boldsymbol{H}} = \mathrm{diag}[\boldsymbol{F}_{N\times L}\hat{\boldsymbol{h}}] \;\;,\;\; \hat{\boldsymbol{H}}_{N_u} = \mathrm{diag}[\boldsymbol{F}_{N_u\times L}\hat{\boldsymbol{h}}]. \tag{15}$$

V. Proposed Equalization Scheme

With the estimates of the channel available both in terms of CIR and CFR, we propose now schemes for channel equalization. Important to keep in mind is the fact that because of insufficient CP, the system model in (11) does not hold for the data blocks and simple one-tap frequency domain equalization can not be performed. Viewed from the frequency domain, we need to design an equalizer filter $\boldsymbol{W} \in \mathbb{C}^{N_u \times N}$ which when pre-multiplied with $\boldsymbol{Y}_k$ (note that because of ISI and ICI we need to consider all the dimensions instead of only N_u) yields an estimate of the transmitted vector $\boldsymbol{S}_k$ as

$$\begin{aligned}\hat{\boldsymbol{S}}_k &= \boldsymbol{W}\boldsymbol{Y}_k \\ &= \boldsymbol{W}\boldsymbol{H}\boldsymbol{X}_k - \boldsymbol{W}\boldsymbol{F}\boldsymbol{H}_{\mathrm{ICI}}\boldsymbol{x}_k + \boldsymbol{W}\boldsymbol{F}\boldsymbol{H}_{\mathrm{ISI}}\boldsymbol{x}_{k-1} + \boldsymbol{W}\boldsymbol{\eta}_k\end{aligned} \tag{16}$$

such that some desired cost function $J(\boldsymbol{S}_k, \hat{\boldsymbol{S}}_k)$ is minimized. In order to get better visualization of the problem at hand we may split the equalizer matrix into three sub-matrices as under

$$\boldsymbol{W} = \left[\boldsymbol{W}^0 \quad \boldsymbol{W}^1 \quad \boldsymbol{W}^2\right], \tag{17}$$

where $\boldsymbol{W}^0, \boldsymbol{W}^2 \in \mathbb{C}^{N_u \times N_0}$ operate on the null sub-carriers at the extremes while $\boldsymbol{W}^1 \in \mathbb{C}^{N_u \times N_u}$ operates on the central N_u sub-carriers.

For the interference-free case received null sub-carriers are zero implying that both $\boldsymbol{W}^0, \boldsymbol{W}^2$ are null matrices and the only design freedom lies in $\boldsymbol{W}^1$ which can be chosen according to the *Zero Forcing (ZF)* or *Minimum Mean Square Error (MMSE)* design criterion as follows,

$$\boldsymbol{W}_{\mathrm{ZF}}^1 = \left(\boldsymbol{H}_{N_u}^{\mathrm{H}}\boldsymbol{H}_{N_u}\right)^{-1}\boldsymbol{H}_{N_u}^{\mathrm{H}} \tag{18}$$

$$\begin{aligned}\boldsymbol{W}_{\mathrm{MMSE}}^1 &= \boldsymbol{R}_S\boldsymbol{H}_{N_u}^{\mathrm{H}}\left(\boldsymbol{H}_{N_u}\boldsymbol{R}_S\boldsymbol{H}_{N_u}^{\mathrm{H}} + \boldsymbol{R}_{\boldsymbol{\eta}}\right)^{-1} \\ &= \left(\boldsymbol{R}_S^{-1} + \boldsymbol{H}_{N_u}^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{\eta}}^{-1}\boldsymbol{H}_{N_u}\right)^{-1}\boldsymbol{H}_{N_u}^{\mathrm{H}}\boldsymbol{R}_{\boldsymbol{\eta}}^{-1},\end{aligned} \tag{19}$$

The MMSE equalizer reduces for the case of uncorrelated noise and uncorrelated transmitted signal i.e. $\boldsymbol{R}_{\boldsymbol{\eta}} = \sigma_\eta^2\boldsymbol{I}$ and $\boldsymbol{R}_S = \sigma_S^2\boldsymbol{I}$, to

$$\boldsymbol{W}_{\mathrm{MMSE}}^1 = \left(\frac{\sigma_\eta^2}{\sigma_S^2} + \boldsymbol{H}_{N_u}^{\mathrm{H}}\boldsymbol{H}_{N_u}\right)^{-1}\boldsymbol{H}_{N_u}^{\mathrm{H}}. \tag{20}$$

Equalizer design for the case of insufficient CP, however, involves determination of all three sub-matrices of (17) in accordance with a particular cost function. The simple ZF criterion has been used in [8] to arrive at the following solution.

$$\left[\boldsymbol{W}_{\mathrm{ZF}}^0 \quad \boldsymbol{W}_{\mathrm{ZF}}^2\right] = -\boldsymbol{W}_{\mathrm{ZF}}^1\boldsymbol{F}^{01}\left(\begin{bmatrix}\boldsymbol{F}^{00} \\ \boldsymbol{F}^{02}\end{bmatrix}\right)^{\dagger}, \tag{21}$$

where

$$\boldsymbol{F} = \left[\boldsymbol{F}_{N\times E}^0 \quad \boldsymbol{F}_{N\times(N-E)}^1\right] \;\;;\;\; \boldsymbol{F}^0 = \begin{bmatrix}\boldsymbol{F}_{N_0\times E}^{00} \\ \boldsymbol{F}_{N_u\times E}^{01} \\ \boldsymbol{F}_{N_0\times E}^{02}\end{bmatrix} \tag{22}$$

and $(.)^\dagger$ denotes pseudo-inversion of a matrix. The solution for $\boldsymbol{W}_{\mathrm{ZF}}^1$ remains the same as in (18). Problems with the ZF approach include its inherent noise amplification property and at least from practical implementation point of view numerical instability arising from numerically low rank matrix to be pseudo-inversed.

We propose here a MMSE based equalizer design and for this, express $\hat{\boldsymbol{S}}_k$ from (16) in a more suitable notation

$$\hat{\boldsymbol{S}}_k = \boldsymbol{W}\boldsymbol{H}\boldsymbol{X}_k - \boldsymbol{W}\boldsymbol{F}\boldsymbol{H}_{\mathrm{ICI}}\boldsymbol{F}^{\mathrm{H}}\boldsymbol{X}_k + \boldsymbol{W}\boldsymbol{F}\boldsymbol{H}_{\mathrm{ISI}}\boldsymbol{F}^{\mathrm{H}}\boldsymbol{X}_{k-1} + \boldsymbol{W}\boldsymbol{\eta}_k. \tag{23}$$

Now in order to fully exploit the structure of the interference matrices (see equations 6 and 7) and that of $\boldsymbol{X}_k = [\boldsymbol{0}_{\boldsymbol{N_o}}{}^{\mathrm{T}}\ \boldsymbol{S_k}{}^{\mathrm{T}}\ \boldsymbol{0}_{\boldsymbol{N_o}}{}^{\mathrm{T}}]^{\mathrm{T}}$ we split the channel matrix as follows.

$$\boldsymbol{H} = \left[\boldsymbol{H}_{N\times N_0}^0 \quad \boldsymbol{H}_{N\times N_u}^1 \quad \boldsymbol{H}_{N\times N_0}^2\right]. \tag{24}$$

Similarly after re-defining, for notational convenience, the inverse fourier matrix $\boldsymbol{F}^{\mathrm{H}}$ as $\boldsymbol{A}$, we split it into various sub-matrices as follows

$$\boldsymbol{A} = \left[\boldsymbol{A}_{N\times N_0}^0 \quad \boldsymbol{A}_{N\times N_u}^1 \quad \boldsymbol{A}_{N\times N_0}^2\right]$$

$$\boldsymbol{A}^1 = \begin{bmatrix}\boldsymbol{A}_{(N-L+1)\times N_u}^{1a} \\ \boldsymbol{A}_{E\times N_u}^{1b} \\ \boldsymbol{A}_{\nu\times N_u}^{1c}\end{bmatrix} \;\;;\;\; \boldsymbol{A}^1 = \begin{bmatrix}\boldsymbol{A}_{(N-E)\times N_u}^{1x} \\ \boldsymbol{A}_{E\times N_u}^{1y}\end{bmatrix} \tag{25}$$

which finally enables us to write $\hat{\boldsymbol{S}}_k$ as

$$\begin{aligned}\hat{\boldsymbol{S}}_k &= \boldsymbol{W}\boldsymbol{H}^1\boldsymbol{S}_k - \boldsymbol{W}\boldsymbol{F}^0\boldsymbol{H}_{\mathrm{I}}\boldsymbol{A}^{1b}\boldsymbol{S}_k + \boldsymbol{W}\boldsymbol{F}^0\boldsymbol{H}_{\mathrm{I}}\boldsymbol{A}^{1y}\boldsymbol{S}_{k-1} + \boldsymbol{W}\boldsymbol{\eta}_k \\ &= \boldsymbol{W}\boldsymbol{C}\boldsymbol{S}_k + \boldsymbol{W}\boldsymbol{D}\boldsymbol{S}_{k-1} + \boldsymbol{W}\boldsymbol{\eta}_k,\end{aligned} \tag{26}$$

where $\boldsymbol{C} = \boldsymbol{H}^1 - \boldsymbol{F}^0\boldsymbol{H}_{\text{I}}\boldsymbol{A}^{1b} \in \mathbb{C}^{N\times N_u}$ and $\boldsymbol{D} = \boldsymbol{F}^0\boldsymbol{H}_{\text{I}}\boldsymbol{A}^{1y} \in \mathbb{C}^{N\times N_u}$. Now we define and evaluate the MMSE cost function as under

$$\begin{aligned}\varepsilon(\boldsymbol{W}) &= J(\boldsymbol{S}_k, \hat{\boldsymbol{S}}_k) = \text{E}[\|\boldsymbol{S}_k - \hat{\boldsymbol{S}}_k\|_2^2] \\ &= \text{tr}\left((\boldsymbol{I} - \boldsymbol{W}\boldsymbol{C})\,\boldsymbol{R}_S\,(\boldsymbol{I} - \boldsymbol{W}\boldsymbol{C})^{\text{H}}\right) \\ &\quad + \text{tr}\left(\boldsymbol{W}\boldsymbol{D}\boldsymbol{R}_S\boldsymbol{D}^{\text{H}}\boldsymbol{W}^{\text{H}}\right) + \text{tr}\left(\boldsymbol{W}\boldsymbol{R}_\eta\boldsymbol{W}^{\text{H}}\right). \end{aligned} \tag{27}$$

Minimizing the cost function with respect to the equalizer matrix $\boldsymbol{W}$, we finally arrive at the following solution for the MMSE equalizer,

$$\boldsymbol{W}_{\text{MMSE}} = \boldsymbol{R}_S\boldsymbol{C}^{\text{H}}\left(\boldsymbol{C}\boldsymbol{R}_S\boldsymbol{C}^{\text{H}} + \boldsymbol{D}\boldsymbol{R}_S\boldsymbol{D}^{\text{H}} + \boldsymbol{R}_\eta\right)^{-1}. \tag{28}$$

The solution although computationally complex as compared to the *Frequency-domain EQualizer* (FEQ) in (19), but is optimal in the MSE sense for the case of insufficient CP. It requires the knowledge of noise covariance matrix, and all the channel impulse response coefficients which can readily be obtained via the proposed channel estimation scheme. For the case of un-correlated Gaussian noise and uncorrelated data symbols we have $\boldsymbol{R}_{\boldsymbol{\eta}} = \sigma_\eta^2\boldsymbol{I}$ and $\boldsymbol{R}_S = \sigma_S^2\boldsymbol{I}$, leading to the simple equalizer expression,

$$\boldsymbol{W}_{\text{MMSE}} = \boldsymbol{C}^{\text{H}}\left(\boldsymbol{C}\boldsymbol{C}^{\text{H}} + \boldsymbol{D}\boldsymbol{D}^{\text{H}} + \frac{\sigma_\eta^2}{\sigma_S^2}\boldsymbol{I}_N\right)^{-1}. \tag{29}$$

VI. Reduced Complexity Equalization

A reduced complexity, sub-optimal, version of the MMSE equalizer can be obtained by using the conventional MMSE one-tap FEQ (as in equation 19) for the central sub-carriers and designing the remaining equalizer portions with a goal of minimizing the interference and noise power. The interference / error term can be given as

$$\begin{aligned}\boldsymbol{e}_k &= \boldsymbol{W}\boldsymbol{F}^0\boldsymbol{H}_{\text{I}}\boldsymbol{d}_k - \boldsymbol{W}\boldsymbol{\eta}_k \\ &= \boldsymbol{W}^{(0+2)}\left(\boldsymbol{F}^{0(0+2)}\boldsymbol{H}_{\text{I}}\boldsymbol{d}_k - \boldsymbol{\eta}_k^{(0+2)}\right) + \boldsymbol{W}^1\left(\boldsymbol{F}^{01}\boldsymbol{H}_{\text{I}}\boldsymbol{d}_k - \boldsymbol{\eta}_k^1\right)\end{aligned} \tag{30}$$

where $\boldsymbol{d}_k = \boldsymbol{A}^{1b}\boldsymbol{S}_k - \boldsymbol{A}^{1y}\boldsymbol{S}_{k-1}$ and the superscript $(.)^{(0+2)}$ denotes the concatenation of two submatrices or vectors i.e.

$$\boldsymbol{W}^{(0+2)} = [\boldsymbol{W}^0 \quad \boldsymbol{W}^2] \in \mathbb{C}^{N_u\times(2N_0)},$$

$$\boldsymbol{F}^{0(0+2)} = \begin{bmatrix}\boldsymbol{F}^{00}\\ \boldsymbol{F}^{02}\end{bmatrix} \in \mathbb{C}^{(2N_0)\times E}, \quad \boldsymbol{\eta}_k^{(0+2)} = \begin{bmatrix}\boldsymbol{\eta}_k^0\\ \boldsymbol{\eta}_k^2\end{bmatrix} \in \mathbb{C}^{(2N_0)}, \tag{31}$$

so that upon minimization of the MSE, $\varepsilon(\boldsymbol{W}^{(0+2)}) = \text{E}[\|\boldsymbol{e}_k\|_2^2]$, we finally arrive at the following solution,

$$\begin{aligned}\boldsymbol{W}_{\text{MMSE}}^{(0+2)} &= -\boldsymbol{W}^1\boldsymbol{F}^{01}\boldsymbol{H}_{\text{I}}\boldsymbol{R}_d\boldsymbol{H}_{\text{I}}^{\text{H}}\boldsymbol{F}^{0(0+2)\text{H}} \\ &\quad \left(\boldsymbol{F}^{0(0+2)}\boldsymbol{H}_{\text{I}}\boldsymbol{R}_d\boldsymbol{H}_{\text{I}}^{\text{H}}\boldsymbol{F}^{0(0+2)\text{H}} + \boldsymbol{R}_{\eta^{(0+2)}}\right)^{-1}\end{aligned} \tag{32}$$

with $\boldsymbol{R}_d = \boldsymbol{A}^{1b}\boldsymbol{R}_S\boldsymbol{A}^{1b\text{H}} + \boldsymbol{A}^{1y}\boldsymbol{R}_S\boldsymbol{A}^{1y\text{H}} \in \mathbb{C}^{E\times E}$. The reduced complexity equalization therefore requires computation of equalizer coefficients for the central sub-carriers from (19) without any matrix inversion and for the null sub-carriers from (32) involving a matrix inverse only of dimensions N_o instead of N in (28).

VII. Simulation Results

Simulation results below are provided for a cyclic prefixed OFDM system, operating with N=2048 sub-carriers over a bandwidth of 20 MHz, with N_o=424 null sub-carriers at both end of the spectrum. The CP length is 127 samples which amounts to approximately 4.1 μs. These simulation parameters are in fact adopted from E-UTRA specs [2].

Vehicular B channel profile [18] about 20 μs (615 taps) long is used to present a comparison of the conventional and proposed transmission, channel estimation and equalization schemes. Results for MMSE channel estimation are presented for $\boldsymbol{R}_{hh}$ based on both, the true Veh-B as well as Uniform *Power Delay Profile* (PDP).

Note that the only possible concern that can be raised about the proposed pilot signal configuration is that the channel

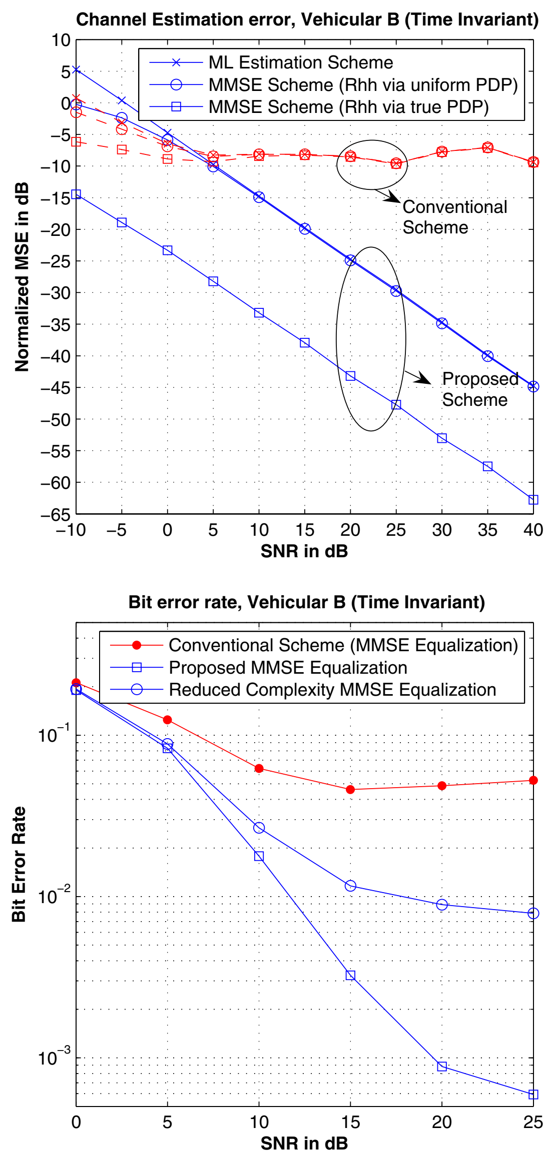
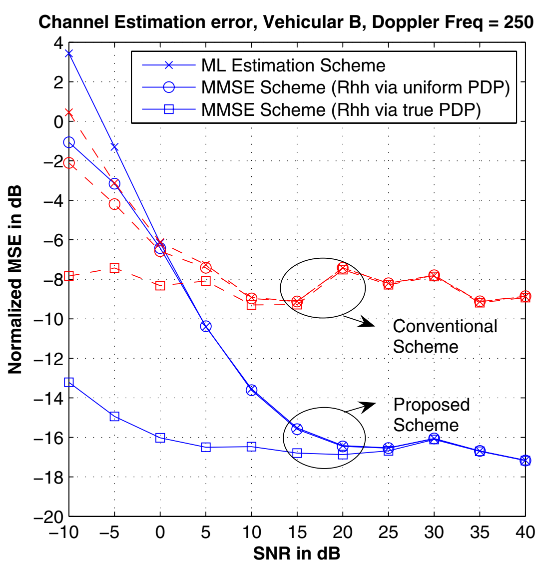


Fig. 1. Comparison of conventional and the proposed scheme in terms of channel estimation MSE and BER for the case of time invariant channel.

tracking performance may suffer in case of rapid channel variations. This stems from the fact that say for a TTI length of eight symbols (as in E-UTRA specifications) the sampling of channel variations now takes place only at the central two symbols of the TTI rather than the two uniformly spaced pilot symbols.

In order to compare the performance of the proposed and conventional schemes for the case of a rapidly time-varying channel, we consider a doppler frequency of 250 Hz which corresponds in this simulation scenario to a vehicular speed of about 100 km/hr. With the simulation results presented below, it is worth appreciating that the reduced channel tracking ability of the proposed scheme is comprehensively overshadowed by the increased accuracy of channel estimation and it emerges to be significantly superior to the conventional scheme in terms of both the channel estimation MSE and the BER even in time variant channels.



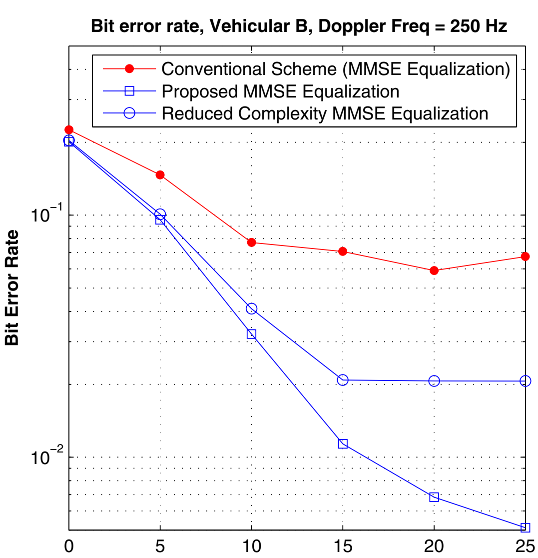


Fig. 2. Comparison of conventional and the proposed scheme in terms of channel estimation MSE and BER for the case of time variant channel.

VIII. Conclusion

In this paper, we analyzed the origin of interferences in channels with their delay spreads longer than the duration of OFDM Cyclic Prefix. We proposed a transmission frame structure with as much transmission efficiency as a conventional one but still enables the estimation of complete CIR (even beyond the CP length) evidenced by the elimination (and reduction) of error floors in estimation MSE for the static (and time variant) channels. We designed, then, the optimal and reduced complexity versions of the MMSE based linear equalizer in order to suppress the insufficient CP generated interferences. Simulation results for the proposed estimation and equalization schemes confirm their significant superiority over the conventional approaches.

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