Feedback of Channel State Information in Wireless Systems

Pedro Tejera, Wolfgang Utschick
Associate Institute for Signal Processing
Arcisstrasse 21, Munich University of Technology, 80290 Munich, Germany
Email: {tejera,utschick}@tum.de

***Abstract*— The problem of sending channel state information from the receiver to the transmitter of a wireless link is investigated in this paper. If the channel state is Gaussian distributed, this problem is equivalent to that of transmission of a Gaussian source over a noisy channel. We focus on a model in which the source outputs are statistically independent and the feedback channel is either AWGN or Rayleigh fading. Due to the strict delay constraints, information theoretic results are hardly applicable to the analysis of such a setting. As a consequence, despite its simplicity, little is known about its fundamental performance limits. Here, two different delay limited digital transmission approaches and a linear analog transmission approach are discussed and compared. If D channel uses per source output are allowed, it is shown that for the AWGN feedback channel delay limited digital approaches can achieve a distortion decay of at least $D/2$ dB per dB of SNR. This decay rate is 1 for the linear analog approach regardless D. For Rayleigh feedback channels the distortion decay rate is shown to be upper bounded by 1 for digital approaches and is asymptotically 1 for the analog approach. This fact and simplicity are good reasons for the use of analog transmission for feedback purposes over fading channels.**

I. Introduction

Channel state information at the transmitter of a wireless link permits the application of adaptivity techniques in order to either boost transmission rate, reliability or decrease transmit power required to achieve certain performance. Channel state information can be obtained at the transmitter exploiting reciprocity in time division duplex schemes. By contrast, in frequency division duplex schemes a feedback channel is typically needed to convey channel state information (CSI) from the receiver to the transmitter. Of late, in the context of multiple-input multiple-output (MIMO) wireless channels, several authors have proposed analog transmission for feedback of CSI [1]–[3]. The main reasons are claimed to be simplicity and mathematical tractability. However, no precise statements are made that justify analog transmission on the grounds of performance. A primary goal of the present work is to analyze and compare performance of digital and analog approaches in order to better understand the basic difference between both techniques and their suitability for feedback purposes.

When studying transmission of CSI over a feedback link, CSI can be thought of as the output of a source. The goal is the reconstruction of the source at the other end of the link with minimum distortion. If high delay and complexity can be tolerated, the optimum performance theoretically achievable (OPTA) can be approximated by a digital approach that first performs an optimum coding (quantization) of the source at a rate close to the capacity of the channel and then performs an almost error free transmission of code vectors by means of a powerful channel coding scheme [4]. That is, always when high delay and complexity can be tolerated a digital approach achieves optimum performance. Unfortunately, feedback of CSI imposes taugh delay constraints in order to avoid outdating. As a result, error free transmission is impossible. Furthermore, given a particular channel state, this must be transmitted before the next channel state becomes available. This forces a causal coding of the source. Under these constraints optimality of digital transmission is not guaranteed and simple analog approaches may be an interesting alternative.

A theoretical framework for the analysis of digital systems under delay, complexity or causality constraints is so far missing. An attempt to elaborate a general framework in [5] ended up with more questions than responses despite the simplifying assumption of a noiseless transmission channel. This lack of theoretical foundation has given rise to a heterogeneous landscape of approaches specifically taylored for particular settings and applications that are commonly referred to as joint source and channel coding schemes [6] [7]. In the setting considered here the source outputs are statistically independent, Gaussian distributed scalar values. Each of these values is transmitted over D channel uses in the feedback link. Both AWGN and Rayleigh fading feedback channels are investigated. As distortion measure mean square error is considered. In the next section, we shall see that this setting describes a number of practically interesting feedback approaches.

Optimum transmission of Gaussian sources over AWGN channels has been investigated in [8] under the assumption of linear receivers. In [9] an algorithm is proposed that jointly optimizes the receiver, joint source and channel encoder and the signal constellation. An extension of this algorithm to Rayleigh fading channels is presented in [10]. Specially the two last algorithms are numerically very complex and do not provide any insight regarding performance of delay constrained digital approaches. Here, the focus is on analysis rather than design. Two simple digital approaches are investigated. The first maps quantizer outputs to a set of signals that is chosen aiming at a maximization of the minimum distance between elements of the set. This scheme is referred to as non topological approach for reasons that will become clear later. The second maps quatizer outputs so that neighborhood relations in the domain are preserved in the range. For this reason we call this scheme topological approach.

For the AWGN feedback link, a general analysis based on high resolution quantizers and random codes shows that $D/2$ is a lower bound on the distortion decay in dBs that can be expected from delay constrained digital approaches when the channel SNR increases 1 dB in the high SNR region. This rate of decay is D for OPTA. This is in contrast with the distortion decay rate of the optimum linear analog approach which is 1 regardless D. It turns out that the non topological scheme behaves as expected from the bounds for digital approaches. On the contrary, performance of the linear analog scheme seems to tightly upper bound performance of the topological scheme.

The picture changes significantly for Rayleigh fading feedback channels. Distortion decay rate of digital approaches is shown to be limited to 1 regardless D. This rate is also achieved by the analog approach at asymptotically large SNR values. Therefore, neither digital approaches nor the analog scheme significantly profit from an increase in D. Simulation results show that analog transmission may deliver better performance than both digital approaches and the topological scheme may outperform the non topological approach. This fact offers a rationale for the use of analog transmission for feedback over fading channels or application of simple digital approaches based on repetition codes as, for instance, the topological scheme considered here.

The remainder of the paper is structured as follows. In Section II the system model is introduced and motivated. In Section III the topological and non topological digital approaches are introduced and briefly discussed. In Section IV performance bounds are derived for the AWGN feedback link. The same is done for the Rayleigh fading feedback link in Section V. Simulation results are shown and commented in Section VI. Finally, conclusions are drawn in Section VII.

II. System Model

The system model is illustrated in Fig. 1. The transmitter sends pilots to the receiver that make possible an estimation of the channel state, which in the general case is represented by a matrix $\boldsymbol{H}$. We assume that this estimate is perfect and constitutes the CSI that is sent back to the transmitter over the feedback link. The feedback link is visualized in Fig. 2. On this link one value $z \in \mathbb{C}$ is transmitted at a time. These values may be regarded as the outputs of a source. For transmission of each source output the channel can be used D times. The encoder is a map of source values onto the set of transmit signals $\boldsymbol{s}_i \in \mathbb{C}^D$, $i \in \{1, \ldots, M\}$. The signals are supposed to satisfy a power constraint,

$$\frac{1}{D} \sum_{i=1}^{M} \|\boldsymbol{s}_i\|^2 P(\boldsymbol{s}_i) \leq \mathrm{SNR} \tag{1}$$

where $P(\boldsymbol{s}_i)$ is the probability that signal $\boldsymbol{s}_i$ is transmitted. The channel is modeled by a channel gain $g \in \mathbb{C}$ and an additive noise $w \in \mathbb{C}$, $w \sim \mathcal{CN}(0, 1)$. We consider two classes of channels: an AWGN channel and a Rayleigh fading channel. For the AWGN channel $g = 1$. For the Rayleigh fading channel $g \sim \mathcal{CN}(0, 1)$ and is constant during transmission of a source value, i.e., a block fading model is assumed with block length of D channel uses. The decoder, estimates the transmitted source value based on the received signal $\boldsymbol{y} \in \mathbb{C}^D$. Distortion is defined as $\epsilon = \mathrm{E}|z - \hat{z}|^2$. A minimum variance estimator is employed at the decoder, which is optimum for this distortion measure. For analysis purposes the source outputs are assumed to be statistically independent $z \sim \mathcal{CN}(0, 1)$.

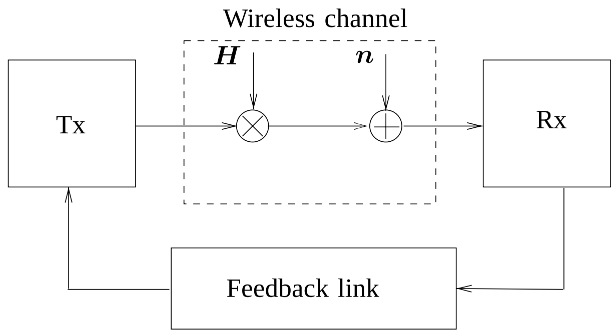

Figure 1. System model.

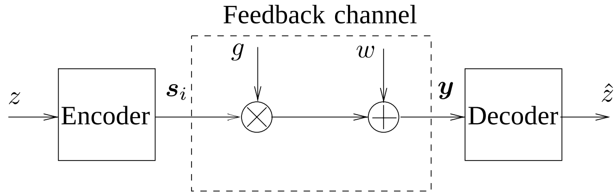

Figure 2. Feedback link.

This model directly applies if the wireless channel is flat fading single input single output (SISO) and successive channel states are uncorrelated. In this case channel states are the source outputs. If correlations exist the model also applies if only innovations are fed back, which are statistically independent [1]. In this case innovations are the output of the source. For the general case of MIMO wireless channels with arbitrary correlation in time, frequency or space the model applies if, for the sake of simplicity, each entry of the channel matrix is independently fed back and the decoder does not make use of correlations. In such case the source outputs are the entries of the channel matrix.

III. Delay Constrained Digital Approaches

Without loss of optimality the encoder may have the structure indicated in Fig. 3. It consists of the concatenation of a quantizer and a mapping of reproduction values to transmit signals. Two different approaches are considered here. Both approaches use quadrature amplitude modulation (QAM) constellation signals for transmission and a quantizer

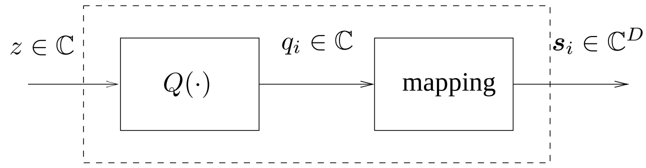

Figure 3. Encoder.

optimized according to the generalized Lloyd algorithm (GLA) [11]. Given a number $M = 2^{2b}$ of reproduction values the starting points for the GLA are chosen to correspond to an optimum uniform scalar quantization of real and imaginary parts separately, i.e., the initial values are elements of the set

$$\{r + \alpha(n + jm) \ : \ n, m \in \mathbb{Z} \wedge -\frac{\sqrt{M}}{2} \leq n, m \leq \frac{\sqrt{M}}{2} - 1\}$$

where $r = \alpha/2 + j\alpha/2$ and α is the optimum distance between reproduction values for a uniform scalar quantizer [12]. The two approaches differ in the mapping block. The first scheme uses a mapping that tries to maximize the distances between any two points of the image. This scheme is called non topological approach. The second scheme uses a mapping that nearly preserves the distance relations between the points of the original set. This scheme will be referred to as topological approach.

Assume that q_i is a final quantizer output that corresponds to the reproduction value $r + \alpha(n + jm)$ at the beginning of the GLA algorithm. To this point the topological scheme assigns the signal $\boldsymbol{s}_i = s_i \mathbf{1}_D$ where $s_i = (1/2 + n + j(1/2 + m))$ and $\mathbf{1}_D$ is a D dimensional vector with unit entries. The map of the non topological scheme is a composition of three mappings. First, to each q_i a sequence of $2b$ bits is assigned according to a Gray mapping, i.e., neighboring points differ at most in one bit. Then, these bits are encoded with a block code of rate $R = 1/2$. Finally, the sequence formed by the $4b$ resulting bits is fragmented in subsequences of $4b/D$ bits and each of these subsequences is Gray mapped to a symbol of a QAM constellation of size $2^{4b/D}$. Each of the D resulting symbols is transmitted in a different channel use. If binary codes with good distance properties are chosen the vectors of QAM symbols selected for transmission by the non topological scheme will be conveniently far apart. However, this convenient placement of transmit signals will in general not preserve neighborhood relations. Prior to transmission, signals $\boldsymbol{s}_i$ are normalized so that the power constraint in (1) is satisfied.

Preserving neighborhood relations is convenient since in that case signals likely to be mutually mistaken do represent close values of the source, leading to mild distortion. On the other hand, maximizing minimum distance between transmit signals makes communication more reliable. Ideally both goals should be combined in order to obtain an optimum map. However, fully reconciliation of both paradigms appear to be impossible [13].

IV. AWGN Feedback Link

A. Analog Transmission

The basic difference between analog and digital transmission resides in the nature of the map performed by the encoder in Fig. 2. In case of analog transmission this map is injective, i.e., the range is uncountable. In case of digital transmission the range is typically a finite countable set. Here, the focus is on simple linear maps.[1] Correspondingly, the transmitted and received signals depend linearly on the source output. The received signal can be written as

$$\boldsymbol{y} = \boldsymbol{\phi}\sqrt{\mathrm{SNR}}z + \boldsymbol{w} \qquad (2)$$

where $\boldsymbol{\phi} \in \mathbb{C}^D$ is an arbitrary vector of norm $\sqrt{D}$ in order to fulfil the power constraint. For this model, distortion is minimized by estimating z with an MMSE estimator. The resulting distortion is given by

$$\epsilon = \frac{1}{1 + D\mathrm{SNR}}. \qquad (3)$$

It is interesting to observe that distortion does not depend on $\boldsymbol{\phi}$. In particular, using the channel only once and transmitting with power $D\mathrm{SNR}$ is equivalent to using the channel D times transmitting each time with power SNR. Expressing ϵ and SNR in dBs we obtain

$$\epsilon(\mathrm{dB}) = -\mathrm{SNR}(\mathrm{dB}) + O(1), \quad \mathrm{SNR} \to \infty,$$

that is, in the high SNR region distortion decays at a rate of 1 with respect to SNR regardless D.

B. OPTA

For an i.i.d. circularly symmetric Gaussian source of unit variance the rate distortion function is easily computed [16] as

$$R(\epsilon) = \begin{cases} \log\left(\frac{\sigma_h^2}{\epsilon}\right) & \text{if} \quad \epsilon < \sigma_h^2 \\ 0 & \text{if} \quad \epsilon \geq \sigma_h^2 \end{cases}. \qquad (4)$$

The maximum number of bits that can be transmitted over the channel per source output is given by

$$C = D\log(1 + \mathrm{SNR}). \qquad (5)$$

According to the joint source and channel coding theorem with a fidelity criterion [4] the minimum achivable distortion can be computed by equating (4) and (5) and solving for ϵ. Doing that, we obtain

$$\epsilon = \frac{1}{(1 + \mathrm{SNR})^D}. \qquad (6)$$

Note that for $D = 1$ (6) and (3) are equal. That is, the analog scheme performs optimally without the infinity delay and complexity required by the optimum digital approach (cf. [17], [18]). Note also that the analog scheme needs not know the channel SNR while the optimum digital scheme needs that knowledge. Expressing ϵ and SNR in dBs, we observe

$$\epsilon(\mathrm{dB}) = -D\mathrm{SNR}(\mathrm{dB}) + O(1), \quad \mathrm{SNR} \to \infty,$$

[1] Some work on non linear mappings has been done in [14] [15].

i.e., for $D > 1$ the gap between OPTA and the linear analog scheme becomes arbitrarily large for increasing SNR. An interesting question is to know whether this statement also holds if delay incurred by the digital approach is constrained. The answer is positive. In the next section we prove that $D/2$ is a lower bound for the decay rate in distortion at high SNR that can be achieved by delay constrained digital approaches.

C. Lower Bound on Asymptotic Distortion Decay

In order to derive this bound an encoder is assumed as that in Fig. 3. The quantizer consists of two optimum scalar quantizers separately quantizing real and imaginary parts of each source output with a resolution of b bits. Let $q_{(n,m)} = \alpha_n + j\beta_m$ denote the reproduction values of the quantizer with $(n,m) \in \{1,\ldots,2^b\} \times \{1,\ldots,2^b\}$. For each transmission a set of $M = 2^{2b}$ signals $\boldsymbol{s}_i \in \mathbb{C}^D$ is randomly generated. Each component of the signal vectors is independently drawn according to a circularly symmetric Gaussian distribution $\mathcal{CN}(0, \mathrm{SNR})$. Index pairs representing quantization levels are randomly mapped to indexes $i \in \{1,\ldots,M\}$ representing transmit signals. The decoder performs maximum likelihood detection and reverses the random mapping in order to retrieve the transmitted quantizer outputs. Let $(\hat{n},\hat{m})$ be the pair of indexes obtained at the receiver upon detection and mapping reversal.

If no transmission errors occur distortion is entirely caused by the quantizer. At high resolution, i.e., high b, distortion is well approximated by [11]

$$\epsilon_{\mathrm{ne}} = \frac{2\pi 3^{3/2}}{12M}. \tag{7}$$

If transmission errors occur, we distinguish three different cases: **e1**) $\hat{n} \neq n$ and $\hat{m} \neq m$, **e2**) $\hat{n} = n$ and $\hat{m} \neq m$, **e3**) $\hat{n} \neq n$ and $\hat{m} = m$. Let P_{e1}, P_{e2} and P_{e3} be the probabilities of occurrence of each of these types of errors and ϵ_{e1}, ϵ_{e2} and ϵ_{e3} the average distortions conditioned on the occurrence of **e1**, **e2** and **e3**, respectively. Using these definitions, the average distortion of the system can be written as

$$\epsilon = \epsilon_{\mathrm{ne}}(1 - P_{\mathrm{e}}) + \sum_{i=1}^{3} P_{\mathrm{e}i}\epsilon_{\mathrm{e}i} \tag{8}$$

where P_{e} is the probability of transmission error. Obviously, $P_{\mathrm{e}} = \sum_{i=1}^{3} P_{\mathrm{e}i}$. Furthermore,

$$P_{\mathrm{e1}} = \frac{(\sqrt{M}-1)^2}{M-1} P_{\mathrm{e}}, \quad P_{\mathrm{e2}} = P_{\mathrm{e3}} = \frac{\sqrt{M}-1}{M-1} P_{\mathrm{e}}.$$

Substituting these expressions in (8) we obtain

$$\epsilon = \epsilon_{\mathrm{ne}}(1 - P_{\mathrm{e}}) + \bar{\epsilon}_{\mathrm{e}} P_{\mathrm{e}} \tag{9}$$

where $\bar{\epsilon}_{\mathrm{e}} = \frac{(\sqrt{M}-1)^2}{M-1}\epsilon_{\mathrm{e1}} + \frac{\sqrt{M}-1}{M-1}(\epsilon_{\mathrm{e1}} + \epsilon_{\mathrm{e2}})$.

The probability of transmission error obtained by using an ensemble of random code books with $M = 2^{DR} = 2^{2b}$ code words is upper bounded by

$$P_{\mathrm{e}} \leq 2^{-DE_r(R)} \tag{10}$$

where $E_r(R)$ is the random coding exponent [19]. For the AWGN channel and the Gaussian input distribution considered here this exponent can be written as

$$E_r(R) = \max_{0 \leq \rho \leq 1} \rho \left(\log_2 \left(1 + \frac{\mathrm{SNR}}{1+\rho} \right) - R \right).$$

Choosing $\rho = 1$ a looser and simpler upper bound is obtained as

$$P_{\mathrm{e}} \leq 2^{-D(R_0 - R)} \tag{11}$$

where $R_0 = \log_2(1+\mathrm{SNR}/2)$ is the so-called cut-off rate [20]. Using (7) and (11), distortion in (9) can be upper bounded as

$$\epsilon \leq K_1(1 - P_{\mathrm{e}})2^{-DR} + \bar{\epsilon}_{\mathrm{e}} 2^{-D(R_0 - R)}$$

where K_1 is a constant independent of R. Now, choosing $R = 2\lfloor DR_0(\mathrm{SNR})/4 \rfloor / D$ and noting $(1 - P_{\mathrm{e}}) \leq 1$ the following upperbound results

$$\epsilon \leq (K_1 + \bar{\epsilon}_{\mathrm{e}}) 2^{-2\lfloor DR_0/4 \rfloor}. \tag{12}$$

Assume that the average distortion conditioned on transmission errors $\bar{\epsilon}_{\mathrm{e}}$ is bounded. In that case from (12) it is easily shown that

$$\epsilon(\mathrm{dB}) = -\frac{D}{2}\mathrm{SNR}(\mathrm{dB}) + O(1), \quad \mathrm{SNR} \to \infty,$$

i.e., distortion of the optimum delay constrained digital approach decays at least at a rate $D/2$ in the high SNR region.

The boundedness assumption on $\bar{\epsilon}_{\mathrm{e}}$ is key for the validity of this result. $\bar{\epsilon}_{\mathrm{e}}$ is bounded if all three $\epsilon_{\mathrm{e}i}$ are bounded. Here, for reasons of space, we only sketch the proof for the boundedness of ϵ_{e1}. The proofs for the other two cases are similar. Let I_n and I_m be the intervals of the scalar quantizers corresponding to the reproduction value $q_{(n,m)}$ and $z = \alpha + j\beta$ with real and imaginary parts in these intervals. If this source value is transmitted the distortion at the receiver conditioned on the occurrence of **e1** is given by

$$\epsilon_{\mathrm{e1},z} = \frac{1}{\sqrt{M}-1} \left(\sum_{\substack{\hat{n}=1 \\ \hat{n} \neq n}}^{\sqrt{M}} |\alpha - \alpha_{\hat{n}}|^2 + \sum_{\substack{\hat{m}=1 \\ \hat{m} \neq m}}^{\sqrt{M}} |\beta - \beta_{\hat{m}}|^2 \right) \tag{13}$$

Note that due to the random mapping the probability of detecting a certain pair of indexes conditioned on **e1** is uniformly distributed over the set $\{1,\ldots,\sqrt{M}\} \times \{1,\ldots,\sqrt{M}\} \setminus \{(n,m)\}$. Averaging over all possible outputs of the source we obtain

$$\epsilon_{\mathrm{e1}} = \frac{2}{\sqrt{M}-1} \sum_{n=1}^{\sqrt{M}} \int_{I_n} \sum_{\substack{\hat{n}=1 \\ \hat{n} \neq n}}^{\sqrt{M}} |\alpha - \alpha_{\hat{n}}|^2 p(\alpha) d\alpha \tag{14}$$

where the fact has been used that both terms in (13) are identically distributed. Expanding the square in (14) and after some simple manipulations we obtain,

$$\epsilon_{\mathrm{e1},z} = 1 + \frac{2\sqrt{M}}{\sqrt{M}-1}\frac{1}{\sqrt{M}}\sum_{n=1}^{\sqrt{M}}\alpha_n^2 +$$
$$+\frac{2}{\sqrt{M}-1}\left(2\sum_{n=1}^{\sqrt{M}}\int_{I_n}\alpha_n\alpha p(\alpha)d\alpha - \sum_{n=1}^{\sqrt{M}}\int_{I_n}\alpha_n^2 p(\alpha)d\alpha\right).$$

The third term is clearly bounded. As for the second term, boundedness is proved by observing

$$\frac{1}{\sqrt{M}}\sum_{n=1}^{\sqrt{M}}\alpha_n^2 \to \int \alpha^2\lambda(\alpha)d\alpha, \quad M\to\infty,$$

where $\lambda(\alpha)$ is the point density function of the optimum scalar quantizer at high resolution [11].

V. Rayleigh Fading Feedback Link

A. Analog Transmission

If the linear analog scheme described in Section IV-A is used for transmission over a Rayleigh fading channel (2) becomes

$$\boldsymbol{y} = g\boldsymbol{\phi}\sqrt{\mathrm{SNR}}z + \boldsymbol{w}.$$

Conditioned on a fixed channel gain g the distortion at the output of the MMSE estimator can be written as

$$\epsilon_g = \frac{1}{1 + D|g|^2\mathrm{SNR}}$$

and computing the mean over all possible channel states the resulting average distortion is given by

$$\epsilon = \frac{1}{D\mathrm{SNR}}E_1\left(\frac{1}{D\mathrm{SNR}}\right)\exp\left(\frac{1}{D\mathrm{SNR}}\right) \qquad (15)$$

where $E_1(\cdot)$ is the exponential integral function.[2] An asymptotic analysis of this expression at high SNR yields [21]

$$\epsilon(\mathrm{dB}) = -\mathrm{SNR(dB)} + O(\log(\mathrm{SNR(dB)})), \quad \mathrm{SNR}\to\infty,$$

i.e., asymptotically distortion decays 1 dB for an increment of 1 dB in SNR. This behavior is independent of D.

B. OPTA

In the Rayleigh fading feedback link the maximum number of bits that can be transmitted without error per source value is given by

$$C = D\mathrm{E}_{\boldsymbol{g}}\left\{\log(1+|g|^2\mathrm{SNR})\right\}.$$

Computation of this expected value yields

$$C = \frac{D}{\log_{\mathrm{e}}2}E_1\left(\frac{1}{\mathrm{SNR}}\right)\ \exp\left(\frac{1}{\mathrm{SNR}}\right). \qquad (16)$$

Now, equating (16) and (4) and solving for ϵ we obtain

$$\epsilon = \exp\left(-DE_1\left(\frac{1}{\mathrm{SNR}}\right)\ \exp\left(\frac{1}{\mathrm{SNR}}\right)\right).$$

[2] $E_1(x) = \int_x^\infty \frac{e^{-t}}{t}dt$

An asymptotic analysis of this expression reveals

$$\epsilon(\mathrm{dB}) = -\eta D\mathrm{SNR(dB)} + O(1), \quad \mathrm{SNR}\to\infty,$$

with $1/2 \le \eta \le 1$. That is, the asymptotic distortion decay rate improves with D as in the AWGN feedback link. However, now there is fundamental difference between the optimum digital approach with unconstrained delay and delay constrained approaches. The former exploits diversity in the fading channel. The latter are unable to profit from diversity as they only see one channel gain during transmission. A tighter upper bound can be derived if this limitation of delay constrained approaches is considered. For a given fixed channel gain g the minimum achievable distortion is given by the OPTA over an AWGN link with signal to noise ratio $|g|^2\mathrm{SNR}$, i.e.,

$$\epsilon_g = \frac{1}{(1+|g|^2\mathrm{SNR})^D}. \qquad (17)$$

Obviously, distortion obtained by averaging (17) over g represents an upper bound on the average performance of delay constrained schemes. Computation of this expected value yields

$$\epsilon = \sum_{i=1}^{D-1}\frac{(-1)^{i-1}(D-i-1)!}{(D-1)!\mathrm{SNR}^i} + \frac{(-1)^{D-1}E_1\left(\frac{1}{\mathrm{SNR}}\right)\ \mathrm{e}^{1/\mathrm{SNR}}}{(D-1)!\mathrm{SNR}^D}.$$

For $D=1$ this bound is achieved by the analog approach (cf. (15)). For $D>1$

$$\epsilon(\mathrm{dB}) = -\mathrm{SNR(dB)} + O(1), \quad \mathrm{SNR}\to\infty,$$

that is, for fading channels, if diversity can not be exploited, digital approaches are uncapable to benefit from the higher dimensionality of the signal space in the way they do over AWGN channels.

VI. Simulation Results

Fig. 4 shows simulation results of both digital approaches discussed in Section III for the AWGN feedback link. The horizontal axis represents channel SNR. The vertical axis represents output SNR defined as $\mathrm{SNR}_{\mathrm{out}} = -\epsilon(\mathrm{dB})$. The variable b indicates the number of bits per real dimension used for the quantization of the source. $D=4$ uses of the channel are made for each source value. In addition to the curves of the delay constrained digital schemes, curves corresponding to the analog approach and OPTA are also plotted. Consistently with the dicussion in Section IV OPTA increases 4 dBs per dB of channel SNR at high SNR values. The slope of the analog approach is 1. The non topological scheme seems to follow the growth rate of OPTA. By contrast, the non topological scheme seems to be upper bounded by the analog approach, i.e., even random codes have the potential to perform much better than this scheme at high SNR. At low SNR, the topological approach benefits from the neighborhood preserving mapping and performs better than the non topological approach that suffers from the well known threshold effect and the fact that mutually mistaken signals are likely to correspond to very distant source values [13]. Performance of the non topological approach critically depends on the choice of resolution bits.

For the topological approach more bits provide a uniform performance improvement over the entire SNR range.

Fig. 5 shows performance over a Rayleigh fading link. The additional curve corresponds to the second bound derived in Section V-B. The growth rates of the analog scheme and both information theoretical bounds are consistent with the analysis performed in the preceding section. Now, analog transmission uniformly outperforms the delay constrained digital schemes. The topological scheme also shows better performance than the non topological scheme over the whole range of SNR values.

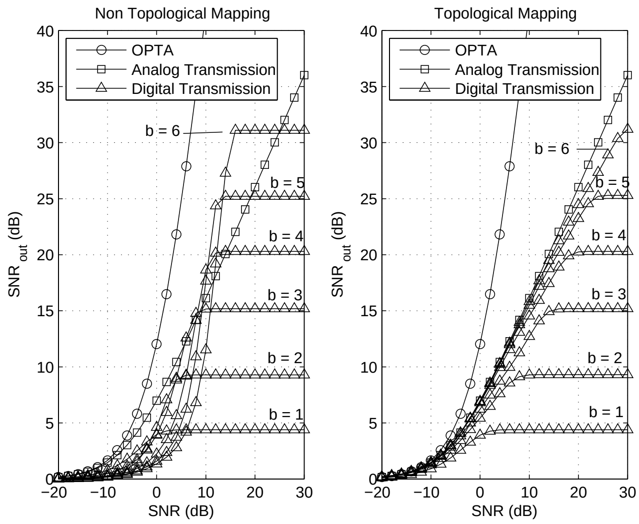

Figure 4. AWGN feeback link. $D = 4$.

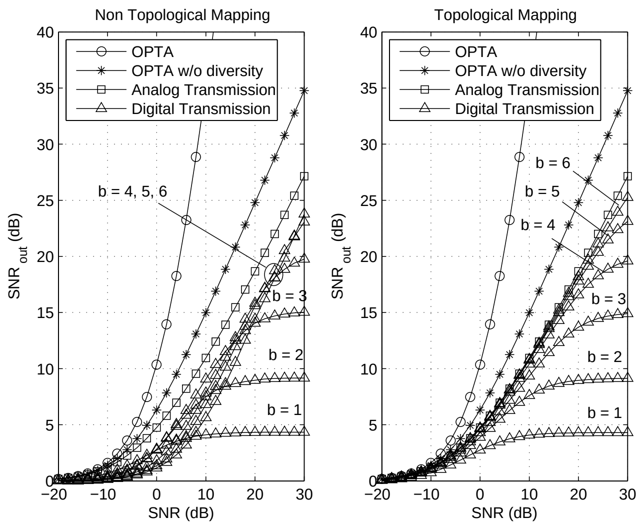

Figure 5. Rayleigh feeback link. $D = 4$.

VII. Conclusion

On an AWGN feedback link, delay constrained digital approaches have the potential to use the higher dimensionality of the signal space in order to make performance gain with respect to a linear analog approach arbitrarily large for increasing SNR. Only in the low SNR region or if only a channel use is made per source value performance of the analog approach is optimum. On a Rayleigh feedback link, digital approaches do not significantly benefit from the higher dimensionality of the signal space if diversity can not be exploited due to delay constraints. In such case a simple linear analog approach or a digital approach preserving neighborhood relations of the source in the signal space may clearly outperform performance achieved by the classical paradigm of using signal sets with large minimum distance.

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