

On Time-Switched Space-Time Transmit Diversity in MISO Systems

Michel T. Ivrlač and Wolfgang Utschick

Institute for Circuit Theory and Signal Processing

Munich University of Technology

80290 Munich, Germany

phone: +49 (89) 289285 {11,24}

email: {ivrlac, utschick}@nws.ei.tum.de

Abstract— We investigate the performance of time-switched space-time transmit diversity schemes applied to a wireless communication system with multiple antennae at the transmitter and single antenna at the receiver. A new method of analysis of time-switched transmit diversity schemes is presented which builds on an information theoretic equivalent channel and a generalized definition of diversity for coded systems. We show that coded time-switched Alamouti space-time block codes actually outperform all coded orthogonal space-time block codes which use more than two antennae. Cutoff rate analysis for modulated signals, proves their performance to be close to optimum in a range of code-rates, which is interesting for wireless communication.

I. INTRODUCTION

In communication systems operating over fading wireless channels, multiple antennae at the transmit side can be used to provide space diversity, which is crucial when the channel is neither time- nor frequency selective enough to provide a reliable link. Currently, a couple of ways are discussed to achieve this aim [1]. Among them are two promising approaches. The first one uses space-time coding which uses all transmit antennae simultaneously to achieve maximum possible diversity advantage [2], [3], [4]. This is referred to as space-time transmit diversity (STTD). In the second approach – time-switched transmit diversity (TSTD) – the data stream is cycled through the transmit antennae in a round ribbon fashion such that only one antenna is used at a time [5]. This leads to a time-selective link. In contrast to STTD, a TSTD scheme can provide diversity only by channel coding which takes advantage of the time selectivity. While TSTD has lower diversity advantage than STTD, the latter has the critical problem, that efficiently decodeable space-time codes suffer from rate loss, if more than two transmit antennae are used [3]. In this paper we look at the combination of STTD and TSTD schemes for digitally modulated signals. We show that a simple time-switched Alamouti scheme actually outperforms all orthogonal space-time block codes which use more than two antennae. The performance is also very close to hypothetical ideal space-time block codes. Besides its simplicity and good performance, a combined TSTD-STTD scheme is also attractive, because the number of transmit antennae can be changed without the receiver having to know, which makes such a scheme fairly independent of standardization.

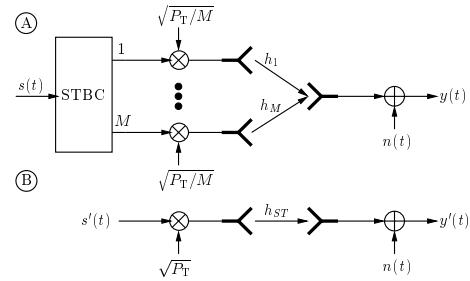


Fig. 1. Space-Time Diversity scheme (A) and equivalent channel (B)

II. SPACE-TIME TRANSMIT DIVERSITY

In the STTD scheme shown in Fig. 1 the data signal is encoded into a space-time block code (STBC) and transmitted simultaneously over M antennae. As the channel is assumed to be unknown to the transmitter, the total transmit power P_T is shared equally among the M transmit antennae. The signals then propagate through a frequency and time non-selective channel with complex fading coefficients h_i , $1 \leq i \leq M$, and arrive at a single receive antenna where they get perturbed by additive, temporally white Gaussian noise with variance σ_n^2 (upper part of Fig. 1). After passing a STBC decoder/combiner, the system behaves like an equivalent single-input single-output (SISO) system [6], with complex fading coefficient h_{ST} , such that

$$|h_{ST}|^2 = \frac{1}{M} \sum_{i=1}^M |h_i|^2. \quad (1)$$

In the equivalent SISO channel the signal is launched with transmit power P_T and gets perturbed with zero-mean complex Gaussian noise of variance σ_n^2 , as is depicted in the lower half of Fig. 1. For i.i.d. symmetric, zero-mean and unity variance complex Gaussian distributed h_i , i.e. Rayleigh fading, the equivalent channel's fading statistics is given by a M -th order Nakagami distribution:

$$p_{\gamma_{ST}}(\gamma_{ST}) = \frac{M^M}{\Gamma(M)} \cdot \gamma_{ST}^{M-1} \cdot \exp(-\gamma_{ST} \cdot M), \quad (2)$$

where we used the abbreviation $\gamma_{ST} = |h_{ST}|^2$. The instantaneous channel capacity reads as

$$C = R_{ST} \cdot \log_2 \left(1 + \frac{P_T}{\sigma_n^2} \gamma_{ST} \right), \quad (3)$$

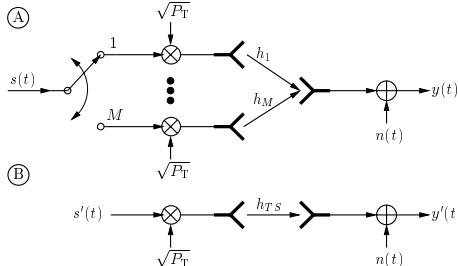


Fig. 2. Time-Switched Transmit Diversity scheme (A) and its equivalent channel (B)

where R_{ST} is the rate of the STBC used. While the Alamouti space-time coding scheme [2] does not involve rate loss, i.e. maintains $R_{\text{ST}} = 1$, orthogonal STBC used with $M > 2$ transmit antennae introduce some loss of rate [3], which may causes severe performance degradation, as we shall see later.

III. TIME-SWITCHED TRANSMIT DIVERSITY

In a TSTD system depicted in Fig. 2 only one antenna is used at a time, transmitting at full transmit power P_{T} . Assuming equal air-time of the antennae - which is the best strategy if the channel is unknown to the transmitter - the instantaneous channel capacity C is the average of the capacities C_i , $1 \leq i \leq M$ of the individual links:

$$C = \frac{1}{M} \sum_{m=1}^M \log_2 \left(1 + \frac{P_{\text{T}}}{\sigma_n^2} \gamma_m \right), \quad (4)$$

where we used the abbreviation $\gamma_m = |h_m|^2$. Unlike in the STTD system, a diversity advantage can not be obtained by linear processing, as the transmit-switching procedure merely introduces an artificial time selectivity, but does not directly alter the fading statistics. However diversity gain is exploitable with proper channel coding which makes use of the introduced time variance, by interleaving the code symbols over successive antenna switches. In the following we will therefore assume a coded TSTD system, and later compare its performance to a coded STTD system.

It is interesting to note, that a coded TSTD system can be described by a non-switched SISO equivalent channel, with complex random fading coefficient h_{TS} , suitably distributed. Again writing $\gamma_{\text{TS}} = |h_{\text{TS}}|^2$, this equivalent SISO channel has capacity

$$C_{\text{TS}} = \log_2 \left(1 + \frac{P_{\text{T}}}{\sigma_n^2} \gamma_{\text{TS}} \right). \quad (5)$$

Comparison of (5) with (4) shows the dependence of γ_{TS} on the γ_m :

$$\gamma_{\text{TS}} = \frac{\sigma_n^2}{P_{\text{T}}} \cdot \left[\left[\prod_{m=1}^M \left(1 + \frac{P_{\text{T}}}{\sigma_n^2} \gamma_m \right) \right]^{1/M} - 1 \right], \quad (6)$$

which is necessary for the equivalent channel to have the same capacity as the original TSTD system, i.e. maintain equivalence of (4) and (5). From (6) the probability density function (pdf)

of γ_{TS} can be computed. For i.i.d. γ_m , it can be shown [7], that the pdf can be expressed as

$$p_{\gamma_{\text{TS}}}(\gamma_{\text{TS}}) = M \cdot \frac{P_{\text{T}}/\sigma_n^2}{1 + \frac{P_{\text{T}}}{\sigma_n^2} \gamma_{\text{TS}}} \int_{-\infty}^{+\infty} F(x) \cdot \left(1 + \frac{P_{\text{T}}}{\sigma_n^2} \gamma_{\text{TS}} \right)^{-j2\pi M x} dx \quad (7)$$

where

$$F(x) = \left(\mathbb{E}_{\gamma_i} \left\{ \left(1 + \frac{P_{\text{T}}}{\sigma_n^2} \gamma_i \right)^{j2\pi x} \right\} \right)^M. \quad (8)$$

Note, that (7) is a function of transmit power, which means that the statistical properties of a coded TSTD system are dependent on transmit power, i.e. the chosen operating point. We refer to such a behavior as *statistical nonlinearity*.

IV. TIME-SWITCHED SPACE-TIME TRANSMIT DIVERSITY

STTD and TSTD can be used in combination as shown in Fig. 3 for the case $M = 8$. Space-time block codes designed for $m \leq M$ antennae are used, and time-switching between T blocks comprised of m antennae is applied, such that $T \cdot m = M$. The instantaneous channel capacity is now

$$C = \frac{R_{\text{ST}}}{T} \sum_{t=1}^T \log_2 \left(1 + \frac{P_{\text{T}}}{\sigma_n^2} \gamma_{m,t} \right), \quad (9)$$

where $\gamma_{m,t}$ with $t = 1, 2, \dots, T$ are the squared equivalent fading coefficient magnitudes, which are provided by the space-time block-code. In the case of Rayleigh fading, we have from (2) the pdf of $\gamma_{m,t}$ given as

$$p_{\gamma_{m,t}}(\gamma_{m,t}) = \frac{m^m}{\Gamma(m)} \cdot \gamma_{m,t}^{m-1} \cdot \exp(-\gamma_{m,t} \cdot m). \quad (10)$$

For i.i.d. $\gamma_{m,t}$ the capacity equivalent channel can be obtained similarly as for the purely time-switched system.

Before beginning the analysis and comparison of TSTD, STTD and combined schemes, let us first introduce the notion of diversity for coded communication systems, which will be helpful in providing some insight into different behavior of TSTD and STTD systems.

V. DIVERSITY IN CODED COMMUNICATION SYSTEMS

The amount of diversity D provided by a coded communication system is measured by the slope of the frame error rate (FER) after channel decoding with respect to transmit power:

$$D := - \frac{d \log \text{FER}}{d \log P_{\text{T}}/\sigma_n^2}. \quad (11)$$

It is in general a function of FER and/or P_{T}/σ_n^2 , which define the operating point the communication system is used in. The *diversity order* D_{\max} is defined as the maximum of D with respect to FER and P_{T}/σ_n^2 . Assuming ideal, i.e. capacity achieving channel codes, the relationship between FER and P_{T}/σ_n^2 is governed by the *outage capacity* $C_{\text{out}}(P_{\text{T}}/\sigma_n^2, p)$, where p

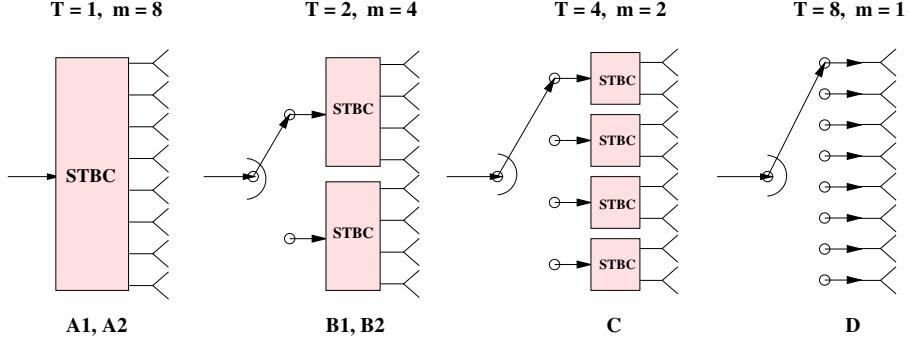


Fig. 3. Time-Switched Space-Time Transmit Diversity. The M transmit antennae are divided into T groups of m antennae each, such that $T \cdot m = M$. At each time instant only one group is transmitting a space-time block code, designed for m antennae, while a switching between blocks occurs in a random, or round robin fashion. The diagram shows different setups of T and m .

is the probability, that the instantaneous channel capacity C is lower than C_{out} , i.e.

$$\text{Prob} \{C < C_{\text{out}}\} = p. \quad (12)$$

Assuming a data rate equal to C_{out} a frame error will occur after channel decoding with probability p , hence (11) can be written as

$$D := - \left. \frac{d \log p}{d \log P_T / \sigma_n^2} \right|_{C_{\text{out}}(P_T / \sigma_n^2, p) = \text{const}} \quad (13)$$

The operating point is specified by either the pair $(p, P_T / \sigma_n^2)$ or the pair $(C_{\text{out}}, P_T / \sigma_n^2)$ or the pair (C_{out}, p) , where the latter seems to be the most descriptive. A coded communication system set up to provide outage capacity C_{out} with outage probability p , can achieve diversity D given by (13), which can also be stated in a more explicit way, by writing $C_{\text{out}}(P_T / \sigma_n^2, p) = \text{const}$ in form of a total differential:

$$0 = dC_{\text{out}} = \frac{\partial C_{\text{out}}}{\partial P_T / \sigma_n^2} dP_T / \sigma_n^2 + \frac{\partial C_{\text{out}}}{\partial p} dp. \quad (14)$$

This leads immediately to

$$\frac{dp}{dP_T / \sigma_n^2} = - \frac{\partial C_{\text{out}}}{\partial P_T / \sigma_n^2} \cdot \left(\frac{\partial C_{\text{out}}}{\partial p} \right)^{-1}, \quad (15)$$

and finally (13) can be written explicitly as

$$D = \frac{P_T / \sigma_n^2}{p} \cdot \frac{\partial C_{\text{out}}}{\partial P_T / \sigma_n^2} \cdot \left(\frac{\partial C_{\text{out}}}{\partial p} \right)^{-1}. \quad (16)$$

For purpose of illustration have a look at a simple example of a Rayleigh fading SISO system where the zero-mean unity variance signal $s(t)$ is launched with transmit power P_T and arrives at the receiver scaled by a Rayleigh distributed path coefficient h , which is normalized to unity average power, and perturbed with additive temporally white zero-mean complex Gaussian noise $n(t)$ with variance σ_n^2 at the receiver:

$$y(t) = h \cdot \sqrt{P_T} \cdot s(t) + n(t).$$

The outage capacity can readily be computed as

$$C_{\text{out}} = \log_2 \left(1 - \frac{P_T}{\sigma_n^2} \log_e (1 - p) \right),$$

and applying (16) we get the diversity

$$D = -(1 - p) \cdot \log_e (1 - p) / p,$$

which is merely a function of the outage probability p and therefore *independent* of the outage capacity or transmit power. While this independency property holds for all STBC diversity schemes, it does not hold for the TSTD case, which is due to the dependency of the effective fading statistics on transmit power. In Section III we referred to this behaviour as *statistical non-linearity*. We will elaborate on this in the following sections. Note, that the maximum of D is obtained for $p \rightarrow 0$ and yields the diversity order of $D_{\max} = 1$, as expected.

VI. INFLUENCE OF INTERLEAVING

Coded communication systems operated over time selective fading channels can be used in conjunction with interleaving, where the code-symbols forming the code-words are spread in time to be affected by different channel situations (time-hopping). In the following we will assume for simplicity a block fading channel, which remains constant during the coherence time T_{coh} and then changes abruptly to take on a new, independent realization. Assuming the interleaver length covers L_{int} coherence times, the decoder is allowed to make its decision after seeing L_{int} independent realizations of the fading channel. The effective channel capacity C' is therefore given as

$$C'(L_{\text{int}}) = \frac{1}{L_{\text{int}}} \sum_{l=1}^{L_{\text{int}}} C_l, \quad (17)$$

where C_l with $l = 1, 2, \dots, L_{\text{int}}$ are the instantaneous capacities corresponding to the different channel realizations during interleaving time. The notion of outage capacity is carried over, such as $\text{Prob} \{C' < C_{\text{smo}}\} = p$, where $C_{\text{smo}}(P_T / \sigma_n^2, p, L_{\text{int}})$ is called *sample-mean outage capacity*

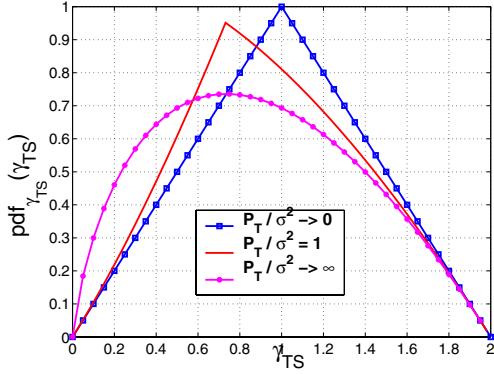


Fig. 4. Transmit power dependent equivalent fading pdf of a coded but non-interleaved $M = 2$ antenna TSTD. The squared magnitude of the fading coefficients are i.i.d. uniformly distributed. In the low transmit power region the pdf equals the one obtained by the Alamouti STTD scheme, while deviations to less favorable distributions occur for high transmit powers.

[8]. Analysis of diversity of coded communication systems employing interleaving can be carried out by substituting C_{smo} for C_{out} in (13) or (16).

VII. ANALYSIS

Let us analyze the behavior of TSTD compared to STTD systems now. Firstly, we will look at the fading statistics of the equivalent channel of coded TSTD systems to see the effect of statistical nonlinearity. Secondly, the influence of this nonlinearity on diversity performance will be addressed, and finally we will use cutoff rate analysis to compare TSTD, STTD and combined schemes including effects of linear digital modulation.

A. Statistical Nonlinearity

From (6) it is evident, that the equivalent fading statistics of a coded TSTD system depends on transmit power. In the low transmit power region however, this dependency vanishes, for

$$\lim_{P_T/\sigma_n^2 \rightarrow 0} \gamma_{\text{TS}} = \frac{1}{M} \sum_{m=1}^M \gamma_m = \frac{1}{M} \sum_{m=1}^M |h_m|^2, \quad (18)$$

i.e. the equivalent fading statistics of TSTD and STTD systems are the same, and only begin to deviate for high transmit power. To illustrate this point, have a look at a simple $M = 2$ antenna TSTD system, where the squared magnitudes of the path coefficients are uniformly distributed, i.e.

$$p_{\gamma_i}(\gamma_i) = \begin{cases} 1/2 & \text{for } 0 \leq \gamma_i \leq 2 \\ 0 & \text{else} \end{cases}, \quad i = 1, 2.$$

In this case, the equivalent fading statistics from (7) can be computed in closed form:

$$p_{\gamma_{\text{TS}}}(\gamma_{\text{TS}}) = \begin{cases} \frac{\sigma_n^2}{P_T} \left(1 + \frac{P_T}{\sigma_n^2} \gamma_{\text{TS}} \right) \log_e \left(1 + \frac{P_T}{\sigma_n^2} \gamma_{\text{TS}} \right) & \text{for } 0 \leq \gamma_{\text{TS}} \leq \delta \\ \frac{\sigma_n^2}{P_T} \left(1 + \frac{P_T}{\sigma_n^2} \gamma_{\text{TS}} \right) \log_e \left(\frac{1+2\frac{P_T}{\sigma_n^2}}{1+\frac{P_T}{\sigma_n^2} \gamma_{\text{TS}}} \right) & \text{for } \delta \leq \gamma_{\text{TS}} \leq 2 \\ 0 & \text{else} \end{cases}$$

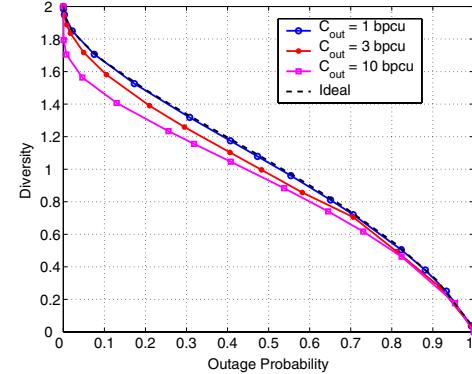


Fig. 5. Capacity dependent diversity of a coded but non-interleaved $M = 2$ TSTD system operated over i.i.d. Rayleigh fading. For outage capacities below 1 bit per channel use, there is essentially no loss of diversity compared to the Alamouti STTD scheme.

with $\delta = \frac{\sigma_n^2}{P_T} \left(\sqrt{1 + 2 \frac{P_T}{\sigma_n^2}} - 1 \right)$. Fig. 4 shows the graphical representation of this transmit power dependent pdf. Note, that for uniformly i.i.d. γ_i the Alamouti STBC scheme would produce a triangularly shaped pdf. As expected from (18) the TSTD system achieves the very same pdf for low enough transmit power. Increase of transmit power however, leads to less favorable distributions, which allow deep fades to happen with higher probability than necessary, hence reducing diversity advantage. Asymptotically, we have

$$\lim_{P_T/\sigma_n^2 \rightarrow \infty} p_{\gamma_{\text{TS}}}(\gamma_{\text{TS}}) = \gamma_{\text{TS}} \cdot \log_e \frac{2}{\gamma_{\text{TS}}}, \quad \text{for } 0 \leq \gamma_{\text{TS}} \leq 2,$$

which exhibits the largest deviation from the ideal shape.

B. Diversity

To make some quantitative assertions on the diversity performance of a coded TSTD system, we use the method derived in Section V by computing outage capacities and applying (16). Again we use $M = 2$ antennae, but this time the fading coefficients are drawn from independent and identical Rayleigh distributions. To this end (16) has to be solved numerically. The results are depicted in Fig. 5, which shows the available diversity as a function of outage probability, i.e. frame error rate, with outage capacity as parameter. The dashed curve represents the maximum obtainable diversity, which is achieved by the Alamouti scheme, independent of outage capacity. We can see from Fig. 5, that the TSTD system competes marvelously as long as the requested outage capacities remain lower than a few bits per channel use, which is actually rather a large value for a MISO system. For $C_{\text{out}} = 1$ bit per channel use, there is essentially no deviation from the ideal. Considerable loss of diversity performance, especially at low frame error rates, occurs only if we ask for rather huge outage capacities, like 10 or more bits per channel use.

C. Cutoff Rate Performance

While capacity is a theoretical limit for infinite block length codes and zero error probability, the cutoff-rate gives a bound

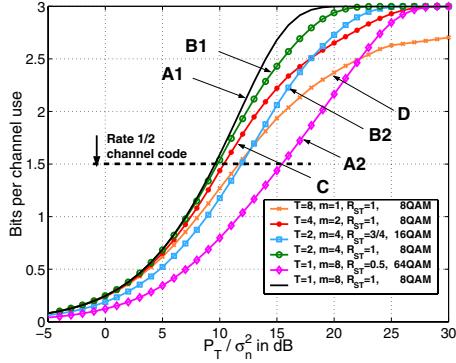


Fig. 6. Outage cutoff rate for different setups of time-switched space-time transmit diversity systems. See also Table I for setup details and Fig. 3 for a schematic diagram.

for finite block length and error probability. The cutoff rate is useful because of the cutoff-rate theorem [9], which states that there exist $(n, k)_q$ block codes, with code-word error probability P_w after maximum likelihood decoding being upper bounded by $P_w < 2^{-n \cdot (R_0 - R_b)}$, provided the binary code rate $R_b := \frac{k}{n} \cdot \log_2 q$ is less than the cutoff-rate

$$R_0 = -\log_2 \int_{\mathcal{C}} \left(\sum_{s \in \mathcal{M}} \frac{1}{q} \sqrt{p(y|s)} \right)^2 dy, \quad (19)$$

where \mathcal{M} , with $|\mathcal{M}| = q$ is the set of code symbols (input alphabet) and $p(y|s)$ is the probability density function of the received signal y given the transmitted code symbol s . By labeling the elements of $\mathcal{M} = \{s_1, s_2, \dots, s_q\}$ the instantaneous cutoff-rate for a AWGN SISO system operated at SNR γ can be written as [8]

$$R_0(\gamma) = \log_2(q) - \log_2 \left(1 + \frac{2}{q} \sum_{p=1}^{q-1} \sum_{t=p+1}^q \exp \left(-\frac{1}{4} \gamma \cdot \frac{P_T}{\sigma_n^2} |s_p - s_t|^2 \right) \right). \quad (20)$$

In the general case of combined STTD-TSTD according to section IV, where time switching between $T \geq 1$ blocks consisting of $m \leq M$ antennae are used in conjunction with space-time block coding, the cutoff rate reads as

$$R_0(m, T) = \frac{R_{ST}}{T} \sum_{t=1}^T R_0(\gamma_{m,t}), \quad (21)$$

where $\gamma_{m,t}$ with $t = 1, 2, \dots, T$ are the squared equivalent fading coefficient magnitudes, which are provided by the space-time block-code. In the case of Rayleigh fading, their distribution is given by (10).

Let us take the example depicted in Fig. 3. We fix the raw, i.e. uncoded data rate to 3 bits per channel use, and look at the outage cutoff rate performance of the combined STTD-TSTD schemes with different (m, T) -setups from Table I. As some of the space-time codes exhibit rate-loss the size of the modulation alphabet is expanded accordingly, as to satisfy the given uncoded data rate (see Table I for details). The *orthogonal*

TABLE I
PARAMETERS FOR DIFFERENT SETUPS OF STDD-TSTD SCHEME

Setup	m	T	R_{ST}	STBC	Modulation
A1	8	1	1	ideal	8QAM
A2	8	1	1/2	orthogonal	64QAM
B1	4	2	1	ideal	8QAM
B2	4	2	3/4	orthogonal	16QAM
C	2	4	1	Alamouti	8QAM
D	1	8	1	none	8QAM

STBC-s refer to the best known STBC-s given in [2], [3] and [4], while the *ideal* ones are hypothetical non-orthogonal STBC-s which do not suffer from rate loss, but need a high complexity decoder. Fig. 6 shows the outage cutoff-rate performance for an outage probability of $p = 0.01$. The time-switched Alamouti scheme (setup C) clearly outperforms *all* other orthogonal STBC-s for code-rates below 0.8. When using codes of rate 1/2, the loss compared to an *ideal* 8 antenna STBC (setup A1) is fairly small. Even the purely time-switched system (setup D) performs fairly well and outperforms all other orthogonal STBC-s for code-rates below about 0.55. Use of orthogonal STBC-s with more than two antennae in a time-switched system seems only useful when operating at rather high code-rates, uncommon in wireless communications.

VIII. CONCLUSION

The performance of time-switched space-time transmit diversity schemes in a wireless MISO channel was investigated by means of a new method of analysis which builds on an information theoretic equivalent channel. An outage capacity based definition of diversity of coded systems was introduced, that provides insight in different behavior of STTD and TSTD systems. Analysis showed, that coded time-switched Alamouti space-time block codes outperform *all* coded orthogonal space-time block codes which use more than two antennae in a range of code-rates, which is interesting for wireless communication.

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