HQC and Ciphertext Quantization

6G Future Lab Bavaria Speaker Series

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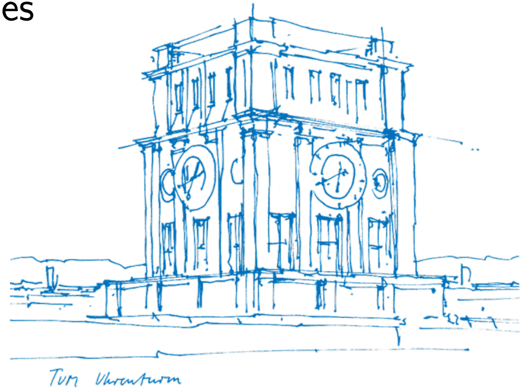
Institute for Communications Engineering

October 14, 2025



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Goal: *Reduce ciphertext length using vector quantization such that DFR is negligible.*

Notation

Vector Space

$$\mathbb{F}_2^n$$

Polynomial Ring

$$\mathcal{R}_n = \mathbb{F}_2[x]/(x^n - 1)$$

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Definition (Linear Code)

An (n, k) linear code $\mathcal{C}$ is a k-dimensional subspace of $\mathbb{F}_2^n$ equipped with two functions -
$\mathcal{C}.\mathtt{enc} : \mathbf{m} \mapsto \mathbf{c} \in \mathcal{C}$

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HQC Primer

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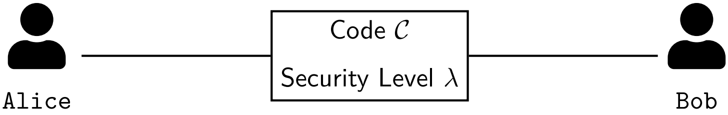

HQC Primer

Code $\mathcal{C}$

Security Level λ

Alice

Bob

KeyGen

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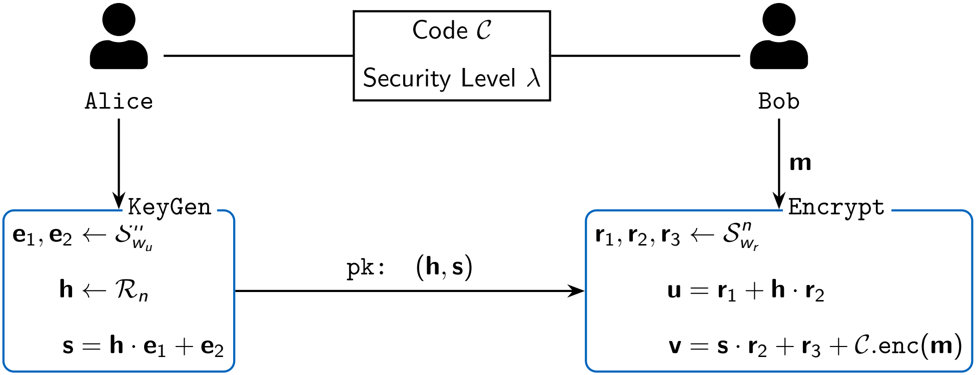
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$\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3 \leftarrow \mathcal{S}^n_{w_r}$
$\mathbf{u} = \mathbf{r}_1 + \mathbf{h} \cdot \mathbf{r}_2$
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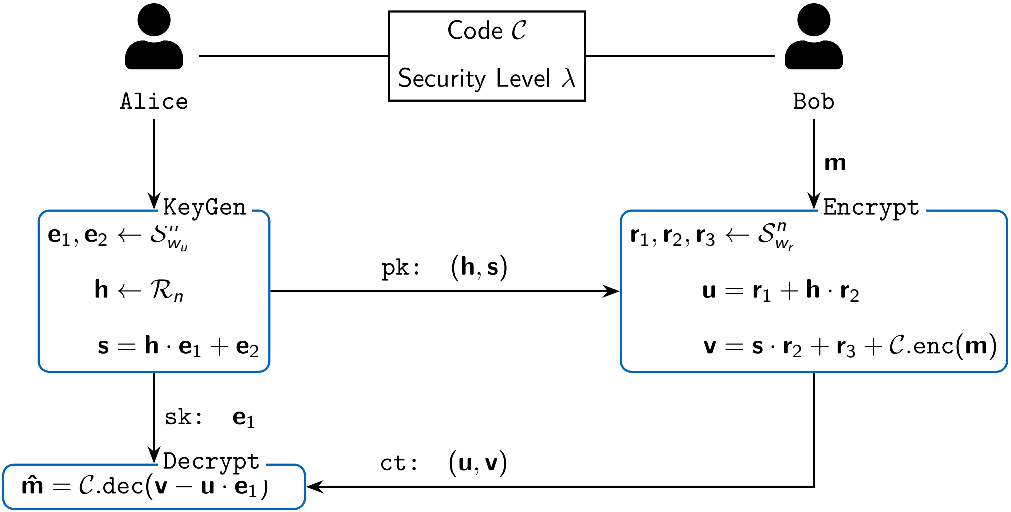

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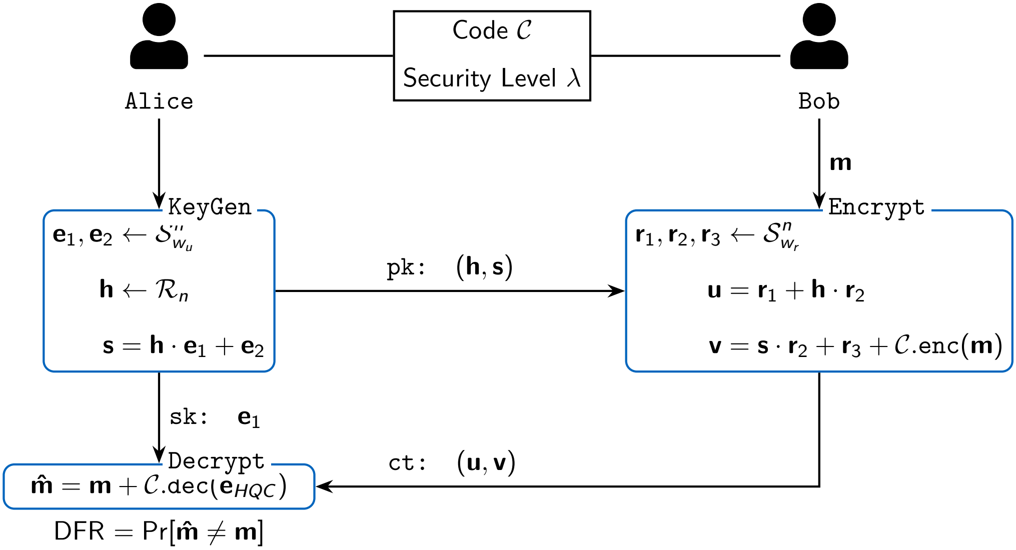

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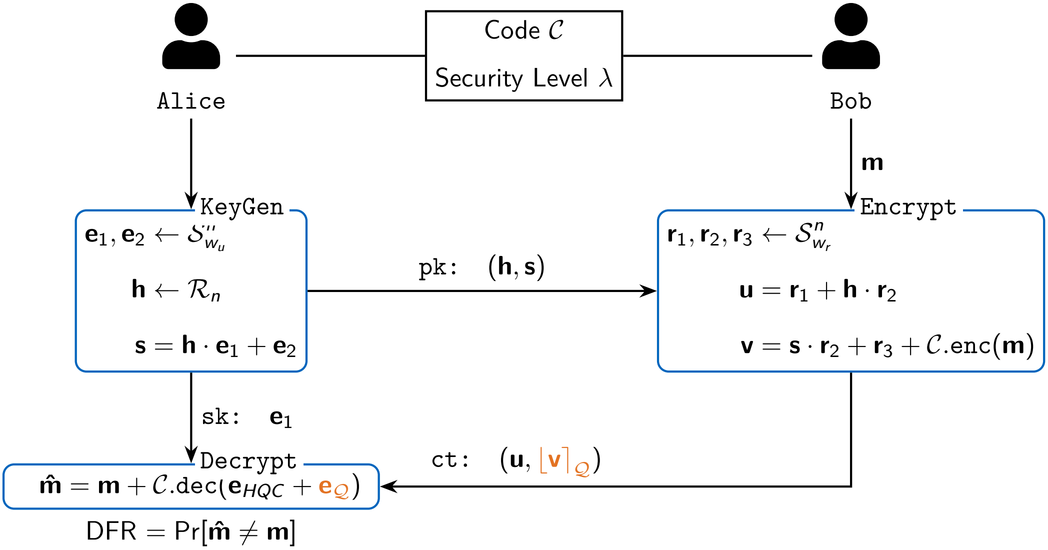

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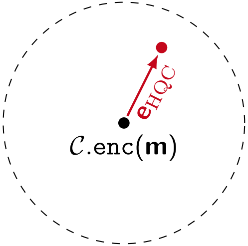
What is Vector Quantization?

$$\begin{aligned}\lfloor\cdot\rceil_{\mathcal{Q}}: \quad & \mathbb{F}_2^n \to \mathcal{Q} \\ & \mathbf{v} \mapsto \lfloor\mathbf{v}\rceil_{\mathcal{Q}} = \mathbf{v} + \{\mathbf{v}\}_{\mathcal{Q}}\end{aligned}$$

$$n\,\text{bit} \to \rho n\,\text{bit}, \quad \rho < 1$$



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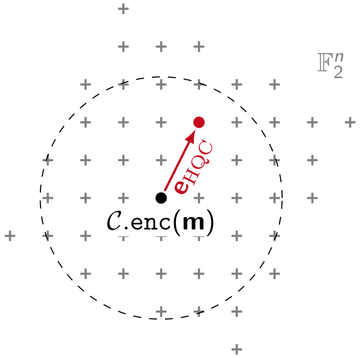
Theorem (Rate-Distortion Bound)

Compression by ρ creates average distortion

$$\mathbb{E}_{\mathbf{v}}\left[\frac{1}{n} wt_{\mathrm{H}}(\{\mathbf{v}\}_{\mathcal{Q}})\right] \geq H^{-1}(1-\rho)$$

[Shannon, 1948]



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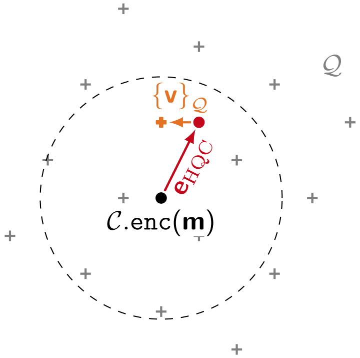

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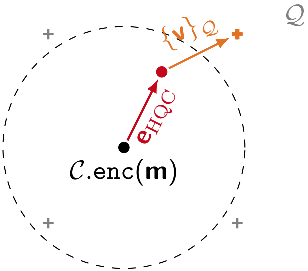

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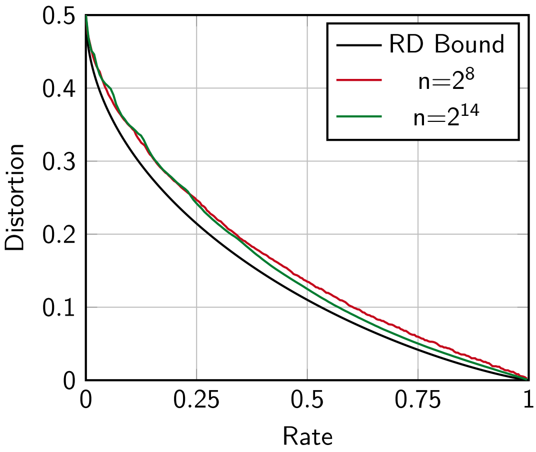
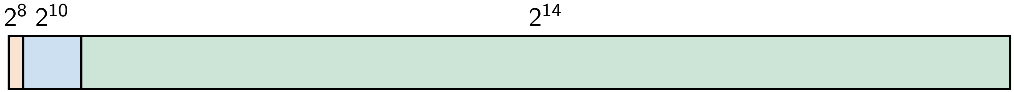


Quantization with Polar Codes (Black-Box Approach)

NIST 1 HQC Ciphertext **v** of length $n_\mathcal{C} = 17664$



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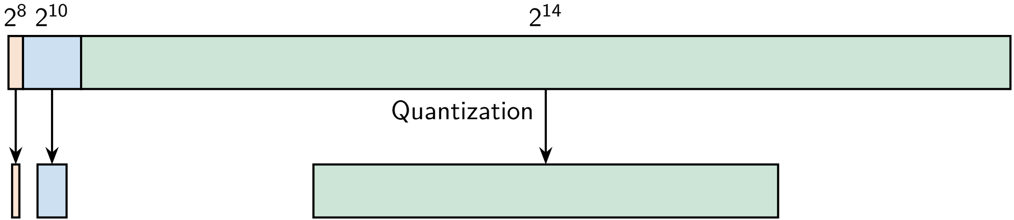

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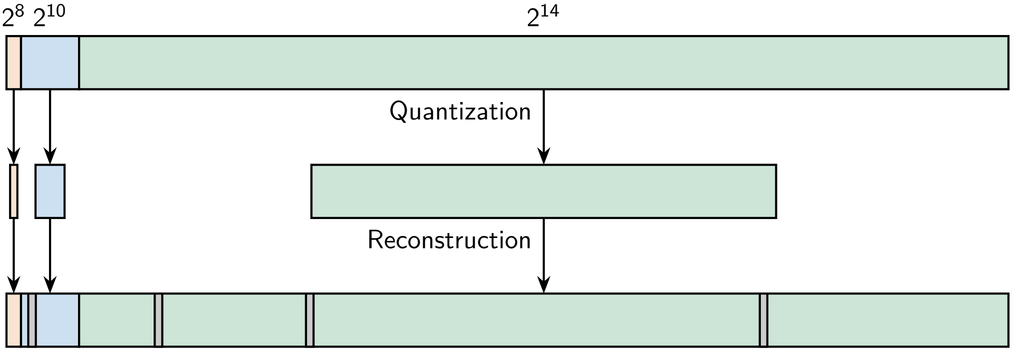
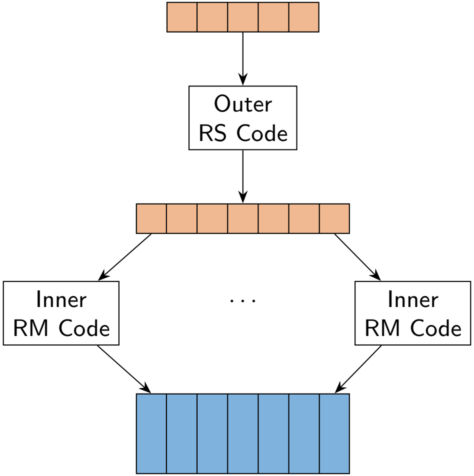


Aligned Quantization Code (White-Box Approach)

Construction of code $\mathcal{C}$



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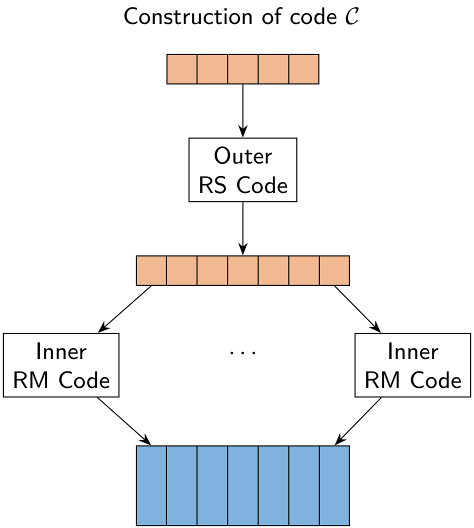

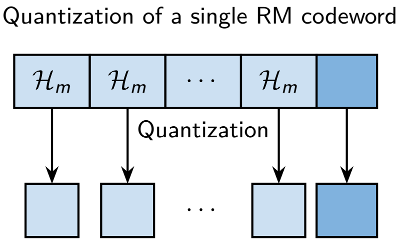

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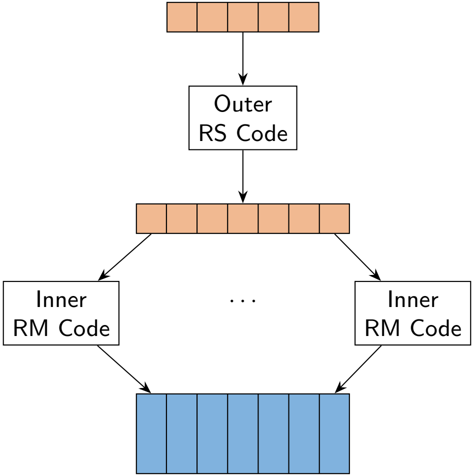
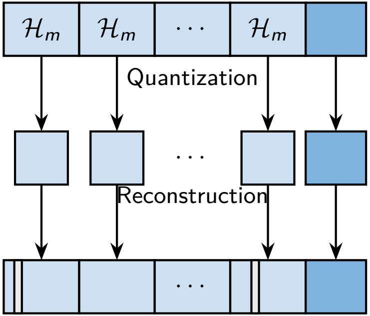
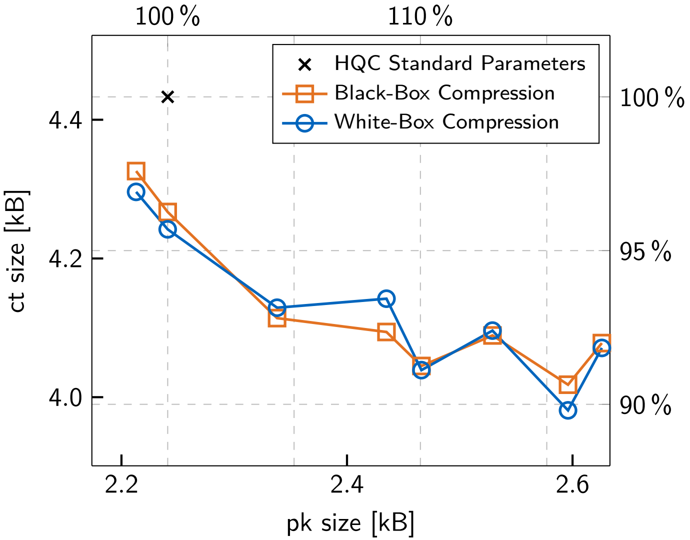


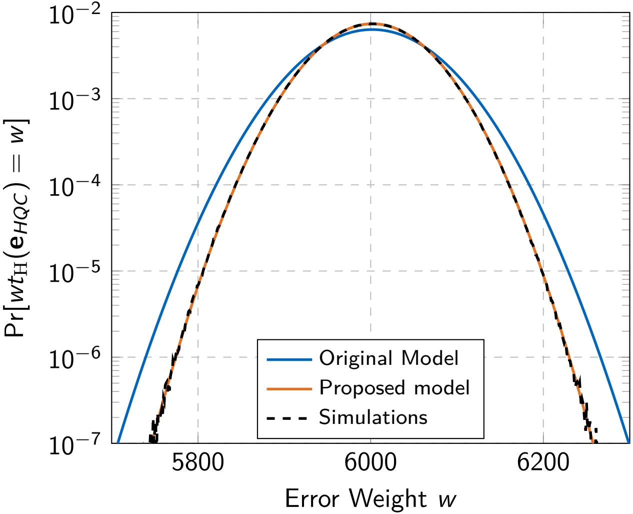
Quantization of a single RM codeword



Simulation Results



New Error Weight Model



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Open Questions

- Other candidates for quantization code $\mathcal{Q}$?
- Better error-correcting code $\mathcal{C}$ with predictable DFR?

References I

Korada, S. B., & Urbanke, R. L. (2010). Polar codes are optimal for lossy source coding. *IEEE Transactions on Information Theory*, *56*(4), 1751–1768. https://doi.org/10.1109/TIT.2010.2040961

Shannon, C. E. (1948). A mathematical theory of communication. *The Bell system technical journal*, *27*(3), 379–423.