The International Height Reference System (IHRS) and its realisation, the International Height Reference Frame (IHRF)

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Towards a global unified height system

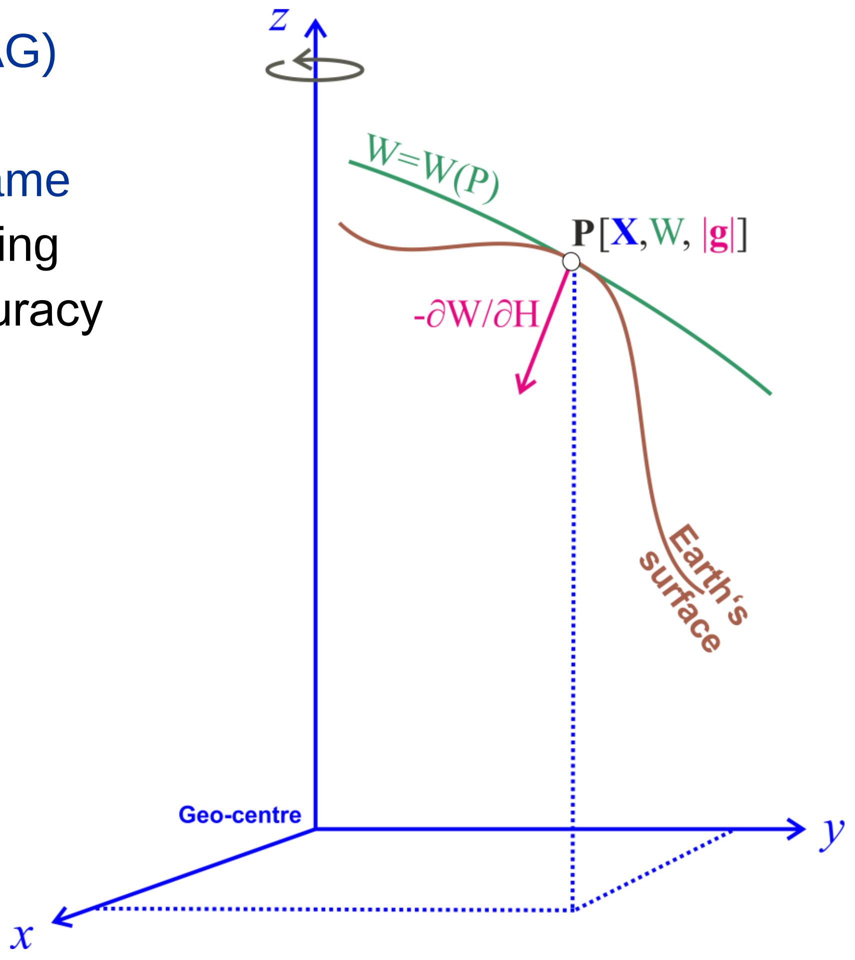
A main objective of the International Association of Geodesy (IAG) and its Global Geodetic Observing System (GGOS) is the implementation of an integrated Global Geodetic Reference Frame (GGRF) that supports the consistent determination and monitoring of the Earth's geometry, rotation, and gravity field with high accuracy worldwide.

The GGRF includes:

- Geocentric Cartesian coordinates $\mathbf{X}$, $\dot{\mathbf{X}}$
- Gravity vector $\mathbf{g}$, $\dot{\mathbf{g}}$
- Potential of the Earth's gravity field W, $\dot{W}$
- Physical height H, $\dot{H}$

Positions of points or objects on or close to the Earth's surface are to be given by:

$W(\mathbf{X})$, $P\{W, \mathbf{X}\}$, or $P\{\mathbf{X}, W, \mathbf{g}\} = P\{\mathbf{X}, W, -\nabla W\}$
and $g = -\partial W/\partial H$



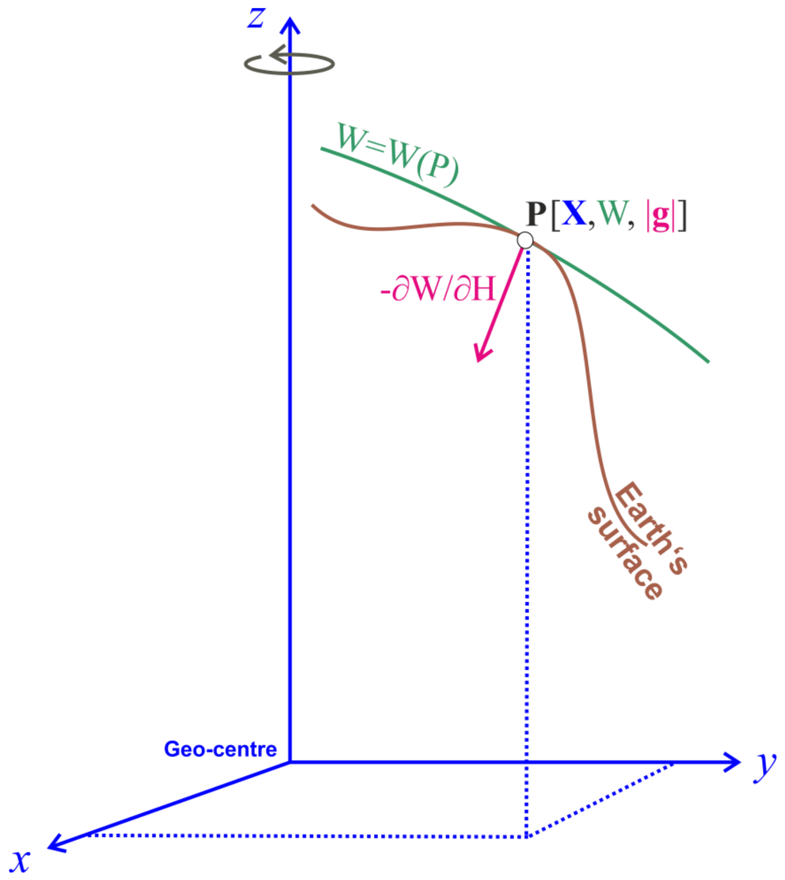
See: IAG (2017) *Description of the Global Geodetic Reference Frame*. Position paper adopted by the IAG Executive Committee in April 2016. J Geod 91,113–116, https://doi.org/10.1007/s00190-016-0994-6.

IAG Resolutions 2015

The establishment of a GGRF demands the implementation of a worldwide-unified (standardised) physical reference system.

A first concrete step oriented to this purpose was the release of two IAG resolutions during the IUGG2015 General Assembly (Prague, July 2015):

- one for the definition and realisation of an International Height Reference System (IHRS), and
- the second one for the establishment of an International Gravity Reference System (IGRS) based on absolute gravity measurements (as replacement of the IGSN71).



→ For the IAG resolutions, see Drewes et al. (2016), *The Geodesist's Handbook 2016*, J Geod, https://doi.org/10.1007/s00190-016-0948-z

→ This presentation concentrates on the IHRS. For more details on the IGRS, please refer to Wziontek et al. (2021), *Status of the International Gravity Reference System and Frame*, J Geod 95, 7, https://doi.org/10.1007/s00190-020-01438-9

Definition of the International Height Reference System (IHRS)

IAG Resolution No. 1, Prague, July 2015

1) Vertical coordinates are potential differences with respect to a conventionally fixed W_0 value:

$$C_P = C(P) = W_0 - W(P) = -\Delta W(P)$$

$$W_0 = const. = 62\,636\,853.4 \text{ m}^2\text{s}^{-2}$$

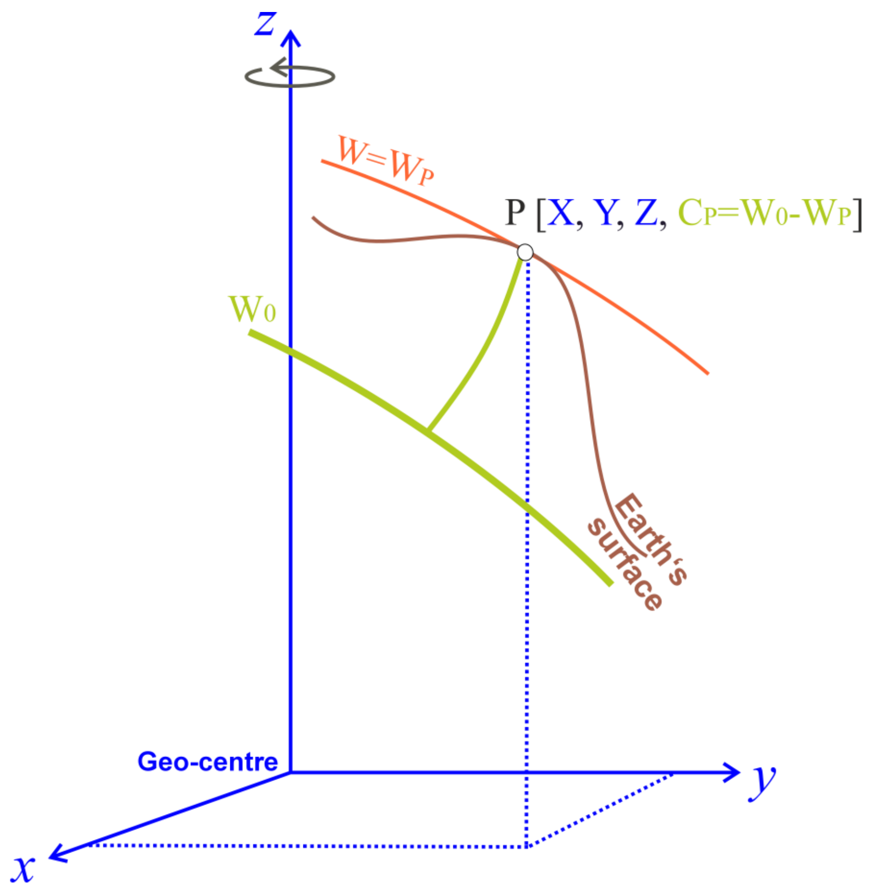
2) The position P is given in the ITRF

$\mathbf{X}_P\ (X_P, Y_P, Z_P)$; i.e., $W(P) = W(\mathbf{X}_P)$

3) The estimation of $\mathbf{X}(P)$, $W(P)$ (or $C(P)$) includes their variation with time; i.e., $\dot{\mathbf{X}}(P)$, $\dot{W}(P)$ (or $\dot{C}(P)$).

4) Coordinates are given in mean-tide system / mean (zero) crust.

5) The unit of length is the meter and the unit of time is the second (SI).



→ Ihde et al. (2017), *Definition and proposed realization of the International Height Reference System (IHRS)*. Surv Geophy 38(3), 549-570, https://doi.org/10.1007/s10712-017-9409-3

→ Sánchez et al. (2016), *A conventional value for the geoid reference potential W_0*, J Geod, 90(9): 815-835, https://doi.org/10.1007/s00190-016-0913-x,

Realisation of the IHRS

A reference frame realises a reference system in two ways:

- physically, by a solid materialisation of points (or observing instruments),
- mathematically, by the determination of coordinates referring to that reference system. The coordinates of the points are computed from the measurements following the definition of the reference system.

The implementation of the IHRS mainly requires:

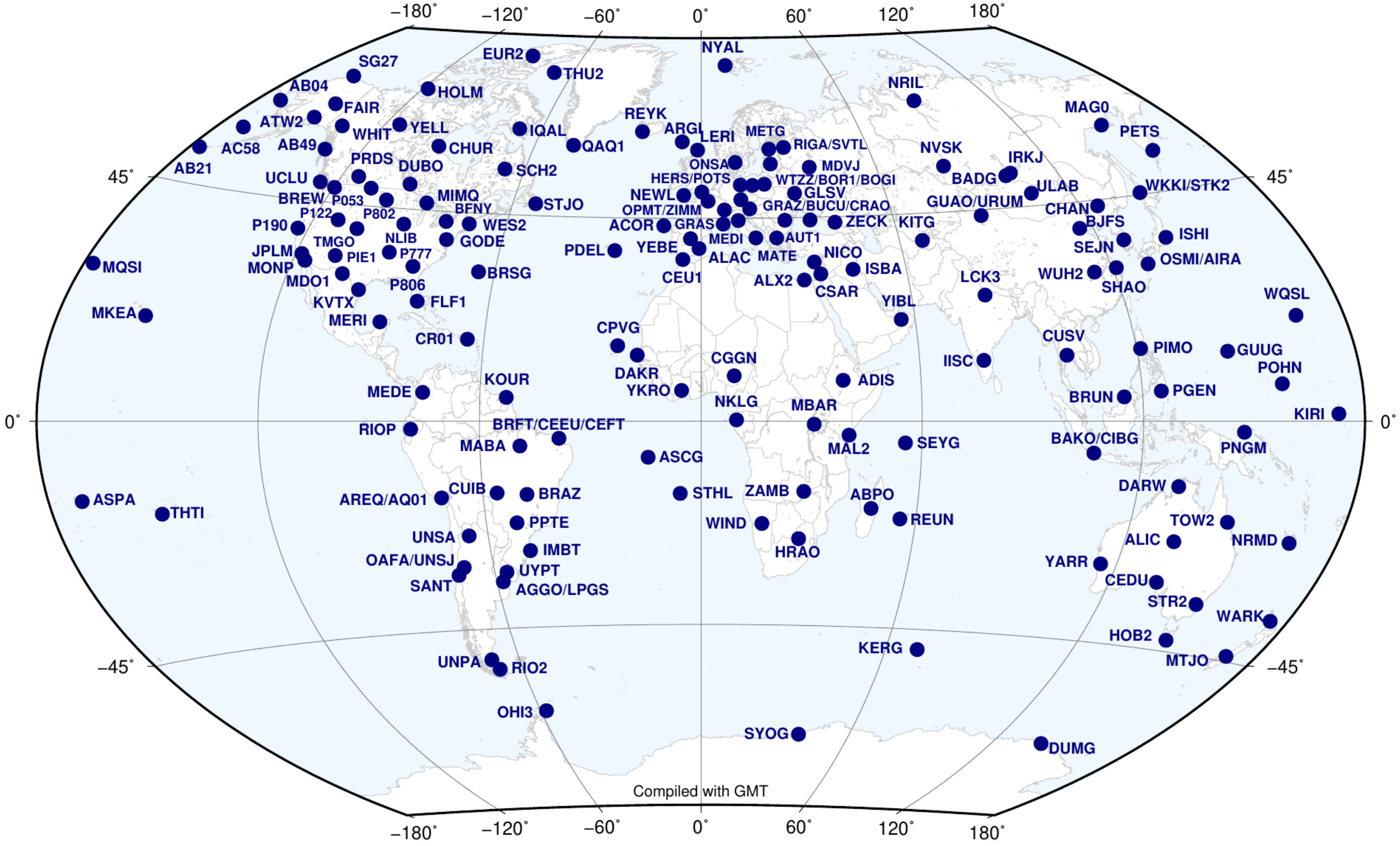
1) A global reference network for the IHRS realisation: the International Height Reference Frame (IHRF)
2) The determination of reference IHRF coordinates (W, $\dot{W}$, $\mathbf{X}$, $\dot{\mathbf{X}}$) at the reference stations
3) Clear standards, conventions and procedures to ensure consistency between the definition (IHRS) and the realisation (IHRF)
4) Operational and organisational infrastructures (reference stations, data centres, analysis centres, combination centres, product centres, etc.) to guarantee maintenance and availability of the IHRF.

Criteria for the IHRF reference network configuration

1) Hierarchy:
   - A global network → worldwide distribution, including
   - A core network → to ensure sustainability and long term stability
   - Regional and national densifications → local accessibility
2) Collocated with:
   - fundamental geodetic observatories → connection between $\mathbf{X}$, W, $\mathbf{g}$ and time realisation (reference clocks);
   - continuously operating reference stations → to detect deformations of the reference frame (preference for ITRF and regional reference stations, like SIRGAS, EPN, APREF, etc.);
   - reference tide gauges and national vertical networks → to facilitate the vertical datum unification;
   - reference stations of the new International Terrestrial Gravity Reference Frame - ITGRF → to integrate the gravity and physical height reference frames.
3) Main requirement: availability of surface gravity data around the IHRF reference stations for high-resolution gravity field modelling (i.e., precise estimation of W).

First proposal for the IHRF reference network (~170 stations)

- Station selection coordinated by the GGOS-FA Unified Height System in agreement with the GGOS Bureau of Networks and Observations, the Bureau Gravimétrique International (absolute gravity stations), as well as with the IAG regional sub-commissions for reference frames and gravity field modelling.
- A living network: new stations and decommission of stations.
- To be extended by regional/national densifications.



Basic considerations on the IHRS/IHRF coordinates

1) The IHRS/IHRF is based on the combination of
   - a geometric component given by the coordinate vector $\mathbf{X}$ in the ITRS/ITRF and
   - a physical component given by the determination of potential values W at $\mathbf{X}$.
2) The determination of $\mathbf{X}$ follows the IERS Conventions and will not be further considered here.
3) To be in agreement with the reliability of the ITRF, the expected accuracy of W is
   - Positions: $\approx \pm 3 \times 10^{-2}\ m^2 s^{-2}$ (about 3 mm)
   - Velocities: $\approx \pm 3 \times 10^{-3}\ m^2 s^{-2}/a$ (about 0.3 mm/a)
4) For the moment, our goal is $\pm 1 \times 10^{-1}\ m^2 s^{-2}$ (about 1 cm)
5) The IHRS/IHRF coordinates include the determination/modelling of time variations. For the moment, we consider static coordinates only.

Approaches for the determination of IHRF potential coordinates

1) Global gravity models of high resolution (GGM-HR), including topography-based synthetic gravity signals (like the model XGM2019e (Zingerle et al., 2019), or the combination of GGMs of degree > 2156 with topography potential models like Earth2014 (Rexer et al., 2016), ERTM2160 (Hirt et al., 2014), etc.)

2) Precise regional gravity field modelling (methods for the geoid/quasi-geoid determination)

$$W(P) = U(P) + \zeta(P) \cdot \gamma_Q + \Delta W_0 \quad [\mathrm{m^2 s^{-2}}] \;\rightarrow\; W(P) = W_0 - \big(h(P) - \zeta(P)\big) \cdot \bar{\gamma}_{QQ_0} \quad [\mathrm{m^2 s^{-2}}]$$

or

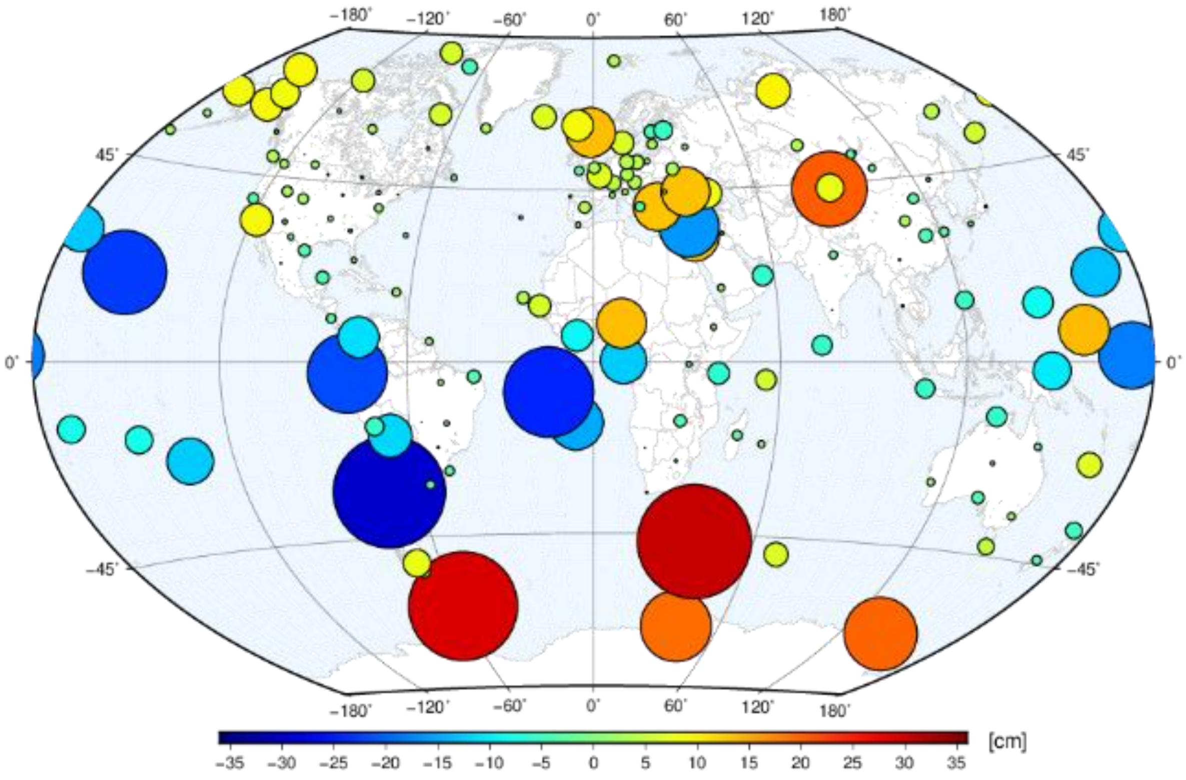
$$W(P) = W_0 - \big(h(P) - N(P)\big) \cdot \bar{g}\,(P) \quad [\mathrm{m^2 s^{-2}}] \text{ with}$$

$$\bar{g}(P) = g(P) + 0.424 \times 10^{-6} \cdot \big(h(P) - N(P)\big) + TC(P) \quad [\mathrm{ms^{-2}}]$$

3) Vertical datum unification of the local height systems into the IHRF (see Sánchez and Sideris (2017), https://doi.org/10.1093/gji/ggx025)

$$W(P) = \big(W_0^{local} + \delta W\big) - C_P^{local} \quad \text{with} \quad \delta W = W_0^{IHRF} - W_0^{local}$$

IHRF potential coordinates based on global gravity models



Mean: 0.5 cm, STD: ± 8.2 cm, Min: −30.5 cm, Max: 31.0 cm

See Sánchez et al. (2021), *Strategy for the realisation of the IHRS*, J Geod 95, 33 (2021). https://doi.org/10.1007/s00190-021-01481-0

- Comparison of normal heights inferred from XGM2019e (degree 5540) with the mean value of the normal heights obtained from EIGEN-6C4 (degree 2190; Förste et al., 2015), GECO (degree 2190; Gilardoni et al., 2016), and SGG-UGM-1 (degree 2159; Liang et al., 2018).
- These differences are mainly attributable to the contribution of surface gravity data posterior to EGM2008 (Pavlis et al., 2012) and the gravity synthetic effects inferred from the Earth2014 topography model.
- To evaluate the reliability of these values, independent data (e.g., levelling+gravimetry) are needed.
- New and better surface gravity data distribution and quality (as in preparation for the EGM2020) will strongly improve the GGM-based estimates.

IHRF potential coordinates based on regional gravity field modelling

- GGMs based on SLR, GRACE and GOCE are very precise (±1 ... ±2 cm @ 100 km)
- Mean omission error globally: ≈ ±45 cm
- Goal is to reduce these ±45 cm to ±1 cm by solving the Geodetic Boundary Value Problem (GBVP) with the combination of a GGM + surface gravity data + topography effects

$$W_P = U_P + T_P \qquad T_P = T_{P,satellite-only} + T_{P,residual} + T_{P,terrain}$$

- The determination of T_P demands a series of approximations, which influence the results; i.e., different methodologies produce different potential values
- A "centralised" computation (like in the ITRF) is quite complicated due to the restricted accessibility to surface gravity data. So, regional/national experts have to be involved in the determination of the potential coordinates in their regions/countries
- A "standard" computation procedure may be not appropriate as
  - different data availability and different data quality exist around the world
  - regions with different characteristics require particular approaches (e.g. modification of kernel functions, size of integration caps, geophysical reductions like GIA, etc.).

Colorado experiment: summary of approaches and models

- GGMs: GOCO05s, XGM2016, XGM2018, xGEOID17B, EIGEN-6C4
- Topographic effects based on SRTM V4.1, EARTH2014, ERTM2160

A

Solutions based on the quasi-geoid computation using FFT integration and a Wong-Gore modification of the integral kernel

(1) Curtin University

(2) DTU

(3) National Geodetic Survey

(4)

B

Solutions based on the quasi-geoid computation with a least-squares modification of Stokes' formula with additive corrections (LSMSA)

(5) CASM

(6) İstanbul Teknik Üniversitesi 1773

(7) LANTMÄTERIET

(8) VÚGTK

C

Solutions based on the quasi-geoid computation using spherical radial basis functions (9) and least-squares collocation (10, 11)

(9) TUM (DGFI)

(10) TUM (IAPG)

(11) POLITECNICO MILANO 1863

D

Solutions based on the geoid computation using the Helmert-Stokes (H-S) scheme and then converted to the quasi-geoid

(12)

(13) Natural Resources Canada

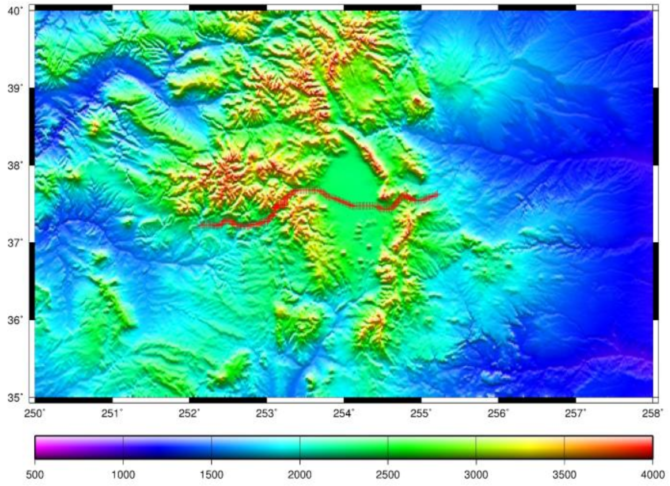
Colorado experiment: comparison of potential values

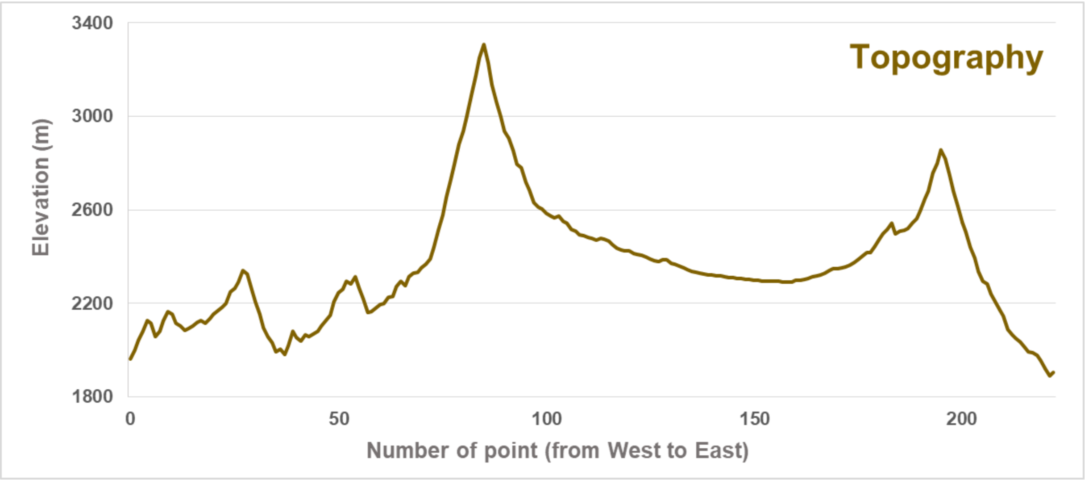
1) The potential values are inferred from the height anomalies computed by each group

$$W(P) = U(P) + \zeta(P) \cdot \gamma_Q + \Delta W_0 \quad [\mathrm{m^2 s^{-2}}] \;\rightarrow\; W(P) = W_0 - \big(h(P) - \zeta(P)\big) \cdot \bar{\gamma}_{QQ_0} \quad [\mathrm{m^2 s^{-2}}]$$

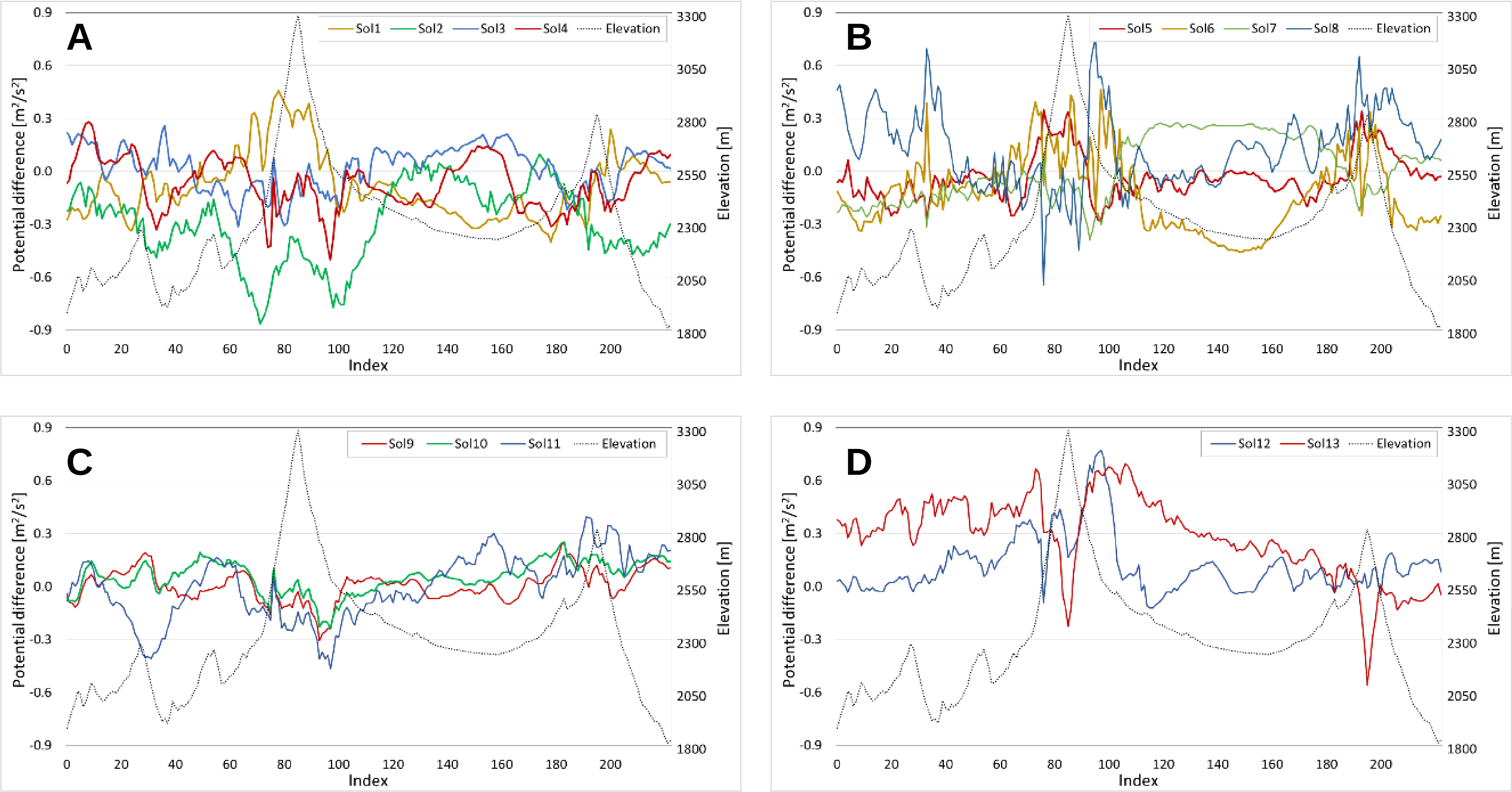
2) The potential values are converted to geopotential numbers with respect to the IHRS W_0 value, $C(P) = W_0 - W(P)$

3) and are compared with each other (to evaluate the consistency between methods) and with the potential differences inferred from levelling + gravimetry (to evaluate the reliability of the results)





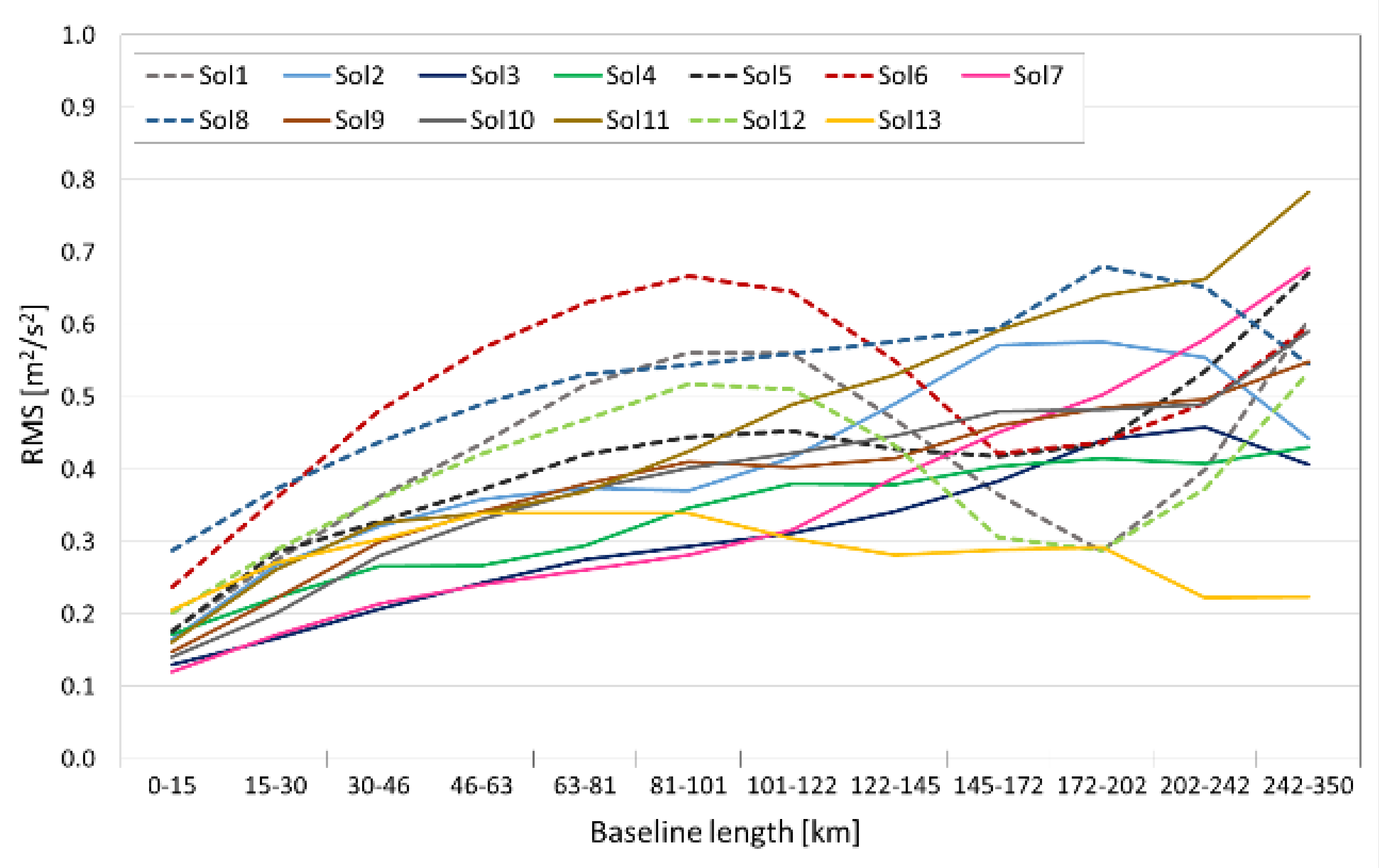
Colorado experiment: comparison of potential values wrt mean value (model − mean)



- Agreement within ±0.09 m^2s^{-2} (±0.9 cm) and ±0.23 m^2s^{-2} (±2.3 cm) in terms of STD wrt mean value.
- The overall differences range from −0.86 m^2s^{-2} (−8.8 cm) to +0.77 m^2s^{-2} (+7.9 cm).
- Discrepancies present a strong correlation with the topography.

See Sánchez et al. (2021), *Strategy for the realisation of the IHRS*, J Geod 95, 33 (2021). https://doi.org/10.1007/s00190-021-01481-0

Colorado experiment: comparison of potential values with levelling (+ gravimetry)



The RMS values of the ΔC_{ij} differences for each interval indicates the consistency between the model-based and levelling-based potential values as a function of the distance.

See Sánchez et al. (2021), *Strategy for the realisation of the IHRS*, J Geod 95, 33 (2021). https://doi.org/10.1007/s00190-021-01481-0

Learnings from the Colorado experiment

The determination of potential values may be classified in three main scenarios:

a) Regions without (or with very few) surface gravity data,

- The only option to determine potential values is the use of GGM-HRs
- Expected mean accuracy values around the $\pm 4.0\ m^2s^{-2}$ ($\pm$40.0 cm) level or even worse in regions with strong topography gradients
- It could be improved for instance to the $\pm 1.0\ m^2s^{-2}$ ($\pm$10.0 cm) level if new and better surface gravity data are included in the GGMs.
- To avoid multiple potential values provided by different GGM-HRs at the same point, it is necessary to select one GGM-HR as reference model.

b) Regions with some surface gravity data, but with poor data coverage or unknown data quality,

- The reliability of the existing (quasi-)geoid models is poor
- Additional gravity surveys around the IHRF stations to increase the accuracy of the geopotential numbers computed at those specific stations.

c) Regions with good surface gravity data coverage and quality.

- Potential values may be inferred from precise (quasi-)geoid regional models

First solution for the IHRF

Computation of a first static solution for the IHRF based on

- the existing regional/national geoid/quasi-geoid models
- GGM-HR + topographic gravity signals

Regional/national geoid/quasi-geoid models play a main role in the realisation of the IHRF because

- they include surface gravity data that are not always available for the determination of GGM-HR,
- they can assimilate new regional/local gravity surveys very quickly, and
- national/regional experts on gravity field modelling have the best possible knowledge about the local conditions (topography, data distribution, geophysical corrections, validation data, etc.) to be considered in the computation of the (quasi-)geoid, or more precisely, in the determination of the disturbing potential T in their countries/regions

A detailed guideline to standardise the recovering of potential values from regional (quasi-)geoid models was prepared and distributed to about 40 colleagues worldwide.

Computation of a first solution for the IHRF

M Véronneau (NRCan)
J Huang (NRCan)
YM Wang (NGS/NOAA)
K Ahlgren (NGS/NOAA)
D Avalos-Naranjo (INEGI)

H Denker (UniHannover)
J Schwabe (BKG)
R Barzaghi (PoLiMi)
J Ågren (U. Gävle, NGK)
H Teitsson (DTU)

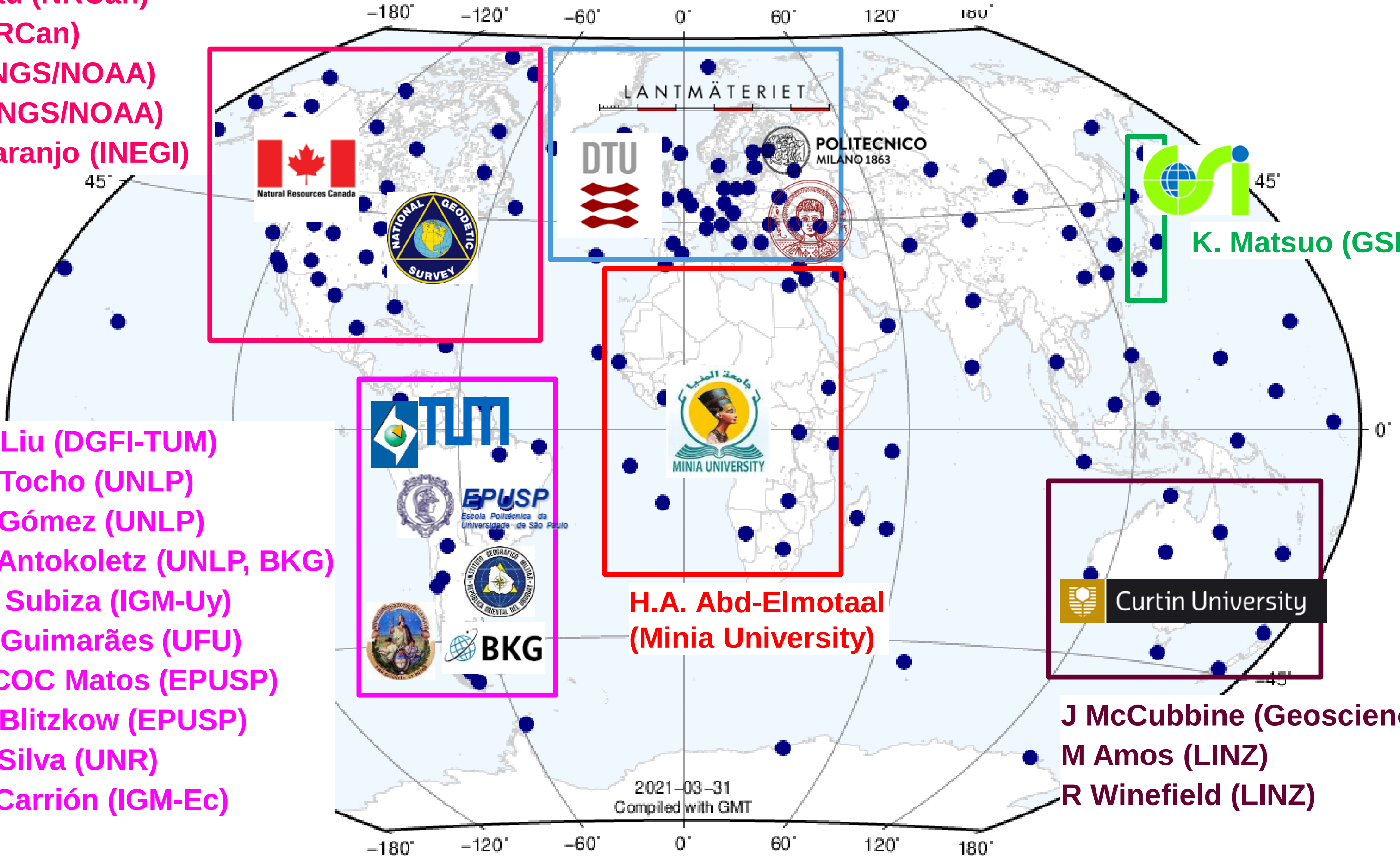
M Bilker-Koivula (NLS)
G. Vergos (AUTH)
U Martí (swisstopo)
C Ullrich (BEV)

K. Matsuo (GSI)

Q Liu (DGFI-TUM)
C Tocho (UNLP)
A Gómez (UNLP)
E Antokoletz (UNLP, BKG)
W Subiza (IGM-Uy)
G Guimarães (UFU)
ACOC Matos (EPUSP)
D Blitzkow (EPUSP)
V Silva (UNR)
J Carrión (IGM-Ec)

H.A. Abd-Elmotaal
(Minia University)

J McCubbine (Geoscience Australia)
M Amos (LINZ)
R Winefield (LINZ)



The rest of the world with the latest GGM + topography signals from Earth2014 and ERTM2160

Computation of a first solution for the IHRF

Input data: Disturbing potential computed for the local/regional (quasi-)geoid models, without fitting to GNSS/Levelling data

$$W = U + T \quad \rightarrow \quad \Delta g = \delta g + \frac{1}{\gamma}\frac{\partial \gamma}{\partial h}T = -\frac{\partial T}{\partial h} + \frac{1}{\gamma}\frac{\partial \gamma}{\partial h}T \quad \rightarrow \quad N \sim \zeta = \frac{T}{\gamma}$$

Quasi-geoid $W(P) = U(P) + \zeta(P)\cdot\gamma_Q + \Delta W_0 \quad [\mathrm{m^2 s^{-2}}] \rightarrow W(P) = W_0 - \big(h(P) - \zeta(P)\big)\cdot\bar{\gamma}_{QQ_0} \quad [\mathrm{m^2 s^{-2}}]$

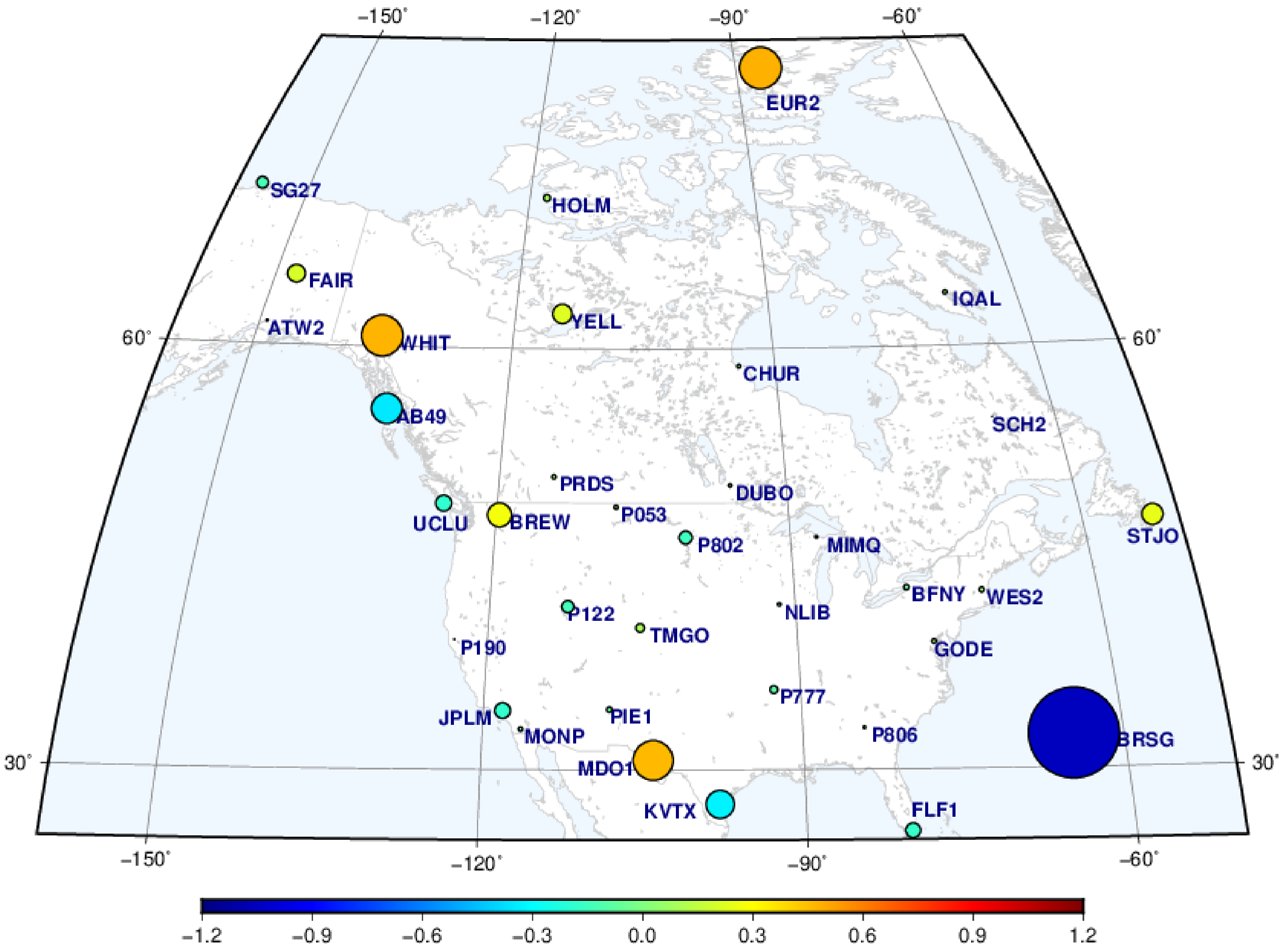
Geoid $W(P) = W_0 - \big(h(P) - N(P)\big)\cdot\bar{g}(P) \quad [\mathrm{m^2 s^{-2}}]$ with

$$\bar{g}(P) = g(P) + 0.424\times 10^{-6}\cdot\big(h(P) - N(P)\big) + TC(P) \quad [\mathrm{ms^{-2}}]$$

Results: Potential values

Station	Latitude[°]	Longitude[°]	h [m]	W[m2m-2]
ACOR 13434M001	43.36438604	-8.39892871	66.874	62636739.38
ALAC 13433M001	38.33892293	-0.48122575	60.317	62636759.06
ALX2 30102M001	31.19706855	29.91099701	57.951	62636437.34
ARGI 10117M002	61.99737250	-6.78352091	110.239	62636330.26
AUT1 12619M002	40.56681956	23.00372276	150.053	62635796.03
BOGI 12207M003	52.47499384	21.03521983	139.917	62635784.52
BOR1 12205M002	52.27695884	17.07346111	124.355	62635983.84
BUCU 11401M001	44.46394666	26.12574534	143.220	62635799.80
CEU1 13449M002	35.89197482	-5.30638778	52.435	62636758.45

Computation of a first solution for the IHRF



Differences between the potential values inferred from the Canadian geoid model PCGG20_21A and the US quasi-geoid model xG20B **(thanks to M Véronneau, J Huang, YM Wang and K Ahlgren)**:

Mean: -0.01 m^2s^{-2} (~ -1 cm)
STD: 0.26 m^2s^{-2} (~ 3 cm)
Min.: -1.05 m^2s^{-2} (~ -10 cm)
Max.: 0.48 m^2s^{-2} (~ 5 cm)

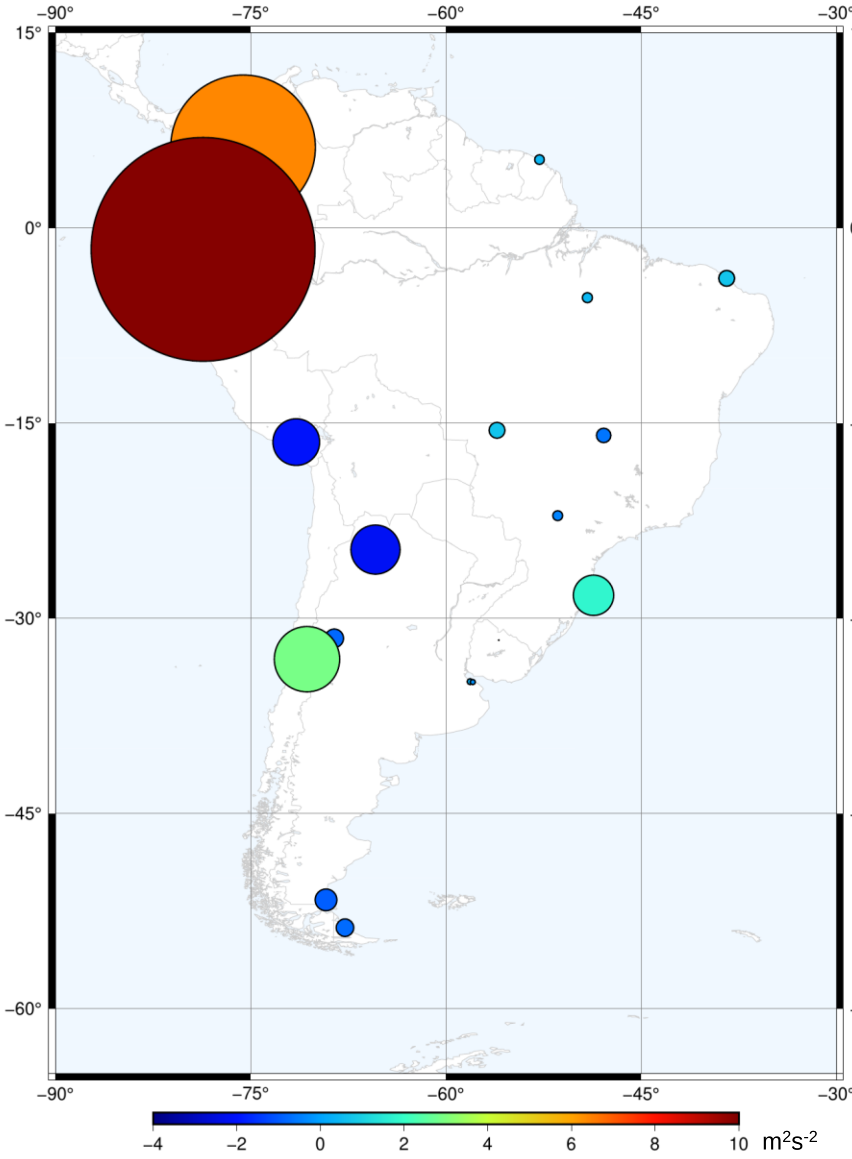
Computation of a first solution for the IHRF

EGG2016	GCG2016	Difference
18032.74	18032.81	0.07
15946.68	15946.73	0.05
15381.77	15381.78	0.02
1019.01	1018.97	-0.04
16690.78	16690.73	-0.04
6070.26	6070.18	-0.08

IHRF geopotential numbers inferred from the European quasi-geoid model EGG2016 and the German quasi-geoid model GCG2016 **(thanks to H Denker and J Schwabe)**

Differences (@ 6 points)
Mean: -0.006 m^2s^{-2} (~ 0 cm)
STD: 0.050 m^2s^{-2} (~ 0.5 cm)
Min.: -0.080 m^2s^{-2} (~ -0.8 cm)
Max.: 0.065 m^2s^{-2} (~ 0.7 cm)

Computation of a first solution for the IHRF



Differences between the geopotential numbers inferred from the South American geoid models GEOIDE2021 and GEOIDE2023 **(thanks to ACOC Matos, D Blitzkow, G Guimarães)**

Differences @ 20 points (@ 18 points)

Mean:	0.661 m^2s^{-2}	(-0.087 m^2s^{-2})
STD:	2.748 m^2s^{-2}	(1.238 m^2s^{-2})
Min.:	-2.175 m^2s^{-2}	(0.661 m^2s^{-2})
Max.:	9.901 m^2s^{-2}	(2.896 m^2s^{-2})

Computation of a first solution for the IHRF

US main territory (30 stations):
Model NAPGD2022 - xG20B
Mean accuracy 0.45 m^2s^{-2}

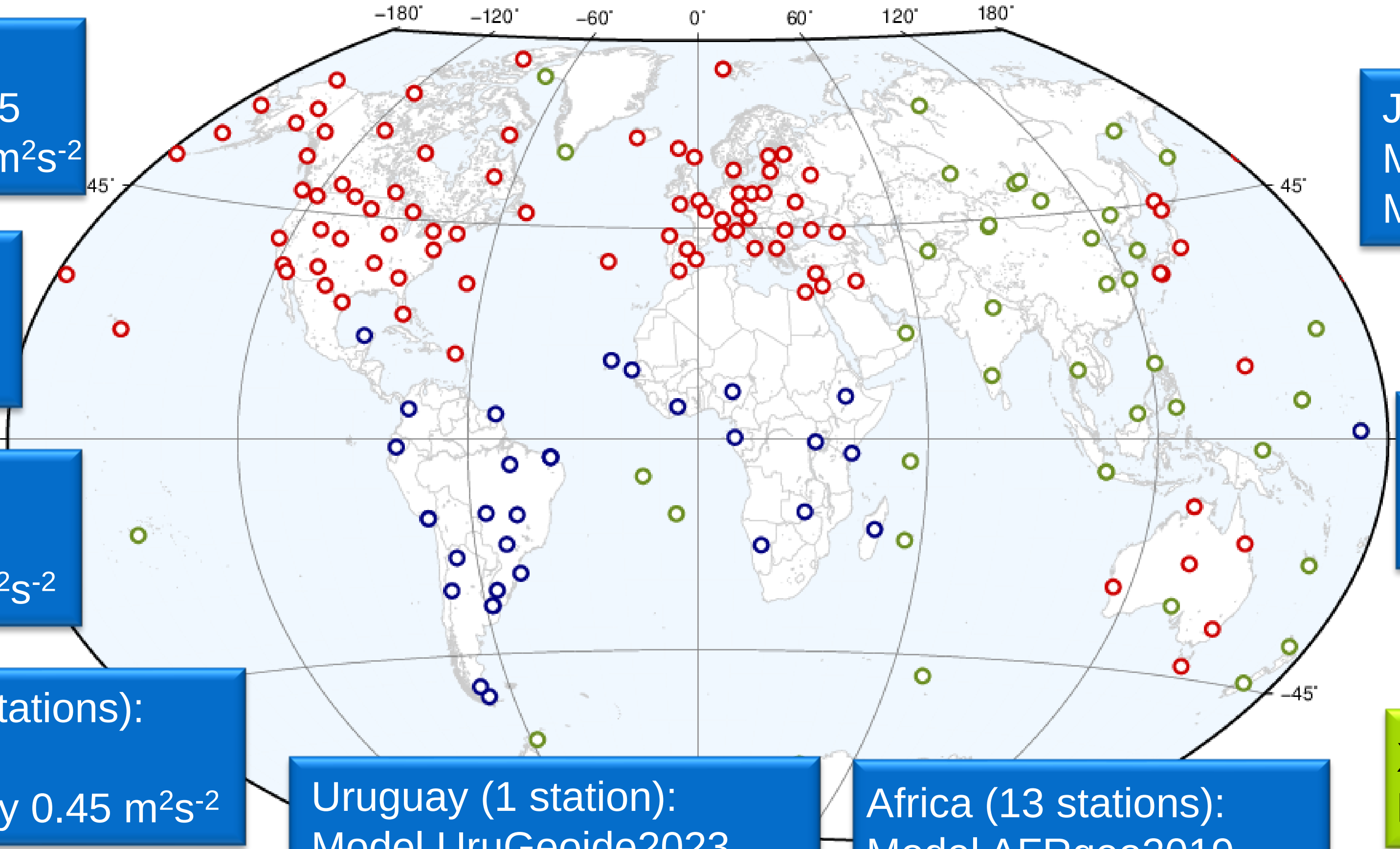
Canada (11 stations):
Model PCGG20_21A
Mean accuracy 0.35 m^2s^{-2}

Europe (40 stations):
Model EGG2016
Mean accuracy 0.50 m^2s^{-2}

Mexico (1 station):
Model GGM-CA 2015
Mean accuracy 2.0 m^2s^{-2}

Japan (5 stations):
Model JGEOID2019
Mean accuracy 0.57 m^2s^{-2}

Colombia (1 station):
Model QGeoidCOL2023
Mean accuracy 1.57 m^2s^{-2}



Australia (6 stations):
Model AGQG2017
Mean accuracy 0.62 m^2s^{-2}

Brazil (6 stations):
Model GEOIDE2023
Mean accuracy 0.50 m^2s^{-2}

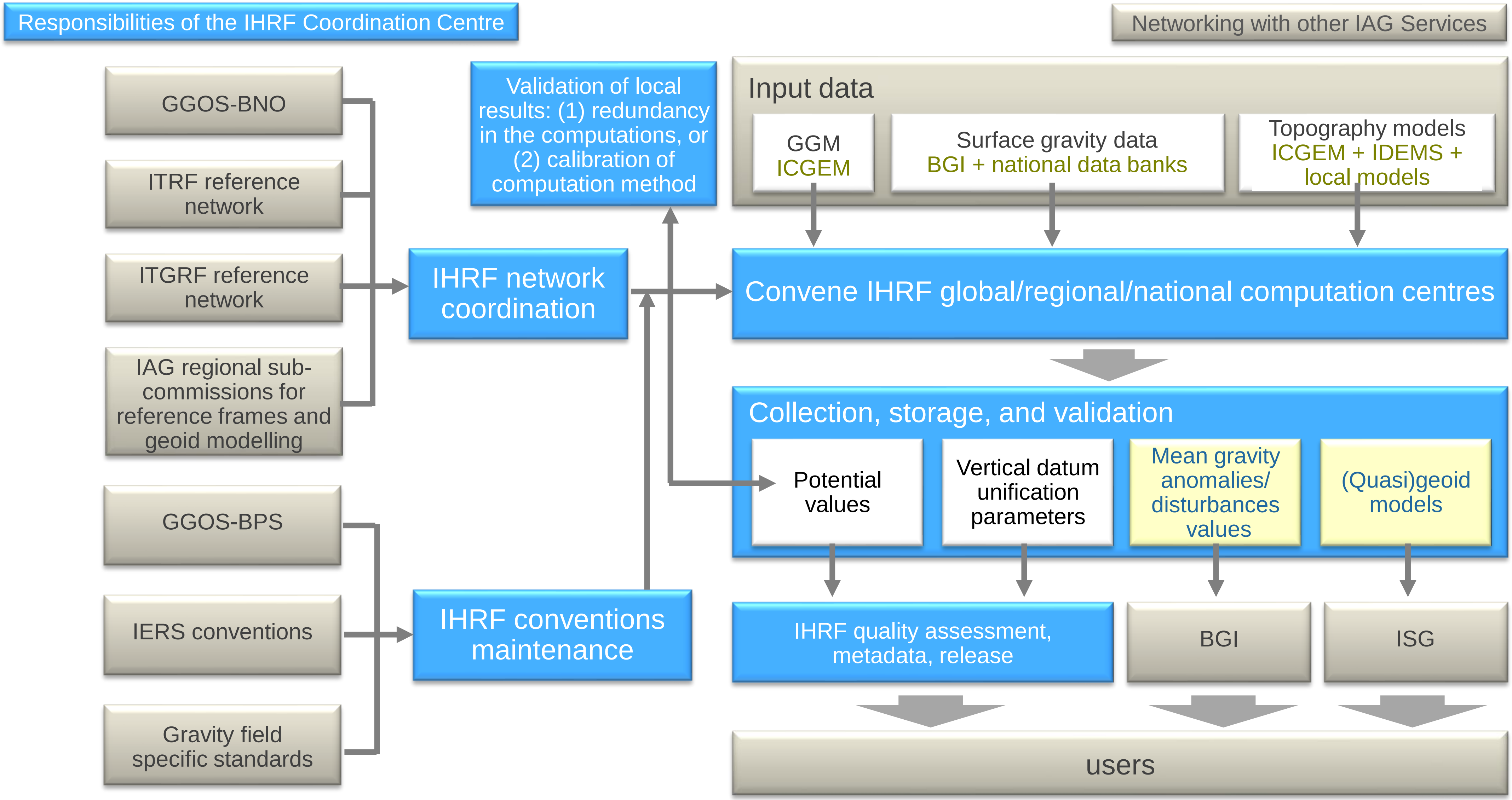
Argentina (5 stations):
Point by point
Mean accuracy 0.45 m^2s^{-2}

Uruguay (1 station):
Model UruGeoide2023
Mean accuracy 0.35 m^2s^{-2}

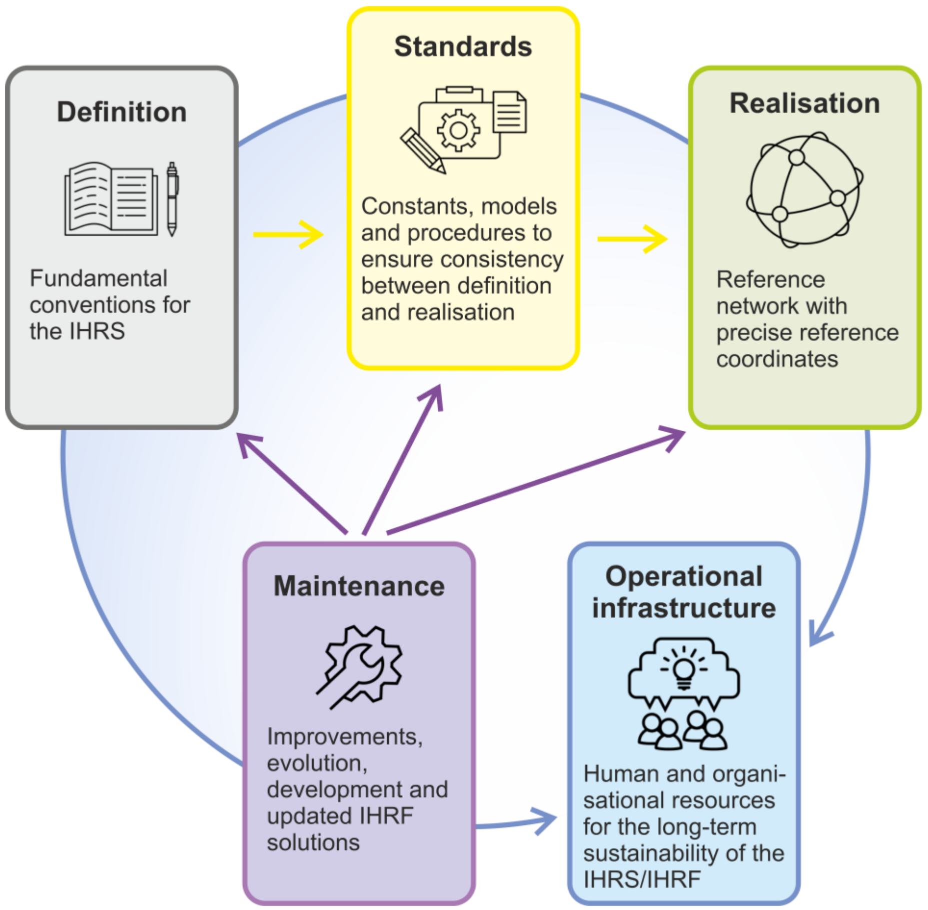
Africa (13 stations):
Model AFRgeo2019
Mean accuracy 2.0m^2s^{-2}

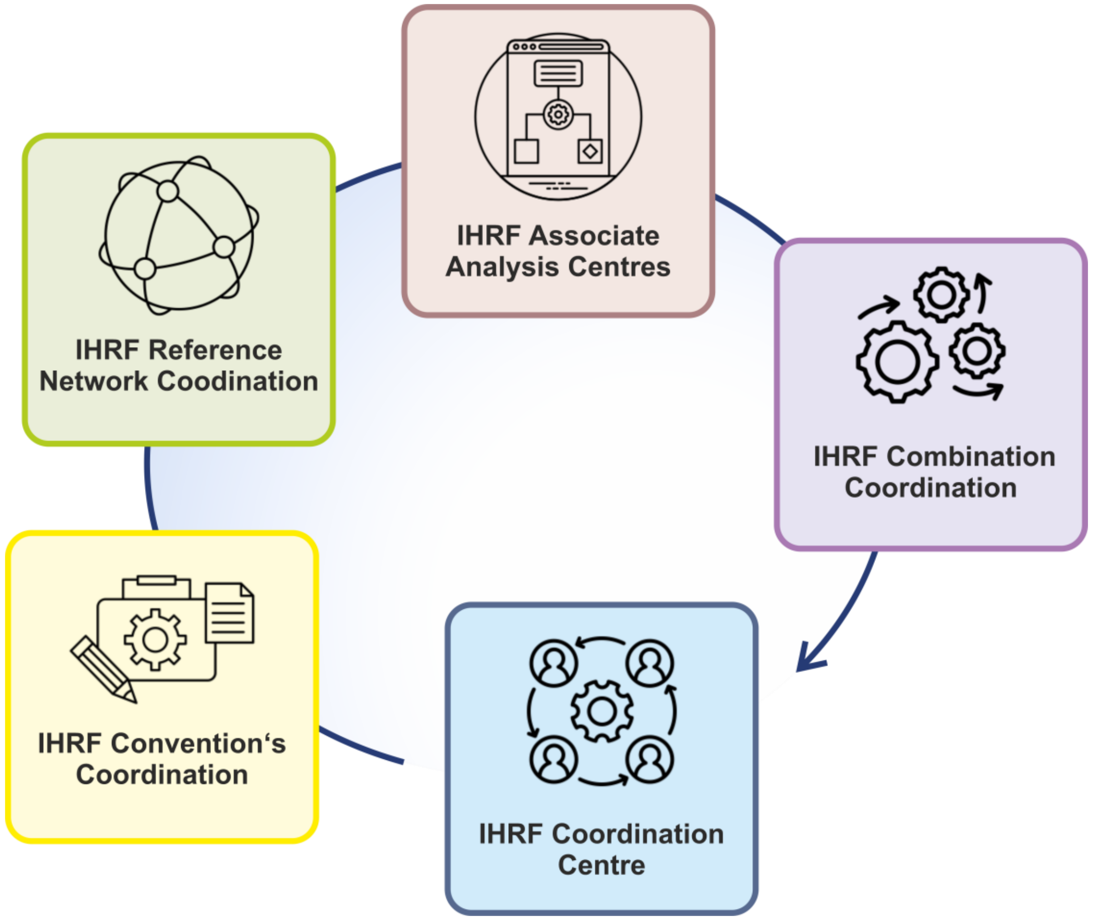
XGM2019
Mean accuracy 4.0m^2s^{-2}

Ensuring a reliable and long-term sustainable realisation of the IHRS



IHRF Coordination Centre of the International Gravity Field Service (IGFS), https://ihrfcc.topo.auth.gr/





Summary

1) The first solution for the IHRF is almost concluded, it contains geopotential numbers with accuracy varying from about 3 cm to 1 m. A detailed computation report with accuracy analysis is in preparation.

2) The determination of the gravity potential $W = U + T$ (thus, geoid or height anomaly) is the core element for the establishment of the IHRS/IHRF or any geopotential/geoid/quasi-geoid-based height system. The IHRF and those systems are as accurate as the geoid or the height anomaly.

3) The comparison of different computation strategies (the Colorado experiment) proves that we can reach an agreement of about 0.2 m^2s^{-2} (~ 2 cm). However, this depends on the availability of surface gravity data. When no gravity data is available, the uncertainty may reach 10 m^2s^{-2} (~ 1 m).

4) Surface gravity data (terrestrial, airborne) acquisition and the precise determination of the (quasi-)geoid belong to the primary geodetic infrastructure. They are the counterpart of the geometric reference frame. Both IHRF and ITRF have to be consistent, be correspondingly developed and provide similar accuracy.

Summary

5) Quality assessment of the W values is very difficult because a proper error propagation and redundancy are practically impossible. External validation data (i.e. GNSS/levelling) are required and sometimes their quality is very poor. In the meantime,
   - → Redundancy is required: two calculations using the same input data and different software, or
   - → Evaluation of the computation procedures used by national/regional experts: determination of potential values using a certain set of input data and comparison with results obtained by other approaches (like in the Colorado experiment)

6) Unresolved challenges (not only in the IHRS/IHRF but also in any geopotential-based height system):
   - to quantify how much of the discrepancies between different computations is related to the treatment of the data (preprocessing), and how much is related to the different approaches
   - quality assessment
   - determination of geopotential changes with time.

7) Under the coordination of the International Gravity Field Service (IGFS), i.e., the IHRF Coordination Centre (IHRF-CC), regular solutions for the IHRF (synchronised with the release of new ITRF solutions) should be determined to take account for new stations, new observations, better standards and models, better computation algorithms, etc.