# Estimation of the topside ionosphere and plasmasphere by an Ensemble Kalman Filter and the simultaneous multiplicative column-normalized method SMART+

**T. Gerzen[1], M. Schmidt[1], D. Minkwitz[2]**

[1] Deutsches Geodätisches Forschungsinstitut der Technischen Universität München (DGFI-TUM), Munich, Germany

[2] Airbus Defence and Space GmbH, Willy-Messerschmitt-Str. 1, Taufkirchen, Germany

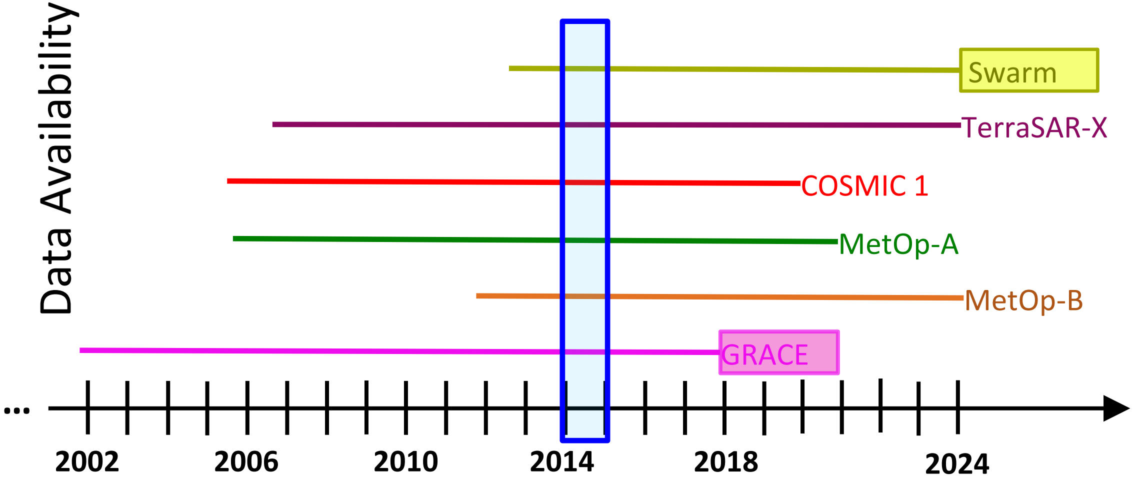
GGOS Topical Meeting on the Atmosphere
Potsdam, Germany

October 7 - 9, 2024

# Why reconstruction of the topside ionosphere and plasmasphere?

- Up to 50 % of the signal delays of L-band signals used in GNSS originate from altitudes above the ionospheric F2 layer.
- Reconstructions ...
    - combine different measurements to one product,
    - provide a contribution to the improved characterization of the topside part of the ionosphere,
    - improve understanding of the coupled system “Magnetosphere – Plasmasphere – Ionosphere”.

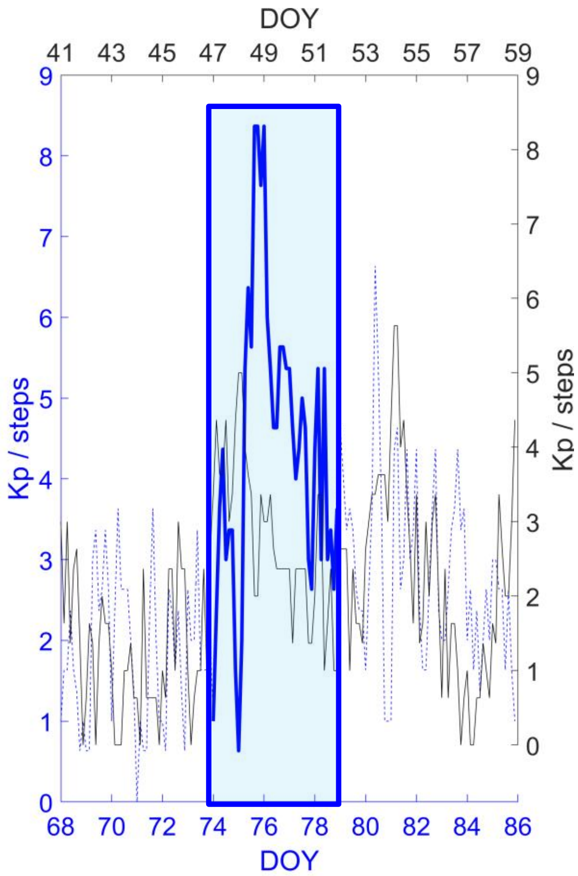
# Data base for the reconstructions: Space-based STEC



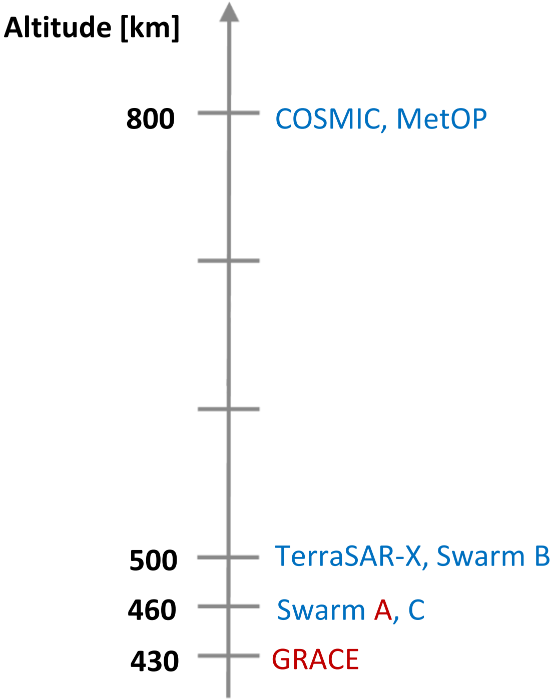

Considered periods within 2015

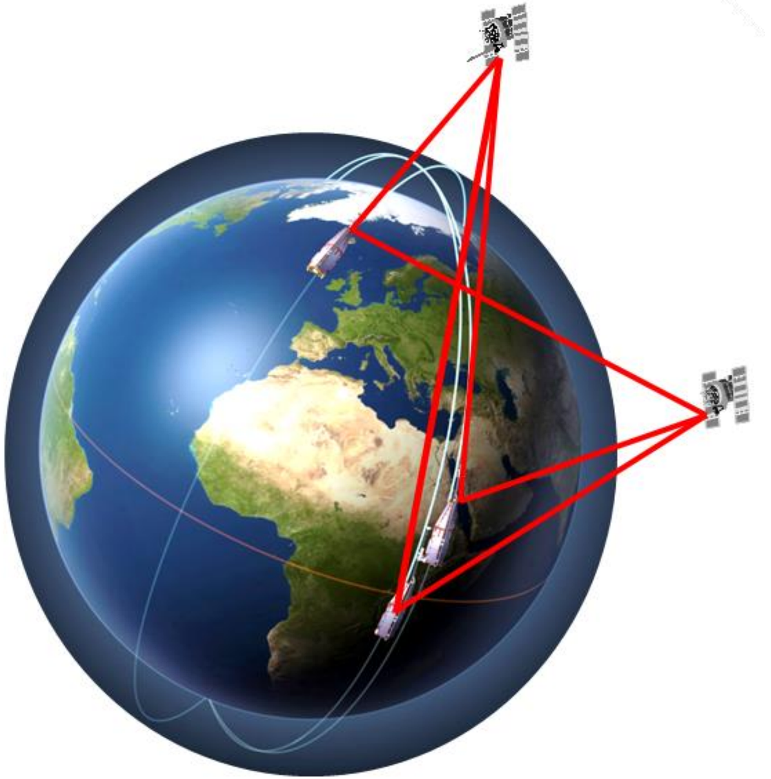
DOY 074 – 079

DOY 041 – 059



# LEO orbit altitudes 2015



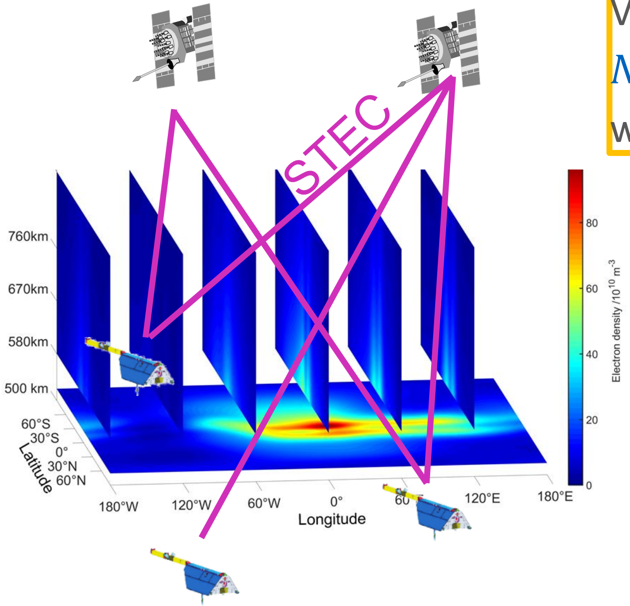



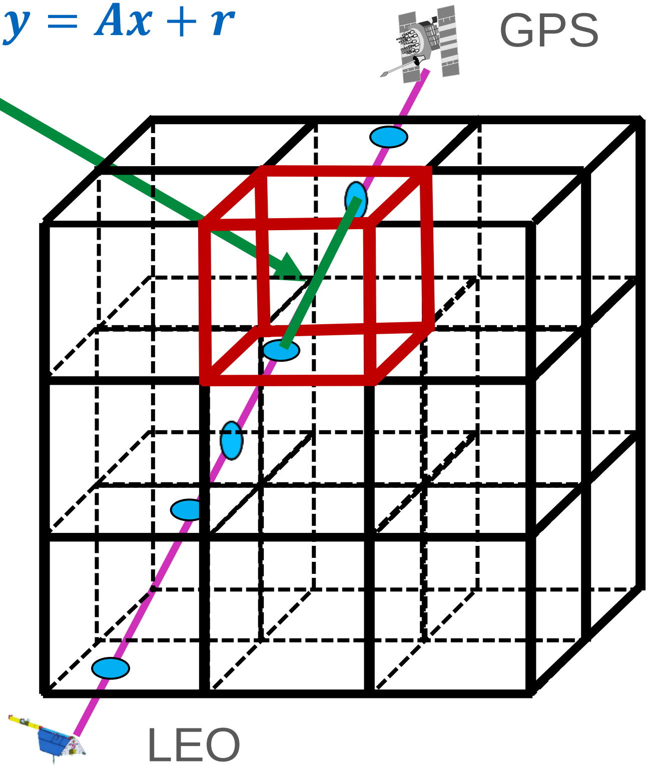
# Problem formulation

$$STEC_j = \int Ne(h,\theta,\varphi)\,dj \;\Rightarrow\; STEC_j \approx \sum_{i=1}^{n} Ne_i \cdot a_{ji} \;\Rightarrow\; \boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{r}$$

?

Voxelisation assuming $Ne(h,\theta,\varphi)$ constant within a voxel



# Reconstruction approaches

$$STEC_j = \int Ne(h,\theta,\varphi)\, dj \;\Rightarrow\; STEC_j \approx \sum_{i=1}^{n} Ne_i \cdot a_{ji} \;\Rightarrow\; \boxed{\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{r}}$$

**Reconstruction of electron densities:**

- Ensemble Kalman Filter **EnKF** [1,2]
- Simultaneous multiplicative column normalized method **SMART+** [3,4]


[1:] Gerzen et al., Adv. Space Res., https://doi.org/10.1016/j.asr.2024.08.017, 2024.

[2:] Gerzen et al., Ann. Geophys., 10.5194/angeo-38-1171-2020, 2020.

[3:] Gerzen et al., Ann. Geophys., doi:10.5194/angeo-35-203-2017, 2017.

[4:] Gerzen and Minkwitz, Ann. Geophys., doi: 10.5194/angeo-34-97-2016, 2016.

# Ensemble Kalman Filter (EnKF)

- **Generation of initial ensembles** $X^f(t_0)$ by the NeQuick model.

- **Propagation model:**

  Empirical model assuming that the electron density differences between the EnKF analysis and the background model form a first order Gauss–Markov sequence. These differences are propagated from one time step to the next by assuming persistence combined with an exponential decay:

$$X^f(t_{n+1}) - X^b(t_{n+1}) \cdot \epsilon_{1\times N} = \exp\left(-\frac{t_{n+1} - t_n}{\tau}\right) \cdot \{X^a(t_n) - X^b(t_n)\}$$

- $X^b(t)$ is the ensemble of electron density vectors calculated by NeQuick,
- $\tau$ is the temporal correlation parameter chosen here as 3 hours.

# SMART+

**… is a combination of**

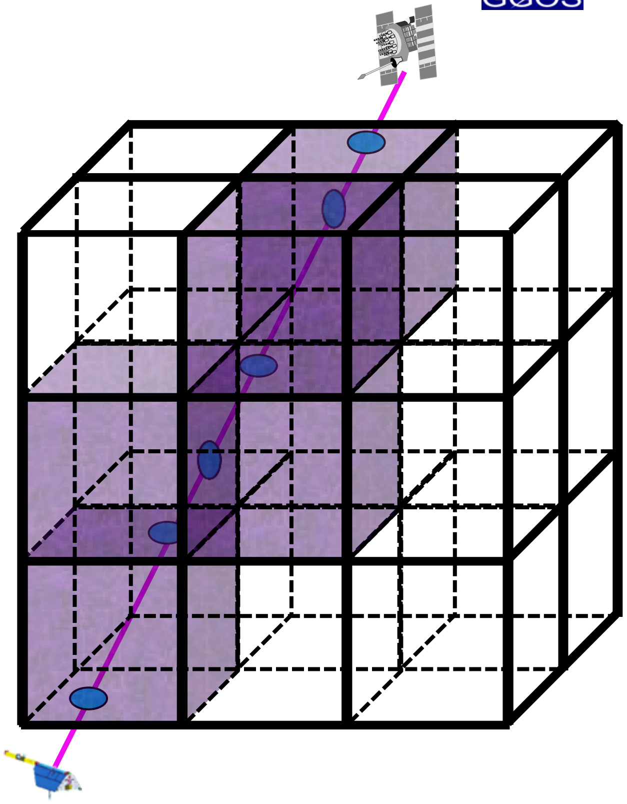
- the iterative simultaneous multiplicative column normalized method (**SMART**) and

- 3D successive correction method (**3D SCM**).

# SMART

- **First step:** iterative simultaneous multiplicative column normalized method (**SMART**):

SMART distributes the STEC measurements among the electron densities in the voxels, which are intersected by at least one measurement ray-path.

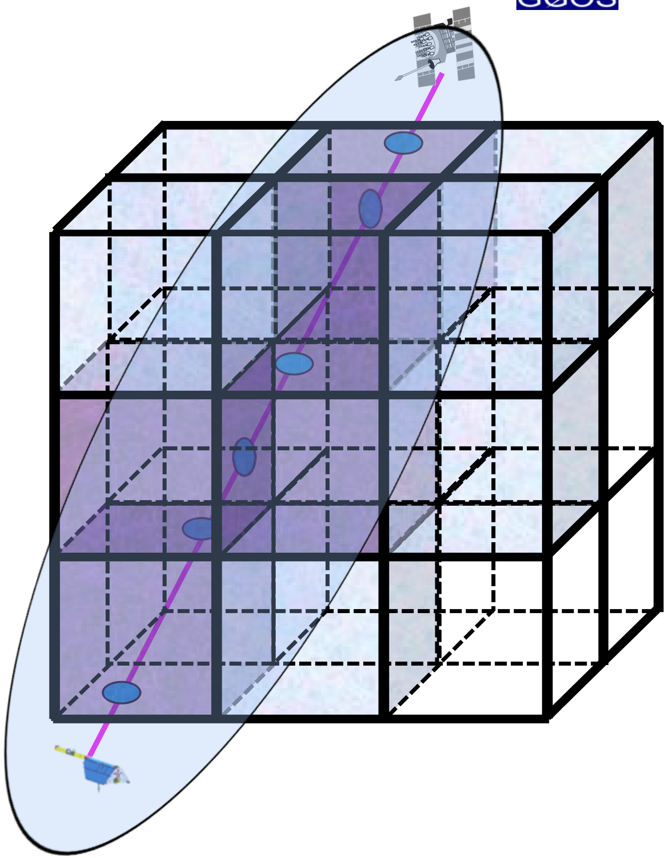


# 3D SCM

- First step: iterative simultaneous multiplicative column normalized method (SMART).

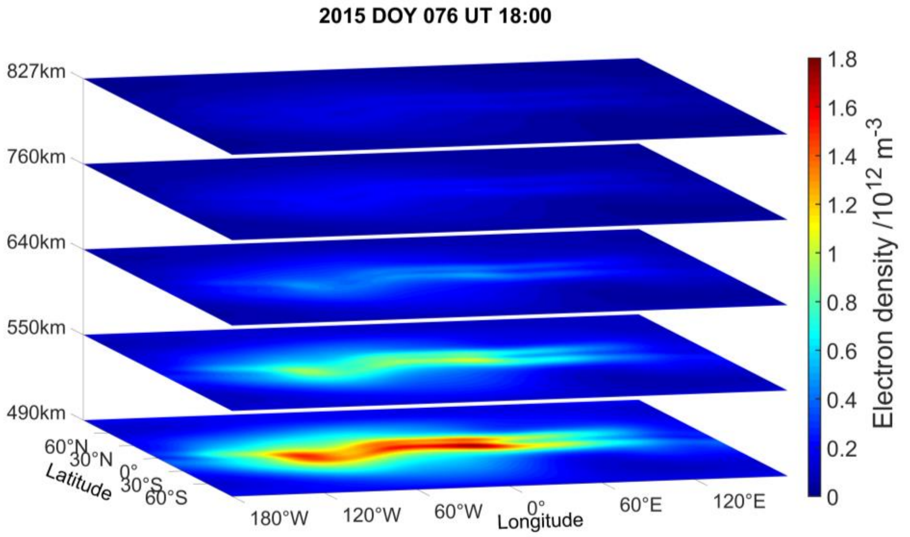
- **Second step:** 3D successive correction method (**3D SCM**):

After a predefined number of SMART iterations are performed, an extrapolation is done from intersected to not intersected voxels, assuming gaussian correlation model with correlation lengths depending on latitude, longitude and altitude.

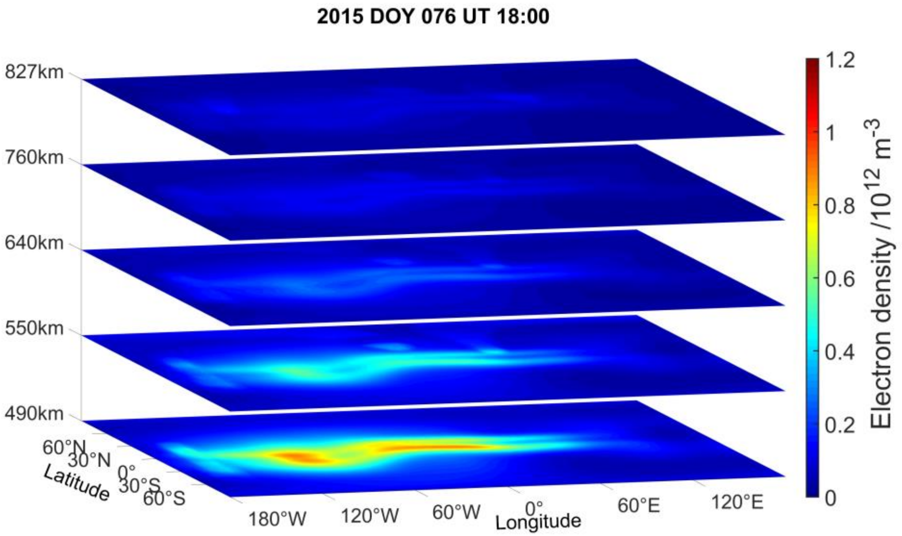


# EnKF estimation for St Patrick's Day storm
# DOY 076, 2015

## EnKF

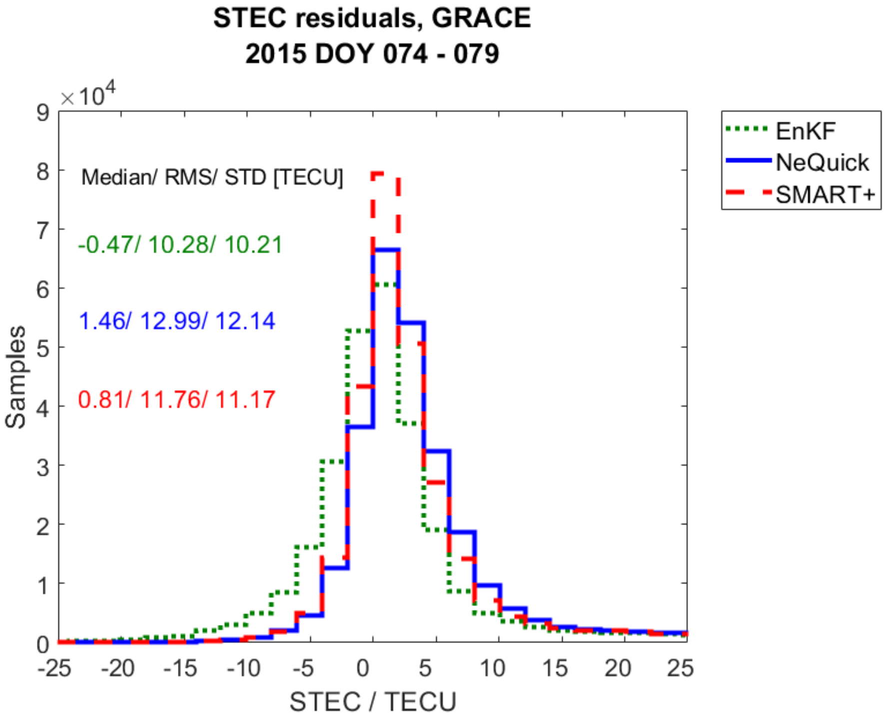



## SMART+



# Validation with independent STEC

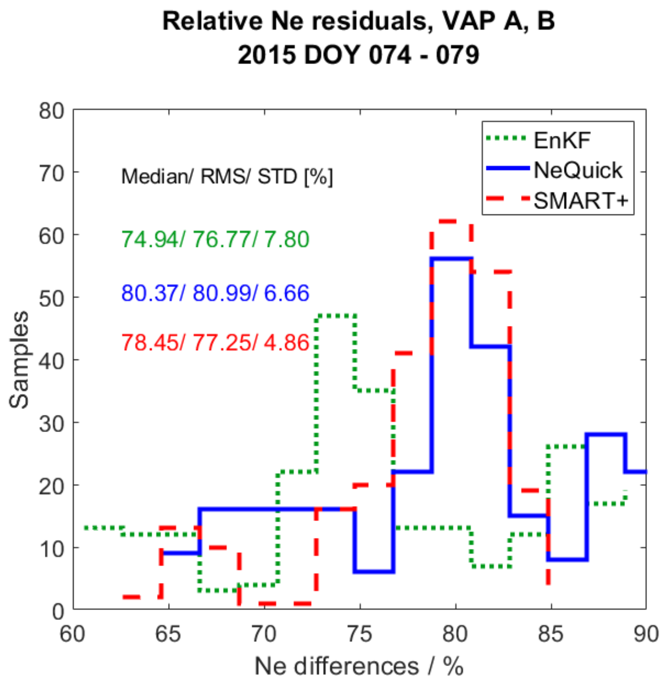
- NeQuick underestimates the measured STEC.
- Both methods reduce the median of the residuals.
- EnKF reduces the absolute value of the background median by ~68%.
- SMART+ reduces the median of the background residuals median by ~44%.




$$STEC_{measured} - STEC_{estimated}$$

# Validation with independent VAP electron density

- EnKF reduces the NeQuick median and RMS by up to ~7 %.
- SMART+ reduces the NeQuick statistics by up to ~17 %.
- However, the residuals of SMART+ and EnKF follow the behavior of NeQuick.
- We assume this is because of the relative low contribution of the low electron densities at the VAP altitudes to the LEO STEC, which represents the only data base for the reconstructions.




$$\frac{Ne_{measured} - Ne_{estimated}}{Ne_{measured}}$$

# Conclusions and next steps

- Both reconstruction approaches are able to reduce the a priori STEC and VAP residuals.
- Validation with VAP underpins the need for a more accurate modelling of the plasmasphere.
- Preconditioning of background model, in terms of F2 layer characteristics is an option to improve the reconstruction [3].

[3:] Gerzen et al., Ann. Geophys., doi:10.5194/angeo-35-203-2017, 2017.

# Thank you very much for your attention!

This study was partly performed in the MuSE project funded by the DFG (project number 273481272) as a part of the Priority Programme "DynamicEarth", SPP-1788 https://gepris.dfg.de/gepris/projekt/273481272?language=en.

**References**

1. Gerzen T., Minkwitz D., Schmidt M., Rudenke S.: Validation of the NeQuick model, Ensemble Kalman filter and SMART+ based estimations of the topside ionosphere and plasmasphere. Adv. Space Res., https://doi.org/10.1016/j.asr.2024.08.017, 2024.

2. Gerzen T., Minkwitz D., Schmidt M., Erdogan E.: Analysis of different propagation models for the estimation of the topside ionosphere and plasmasphere with an ensemble Kalman filter. Ann. Geophys., doi:10.5194/angeo-38-1171-2020, 2020.

3. Gerzen, T., V. Wilken, D. Minkwitz, M. Hoque, S. Schlüter, Three-dimensional data assimilation for ionospheric reference scenarios. Ann. Geophys., doi:10.5194/angeo-35-203-2017, 2017.

4. Gerzen, T. and D. Minkwitz, Simultaneous multiplicative column normalized method (SMART) for the 3D ionosphere tomography in comparison with other algebraic methods. Ann. Geophys., doi: 10.5194/angeo-34-97-2016, 2016.