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MULTILEVEL CROSS-ENTROPY METHOD FOR DATA-BASED STRUCTURAL MODEL UPDATING AND RELIABILITY ASSESSMENT

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Performance prediction and reliability assessment of engineering structures play an important role in optimizing investment decisions for the upkeep of the built environment. An accurate performance evaluation requires appropriate characterization of all uncertainties influencing the system response considering any available data. The uncertainty could be inherent in the external forces, e.g., load due to environmental actions like earthquake and wind, as well as the computational model representing the structure, for instance, in the parameters linked to material properties, boundary conditions, structural member geometry, and degradation processes. Ideally, these uncertain quantities are modeled probabilistically using a set of random variables, which can be learned from data through Bayesian analysis. The uncertainty in structural performance is then quantified through statistical postprocessing of the structural response variations, e.g., by estimating failure probabilities.

This contribution focuses on Monte Carlo simulation-based strategies for Bayesian inference of structural model parameters and reliability. In the Bayesian framework, the learning process is performed using Bayes' theorem which estimates the distribution of the uncertain model parameters, say $\boldsymbol{X}$, in terms of an ensemble of data sets and prior information: $p(\boldsymbol{x}|d) = p(d|\boldsymbol{x})p(\boldsymbol{x})/C$. Here, $p(\boldsymbol{x})$ is a probability density function (PDF) which denotes the prior belief of $\boldsymbol{X}$, d denotes the data, $p(\boldsymbol{x}|d)$ is the posterior PDF, C is the normalization constant of $p(\boldsymbol{x}|d)$, known as the evidence, and $p(d|\boldsymbol{x})$ is the likelihood function. The reliability of the structure is assessed by evaluating its probability of failure: $P_{F|d} = \int_{-\infty}^{\infty} I_F(\boldsymbol{x})p(\boldsymbol{x}|d)\mathrm{d}\boldsymbol{x}$, where the function $I_F(\cdot)$ is the indicator of the failure event F. Over the past decades, several specialized sampling algorithms have been proposed to estimate $p(\boldsymbol{x}|d)$ and $P_{F|d}$. Nonetheless, the development of simulation strategies that are robust and computationally efficient remains important.

We present our ongoing works on a class of multilevel Monte Carlo sampling techniques for estimating the posterior PDF and the failure probability. These methods solve the problem by means of an adaptive importance sampling strategy based on the cross-entropy method [1]. Herein, a set of tempered posteriors is introduced, which defines a smooth transition between the prior and posterior densities of the uncertain parameters [2]. An approximation to the posterior PDF is determined by fitting a sequence of parameterized PDFs that minimize the Kullback-Leibler divergence from these intermediate densities. An estimate of the evidence C is also obtained as a byproduct. The parametric density approximating the posterior PDF is subsequently used to construct an effective IS density to estimate the probability of failure by importance sampling [3]. This alternate sampling density is obtained as a close approximation of the optimal IS density for $P_{F|d}$, through a second round of cross-entropy minimization. Hence, it enables the generation of frequent samples of the uncertain parameters from the failure domain and yields a probability of failure estimator with accelerated statistical convergence. Through numerical studies, we demonstrate that the proposed approach outperforms state-of-the-art sampling methods in various settings. Further studies to extend these developments to enable risk- and reliability-based optimal decision-making in structural integrity management are currently being pursued by the authors.

References

[1] R. Y. Rubinstein, D. P. Kroese. *Simulation and the Monte Carlo method*, John Wiley and Sons, 2016.
[2] M. Engel, O. Kanjilal, I. Papaioannou, D. Straub. *Bayesian updating and marginal likelihood estimation by cross entropy-based importance sampling*, Journal of Computational Physics 473, 111746, 2023.
[3] O. Kanjilal, I. Papaioannou, D. Straub. *Bayesian updating of reliability by cross entropy-based importance sampling*, Structural Safety 102, 102325, 2023.