ACTIVE LEARNING SURROGATES FOR ENHANCED RELIABILITY ASSESSMENT OF ENGINEERING SYSTEMS

Bouattour, O[1], Kanjilal, O[2], and Ghoshdastidar, D[3]

[1]School of Computation, Information and Technology, TU Munich
[2]Georg Nemetschek Institute Artificial Intelligence for the Built World, TU Munich
[3]School of Computation, Information and Technology, TU Munich
Corresponding E-mail: omar.bouattour@tum.de

Introduction

In engineering reliability assessment, the failure probability of a system is calculated as

$$P_F = \int_{\boldsymbol{x} \in F} f_{\boldsymbol{X}}(\boldsymbol{x})\, d\boldsymbol{x} \qquad (1)$$

in which $\boldsymbol{X}$ is a random vector with probability density function (PDF) $f_{\boldsymbol{X}}(\boldsymbol{x})$ and F is the failure domain defined by $F = \{\boldsymbol{x}: G(\boldsymbol{x}) \leq 0\}$, where $G(\cdot)$ is the performance function. The components of $\boldsymbol{X}$ are random variables representing the system uncertainties, e.g. loads, material properties, boundary conditions, deterioration parameters etc. With very few exceptions, the integral in equation 1 has to be evaluated numerically. The standard Monte Carlo simulation (MCS) method provides a simple and versatile approach to estimate the reliability (Rubinstein & Kroese, 2016). Although the method is straight-forward to apply, accurate estimation of P_F can require several evaluations of $G(\cdot)$ when the failure domain is complex, such as being highly nonlinear, discontinuous, or represented through multiple constraints. If the evaluation of $G(\cdot)$ is expensive, e.g. when the computational model of the system resides in a finite element software, the procedure becomes extremely tedious.

One way to address this pitfall of standard MCS is to approximate $G(\cdot)$ with computationally cheaper surrogates (Echard, et al., 2011, Pan & Dias, 2017). Using a random sample to train a surrogate may not produce a good approximation. This necessitates the incorporation of active learning techniques to efficiently select the training points. The objective of this study is to enhance standard MCS with active learning surrogate models to efficiently predict the reliability of engineering systems. To this end, we focus on the Kriging surrogate (Echard, et al., 2011) and the artificial neural network (ANN) (Bishop, 2006). In addition to the U-function and the expected feasibility function (EFF), which have been widely used in previous studies, we investigate an alternative learning framework with new learning functions and stopping criteria for surrogate training. A novel strategy to estimate the statistics of the surrogate model predictions is employed during the training process. The efficacy of the methods with respect to standard MCS is illustrated through numerical studies.

Methods

The standard Monte Carlo simulation (MCS) method evaluates the reliability integral through random sampling to yield an unbiased estimator of P_F given by:

$$\hat{P}_F = \frac{1}{n_{MC}} \sum_{i=1}^{n_{MC}} I\{\, G(\boldsymbol{x}^{(i)}) \leq 0\} \tag{2}$$

where $\{\boldsymbol{x}^{(i)}, i = 1, \ldots, n_{MC}\}$ are independent realizations of $\boldsymbol{X}$ generated from the PDF $f_{\boldsymbol{X}}(\boldsymbol{x})$, n_{MC} is the sample size and $I\{\cdot\}$ is the indicator function of the failure event. The accuracy of estimation is inversely proportional to the coefficient of variation (CoV) of $\hat{P}_F$, given by $\delta_{\hat{P}_F} = \sqrt{(1-P_F)/n_{MC}P_F}$. We use Kriging surrogate and ANN, enriched with active learning, to devise an approximation $\hat{G}(\cdot)$ of the performance function before using with MCS. The final estimate of the failure probability is obtained by replacing $G(\cdot)$ with $\hat{G}(\cdot)$ in equation 2.

The training of the surrogate models is done on a limited set of samples, which constitute the design of experiment (DoE). The DoE is updated iteratively during the training process based on the learning functions. These functions aim at choosing the best points from the population S to be added to the DoE. This approach ensures that a relatively small DoE, or equivalently the number of evaluations of the computational model, can adequately train the surrogate model to perform sufficiently well in predicting the values of the performance function of all samples in the population. Learning functions can be designed based on different criteria. We consider distance-based learning functions, defined based on the spatial positioning of the samples, and compare the performance with the U-function and the EFF for both surrogates. A schematic of the reliability estimation procedure is shown in Figure 1.

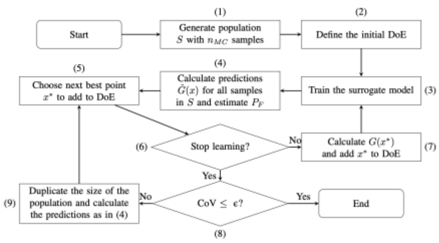

Figure 1: Flow chart of active learning algorithm for reliability assessment.

Results and discussions

We consider the series system with four branches studied in (Echard, et al., 2011). The results from the numerical study are summarized Table 1. The size of the population S is set to $n_{MC} = 10^6$. The design of experiments at the initial state and at convergence for the Kriging model using U- and distance-based-functions are visualized in Figure 2.

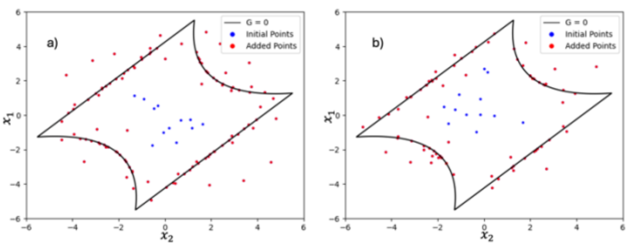

Figure 2: DoE using a) U- and b) distance-based-functions. $G(\cdot)$ is defined in terms of two independent standard Gaussian random variables, denoted by x_1 and x_2.

We use the results from the standard Monte Carlo Simulation as benchmark, where the estimated probability of failure is $P_F = 4.45.10^{-3}$. In Table 1, K refers to the Kriging model and DL to the distance-based learning function. U-DL combines both U-and the distance-based learning functions. Indeed, the active learning for adaptive designs reduces the number of $G(\cdot)$-function calls. In terms of the number of function evaluations for convergence, the distance-based learning functions outperform the EFF. For a specific choice of the surrogate model, the estimate obtained with the U-function shows best agreement with the reference solution.

Table 1: Performance of different methods for the four-branched series system

Method	$\widehat{P}_F$	**#G(.) calls**	$100\,\lvert\widehat{P}_F - P_F\rvert/P_F$
K-U	$4.47.10^{-3}$	132	0.54
K- EFF	$4.49.10^{-3}$	136	0.89
K-DL	$4.39.10^{-3}$	96	1.35
K-DL-U	$3.93.10^{-3}$	75	11.69
ANN-U	$4.40.10^{-3}$	84.8	1.12
ANN-EFF	$4.55.10^{-3}$	171.8	2.25
ANN-DL	$4.37.10^{-3}$	110.8	1.8
ANN-DL-U	$4.37.10^{-3}$	108.4	1.8

Conclusions

This study shows that active learning surrogate models can significantly improve the efficiency of the standard Monte Carlo simulation method for reliability estimation. The choice of the learning function, in combination with the surrogate model, influences the convergence of the reliability estimation algorithm and accuracy of the failure probability estimate. Further numerical investigations are currently being pursued by the authors.

Acknowledgements

Omar Bouattour is supported by the DAAD program Konrad Zuse School of Excellence in Reliable AI. Oindrila Kanjilal is supported by the Nemetschek Innovation Foundation.

References

1. Bishop, C.M. (2006). Pattern Recognition and Machine Learning, Springer.
2. Echard, B., Gayton, N., & Lemaire, M. (2011). AK-MCS: an active learning reliability method combining Kriging and Monte Carlo simulation. Structural safety, 33(2), 145-154.
3. Pan, Q., & Dias, D. (2017). An efficient reliability method combining adaptive support vector machine and Monte Carlo simulation, Structural Safety 67, 85-95.
4. Rubinstein, R.Y., & Kroese, D.P. (2016). Simulation and the Monte Carlo Method, John Wiley & Son, Inc.