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Advanced Signal Processing and Reconstruction Algorithms for Dark-Field Computed Tomography

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Advanced Signal Processing and Reconstruction Algorithms for Dark-Field Computed Tomography

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Abstract

X-ray computed tomography is an essential imaging technique for non-invasive medical diagnosis. Grating-based phase- and dark-field-contrast X-ray imaging is a novel technology that aims to extend conventional attenuation-based X-ray imaging by unlocking two additional contrast modalities: The so-called phase-contrast and dark-field channels provide enhanced soft tissue contrast and additional microstructural information. Initial studies have shown that the dark-field channel, in particular, significantly complements and enhances conventional thoracic radiography. Accessing this additional information comes at the expense of a more intricate measurement setup using a so-called Talbot-Lau interferometer and necessitates sophisticated data processing. A big challenge for translating grating-based dark-field computed tomography to medical applications lies in minimizing the data acquisition time. In this thesis, I present the algorithmic basis for processing the acquired data of a Talbot-Lau interferometer mounted on a continuously rotating clinical CT gantry. While the continuously moving source/detector arrangement is ideal in terms of speed, it precludes conventional phase stepping techniques that require multiple projections under the same angle with different grating positions. Furthermore, the dynamic environment of the fast rotating gantry compromises the interferometer's stability, which leads to vibration-induced artifacts when ignored during processing. Therefore, the proposed data processing pipeline contains correction algorithms for the occurring interferometer fluctuations as well as novel approaches for signal retrieval.

Zusammenfassung

Die Röntgen-Computertomographie ist ein wichtiges bildgebendes Verfahren für die nicht-invasive medizinische Diagnose. Die gitterbasierte Phasen- und Dunkelfeldkontrast-Röntgenbildgebung ist eine neuartige Technologie, die darauf abzielt, die konventionelle, auf Abschwächung basierende Röntgenbildgebung zu erweitern, indem zwei zusätzliche Kontrastmodalitäten erschlossen werden: Der sogenannte Phasenkontrast- und der Dunkelfeldkanal liefern einen verbesserten Weichteilkontrast und zusätzliche mikrostrukturelle Informationen. Erste Studien haben gezeigt, dass insbesondere der Dunkelfeldkanal die konventionelle Thorax-Radiographie deutlich ergänzt und verbessert. Der Zugang zu diesen zusätzlichen Informationen geht jedoch auf Kosten eines aufwändigeren Messaufbaus mit einem so genannten Talbot-Lau-Interferometer und erfordert eine anspruchsvolle Datenverarbeitung. Eine große Herausforderung bei der Übertragung der gitterbasierten Dunkelfeld-Computertomographie auf medizinische Anwendungen liegt in der Minimierung der Datenerfassungszeit. In dieser Arbeit stelle ich die algorithmische Grundlage für die Verarbeitung der Daten eines Talbot-Lau-Interferometers vor, das auf einer kontinuierlich rotierenden klinischen CT-Gantry montiert ist. Die sich kontinuierlich bewegendende Quelle/Detektor-Anordnung ist zwar in Bezug auf die Geschwindigkeit ideal, schließt aber herkömmliche Phase-Stepping-Verfahren aus, die mehrere Projektionen unter demselben Winkel mit unterschiedlichen Gitterpositionen erfordern. Darüber hinaus beeinträchtigt die dynamische Umgebung der schnell rotierenden Gantry die Stabilität des Interferometers, was zu vibrationsbedingten Artefakten führt. Die vorgeschlagene Datenverarbeitungspipeline enthält daher Korrekturalgorithmen für die auftretenden Interferometerfluktuationen sowie neuartige Ansätze zur Signalextrahierung.

*“Quidquid praecipies, esto brevis,
ut cito dicta percipiant animi dociles teneantque fideles:
omne supervacuum pleno de pectore manat.”*

—Horaz, *Ars poetica*

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Chapter 1

Introduction

Gallia est omnis divisa in partes tres

Caesar, *De Bello Gallico*

X-ray computed tomography (CT) is an essential medical imaging technique that provides 3D visualizations of a patient's anatomy and allows for rapid quantitative measurements. However, traditional CT imaging relies solely on attenuation-based contrast and cannot take advantage of the wave properties of X-rays. Phase-contrast and dark-field X-ray imaging offers a promising approach to unlock additional contrast channels based on the wave properties of X-rays for medical applications. These additional contrast mechanisms can be accessed by Talbot-Lau interferometry [1–3] or other methods such as speckle or edge-illumination-based imaging [4, 5].

In Talbot-Lau interferometry, the introduction of diffraction gratings modulates the X-ray wavefront and creates an interference pattern on the detector, serving as a reference. When exposed to the X-ray beam, a sample interacts with the incident wavefront in three significant ways. Firstly, the sample's attenuation reduces the overall intensity of the X-rays. Secondly, porous materials cause small-angle scattering, which blurs the observed interference pattern and reduces its visibility. Lastly, the interaction with the sample induces a phase shift, resulting in a displacement of the observed pattern. A proper analysis of the interferometer's interference pattern thus yields two additional imaging contrasts, the so-called dark-field and phase contrast channels. Conventionally, the process to separate these three contrasts is called *phase retrieval*.

While phase-contrast imaging enhances the visibility of soft tissues [2, 6, 7], the dark-field signal, in particular, has emerged as a promising tool in diagnosing lung diseases, as it can reveal microstructural changes in the lung parenchyma [8–14]. A second application of the technique is in bone imaging, where the added microstructural information has been shown to correlate with bone health [15, 16]. A third application of grating-based imaging lies in breast imaging, where both the phase contrast and the dark-field contrast

have been shown to be of diagnostic value [17–20]. Recent clinical studies have demonstrated the potential of dark-field imaging for pulmonary disease diagnosis using a prototype radiography system [21–23]. However, this system only captures chest radiographs and lacks 3D information. To address this limitation and to combine dark-field with large field-of-view tomographic imaging for clinical use, a human-sized dark-field CT prototype has been introduced in [24].

Previous dark-field CT systems had limitations such as small fields-of-view suited only for small animal studies, slow acquisition times of several minutes, and the absence of continuous gantry rotation [25]. This, in turn, allowed for the so-called step-and-shoot acquisition of multiple shots under the same detector and sample position, but with varying pre-determined grating positions. In contrast, the presented prototype achieves scan times comparable to existing clinical CT scanners by employing continuous gantry rotation. This necessitates a departure from the conventional stepping-based data acquisition since the detector's position moves from shot to shot, and a controlled grating movement is barely possible. The necessary fast sampling of the stepping curve is instead achieved by vibrations induced by the motors on the gantry [26]. While this allows for rapid data acquisition, the induced grating vibrations and other system perturbations cause significant fluctuations in the interferometer state that have to be addressed during data processing. Furthermore, the continuous rotation precludes traditional phase retrieval algorithms as only one view of the sample is available per gantry angle, requiring novel approaches for signal extraction.

In this thesis, I present the algorithmic basis to process the acquired sample scans of this first human-sized dark-field CT prototype. It is divided into three parts:

First, the problem of phase retrieval for continuous acquisition schemes is analyzed. Based on this analysis, I present two projection-based strategies for sample information extraction. The first one is an extension of the so-called sliding window method, where phase retrieval is performed in local patches. The second one

is a global analysis of the sinogram using Tikhonov regularization.

Second, as the prototype employs novel strategies for sampling the stepping curve, a theoretical framework for the analysis of the overall sampling quality is developed. Subsequently, this framework is applied to data from the prototype scanner to assess where optimization of the scanner's performance is possible.

Third, I present a robust processing pipeline for sample scans that employs a fluctuation model developed by [27], to first determine the interferometer state and then perform phase retrieval and reconstruction of scans with arbitrary samples. Determining the interferometer state is especially challenging in sample scans with large samples since substantial portions of the detector are covered by the sample's projection in each view, hindering an unambiguous separation of sample information and interferometer fluctuations. I solve this problem by a thorough analysis of the occurring fluctuations and by using prior knowledge about the vibration characteristics obtained from air scans.

Chapter 2

Background and Prior Work

Umbra et imaginibus utimur.

Cicero, *De Officiis*

In this chapter, I give a brief overview of the theoretical background this thesis is built upon. First, the optical theory underlying X-ray interaction with matter is explored. Second, the principles of grating-based phase-contrast and dark-field imaging are outlined. Next, an introduction is provided to the field of tomographic reconstruction. Finally, the prototype dark-field CT scanner utilized for the measurements conducted for this thesis is introduced.

2.1 X-ray Interactions with Matter

X-ray radiation is electromagnetic radiation in the approximate energy range of 1 keV to 1 MeV. For clinical imaging, a macroscopic description of the influence of a sample on an incident X-ray beam is adequate, i.e., only the compound effect of individual photonic events is of interest to describe the imaging contrasts. The main photonic effects for the energy ranges of X-ray imaging, are photoelectric absorption, Compton scattering, and Rayleigh scattering.

For the purpose of this thesis, it is sufficient to describe the optical properties of a material by its complex refractive index n that combines the aforementioned physical effects into one macroscopic quantity. More extensive discussions of X-ray physics are given by [28]. Commonly, in the energy range of X-rays, n is defined as

$$n = (1 - \delta) + i\beta, \quad (2.1)$$

where δ is called the *refractive index decrement* and β the *extinction coefficient*. While the refractive index' real part leads to a phase shift of the incident wave, the imaginary part describes the medium's attenuation. Neglecting the temporal oscillation of the wave, an electromagnetic plane wave propagating through a

homogeneous medium with refractive index n is modeled by

$$\Psi(\mathbf{r}) = \Psi_0 e^{i\mathbf{k}\mathbf{r}} = \Psi_0 e^{i(1-\delta)\mathbf{k}\mathbf{r}} e^{-\beta\mathbf{k}\mathbf{r}}, \quad (2.2)$$

where $\mathbf{r}$ is the position vector, $\mathbf{k}$ the wave vector pointing in the wave's direction of propagation, and Ψ_0 the wave's amplitude vector.

Attenuation

As the wave propagates through a medium, part of its energy is absorbed, leading to attenuation of the wave. Using Equation 2.2, the magnitude of this attenuation over a distance d can be determined. Specifically, the intensity I of the wave along the propagation direction is given by:

$$I(d) = |\Psi\Psi^*| = \Psi_0^2 e^{-2\beta kd} = I_0 e^{-2\beta kd} = I_0 e^{-\mu d}. \quad (2.3)$$

This demonstrates that the wave intensity decreases exponentially as it travels through the medium. The exponential decay is characterized by the *linear absorption coefficient* μ , which is defined as:

$$\mu = 2\beta k. \quad (2.4)$$

This relationship is known as Lambert-Beer's law, describing the attenuation of electromagnetic waves in a homogeneous medium. For inhomogeneous media, the absorption coefficient becomes spatially dependent. In this case, the intensity is given by:

$$I = I_0 e^{-\int_l \mu(\mathbf{r}) dl}, \quad (2.5)$$

where the integral represents a line integral along the propagation path of the wave.

Refraction

Equation 2.2 states that the phase of a wave that traversed a medium by a length of d is shifted relative to one that traveled through vacuum with $n = 1$. For a homogeneous medium, this phase shift amounts to

$$\Delta\Phi = \delta kd. \quad (2.6)$$

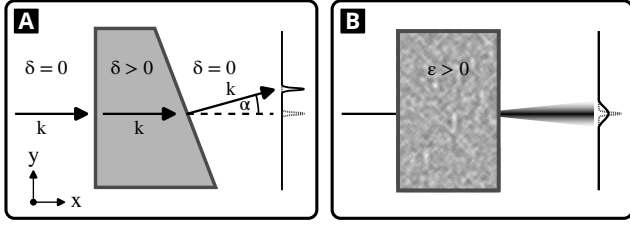


Figure 2.1: **A:** Refraction in a wedge shaped object with $\delta > 0$. The phase gradient at the object's interface causes a deflection in the direction of $\mathbf{k}$. **B:** Beam widening due to small angle scattering in a medium with $\epsilon > 0$. Scattering events within the medium broaden the probability distribution describing the location where a photon will be detected after passing through the sample. Figure adapted from [29].

For inhomogeneous media, again, the spatial dependency of the refractive index has to be accounted for, which results in the general expression for the phase shift:

$$\Delta\Phi = \frac{2\pi}{\lambda} \int_l \delta(\mathbf{r}) dl, \quad (2.7)$$

where $k = \frac{2\pi}{\lambda}$ was used, with λ being the wave's wavelength. If this phase shift varies across directions perpendicular to the wave propagation, it results in a change in the wave vector's direction. For instance, such behavior is observed when a wave passes through a wedge-shaped object as depicted in Figure 2.1A. In this scenario, the change in direction can be approximated by the refractive angle α :

$$\alpha \approx \tan \alpha = \frac{\lambda}{2\pi} \frac{\partial \Phi}{\partial y} = \frac{\partial}{\partial y} \int_l \delta(\mathbf{r}) dl, \quad (2.8)$$

where y denotes the coordinate perpendicular to the undisturbed wavefront, and a small-angle approximation was used.

Small-Angle Scattering

The third macroscopic interaction mechanism important for grating-based imaging is small-angle scattering. In this context, scattering denotes small deflections of the X-ray beams at material interfaces, i.e., microscopic changes in the refraction index. At every interface within the medium, the beam undergoes refraction. In porous media with numerous interfaces oriented in varying directions, this process occurs repeatedly. Due to the repeated refraction events, a sharp X-ray beam traveling through a porous medium gets slightly widened and shows a probabilistic angular distribution of its intensity, which leads to a local loss of lateral coherence. This is depicted in Figure 2.1B.

For isotropic small-angle scattering, the total widening of a sharp X-ray beam after traversing a porous sample can be described by convolving it with a Gaussian

angular distribution

$$A(\alpha) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\alpha^2}{2\sigma^2}\right), \quad (2.9)$$

where σ is a material-dependent parameter describing the second moment of the scatter angle distribution, and α is the scattering angle. Similar to a spatial change in the sample's attenuation coefficient, the width of the scattering distribution may vary within the sample while the beam is traversing the sample. A more general description thus assumes a spatially dependent scattering power density $\epsilon(\mathbf{r})$ and relates it to the effective total distribution width after traversal by

$$\sigma^2 = \int_l \epsilon(\mathbf{r}) dl. \quad (2.10)$$

More commonly, ϵ is also called the *linear diffusion coefficient*. It can be interpreted as a medium's scattering strength, and for dark-field imaging, it is analogous to the linear absorption coefficient.

2.2 X-ray Grating Interferometry

Conventional X-ray imaging relies solely on the attenuation of radiation to create a visual representation of a sample. Consequently, such images correspond to a spatially resolved measurement of the attenuation coefficient μ . In contrast, X-ray grating interferometry extends this capability by enabling the measurement of both the refractive index decrement δ , and the scattering strength ϵ . To achieve this, the relative phase shift and small-angle scattering caused by the sample must be measured. Since X-ray detectors can only measure intensities, these effects must first be translated into detectable intensity changes. Grating-based phase-contrast imaging addresses this by using gratings to create an interference pattern that serves as a reference. When a sample is introduced into the beam path, it distorts the incoming wavefront, altering the resulting interference pattern. By comparing the distorted pattern with the reference, the attenuation coefficient μ , ϵ , and δ can be extracted.

2.2.1 Talbot Effect

In grating-based imaging, the so-called *Talbot effect* is exploited to create the necessary reference fringe pattern. This effect describes how a periodic coherent wavefront replicates itself at specific propagation distances known as Talbot distances d_t . When a periodic structure, such as a grating, is illuminated, it produces self-images of the grating at intervals of d_t behind it. For an absorbing grating G_1 with a periodicity p_1 , the first Talbot distance depends on the wavelength λ and

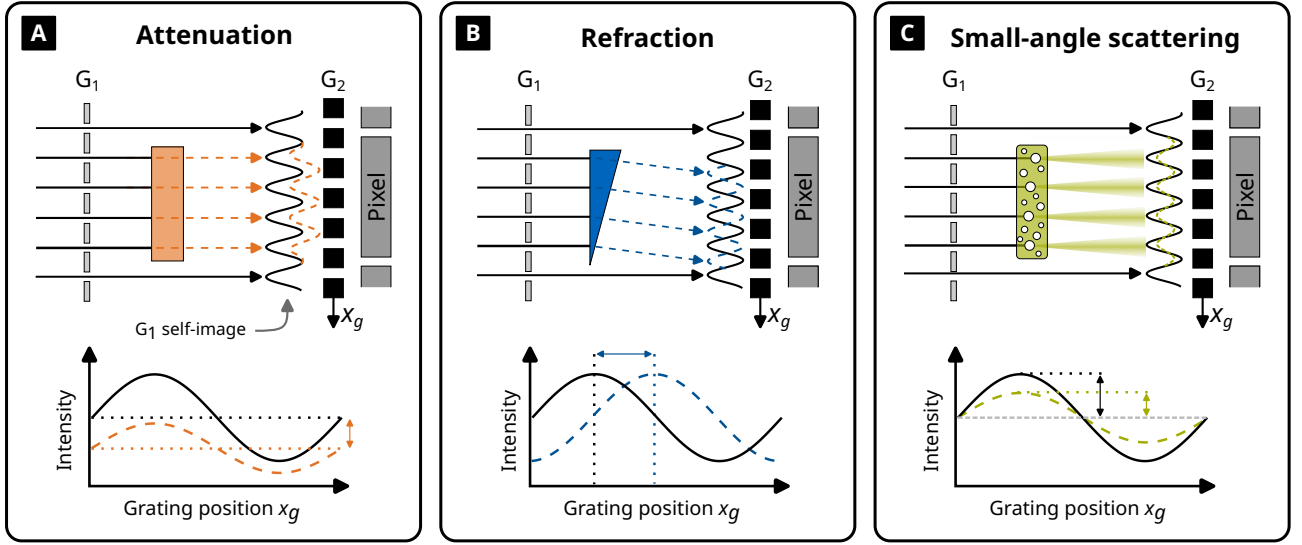


Figure 2.2: Functional principle of a Talbot-Lau interferometer and contrast mechanisms: The Talbot effect generates a self-image of the grating G_1 at the detector plane when no sample is present in the beam path. The resulting interference pattern is analyzed by moving an analyzer grating G_2 across the detector, yielding the stepping curve. **A:** If a purely absorbing object is introduced, it reduces the average intensity of the pattern and thus the mean of the local stepping curve. **B:** Refraction within the sample causes a lateral shift in the pattern. **C:** Small-angle scattering diminishes the modulation amplitude by blurring the pattern.

is given by:

$$d_t = \frac{2p_1^2}{\lambda}. \quad (2.11)$$

The Talbot effect also applies to gratings that periodically modulate the phase of the incoming waves, though the self-imaging occurs at different distances in this case. It is important to note that this simplified description holds only for coherent and plane waves. A more detailed analysis of the Talbot effect and its application to grating-based imaging is discussed in [30].

2.2.2 Image Formation

Typically, the reference pattern generated by the G_1 grating is too small to be resolved directly. To sample the pattern, a second analyzer grating G_2 in front of the detector is necessary: By matching the periodicity p_2 of G_2 to the periodicity of the self-image of G_1 , a so-called *stepping curve* can be recorded by shifting one of the gratings laterally. A proper analysis of this stepping curve then yields the different contrast channels. Figure 2.2 shows a schematic of a Talbot-Lau interferometer and the three contrast-generating effects.

Grating-based Talbot-Lau X-ray dark-field imaging with incoherent sources employs a third grating G_0 placed in front of the X-ray source. This creates an array of partially coherent slit-sources that can interfere with each other and produce a Talbot-carpet in spite of the lacking coherence of conventional X-ray sources.

In good approximation, the stepping curve recorded at the detector can be modeled as a truncated Fourier series. A detailed derivation can be found in [31]. Therefore, a reference air scan y^r can be expressed as:

$$y^r = I + \sum_{n=1}^{\infty} IV_n \cos(n \cdot \phi), \quad (2.12)$$

where the state of the interferometer is described by the parameters I , V_n , and ϕ , which denote the mean intensity, the fringe amplitude (visibility) of order n , and the phase of the fringe pattern, respectively. Generally, these parameters depend on the individual pixel, shot, and grating position. A sample in the beam attenuates, small-angle scatters, and phase-shifts the incident wave, which alters the measured fringe pattern. While the phase-shift of the wave leads to an offset in the recorded interferometer phase of Φ , the effects of isotropic small-angle scattering and sample attenuation can be described by a convolution of the reference fringe pattern with a scattering kernel k that attenuates and smears out the stepping curve [32, 33]:

$$k(\Delta\phi) = \frac{T}{\sqrt{2\pi}\sigma} e^{-\frac{\Delta\phi^2}{2\sigma^2}}, \quad (2.13)$$

where T is the sample's transmission, and σ is a material dependent parameter describing the second moment of the scatter angle distribution. The resulting fringe pattern y^s of a sample measurement is then

given by:

$$y^s = (y^r * k) [\phi + \Phi], \quad (2.14)$$

$$= TI + \sum_{n=1} TIV_n e^{-\frac{(\sigma n)^2}{2}} \cos(n(\phi + \Phi)), \quad (2.15)$$

$$= TI + \sum_{n=1} TIV_n D^{n^2} \cos(n(\phi + \Phi)), \quad (2.16)$$

where in the last row the sample's dark-field signal as a measure of its small-angle scattering strength was introduced as:

$$D = e^{-\frac{\sigma^2}{2}}. \quad (2.17)$$

In most Talbot-Lau setups, the contributions of the higher-order terms can be neglected due to the effective blurring from large focal spots [30], and the series is commonly truncated after the first order $n = 1$, which results in the canonical forward model of grating-based Talbot-Lau imaging [2]:

$$y^s = TI + TIVD \cos(\phi + \Phi), \quad (2.18)$$

where V is simply referred to as the visibility of the interferometer. The method's different contrast modalities are thus represented by the sample transmission T , the dark-field D , and the (differential) phase Φ .

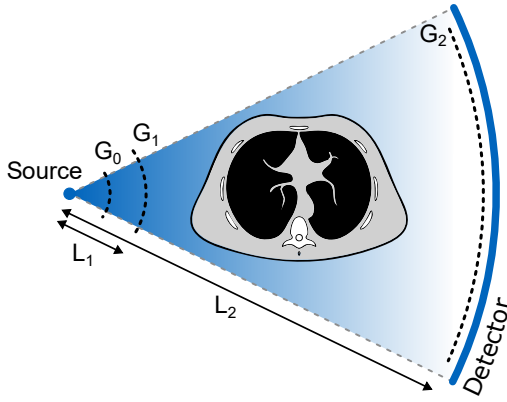


Figure 2.3: Talbot-Lau interferometer with an inverse geometry. The sample is positioned in between G_1 and G_2 . To account for the divergent beam, all gratings are bent.

Based on the interaction effects described in Section 2.1 the three sample parameters are related to the sample via

$$T = \exp \left(- \int_{L_1}^{L_2} \mu(\mathbf{S} + l\mathbf{n}) dl \right), \quad (2.19)$$

$$D = \exp \left(- \int_{L_1}^{L_2} s_D(l) \varepsilon(\mathbf{S} + l\mathbf{n}) dl \right), \quad (2.20)$$

$$\Phi = \partial_y \int_{L_1}^{L_2} s_\Phi(l) \delta(\mathbf{S} + l\mathbf{n}) dl, \quad (2.21)$$

where the linear attenuation coefficient μ , the refractive index decrement δ , and the linear diffusion coefficient ε are the material specific parameters of the sample. Line integrals are computed from the source position $\mathbf{S}$ in the direction $\mathbf{n}$ towards the detector pixel, where the object is known to be located between the gratings G_1 and G_2 , which have distances L_1 and L_2 from the source, respectively. A schematic of the interferometer geometry is depicted in Figure 2.3. The partial derivative ∂_y is taken perpendicular to the grating lamellae in the projection domain. Finally, s_D and s_Φ are setup geometry-specific sensitivities, which are 0 at L_2 and grow linearly towards L_1 . This is because signal strength for these two channels is generally strongest for objects right at the G_1 and drops to zero at the G_2 due to the lacking propagation distance of the wavefront. The assumption of a linear sensitivity gradient is exact for the differential phase signal [34], but only approximate for the dark-field signal [35, 36]. For a discussion and an experimental demonstration of the resulting position dependency of the dark-field signal, I refer to [36], where the linear approximation of s_D is motivated further. For the presented dark-field prototype, this effect has to be included since the sample dimensions are at a similar scale as the distance between G_1 and G_2 .

2.2.3 Signal Extraction

After acquiring measurements with a Talbot-Lau interferometer, the recorded stepping curves need to be processed and analyzed to extract the sample channels T , D , and Φ . If the phase stepping $\phi(t)$ is known to be equidistant, these parameters can be extracted by a simple Fourier analysis of the measured intensities. However, for an arbitrary phase stepping scheme, phase retrieval has to be performed by solving an optimization problem for each stepping curve that fits the model (2.18) to the measured data y^{meas} by minimizing a negative log-likelihood function. For simplicity, Gaussian distributed measurements are assumed, which leads to a least-squares problem:

$$-\log \mathcal{L} = \sum_t w(t) \left(y^{\text{meas}}(t) - y^{\text{model}}(t) \right)^2. \quad (2.22)$$

Here, one sums over all recorded grating positions t . The results of this projection-based approach are statistical estimators for the sample projections $\hat{T}$, $\hat{D}$, and $\hat{\Phi}$:

$$\hat{T}, \hat{D}, \hat{\Phi} = \arg \min_{T, D, \Phi} \{ -\log \mathcal{L} \}. \quad (2.23)$$

As statistical information from the measured data is included in the optimization by means of the statistical weights w , this algorithm is called statistical phase retrieval (SPR) [37]. In practice, the optimization process can be simplified significantly by first linearizing

the forward model through a change of variables. To achieve this, one employs the trigonometric identity

$$\cos(\Phi + \phi) = \cos(\Phi) \cos(\phi) - \sin(\Phi) \sin(\phi), \quad (2.24)$$

and introduces new variables:

$$M = T, \quad C = TD \cos(\Phi), \quad S = -TD \sin(\Phi). \quad (2.25)$$

In these variables, the forward model is now linear:

$$y = MI + CIV \cos(\phi) + SIV \sin(\phi). \quad (2.26)$$

Introducing matrix notation, the forward model (2.18) can thus be expressed as

$$\mathbf{y} = \mathbf{X}\mathbf{b}, \quad (2.27)$$

with the observation vector containing the N measurements:

$$\mathbf{y} = \left(y(t_1), \dots, y(t_N) \right)^T \in \mathbb{R}^N, \quad (2.28)$$

the parameter vector:

$$\mathbf{b} = \left(M, C, S \right)^T \in \mathbb{R}^3, \quad (2.29)$$

and the model matrix containing the interferometer parameters:

$$\mathbf{X} = \begin{pmatrix} I & IV \cos(\phi(t_1)) & IV \sin(\phi(t_1)) \\ \vdots & \vdots & \vdots \\ I & IV \cos(\phi(t_N)) & IV \sin(\phi(t_N)) \end{pmatrix} \in \mathbb{R}^{N \times 3}, \quad (2.30)$$

where the dependency of I and V on t has been omitted to simplify the notation. The optimization problem 2.23 can then be written as:

$$\hat{\mathbf{b}} = \arg \min_{\mathbf{b}} \{ (\mathbf{y} - \mathbf{X}\mathbf{b})^T \mathbf{W} (\mathbf{y} - \mathbf{X}\mathbf{b}) \}, \quad (2.31)$$

where $\mathbf{W} \in \mathbb{R}^{N \times N}$ is the diagonal weight matrix with $W_{ii} = w(t_i)$. The solution of this weighted least-squares problem is given by:

$$\hat{\mathbf{b}} = (\mathbf{X}^T \mathbf{W} \mathbf{X})^{-1} \mathbf{X}^T \mathbf{W} \mathbf{y}. \quad (2.32)$$

However, in practice, a more efficient approach to determine $\hat{\mathbf{b}}$ is to solve the linear problem via matrix decomposition, avoiding an explicit calculation of the inverse system matrix. Once the optimal sample parameters are found in the linearized model, they have

to be transformed back to yield the attenuation, dark-field, and phase channels using:

$$\begin{pmatrix} T \\ D \\ \Phi \end{pmatrix} = \begin{pmatrix} M \\ \frac{\sqrt{C^2 + S^2}}{M} \\ -\arctan(\frac{S}{C}) \end{pmatrix}. \quad (2.33)$$

2.3 X-ray Computed Tomography

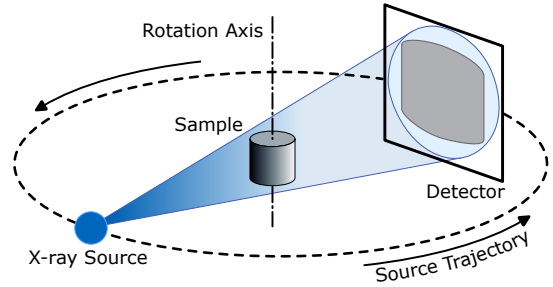


Figure 2.4: Schematic illustration of a cone beam CT configuration. A stack of projective measurements is acquired while either the sample or the X-ray source and the detector are rotating around the iso-center.

X-ray Computed Tomography (CT) is an indispensable medical imaging technique where volumetric data of a sample is acquired by taking multiple projective measurements under different projection angles. The major advantage of CT imaging in comparison to radiography is that it provides unobstructed views of internal structures and allows for quantitative imaging. The higher information content of CT images is, however, paid for with higher patient doses and more complex imaging setups. A schematic depiction of a cone beam CT setup is shown in Figure 2.4. Here, a stack of projective measurements is acquired while either the sample or the X-ray source and the detector are rotating around the iso-center.

Tomographic Reconstruction The extraction of the volumetric information from the stack of projections p and recovering the object function of the sample poses an inverse problem. Tomographic reconstruction is concerned with the solution of this problem. Naturally, it depends on the imaging system's geometry, i.e., the detector and the beam shape. Mathematically, at each detector position and projection angle i the acquired projections represent so-called line integrals $p_i \in \mathbb{R}$ through the object function $f \in \mathcal{S}(\mathbb{R}^3)$ that is assumed to be in the Schwartz space on $\mathbb{R}^3$:

$$p_i = \int_{L_i} f(\mathbf{r}) d\mathbf{r}, \quad (2.34)$$

where L_i is a straight line from the source position through the object towards the detector. The exact path of these line integrals depends on the aforementioned geometry parameters of the system. For conventional

attenuation-based CT, line integrals are obtained from Beer's law (2.19), and the reconstructed object function equals the attenuation coefficient μ . In grating-based dark-field CT, the linear diffusion coefficient ε is reconstructed using a similar relationship (2.20).

In general, the process of projecting the object function forward into projection space can be described by a linear projection operator $\mathcal{P} : \mathcal{S}(\mathbb{R}^3) \rightarrow \mathcal{S}(\mathbb{R}^3)$ that maps an object function into the set of its line integrals p with

$$\mathcal{P}f = p. \quad (2.35)$$

Like the object function, the projection stack also lies in the Schwartz space on $\mathbb{R}^3$. Here, two dimensions represent the detector dimensions, and the third is the variation in acquisition angle. It is commonly also referred to as the *sinogram*. The goal of tomographic reconstruction is to find the inverse of the Projection operator $\mathcal{P}$. It can be shown that this is an ill-posed problem, and thus, the solution is very sensitive to noise in the measured projections.

Filtered Back-Projection In clinical practice, the standard method of reconstruction is the so-called *filtered back-projection* (FBP) approach. Here, reconstructions take the form

$$f = \mathcal{P}^* \mathcal{H} p, \quad (2.36)$$

where $\mathcal{H} : \mathcal{S}(\mathbb{R}^3) \rightarrow \mathcal{S}(\mathbb{R}^3)$ is a suitable high-pass filtering operator and the adjoint of the projection operator $\mathcal{P}^*$ represents the so-called back-projection operator. $\mathcal{P}^*$ can be understood as “smearing” back the projection images p back over the reconstruction volume according to the respective line integrals. That is, at each position $\mathbf{r}$ one sums up the values of all line integrals passing through the point:

$$(\mathcal{P}^* p)(\mathbf{r}) = \int_{\mathbf{r} \in L_i} p_i d\mathbf{i} \quad (2.37)$$

Commonly, the filtering process is performed in Fourier space due to the performance advantages of the fast Fourier transform algorithm and the fact that the filter function takes a simpler shape there. Due to the ill-posedness of the underlying problem, it is furthermore common not to use the mathematical correct filter $\mathcal{H}$ where $\mathcal{P}^* \mathcal{H} \mathcal{P}$ is equal to the identity, but rather an augmented version that suppresses high-frequency noise more strongly in the reconstructed images.

Modern clinical CT imaging employs scanners with a cone beam geometry and bent detector designs, rendering the reconstruction problem quite complicated in practice. An in-depth account of cone beam reconstruction techniques is given by [38], while a more general introduction to CT imaging is given by [39]. For a

thorough and rigorous treatment of the mathematics behind the reconstruction problem, I refer to [40].

2.4 Prototype for Clinical Dark-Field Computed Tomography

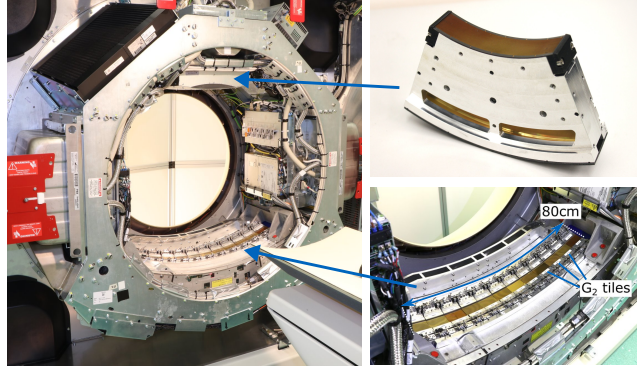


Figure 2.5: Photograph of the dark-field CT prototype's gantry and the two grating mounts. The collimator box contains the G_0 and G_1 mount, while the tiled G_2 is positioned in front of the detector. Figure adapted from [41]

All measurements performed in this thesis were conducted with a human-scale dark-field CT prototype. The hardware design and a characterization of the setup have been published in [24, 26, 42]. An even more detailed account of these topics is given in [41]. Here, only a brief overview of the setup and its working principle is given.

The prototype scanner was built by modifying a commercial clinical CT scanner from Koninklijke Philips N.V.. The modified platform is a Brilliance iCT SP unit that features 64 detector rows and 672 columns with frame rates of up to 4800 frames per rotation. The scanner's gantry is mounted on radial air bearings that allow fast rotation speeds up to 0.27 seconds rotation time. For the dark-field CT prototype, the air bearings are assumed to be advantageous as they decouple the gantry from outside sources of perturbation that might compromise the interferometer's stability.

The main modification of the scanner is the addition of a Talbot-Lau interferometer on the gantry. A photograph of the scanner's gantry and gratings is shown in Figure 2.5 and a schematic of the interferometer geometry is depicted in Figure 2.6. Three cylindrically bent gratings G_0 , G_1 , and G_2 are mounted with custom grating holders that allow for precise and stable alignment of the interferometer. G_0 and G_1 are attached to a grating mount that is pre-aligned in a laboratory setup and housed in the scanner's collimator box, directly in front of the X-ray source's exit window. The large G_2 grating is stitched together from 13 individual tiles and only covers the central 32 detector rows. While G_0 and G_2 are both binary absorption gratings, the G_1 grating is a phase grating with a triangular profile.

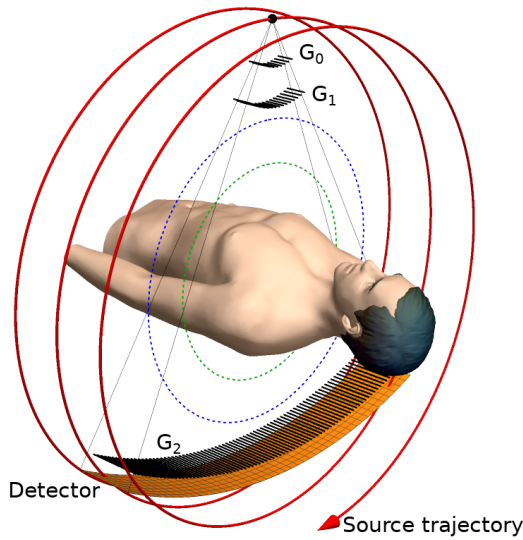


Figure 2.6: Sketch of the Talbot-Lau interferometer in the dark-field CT prototype's gantry. It consists of three cylindrically bent gratings G_0 , G_1 , and G_2 that are subjected to vibrations to achieve phase sampling.

One major challenge in adapting grating-based dark-field computed tomography for medical applications is to keep the data acquisition time within some seconds. An ideal setup involves a continuously moving source/detector arrangement. Therefore, for the dark-field CT prototype, clinical protocols with a continuously rotating gantry are used for data acquisition. However, this approach precludes conventional phase retrieval and stepping techniques that require multiple projections acquired at the same angle with different grating positions. This necessitates a departure from the conventional stepping-based data acquisition since the detector's position moves from shot to shot, and a controlled grating movement is barely possible. The necessary fast sampling of the stepping curve is instead achieved by vibrations induced by the motors on the gantry: The primary source of phase stepping is the vibration induced by the rotating anode's motor that is translated to the gratings and induces an interferometer phase oscillation with a frequency of approximately 176 Hz.

This has significant consequences for the data processing at the prototype since it requires correction algorithms for the occurring vibration- and perturbation-induced artifacts and novel techniques for phase retrieval. Both are discussed in the remainder of this thesis.

Chapter 3

Phase Retrieval for Continuous Acquisition Schemes

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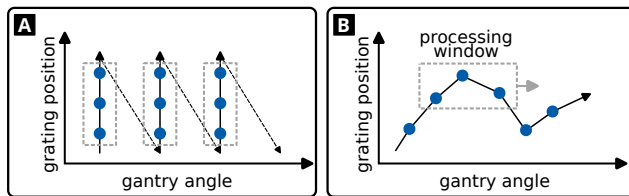


Figure 3.1: **A:** Conventional stepping-based acquisition protocol, where the sample is stationary while a grating is shifted to sample the stepping curve. **B:** Continuous acquisition protocol, where sample and grating move simultaneously. Figure adapted from [43].

This chapter, discusses phase retrieval under continuous acquisition conditions. First, the problem is analyzed theoretically and next two novel approaches for phase retrieval are introduced and applied to data from the DFCT prototype scanner. Parts of this chapter have been published in [43] and [44].

3.1 Introduction

A major challenge for translating grating-based dark-field computed tomography to medical applications lies in minimizing the data acquisition time. While a continuously moving source/detector arrangement is ideal, it prohibits the use of conventional stepping-based data acquisition schemes as depicted in Figure 3.1A since they depend on multiple projections at the same angle with varied grating positions. Instead, for the dark-field CT prototype, a continuous acquisition protocol as sketched in Figure 3.1B is used, where both the gratings and the gantry are moving simultaneously. This enables scan times comparable to existing clinical CT scanners but requires advanced phase-retrieval techniques.

There exist two conceptual approaches to perform phase retrieval for a system with a continuously rotating gantry. In the context of continuously acquired data, I will also refer to this process as *signal demodulation*.

The first and more involved one is to combine the problems of phase retrieval and tomographic reconstruction: A single statistical iterative reconstruction considers the full interferometric image formation process and reconstructs all three imaging channels concurrently [45–47]. However, this approach is significantly hindered by the non-linear and even non-convex, ill-posed nature of the underlying optimization problem, which also exhibits problematic convergence behavior.

In the second approach, phase retrieval is performed in projection space, and some notion of smoothness is assumed about the sinogram to be recovered to combine the information from adjacent shots and/or pixels [43, 48–50]. The so obtained projections are then reconstructed, for example, by filtered back projection (FBP). Naturally, the obtained reconstructions can be used as is or to initialize the more sophisticated iterative algorithms.

One projection-based approach is the so-called sliding window method introduced by [50]. Here, adjacent shots are grouped together and processed jointly with conventional stepping-based phase retrieval algorithms in the projection domain. Consequently, the sample's projection is approximated to be stationary within one processing window.

Another projection-based approach uses carrier fringes of high spatial frequency and derives the fringe phase by Fourier analysis [48] or by processing local patches [49]. Similar to the sliding window approach, the assumption here is that the measured differences in intensity within the patch are mostly due to the different fringe phase and not due to changes in the underlying sample features. A suitable fringe pattern for this approach can be achieved by de-tuning the gratings, which results in Moiré fringes on the detector.

In this chapter, I discuss and present extensions of the

sliding window and carrier fringe method, as well as report on a novel algorithm for phase retrieval under continuous acquisition conditions. Unlike the conventional sliding window phase retrieval method, which operates locally, the new algorithm performs a global analysis of the entire sinogram using Tikhonov regularization. With the global approach, a substantially better performance for the extraction of sharp edges in the sinogram is observed. This reduces streaking in the final reconstructions without a loss in sharpness.

3.2 Problem Statement

For a continuously moving gantry, the linearized model (2.27) can be expressed for each shot t as

$$y_t = \mathbf{x}_t \mathbf{b}_t. \quad (3.1)$$

Here, $\mathbf{x}_t$ contains the interferometer information of the respective shot:

$$\mathbf{x}_t = \begin{pmatrix} I, & IV \cos(\phi(t)), & IV \sin(\phi(t)) \end{pmatrix} \in \mathbb{R}^{1 \times 3}, \quad (3.2)$$

and the sample vector $\mathbf{b}_t \in \mathbb{R}^3$ contains the sample information:

$$\mathbf{b}_t = \begin{pmatrix} M(t), & C(t), & S(t) \end{pmatrix}^T \in \mathbb{R}^3, \quad (3.3)$$

while $y_t \in \mathbb{R}$ is the model intensity for shot t . For readability, the dependency of I and V on t and the general dependency on the respective pixel have been omitted.

Given a set of N measurements y_t^{meas} , we want to find an optimal set of sample vectors $\mathbf{b}_t$ that minimizes the cost function

$$\mathcal{C} = \sum_{t=1}^N (y_t^{\text{meas}} - \mathbf{x}_t \mathbf{b}_t)^2. \quad (3.4)$$

Using vector notation, this can be expressed as:

$$\mathcal{C}(\mathbf{B}) = \|\mathbf{y}_{\text{meas}} - \mathbf{X}\mathbf{B}\|^2, \quad (3.5)$$

where the stacked vectors/matrices $\mathbf{y}_{\text{meas}}$, $\mathbf{B}$ and $\mathbf{X}$ are introduced:

$$\mathbf{y}_{\text{meas}} = \begin{pmatrix} y_1^{\text{meas}}, & \dots, & y_N^{\text{meas}} \end{pmatrix}^T \in \mathbb{R}^N, \quad (3.6)$$

$$\mathbf{B} = \begin{pmatrix} \mathbf{b}_1^T, & \dots, & \mathbf{b}_N^T \end{pmatrix}^T \in \mathbb{R}^{3N}, \quad (3.7)$$

$$\mathbf{X} = \begin{pmatrix} \mathbf{x}_0 & & \\ & \ddots & \\ & & \mathbf{x}_N \end{pmatrix} \in \mathbb{R}^{N \times 3N}. \quad (3.8)$$

Since there are three times more sample parameters than observations y_t , the system matrix $\mathbf{X}^T \mathbf{X}$ of the linear problem (3.5) is singular, and it does not have a unique solution. To circumvent this problem, prior knowledge about the sample parameters can be introduced explicitly into the optimization in the form of a suitable regularization term $\mathcal{R}$:

$$\mathcal{L}(\mathbf{B}) = \mathcal{C}(\mathbf{B}) + \lambda \mathcal{R}(\mathbf{B}). \quad (3.9)$$

Here, $\lambda \in \mathbb{R}$ describes the regularization strength and the goal is to minimize the combined cost $\mathcal{L}$.

In the following, two approaches for regularization are discussed. Both assume some degree of smoothness for $\mathbf{b}_t$. While this is mostly a sensible assumption, it can introduce artifacts at positions of sharp edges in the measured sinogram.

3.3 Sliding-Window-Based Phase Retrieval

The oldest approach for demodulation of continuous data is the so-called sliding window method introduced by [50]. Here, adjacent shots are grouped together and processed jointly with conventional stepping-based phase retrieval algorithms in the projection domain. Again, it can be motivated by assuming smooth sample projections that do not feature strong high-frequency information, and, therefore, adjacent sample parameters look similar. The squared residuals can thus be approximated by the following convolution with a normalized smoothing kernel w .

$$R(t)^2 = (y_t^{\text{meas}} - \mathbf{x}_t \mathbf{b}_t)^2 \quad (3.10)$$

$$\approx \int (y_t^{\text{meas}} - \mathbf{x}_t \mathbf{b}_s)^2 w(s - t) ds. \quad (3.11)$$

Inserting this into the cost function yields:

$$\mathcal{C} = \int R(t)^2 dt \quad (3.12)$$

$$\approx \iint (y_t^{\text{meas}} - \mathbf{x}_t \mathbf{b}_s)^2 w(s - t) ds dt, \quad (3.13)$$

which is discretized by defining $w_{ij} = w(t_i - t_j)$:

$$\mathcal{L}_{\text{SW}} = \sum_{ij} w_{ij} (y_i^{\text{meas}} - \mathbf{x}_i \mathbf{b}_j)^2 \quad (3.14)$$

$$= \sum_i w_{ii} (y_i^{\text{meas}} - \mathbf{x}_i \mathbf{b}_i)^2 + \sum_{i \neq j} w_{ij} (y_i^{\text{meas}} - \mathbf{x}_i \mathbf{b}_j)^2. \quad (3.15)$$

Therefore, one can identify the second term as a regularization term of the original cost function:

$$\mathcal{R}_{\text{SW}} = \sum_{i \neq j} w_{ij} (y_i^{\text{meas}} - \mathbf{x}_i \mathbf{b}_j)^2. \quad (3.16)$$

However, unlike traditional regularization terms, which usually represent prior knowledge about the solution that is independent of the measurement, $\mathcal{R}_{SW}$ depends on the observations themselves. While this could be expressed in terms of a global $3N \times 3N$ Thikonov matrix that depends on the observation vector $\mathbf{y}^{\text{meas}}$, the resulting optimization problem can be solved more elegantly. For this, the combined cost function is expressed as a sum of costs $\mathcal{C}_j$ that only depend on the parameter vector $\mathbf{b}_j$, splitting the problem into so-called windows:

$$\mathcal{L}_{SW} = \sum_j \mathcal{C}_j, \quad (3.17)$$

where $\mathcal{C}_j$ is the cost function of the j th window:

$$\mathcal{C}_j = \sum_i w_{ij} (y_i^{\text{meas}} - \mathbf{x}_i \mathbf{b}_j)^2. \quad (3.18)$$

Therefore, each window can be treated separately and in parallel: To find the value of $\hat{\mathbf{b}}_j$ that minimizes the total cost, one only has to minimize $\mathcal{C}_j$. Since $\mathcal{C}_j$ depends solely on the observations $\mathbf{y}^{\text{meas}}$ in a neighborhood of y_j^{meas} , which is defined by the width of weighting kernel, the sliding window demodulation is a local approach.

3.4 Advanced Sliding-Window-Based Phase Retrieval

While the projection-based demodulation approaches yield faster reconstruction results, they are prone to artifacts, which appear if the projection of samples moves too quickly on the detector (in the sliding window approach) or if the sample's projection changes spatially too rapidly (in the carrier fringe approach). These artifacts can be traced back to cross-talk between the three imaging domains; hence, I refer to them as *cross-talk-artifacts*.

In the following section, local projection-based approaches are discussed with a focus on cross-talk-artifact mitigation. For this, a new interpretation of the phase retrieval problem for continuous acquisitions as a demodulation problem is introduced. In this interpretation, the origin of the cross-talk-artifacts is identified as partially overlapping modulation sidebands. Furthermore, three algorithmic extensions are presented, including a combination with the carrier fringe method, that improve the conventional sliding-window-based phase retrieval and mitigate cross-talk-artifacts. The presented algorithms are tested in a simulation study and on experimental data from the human-scale dark-field CT prototype. In both cases, they achieve a substantial reduction of the occurring cross-talk-artifacts.

3.4.1 Model Analysis

For a data acquisition scheme utilizing a continuously moving detector and in the presence of vibrations and other fluctuations, all the introduced parameters in model (2.18) generally depend on the individual detector pixel $\mathbf{x}$ and time t .

It is, however, instructive to first consider the case of a constant intensity I_0 and visibility V_0 , and focus on one pixel. In this case, Equation (2.18) can be re-written as:

$$y(t) = M(t)I_0 + I_0V_0 [C(t)\cos(\phi(t)) - S(t)\sin(\phi(t))], \quad (3.19)$$

where definition (2.25) was used. While the time-dependence of the interferometer phase ϕ stems from the motion of the gratings, there is also a time-dependence of the sample parameters M , C , and S due to the continuous gantry motion.

The measured signal can be thus thought of as a superposition of three amplitude modulated carrier signals. The first carrier is the constant (DC) function that is modulated by $M(t)$. The second and third carriers are the orthogonal phase stepping carrier signals $\sin(\phi(t))$ and $\cos(\phi(t))$, which depend on the grating movement. Their respective modulation encodes the signals $S(t)$ and $C(t)$ in a shared frequency band. The Fourier spectrum of a qualitative example signal y is illustrated in Figure 3.2. For the example, 2400 shots with a total acquisition time of 1 s were simulated. Furthermore, a linear change of the interferometer phase in time was assumed with a carrier frequency of $f_\phi = 600$ Hz:

$$\phi(t) = 2\pi f_\phi t. \quad (3.20)$$

Such a phase movement corresponds to a shift of the analyzer grating with a constant velocity. The depicted Fourier spectrum illustrates how the different channels' information is distributed in frequency space: While the information of $M(t)$ is decoded in the low frequency band around 0 Hz (DC), the signals $C(t)$ and $S(t)$ are decoded in the sidebands around the phase stepping carrier frequency f_ϕ . For the example shown in Figure 3.2, a strong modulation peak can be observed at $f_\phi = 600$ Hz. If this peak is positioned too close to DC, the information sidebands overlap, resulting in cross-talk during demodulation.

Sliding window phase retrieval consequently consists of three steps: (a) First, the amplitude modulation of the DC carrier needs to be separated from the modulated phase stepping carriers. In the case of a linear phase stepping, this can be accomplished by low-pass filtering the signal, which directly yields $M(t)$. (b) Next, demodulation has to be performed on the remaining high-pass filtered signal to extract $C(t)$ and $S(t)$. (c) Third, the parameters M , C and S have to be transformed back into T , D and Φ using (2.33).

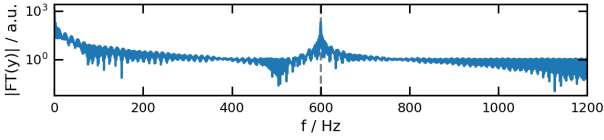


Figure 3.2: Example Fourier spectrum of a test signal acquired with linear phase stepping. The modulation channels C and S are decoded in the sidebands around the modulation peak at $f_\phi = 600$ Hz. The DC modulation M causes a response in the low frequency bands. If the modulation peak is positioned too close to DC, the information sidebands overlap, resulting in cross-talk-artifacts after demodulation. Figure adapted from [43].

Achieving step (a) depends on a sufficient spectral separation of the modulated DC carrier and the phase stepping carriers, i.e., f_ϕ needs to be sufficiently high to separate the modulation sidebands around f_ϕ from the low frequency band around DC. Otherwise, the bands overlap and cross-talk-artifacts emerge in the demodulated channels because an unambiguous separation of the channels is no longer possible. Typically, high-frequency information from the DC modulation blends into the modulation channels C and S as the attenuation signal decoded in M is the strongest signal in setups suitable for imaging humans due to constrained geometry and limited sensitivity. This leads to the emergence of movement-induced cross-talk-artifacts in the dark-field and phase channels at positions of moving sharp edges in the transmission.

In general, the design of the interferometer phase ϕ is thus vital for achieving a good demodulation as it governs the properties of the carrier signal. While linear phase stepping is favorable due to its simplicity, in some settings, fast data acquisition cannot be performed easily with a controlled stepping procedure. In this case, the current grating position has to be estimated for each shot in a pre-processing step and $\phi(t)$ can have a more complex shape. The dark-field CT prototype discussed in [24], for example, achieves a fast phase movement by exploiting system vibrations, which lead to a sinusoidal phase movement [26]. Optimizing the phase stepping for sliding window phase retrieval to maximize the spectral separation of the different carrier terms in this general setting is not trivial and is not discussed here. Here, I focus on improving the demodulation algorithm itself, even for signals with non-optimal phase stepping. Nevertheless, I want to highlight the importance of proper phase sampling as a major factor in avoiding cross-talk-artifacts. A discussion of this topic is given in chapter 4.

3.4.2 Extensions of the Sliding-Window Approach

Conventional sliding-window-based demodulation uses the SPR algorithm and the assumption of a stationary sample within each window. Depending on the employed smoothing kernel, the approach performs a filtering operation on the measured data. In this section, three extensions of the algorithm are introduced that change its implicit filter characteristics and improve the demodulation of sliding window measurements.

Conventional Sliding Window

The general concept of the sliding window approach was introduced in Section 3.2. In summary, demodulation is achieved by defining windows of consecutive shots centered around a respective central shot t_c and performing a least-squares optimization in each window. The goal is to find the sample parameters that minimize the cost function $\mathcal{C}_{\text{window}}$. This cost function is defined as:

$$\mathcal{C}_{\text{window}} = \sum_{t \in \text{window}} w(t - t_c) \left(y^{\text{meas}}(t) - y^{\text{window}}(t) \right)^2, \quad (3.21)$$

with weights w that can be used to introduce a distance weighting in the window to weight shots less towards the borders of the window. Generally, all parameters of the forward model (2.18) depend on the time t and the individual pixel x . However, demodulation in the conventional case is performed for each pixel separately. The pixel dependency is thus omitted in the following. Furthermore, it should be noted that I assume that the interferometer state $\mathbf{X}$ is known from previous processing steps and only the sample parameters need to be determined.

In each window the sample parameters $M(t)$, $C(t)$, and $S(t)$ are approximated to be constant and are denoted by:

$$\tilde{\mathbf{b}} = \left(\tilde{M}, \tilde{C}, \tilde{S} \right)^T. \quad (3.22)$$

Consequently, the model function y_{window} assumed in each window is given by:

$$y^{\text{window}}(t) = \mathbf{x}_t \tilde{\mathbf{b}}. \quad (3.23)$$

This model is linear in the sample parameters. Therefore, the least-squares optimization is a linear regression, which possesses an analytical solution that yields the optimal value of $\tilde{\mathbf{b}}$:

$$\tilde{\mathbf{b}}^{\text{opt}} = \arg \min_{\tilde{\mathbf{b}}} \mathcal{C}_{\text{window}}. \quad (3.24)$$

The full demodulation of the data sinogram is achieved by iterating over all shots and pixels and at each step processing the window centered around t_c . The sample parameters at the current shot are then given by the least-squares optimization results.

This type of processing implicitly employs a low-pass filter that is defined by the window-size and window-weights w . Constant weights, for example, perform an implicit moving average filter during the sample data extraction. By changing the weights, the filter response can be adapted. Generally, a wider weight kernel performs worse at object edges but is more robust against noise and bad phase sampling. For choosing a suitable window size, one can analyze the width of the magnitude of the Fourier transform of the weight kernel $w(t)$. Ideally, it should fall below the noise level between DC and the frequency of the first phase stepping carrier peak. By changing the window size, this cutoff frequency can be shifted to adapt for the different bandwidths of the sample signals. In practice, it is favorable to shift the cutoff closer to the carrier peak than towards DC due to the stronger attenuation signal and thus its wider bandwidth above the noise level.

Local Polynomial Sliding Window

An alternative way of changing the implicit filtering of the sliding window processing is to combine it with a so-called Savitzky-Golay filter [51]. This filter locally approximates the data by fitting polynomials up to a given order within a specified window. Since the Savitzky-Golay filter is also based on a sliding window approach, it can be integrated easily in the sliding window phase retrieval algorithm. For this, Equation (3.26) is extended, and each sample parameter is represented locally as a linear combination of known basis functions and their a priori unknown coefficients:

$$\tilde{\mathbf{b}}^{\text{poly}}(t) = \begin{pmatrix} \sum_m a_m \alpha_m(t) \\ \sum_n b_n \beta_n(t) \\ \sum_k g_k \gamma_k(t) \end{pmatrix} \quad (3.25)$$

For simplicity, the basis functions α_m , β_n , and γ_k are chosen to be polynomials of t . By selecting different polynomial orders the filter roll off and pass band response can be adjusted [52] and made more suitable for the task of signal demodulation. In practice, this adjustment of the polynomial orders is performed empirically to yield the best visual results. Intuitively, the local polynomial fitting of the sample channels can also be thought of as approximating the movement of the sample's projection within a window. Since this approach improves the wrong assumption of a stationary projection within one processing window, movement-induced cross-talk-artifacts are reduced.

The model function for the polynomial SPR $y_{\text{window}}^{\text{poly}}$

assumed in each window is given by:

$$y^{\text{poly}}(t) = \mathbf{x}_t \tilde{\mathbf{b}}^{\text{poly}}(t). \quad (3.26)$$

Again, this model is linear in the basis function coefficients. Therefore, a linear regression yields the optimal coefficients for the respective window:

$$a_m^{\text{opt}}, b_n^{\text{opt}}, g_k^{\text{opt}} = \arg \min_{a_m, b_n, g_k} \mathcal{C}_{\text{window}}. \quad (3.27)$$

The sample parameters at the window's center position t_c are then given by evaluating Equations (3.25) at t_c using the least-squares optimization results $a_m^{\text{opt}}, b_n^{\text{opt}}, g_k^{\text{opt}}$.

For completeness, I would like to mention that this local polynomial approach could be extended to a global method. For this, the basis functions used in Equations (3.25) have to be defined on the full measured sinogram. This enables, for example, the use of B-spline basis functions to generate a continuous spline estimate for each sample channel. The full demodulation of the data sinogram is then again achieved by solving only one large linear system of equations instead of splitting it into multiple smaller ones for each window.

Dual-Pass Sliding Window

Experience shows that cross-talk-artifacts are mainly caused by the varying attenuation within the window. Including an estimate of this attenuation variation into the phase retrieval algorithm is therefore expected to reduce the cross-talk substantially. Therefore, a second approach to avoid the emergence of cross-talk-artifacts is to perform a dual pass procedure and execute the sliding window algorithm twice. In the first sliding window iteration, a guess for the transmission channel T is found. Next, an attenuation corrected measurement y' is calculated:

$$y' = \frac{y}{T}. \quad (3.28)$$

Finally, the sliding window algorithm is applied again to y' and D' and Φ' are extracted. In this iteration, the extracted mean values M' are discarded and set to 1 for the inverse variable transformation (2.33), resulting in

$$D' = \sqrt{S'^2 + C'^2}. \quad (3.29)$$

As this approach explicitly performs a separation of the DC modulation information and the phase stepping carrier terms, cross-talk-artifacts between the attenuation and the modulation channels are mitigated. Furthermore, it enables one to use different algorithmic parameters during each sliding window step, for example, it is possible to use a smaller window size

for the first pass, to extract a sharp attenuation signal, and then use a larger window to extract the dark-field channel when the resolution is of lower priority and a stronger filtering is beneficial to combat noise.

Similarly to the local polynomial approach, this algorithm should be insensitive to slow attenuation changes because in the second pass the attenuation is no longer incorrectly assumed to be stationary within one window. Therefore, it should perform well in windows where the attenuation changes slowly enough to be correctly identified during the first pass. However, in windows where the attenuation changes too rapidly, e.g. due to a sharp edge, the first-pass attenuation does not correctly describe the attenuation and cross-talk-artifacts can still emerge at the position of the edge.

Patch-Wise Sliding Window

Furthermore, as means to mitigate cross-talk-artifacts, the sliding window concept from [50] can be extended from pure temporal to spatio-temporal domain. This is done by defining patches of pixels on the detector and consecutive shots in the measured sinogram. In each patch one optimizes the cost function $\mathcal{C}_{\text{patch}}$:

$$\mathcal{C}_{\text{patch}} = \sum_{x,t \in \text{patch}} w(x,t) \left(y^{\text{meas}}(x,t) - y^{\text{patch}}(x,t) \right)^2, \quad (3.30)$$

while assuming the general model:

$$y^{\text{patch}}(x,t) = \mathbf{x}_{x,t} \tilde{\mathbf{b}}^{\text{poly}}(x,t). \quad (3.31)$$

Here, the basis functions can again be chosen to be polynomials in x and t to combine the patch-wise and polynomial approach or can be simple constant functions. Apart from a higher statistical sampling, this approach has the main advantage that the phase information of neighboring pixels can also be exploited during the optimization step. Therefore, a spatial modulation of the measurement can be used to enhance the effective phase sampling leading to further separation of the sample signals in the frequency domain. This is because the phase stepping carrier peak is then not only shifted along the time direction in the Fourier space of the data sinogram but also orthogonally in spatial domain like in the carrier fringe approach. Therefore, the signals are separated along an additional axis.

Consequently, this method benefits from a careful adjustment of the Moiré fringe pattern on the detector that results in a gradient in the interferometer phase $\phi(x)$. Similar to linear per-shot stepping, this “spatial stepping” benefits from dense fringes and thus a steep phase gradient over the detector.

Table 3.1: Simulation Sliding Window Parameters

	window size	patch size	polynomial orders
Conventional SW	15	1×1	$m = n = k = 0$
Dual-pass SW 1st	9	1×1	$m = n = k = 0$
Dual-pass SW 2nd	15	1×1	$m = n = k = 0$
Polynomial SW	15	1×1	$m \leq 3, n, k \leq 2$
Patch-wise SW	11	5×1	$m = n = k = 0$
Combined SW 1st	15	1×1	$m \leq 3, n, k \leq 2$
Combined SW 2nd	11	5×1	$m = n = k = 0$

Table 3.2: Normalized root-mean-squared errors (NRMSE) for simulation of the head phantom

	NRMSE in %		
	Attenuation	Dark-field	Phase
Conventional SW	2.7	75.7	38.8
Dual-pass SW	1.2	26.6	9.2
Polynomial SW	0.7	25.3	7.2
Patch-wise SW	1.6	3.9	23.4
Combined SW	0.7	3.1	1.6

3.4.3 Simulation Study

To evaluate and demonstrate the performance of the introduced extensions of the conventional sliding window approach for CT imaging, two simulation studies were performed. In both studies, a cylindrical detector and a cone beam geometry with a voxel size of 0.625 mm and 800x800x48 voxels was used, the same as used for reconstructions at the setup described in [24]. Furthermore, the simulated grating and geometry parameters coincide with the values given in [42]. The simulated system geometry thus matches the modified Philips Brilliance iCT that was used for the experimental measurements in Section 3.4.4. In all examples, 2400 shots were simulated in an axial scan assuming a continuous rotation with a total acquisition time of 1.5 s. For the interferometer parameters, an interferometer visibility of $V = 20\%$ was assumed. The simulated fringes were assumed as uniform and horizontal with a period of 5.3 pixels in z -direction, which defines the interferometer pixel phase-map $\phi_p(x)$. Furthermore, a uniform intensity of $I = 1$ was assumed. In both studies, a ground truth phantom was designed, and each channel was forward projected to generate the ground truth projections T , D , and Φ . Finally, the

Table 3.3: Normalized root-mean-squared errors (NRMSE) for simulation of the thorax phantom

	NRMSE in %		
	Attenuation	Dark-field	Phase
Conventional SW	4.1	300.7	120.9
Dual-pass SW	3.7	295.7	118.8
Polynomial SW	4.2	454.2	175.5
Patch-wise SW	2.4	185.1	68.1
Combined SW	4.2	208.8	75.1

simulated measurement y was calculated according to Equation (2.18). All used projectors are based on blob-footprints as described in [53], which were adapted for the cone beam geometry of the setup. For the reconstruction of the phase channel, the Hilbert kernel was used during FBP. To demonstrate the capabilities of the various demodulation approaches under non-ideal conditions, a non-optimal phase sampling was chosen for the simulation: For the phase sampling, the carrier information sidebands overlap significantly with the DC modulation information, even though a better separation could be achieved by a more optimized grating movement. Based on the phase movement observed by [24, 26], the phase was modeled as an oscillation

$$\phi_s(t) = z \sin(2\pi f_\phi t), \quad (3.32)$$

with an amplitude $z = 1.5\pi$ and a frequency of $f_\phi = 175$ Hz. The total interferometer phase is given by:

$$\phi(x, t) = \phi_p(x) + \phi_s(t). \quad (3.33)$$

In the first study, the FORBILD head phantom is used as a sample in all three channels [54]. To clearly isolate the effect of channel cross-talk, in each channel the phantom was rotated with respect to the other two channels. For the attenuation and the phase channel, the phantom was scaled to have voxel values of $1.4 \cdot 10^{-2} \text{ mm}^{-1}$ in the brain matter. In the dark-field the voxels of the brain matter are scaled to have a value of $3.3 \cdot 10^{-3} \text{ mm}^{-1}$. This results in a smallest transmission T value of approximately 0.23%, a smallest dark-field value D of 46% and a maximum differential phase shift Φ of 0.2 rad. These projection values are motivated by prior phantom measurement results with the human sized dark-field CT prototype, which are discussed in [24]. The results of this simulation are depicted in Figure 3.3.

To assess the algorithms' performance for more realistic samples, the study was repeated with a modified FORBILD thorax phantom, where additionally Poisson noise was applied to the simulated data, assuming a total incident per pixel photon count of 10^8 . This value was chosen to match the experimental data in Section 3.4.4. For the attenuation channel, the phantom was scaled to have a value of $2 \cdot 10^{-2} \text{ mm}^{-1}$ in the soft tissue. The phase channel was scaled to have a soft tissue value of $1.6 \cdot 10^{-2} \text{ mm}^{-1}$. The bones of the phase channel were furthermore set to have 1.5 times the value of the soft tissue. For the dark-field channel, only the phantom's lungs and bones with respective values of $5 \cdot 10^{-3} \text{ mm}^{-1}$ and $1.2 \cdot 10^{-2} \text{ mm}^{-1}$ were used. These values are motivated by prior phantom measurement results with the human sized dark-field CT prototype, which are discussed in [24] and should resemble values of the human thorax phantom used in Section 3.4.4. The results of this test case are summarized in Figure 3.4 and Table 3.3.

For the demodulation of the "conventional SW" and the "polynomial SW" cases a window size of 15 was used. To reduce the movement-induced cross-talk-artifacts, in all examples von Hann window weighting [55] was performed in Equation (3.21). Additionally, in the polynomial case the basis functions $\alpha_n(t)$ were chosen to be monomials in t up to third order and $\beta_n(t)$ and $\gamma_k(t)$ up to second order. For the "dual-pass" example, first a demodulation with a window size of 9 was performed to estimate the transmission. In the second pass, a window size of 15 was used on the attenuation corrected data according to Equation (3.43). The narrower weight kernel in the first pass has a frequency response that falls below 10% right at the first carrier peak to maximize the retrieval of high-frequent transmission information. Then, in the second pass for the wider window size of 15 the frequency response drops below 10% approximately halfway between the phase stepping carrier peak and DC to allow for a better retrieval of C and S. Furthermore, for the "patch-wise" example the horizontal fringe pattern was exploited to achieve additional spatial phase sampling and patches of 11 shots and five pixels in axial direction were used for demodulation. Finally, for the "combined" example, the "polynomial" case was treated as a first pass and used for the attenuation correction. In the second pass, a patch-wise demodulation was performed with the same parameters as in the "patch-wise" example. The parameters of the sliding window algorithm for each test case are summarized in Table 3.1. The window and patch sizes of all test cases were chosen empirically to achieve the best visual results.

For a quantitative analysis, the root-mean-squared errors were calculated for each channel and example, and normalized to the ground truth mean value (NRMSE). To remove the influence of the FBP reconstruction error, the ground truth images were first forward projected and afterward reconstructed using the FBP implementation. For the calculation of the NRMSE values given in Tables 3.2 and 3.3, these ground truth reconstructions were used.

3.4.4 Experimental Results

To further demonstrate the capabilities of the shown algorithmic extensions for real measurements, they were applied to data from the dark-field CT prototype presented in [24]. At this setup, the dark-field channel is of primary interest, and it is therefore the focus of the following analysis. The measurement consists of an axial scan of a thorax phantom that was performed with a tube voltage of 80 kVp and a current setting of 550 mAs. Over a rotation period of 1.5 s a total of 4800 projections were acquired. The effective detector size that is covered by gratings is 32 by 590 pixels. Based on the manufacturer's information about the unmodified scanner and assuming an absorption of

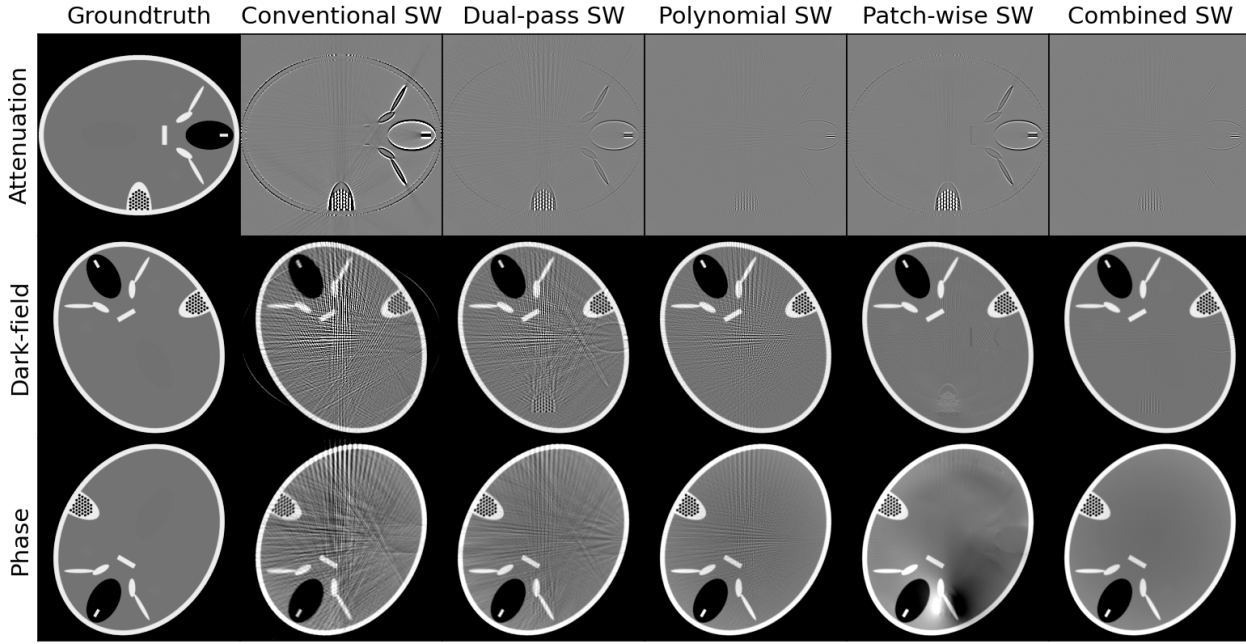


Figure 3.3: Comparison of different sliding window demodulation methods. The rows from top to bottom feature reconstructions of a phantom's attenuation and the reconstructed dark-field and phase channels. Each column features a different demodulation method as outlined in Table 3.1. In the attenuation channel, difference images to the ground truth are depicted. Figure adapted from [43].

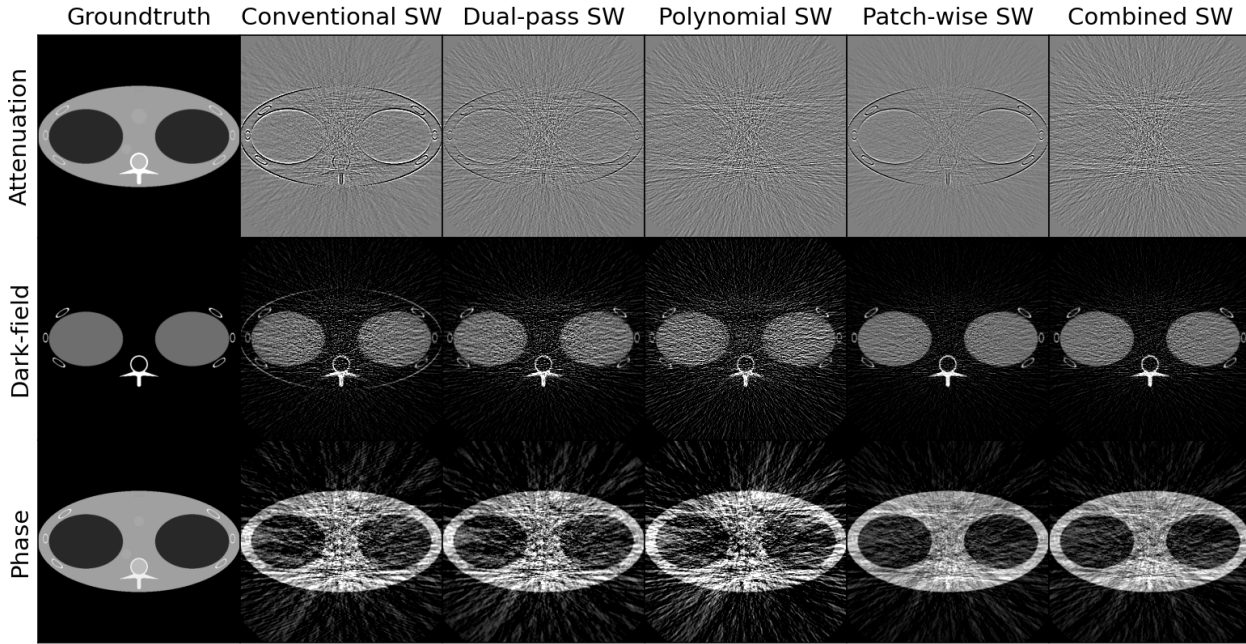


Figure 3.4: Comparison of different sliding window demodulation methods for the FORBILD thorax phantom. Except for the application of Poisson noise to the simulation, all parameters have been kept the same as in Figure 3.3. Figure adapted from [43].

50% for each the G0 and the G2 grating, the per shot photon count is estimated to be $2 \cdot 10^4$ for this measurement. To be able to exploit the spatial modulation of the phase, a horizontal fringe pattern was tuned on the detector. A section of this pattern is depicted in

Figure 3.7. The phase stepping carrier was obtained by the blank-scan processing described by [27], it is driven by multiple oscillations. The primary high-frequency phase oscillation (see Equation (4.49)) has a frequency of approximately $f_\phi = 175$ Hz.

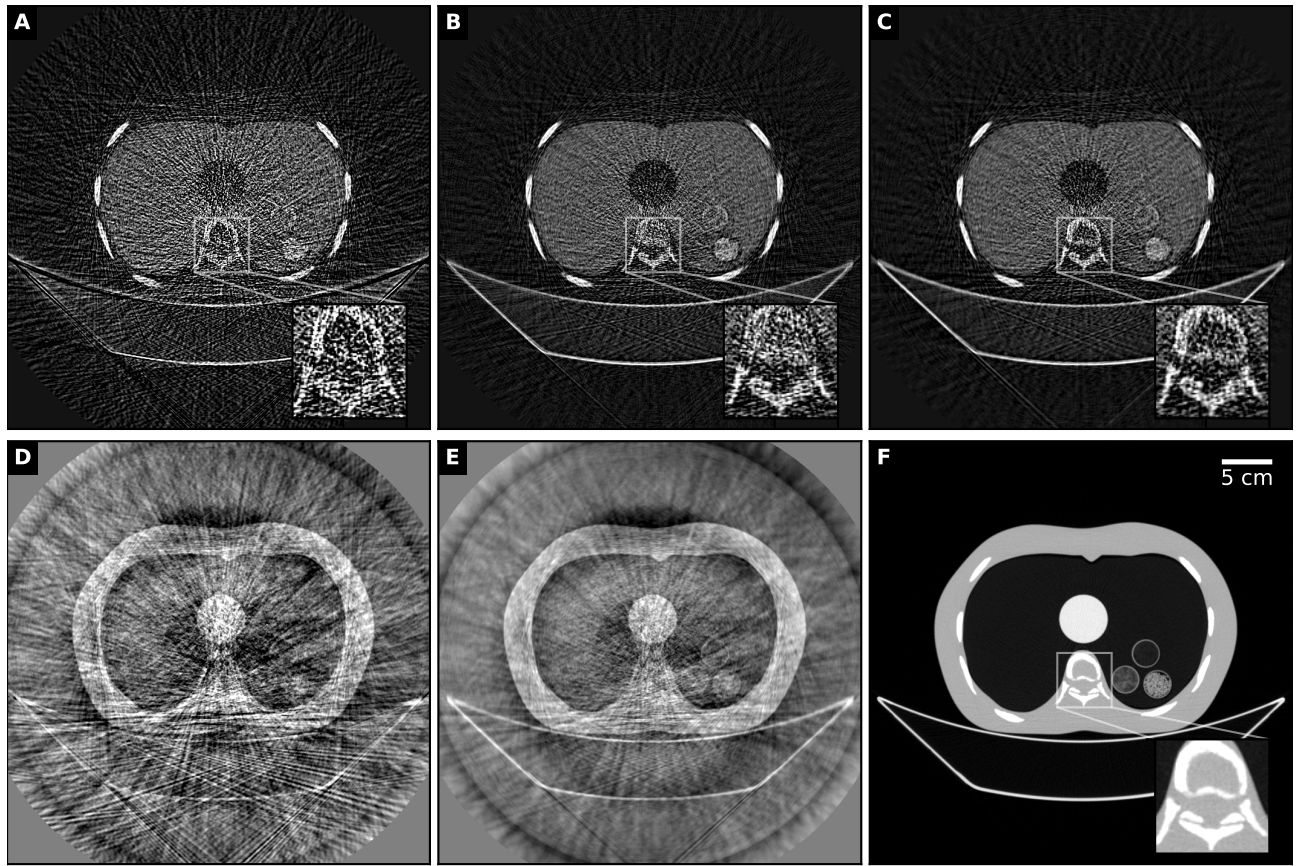


Figure 3.5: Reconstructed dark-field channels that were acquired with **A:** the conventional sliding window algorithm. **B:** a combination of the proposed polynomial and patch-wise demodulation approach. **C:** a combination of the proposed dual-pass and patch-wise demodulation approach. **D:** Reconstructed phase channel acquired with the same algorithm as A. **E:** Reconstructed phase channel acquired with the same algorithm as C. **F:** Reconstructed attenuation channel (window level, -420 HU; window width, 580 HU). Figure adapted from [43].

Three demodulation techniques were tested for this data set, where the demodulation parameters were optimized for the experimental system and measurement settings, and thus differ slightly from the simulation examples. First, a conventional sliding window phase retrieval was performed with a window size of 21 and von Hann window weights. Next, a patch-wise demodulation with 5 pixels in axial direction, i.e., parallel to the rotation axis, and a window size of 21 was performed. Additionally, polynomials of the first order in t were used to model the movement of the sample's projection in the channels C and S , respectively. For the DC channel M , the sample's change in axial direction was modeled with the following set of basis functions: $\{1, t, z, z^2, tz, tz^2\}$. No window weighting was used in this case due to the high noise level. In the third case, the dual-pass approach was applied with a first pass window size of 17 with von Hann window weights. The second pass was performed with a patch-wise demodulation with 5 pixels in axial direction and a window size of 21 without any window weights. The window and patch sizes of all test cases were chosen empirically to achieve the best visual results.

Apart from a beam hardening correction that is described in [26] no major post-processing was applied after reconstruction in any case. The resulting dark-field and phase channels are depicted in Figure 5.6 and Figure 3.6 along with the reconstructed attenuation channel derived from the dual pass demodulation.

3.4.5 Discussion

Multiple observations can be made by analyzing the simulation results in Figure 3.3 and Table 3.2. First, there are strong cross-talk-artifacts for the results of the conventional sliding window demodulation. In the dark-field channel of the conventionally demodulated case, the occurring cross-talk-artifacts are especially severe at the high contrast edges of the attenuation image which produces strong streaking artifacts. In the phase channel, the artifacts are less localized, but the overall reconstruction quality is poor as well.

By contrast, all the presented extensions of the sliding window approach reduce the occurring artifacts substantially. Each of the approaches possesses individual strengths and weaknesses and in our opinion have

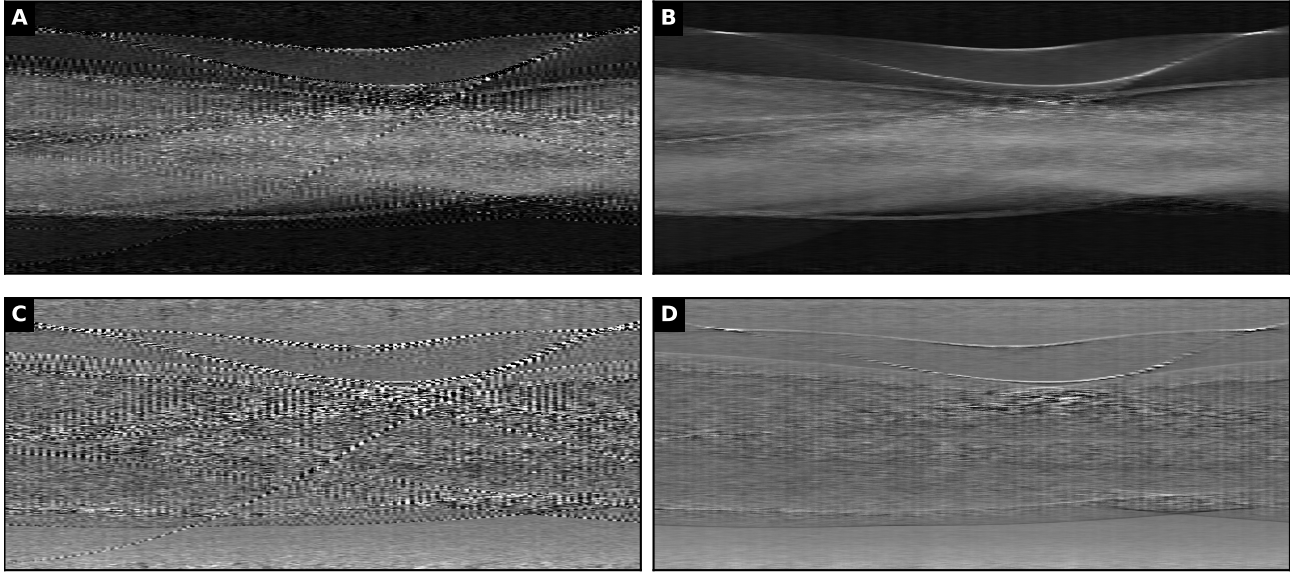


Figure 3.6: **A:** Sinogram section of the dark-field channel depicted in Figure 5.6A. **B:** Sinogram section of the dark-field channel depicted in Figure 5.6C. **C:** Sinogram section of the phase channel depicted in Figure 5.6D. **D:** Sinogram section of the phase channel depicted in Figure 5.6E. Figure adapted from [43].

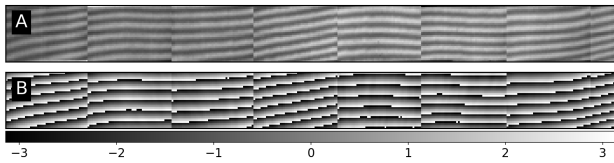


Figure 3.7: **A:** Measured intensity y on a detector section that features Moiré fringes due to a de-tuning of the interferometer gratings G2 and G1. **B:** The corresponding phase ϕ shows a phase gradient in axial direction that can be exploited for additional phase sampling during demodulation. Figure adapted from [43].

use-cases depending on the setup parameters: While the dual-pass approach shows slightly stronger cross-talk in the simulation example than the polynomial demodulation, it is the easiest to implement and is more robust against noise. The polynomial approach yields visually better results for the noise free simulation and the best NRMSE value in the attenuation channel of all simulation examples. However, for the dark-field and phase channels it is more susceptible to noise and therefore less suited for noisy data. This becomes evident, e.g., in the results of the noise study and the high NRMSE value of the dark-field channel in Table 3.3 which can be attributed to the high noise level in the reconstruction. This poor behavior for noisy data is a well known problem of polynomial regression methods as they tend to overfit noise. The patch-wise approach achieves better results for the dark-field channel with less streaking for both the noise-free and the noisy simulations. However, it also features two drawbacks. First, the higher statistical and phase sampling power comes at the price of reducing the resolution in

axial direction, due to the wider demodulation kernel. Second, sharp edges of the sample in axial direction introduce additional cross-talk-artifacts similar to sharp edges in angular direction. This, however, can be mitigated by combining the patch-wise with the polynomial and/or dual-pass approach, which can be seen in the combined example. Such a combined method was therefore applied to the experimental data.

Generally, it should be emphasized that all the presented techniques can be combined for best results. For example, it is possible to perform the first pass step with the polynomial approach and then in the second pass use the patch wise approach to extract the dark-field and phase, benefitting from additional spatial phase sampling. As presented in the “combined” case, this yields the best visual results for the simulation of the first test case.

For the attenuation channel, it can be observed that in all test cases, the most prominent artifacts are related to blurring due to the sliding window approach that averages adjacent shots. This results in a loss of resolution that depends on the distance to the iso-center because the projection of edges further away from the center move faster over the detector. In the simulation, both the polynomial and the combined method achieve a reduction of this blurring by either modeling the movement or using smaller processing windows. Naturally, the stronger blurring also reduces noise, which explains the low NRMSE values of the patch-wise demodulation in the noisy test case. Therefore, it achieves a better value than the polynomial or combined cases, even though it performs worse at retrieving edge information.

For the experimental data, it is immediately visible in Figure 5.6B and C that both advanced demodulation techniques suppress the occurring cross-talk-artifacts in the dark-field channel successfully and reduce streaking considerably. This is especially noticeable at sharp edges far away from the iso-center, like the patient couch, that produce a high-frequency signal in the sinogram [56]. Furthermore, the overall image quality benefits from the stronger implicit filtering of the patch-wise demodulation and the reconstructions appear less noisy. However, this stronger filtering also leads to a loss of resolution. This can be observed, for example, at the vertebrae of the phantom. In the attenuation, it is clearly visible that in the shown slice the vertebrae is hollow. However, due to its change in axial direction it appears more solid in the second test case depicted in Figure 5.6B because this approach averages adjacent slices. The dual-pass approach can mitigate this problem by not employing a patch-wise demodulation in the first pass. The guessed attenuation is therefore better suited to describe changes in axial direction during the second-pass. Consequently, for this example the patch-wise dual-pass demodulation yields better results. In the phase channel, both advanced demodulation results yield very similar results. Therefore, only the result from the combined dual-pass and patch-wise demodulation is shown in Figure 5.6E. Again, both techniques lead to a reduction in the movement-induced streaking artifacts compared to the conventional approach. The remaining artifacts in the phase channel can be related to the scanners low sensitivity for phase-contrast imaging as it is primarily designed and optimized for dark-field imaging.

The demodulation quality of the proposed algorithm in the third test case can also be observed in the sections of the phase and the dark-field sinograms – $\log(D)$ depicted in Figure 3.6B and Figure 3.6D. In the conventionally demodulated sinograms shown in Figure 3.6A and Figure 3.6C there are strong cross-talk-artifacts that are especially prominent at sharp edges. These artifacts resemble the shape of the employed fringe pattern and show an oscillating behavior. For the demodulation that has been performed with the proposed advanced techniques, these artifacts have been reduced significantly and the sinograms appears smoother.

3.4.6 Conclusion

In summary, this section addresses the challenge of improving sliding window phase retrieval to enable fast data acquisitions for clinical phase contrast and dark-field CT. A continuous tomographic data acquisition shows the sample under a different angle for each projection. This leads to movement-induced cross-talk-artifacts when trying to demodulate the signal with a conventional sliding window approach.

I introduced a new interpretation of the phase retrieval problem for a sliding window acquisition as a demodulation problem. In this interpretation, the occurring cross-talk is attributed to partially overlapping modulation sidebands. To mitigate this effect, three advanced extensions of the sliding window SPR algorithm were presented. Since the proposed model extensions still lead to a linear optimization problem, demodulation can be performed in a fast and highly parallelized manner. For each test case for the experimental data, the respective implementation of the proposed algorithm took less than 30 s to demodulate the full data sinogram. In both the simulation study and on experimental data, an improvement in image quality could be shown with the proposed demodulation techniques. While the focus of this section lies on tomographic applications, it should be noticed that the concepts presented here also apply to other scanning-based techniques that employ grating-based imaging and a continuously moving detector or sample.

3.5 Global Tikhonov-Based Phase Retrieval

While the presented extensions of the sliding window method for phase retrieval significantly mitigate cross-talk-artifacts, there are still remaining artifacts at sharp edges that emerge when the projections of the sample move fast over the detector. Therefore, as an alternative to the sliding window method, I developed a second projection-based demodulation approach by using a different regularization function in Equation 3.9.

In the following, this global Tikhonov-based approach for regularization is introduced. Here, Tikhonov regularization is used to directly penalize high-frequency information in the sample vector $\mathbf{B}$. Therefore, like the sliding window method, it assumes some degree of smoothness for $\mathbf{b}_t$. Unlike the sliding window method, this leads to a global linear system that has to be solved, which cannot be split into smaller subsystems as the resulting system matrix is not block diagonal.

3.5.1 Method

For the global Tikhonov method, I suggest the regularization function in Equation (3.9) to be defined as:

$$\mathcal{R}_{\text{Tikhonov}} = \sum_{t=1}^N |\mathbf{b}_t - k * \mathbf{b}_t|^2, \quad (3.34)$$

where k is a symmetric smoothing kernel and the convolution is to be understood as component-wise. The expression $\mathbf{b}_t - k * \mathbf{b}_t$ effectively denotes a high-pass filter and the regularization thus penalizes high frequencies in the sample parameters. This can be motivated

by the demodulation interpretation of the phase retrieval problem introduced in [43], where each sample parameter in $\mathbf{b}_t$ can only occupy a limited bandwidth in Fourier space.

Equation (3.34) can be reformulated in matrix notation and for each component in $\mathbf{b}_t$ the convolution with the kernel $k(t)$ can be represented by a Toeplitz convolution matrix $\mathbf{K} \in \mathbb{R}^{N \times N}$. For axial CT, this matrix is a circulant matrix. By interleaved stacking of the three convolution matrices for each respective component (M , C , and S) into one large matrix $\mathbf{K} \in \mathbb{R}^{3N \times 3N}$, the component-wise convolution can be expressed as:

$$k * \mathbf{b}_t \equiv \mathbf{K}\mathbf{B}. \quad (3.35)$$

Writing (3.34) in matrix notation thus leads to:

$$\mathcal{R}_{\text{Tikhonov}}(\mathbf{B}) = \|\mathbf{B} - \mathbf{K}\mathbf{B}\|^2 \quad (3.36)$$

$$= \|(\mathbf{I} - \mathbf{K})\mathbf{B}\|^2 \quad (3.37)$$

$$= \|\mathbf{\Gamma}\mathbf{B}\|^2, \quad (3.38)$$

where $\mathbf{I}$ is a conformal identity matrix and the Tikhonov matrix $\mathbf{\Gamma} = \mathbf{I} - \mathbf{K}$ was introduced. The total cost function is then

$$\mathcal{L}_{\text{Tikhonov}}(\mathbf{B}) = \|\mathbf{y}_{\text{meas}} - \mathbf{X}\mathbf{B}\|^2 + \lambda \|\mathbf{\Gamma}\mathbf{B}\|^2. \quad (3.39)$$

The sample information vector $\hat{\mathbf{B}}$ that minimizes $\mathcal{L}_{\text{Tikhonov}}$ is given by the solution of the following linear system:

$$(\mathbf{X}^T\mathbf{X} + \lambda\mathbf{\Gamma}^T\mathbf{\Gamma})\hat{\mathbf{B}} = \mathbf{X}^T\mathbf{y}_{\text{meas}}. \quad (3.40)$$

As this approach connects all observed shots into one large linear $3N \times 3N$ system that is not block diagonal, it is a global method of phase retrieval, which is in contrast to the local sliding window approach.

While for simplicity I have so far only focused on a pixel-wise treatment of the phase retrieval problem, the Tikhonov method can be furthermore extended by using a two- or three-dimensional smoothing kernel. Analogous to the patch-wise sliding window method we described in [43], this enables one to exploit the additional spatial phase sampling of a Moiré fringe pattern on the detector. Furthermore, statistical weights can be added to the optimization problem, by including a diagonal weight matrix $\mathbf{W} \in \mathbb{R}^{N \times N}$ to the linear system (3.40):

$$(\mathbf{X}^T\mathbf{W}\mathbf{X} + \lambda\mathbf{\Gamma}^T\mathbf{\Gamma})\hat{\mathbf{B}} = \mathbf{X}^T\mathbf{W}\mathbf{y}_{\text{meas}}. \quad (3.41)$$

In this work, I choose

$$\mathbf{W}_{ii} = \frac{1}{y_i^{\text{meas}}} \quad (3.42)$$

for the statistical weights, which represents an estimate for the inverse of the observations' variances. The

weights can be used additionally to mask out invalid pixels that are either dead or that lie in regions on the detector that are not covered by gratings. This is for example the case at the dark-field CT prototype, where the G2 grating is stitched together from individual grating tiles [42]. Pixels at the border between two grating tiles are only partially covered by the G2 grating and are therefore discarded for phase retrieval.

In our prior work on the sliding window method [43], we found it beneficial to perform a dual-pass phase retrieval: In the first pass, the sample transmission T is extracted using a narrow kernel size and consecutively an attenuation corrected measurement y' is calculated:

$$y' = \frac{y^{\text{meas}}}{T}. \quad (3.43)$$

Next, in the second pass phase retrieval is performed on y' to yield D and Φ using a wider kernel and $1/y'$ as weights. I adapt this for the global phase retrieval method and perform two passes using the global phase retrieval method.

3.5.2 Implementation Details

In practice, the amount of observations, i.e., the number of pixels in the sinogram, can be quite large, which makes a direct inversion of the system matrix $(\mathbf{X}^T\mathbf{W}\mathbf{X} + \lambda\mathbf{\Gamma}^T\mathbf{\Gamma})$ unfeasible. However, since $\mathbf{X}^T\mathbf{W}\mathbf{X}$ is a block diagonal matrix with 3×3 blocks and $\mathbf{\Gamma}^T\mathbf{\Gamma}$ is a banded Toeplitz matrix with the bandwidth depending on the width of the kernel function k , the system matrix is very sparse. Therefore, the linear system can be solved using matrix-free methods.

An effective way to solve the large sparse linear system (3.40) is to use the conjugate gradient (CG) method, as the system matrix is symmetric and positive-definite. For this, the occurring matrices are implemented as sparse linear operators. In practice, a computationally efficient way to perform the component-wise convolution expressed by $\mathbf{\Gamma}^T\mathbf{\Gamma}$ is to use the Fast Fourier Transform algorithm and perform the convolution in Fourier space: Each component in $\mathbf{b}_t$ has to be convolved with a kernel η given by:

$$\eta = (\delta - k) * (\delta - k), \quad (3.44)$$

where δ is the dirac delta function.

Furthermore, to ensure faster convergence of the iterative CG method, a Jacobi preconditioner is used, i.e., one solves the preconditioned system

$$\mathbf{P}(\mathbf{X}^T\mathbf{W}\mathbf{X} + \lambda\mathbf{\Gamma}^T\mathbf{\Gamma})\hat{\mathbf{B}} = \mathbf{P}\mathbf{X}^T\mathbf{W}\mathbf{y}_{\text{meas}}. \quad (3.45)$$

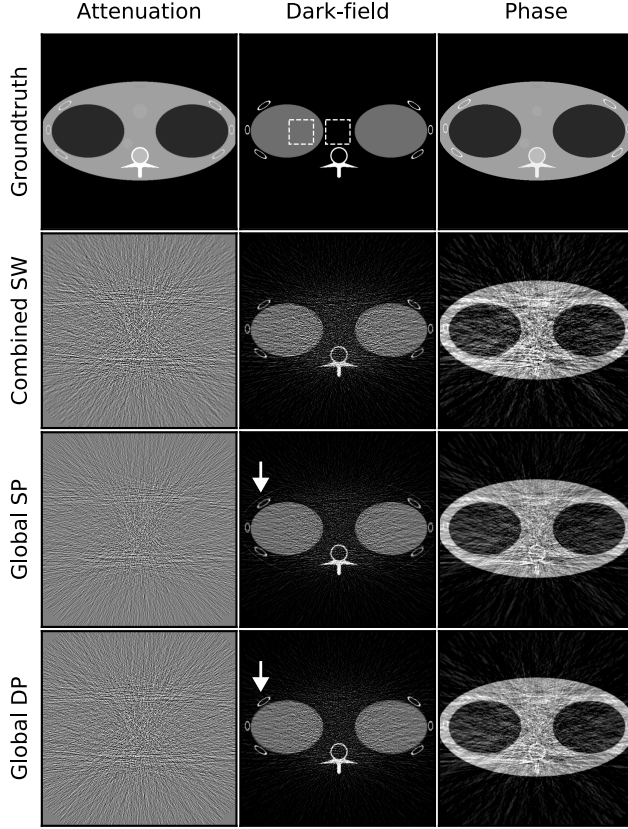


Figure 3.8: Comparison of different phase retrieval methods for the FORBILD thorax phantom simulation. The columns from left to right feature reconstructions of the phantom's attenuation and the reconstructed dark-field and phase channels. Each row features a different demodulation method. In the attenuation channel, difference images to the ground truth are depicted. In the dark-field channel's ground truth image, regions of interest for the analysis of the noise levels are indicated. Figure adapted from [44].

Here, a diagonal preconditioner $\mathbf{P}$ with elements:

$$P_{ij} = \frac{\delta_{ij}}{(\mathbf{X}^\top \mathbf{W} \mathbf{X} + \lambda \mathbf{\Gamma}^\top \mathbf{\Gamma})_{ii}} \quad (3.46)$$

$$= \frac{\delta_{ij}}{\mathbf{X}_{ii}^2 \mathbf{W}_{ii} + \lambda \eta(0)} \quad (3.47)$$

is used, where δ_{ij} is the Kronecker delta.

To optimize run-time of the algorithm, the GPU accelerated implementation of the conjugate gradient algorithm of Google Jax was used [57].

3.5.3 Simulation Study

First, the performance of the global phase retrieval method was tested in a simulation study. For this, the simulation described in [43] using the FORBILD thorax phantom was repeated to generate a simulated axial dark-field CT measurement. The simulation details can be found in our aforementioned publication. Next, three phase retrieval strategies were tested: First, the

Table 3.4: Normalized root-mean-squared errors (NRMSE) for simulation of the thorax phantom

	NRMSE in %		
	Attenuation	Dark-field	Phase
Combined SW	4.2	207.6	75.3
Global Phase Retrieval SP	3.8	160.2	56.7
Global Phase Retrieval DP	4.6	154.3	56.3

Table 3.5: Noise standard deviation in the dark-field channel for simulation of the thorax phantom in a.u.

	Lung	Center
Combined SW	0.15	0.21
Global Phase Retrieval SP	0.11	0.16
Global Phase Retrieval DP	0.11	0.16

combined sliding window method, which we found to be the best sliding window method in [43]. Second, a single pass (SP) global phase retrieval using a 2D kernel consisting of Hanning weights with a width of 3×5 pixels in angular and z-direction and a regularization strength of $\lambda = 0.7$. Third, a dual pass (DP) global phase retrieval. In the first pass, we used a 1D Hanning kernel with a width of 3 pixels in angular direction and $\lambda = 0.5$. In the second attenuation adjusted pass, the kernel function of the global phase retrieval method was chosen to be a 2D kernel consisting of Hanning weights with a width of 3×5 pixels in angular and z-direction and a regularization strength of $\lambda = 0.7$. The resulting reconstructions are depicted in Figure 3.8.

For a quantitative analysis, the root-mean-squared errors were calculated for each channel and test case and normalized to the ground truth mean value (NRMSE). To remove the influence of the FBP reconstruction error, the ground truth images were first forward projected and afterward reconstructed using the FBP implementation. For the calculation of the NRMSE values given in Table 3.4, these ground truth reconstructions were used. To analyze and compare the noise levels in the dark-field channels, the noise standard deviation was calculated in two regions of interest marked in Figure 3.8. These noise levels are summarized in Table 3.5.

Comparing the sliding window results with both the global SP and DP phase retrieval, it is evident that the latter methods perform substantially better in extracting the dark-field and phase channels. In both channels the NRMSE value is lower compared to the sliding window case. While in the SP case there are remaining cross-talk artifacts in the dark-field channel at edges far away from the iso-center, these are gone in the DP reconstruction and the NRMSE value is lower as a result. As summarized in Table 3.5, the noise standard deviations in both the lung and the center of the phantom are lower for both global methods compared to the

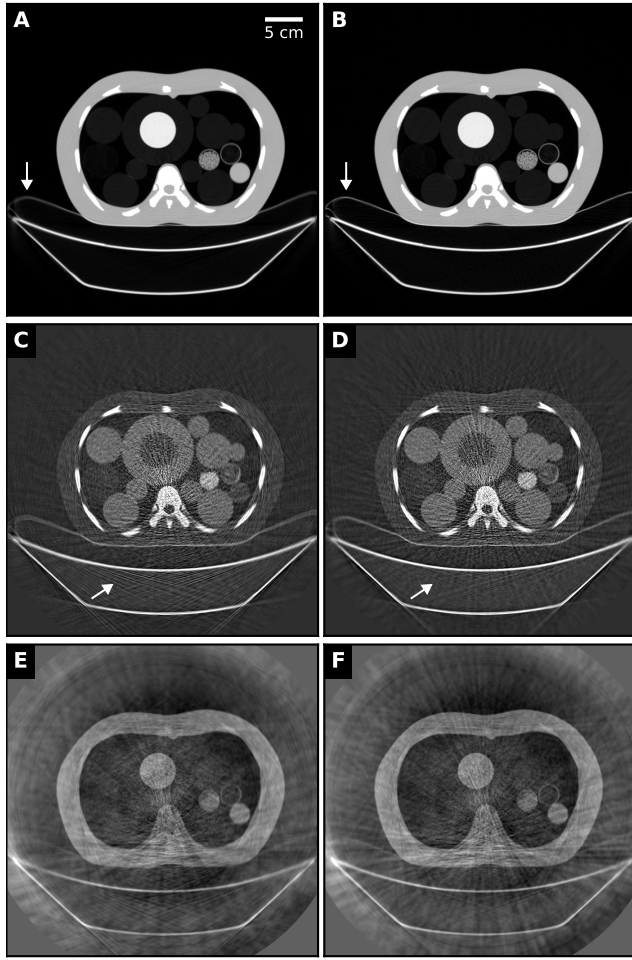


Figure 3.9: Reconstructions of an anthropomorphic thorax phantom acquired with the dark-field CT prototype. No further post-processing was applied after phase retrieval. **A:** attenuation channel extracted with the patch-wise SW method **B:** attenuation channel extracted with the global phase retrieval method **C:** dark-field channel extracted with the patch-wise SW method **D:** dark-field channel extracted with the global phase retrieval method **E:** phase channel extracted with the patch-wise SW method **F:** phase channel extracted with the global phase retrieval method. Figure adapted from [44].

sliding window case. This can be explained by the better implicit filtering of the regularization for the global phase retrieval. The main difference between the SP and the DP method is thus the reduction in cross-talk artifacts for the DP case. In the attenuation channel, all methods perform similarly, by a visual comparison of the difference images in Figure 3.8. However, in terms of NRMSE the SP method shows a slight advantage over the other two methods.

3.5.4 Experimental Results

To test the algorithm on experimental data, it was applied to data from the dark-field CT prototype presented in our prior work [24]. For this, an axial

scan of an anthropomorphic thorax phantom was performed using a tube voltage of 80 kVp and a current of 550 mA. 2400 projections were acquired over a rotation period of 1.5 s resulting in a sinogram with $N = 2400 \times 32 \times 590$ pixels. To simulate porous lung tissue, the phantom was filled with various neoprene cylinders as well as polyoxymethylene and water cylinders to simulate homogeneous non-scattering tissue. For the parameters of the global phase retrieval in the first pass, a 1D Hanning kernel with a width of 3 pixels in angular direction and $\lambda = 0.5$ was used. In the second pass, the kernel function of the global phase retrieval method was chosen to be a 2D kernel consisting of Hanning weights with a width of 7×7 pixels in angular and z-direction. Here, the spatial modulation of the interferometer phase ϕ in z-direction of the Moiré pattern is exploited for improved phase sampling, analogous to the patch-wise sliding window method discussed in [43]. The kernel sizes of both passes were chosen empirically to achieve the best visual results. The regularization strength was chosen as $\lambda = 0.7$ based on a parameter sweep (see Section 3.5.4).

Comparison with the Sliding Window Method

For comparison, the global phase retrieval method was tested against the combined sliding window approach introduced in our prior work [43]. Here, a patch size of 13 pixels in angular direction for the first pass and 15×7 pixels in angular and z-direction for the second pass was used. Both passes use Hanning weights in angular direction. The parameters of the sliding window method were chosen empirically to yield the best visual results. Figure 3.9 depicts the FBP reconstructed volumes and Figure 3.10 the extracted dark-field and phase sinograms after phase retrieval without further post-processing.

In the sinograms of both the dark-field and phase channels, it is evident that the patch-wise sliding window method yields corrupted object edges, since the assumption of a stationary sample is violated in processing patches containing moving edges with high contrast. In the projections in Figure 3.10E, F this results in a checkerboard pattern emerging at sharp edges with the frequency of the Moiré fringe pattern tuned on the detector. For the global phase retrieval method, these artifacts are mitigated substantially and edges are extracted artifact free without a loss in sharpness as evident in the projections shown in Figure 3.10G, H.

In the reconstructed volumes of the dark-field and the phase channel, this leads to a reduction in streaking artifacts, which can be noticed, e.g., inside the patient couch. Furthermore, edges far away from the iso-center are extracted more faithfully and less blurred. This is evident when comparing the edges of the patient couch and cushion at the border of the reconstruction volume, which appear less sharp in the sliding

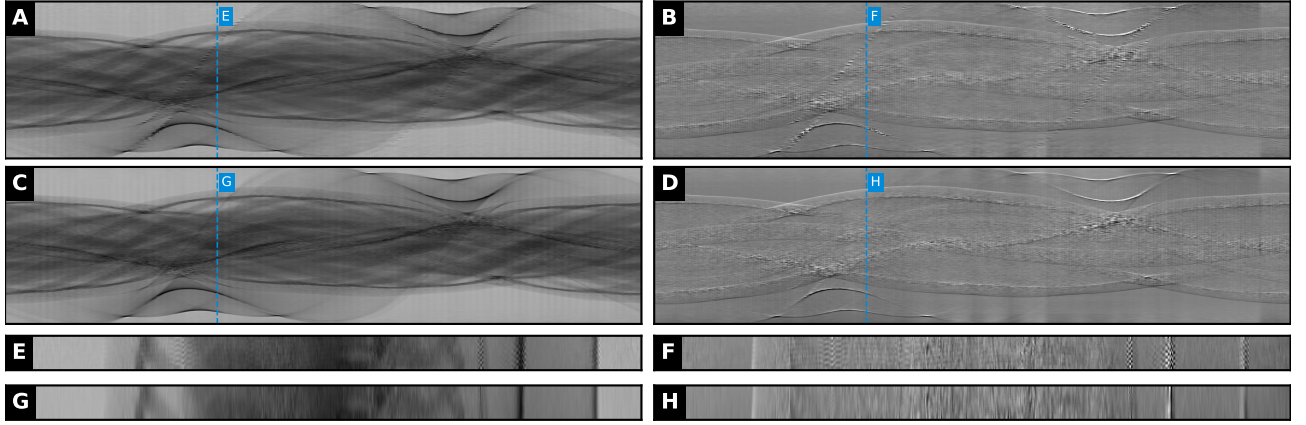


Figure 3.10: Dark-field and phase sinograms of a central slice from the reconstructions shown in Figure 3.9. **A:** dark-field sinogram extracted with the patch-wise SW method **B:** phase sinogram extracted with the patch-wise SW method **C:** dark-field sinogram extracted with the global phase retrieval method **D:** phase sinogram extracted with the global phase retrieval method **E-H:** Projections of the extracted channels as indicated in the respective sinograms. Figure adapted from [44].

window reconstructions than in the global phase retrieval reconstructions.

In terms of run-time, the performance of both algorithms is similar: Both implementations of the patch-wise sliding window method and the global phase retrieval take approximately ten seconds to demodulate the measured sinogram using a Nvidia Titan RTX GPU.

Influence of the Regularization Strength

To choose and analyze the influence of the regularization strength λ for the second pass, a parameter sweep was performed over the full range of the problem's L-curve [58]. For this the second pass was performed with 36 different logarithmically spaced values of λ ranging from 10^{-5} to 1.2×10^3 . For each resulting sample vector $\hat{\mathbf{B}}$ the value of the data term $\|\mathbf{w}\mathbf{X}\hat{\mathbf{B}} - \mathbf{w}\mathbf{y}\|$ and the regularization term $\|\mathbf{\Gamma}\hat{\mathbf{B}}\|$ were calculated. Here, I defined $\mathbf{w} \in \mathbb{R}^{N \times N}$ to be the factorized weight matrix:

$$\mathbf{w}_{ij} = \frac{\delta_{ij}}{\sqrt{\mathbf{W}_{ii}}}. \quad (3.48)$$

A plot of the L-curve alongside example reconstructions of the resulting dark-field channels is depicted in Figure 3.11. Here, one can observe that a stronger regularization strength λ results in stronger blurring of the final reconstructions. This blurring increases in strength with the distance to the iso-center. For extreme values of λ one thus notices either very noisy reconstructions (for very low values of λ) or over-smoothed reconstructions (for very high values). A compromise between these two extremes has to be found. While the L-curve criterion suggests the value of λ at the point of maximal curvature in the corner of the curve

($\lambda = 7 \times 10^{-4}$), I find that additional smoothing improves the visual quality of the reconstruction. Therefore, for the reconstructions depicted in Figure 3.9 I chose a higher value of $\lambda = 7 \times 10^{-1}$. However, this stronger implicit filtering of the global method could also be replaced by additional post-processing and filtering steps after phase retrieval. An advantage of filtering in post-processing is that it can be performed in image space, which avoids the radial dependency of the blurring strength.

3.5.5 Discussion

Applying the global phase retrieval method to the simulated and the measured scan of the thorax phantom showed a clear superiority of the method in comparison to the sliding window phase retrieval. In all three imaging channels, a better or similar image quality could be achieved. The main difference is the better edge retrieval of the global method, which leads to less streaking artifacts and sharper edges far away from the iso-center.

In terms of computational run-time, similar requirements for the global and the patch-wise sliding window method can be observed. For the dark-field CT prototype, the global phase retrieval method can thus fully replace the sliding window phase retrieval without any drawbacks.

Analyzing the influence of the regularization strength, I find that for a good retrieval of the dark-field channel a stronger λ than suggested by the L-curve criterion is advantageous. This is due to the stronger smoothing and high noise level of the dark-field channel. In practice, I thus recommend finding the regularization strength via a parameter sweep and assessing the image quality for each value of λ .

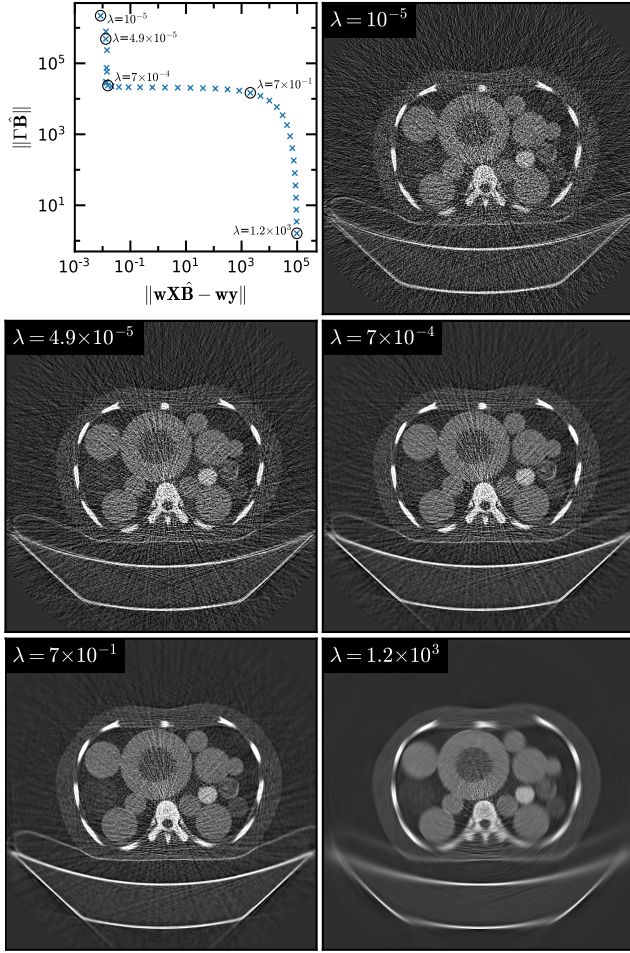


Figure 3.11: L-curve of the second pass of the regularization problem alongside example dark-field reconstructions at selected regularization strengths λ . Stronger regularization strengths yield stronger blurring of the three sinograms M, C, S and consequently in the final reconstructions. Figure adapted from [44].

when the projections of the sample move fast over the detector.

With the proposed global phase-retrieval algorithm, I presented a robust and fast procedure to extract the different sample channels in continuously acquired grating-based imaging. The global Tikhonov-based phase retrieval method is capable of substantially mitigating these cross-talk artifacts, even at sharp edges. In this regard, it outperforms the previous advanced sliding window methods and retrieves edges in the sinogram artifact-free, without compromising sharpness or run-time speeds. Since the proposed algorithm leads to a linear optimization problem with a symmetric and positive definite system matrix, demodulation can be performed fast and efficiently using existing implementations of the conjugate gradient method. While I have focused on the application of the global phase retrieval method for grating-based dark-field CT imaging in this work, I also note that the method can be applied to any scanning-based method in grating-based or coded aperture imaging.

Finally, I'd like to note that the global regularization approach could also be extended further by including additional regularization terms. These could, for example, enforce tomographic consistency in the extracted sinograms or add additional prior information one might have about the measured sample signal. Additionally, the approach could be modified to use a Poissonian likelihood term to better account for the low counting statistics. However, this would require a gradient based approach for optimization.

3.5.6 Conclusion

To enable rapid data acquisition for clinical applications of grating-based CT, a continuous acquisition protocol using a fast rotating gantry is vital. However, this necessitates a departure from conventional stepping-based phase retrieval algorithms. Previous approaches using the sliding window method for phase retrieval result in cross-talk artifacts at sharp edges that emerge

Chapter 4

Optimization of Phase Sampling

In plerisque rebus mediocritas optima est.

Cicero, *De Officiis*

In grating-based imaging, the quality of the extracted sample channels is substantially influenced by the sampling of the interferometer phase ϕ and the stepping curve. I will refer to this process as *phase sampling*. This chapter begins by establishing a theoretical framework to assess the impact of poor sampling. Following this, the phase sampling of the DFCT prototype is examined and optimized to minimize artifacts and noise in the extracted sample channels.

4.1 Theoretical Basis

There are two main aspects of the phase sampling that I will consider here. First, its influence on the variance of the extracted sample channels. Second, specific to the continuous acquisition at the dark-field CT prototype, the spectral separation of the carrier signals, as explained in the previous chapter. This influences the amount of cross-talk between the sample channels and has to be accounted for to optimize image quality.

4.1.1 Covariances of the SPR Estimator

The basis for all phase retrieval algorithms presented in this thesis is the SPR algorithm introduced in Section 2.2.3. In the following, an analysis of the effect of phase sampling on the covariance of the SPR estimator is performed. This estimator is obtained by first solving the linear system (2.31) and then performing a change of variables (2.33). To calculate its covariances, one first has to determine the covariances of the linear estimator and then perform Gaussian error propagation to account for the variable transformation. The latter is achieved by first defining the transformation as:

$$\mathbf{f}(\mathbf{b}) = \begin{pmatrix} T \\ D \\ \Phi \end{pmatrix} = \begin{pmatrix} M \\ \frac{\sqrt{C^2+S^2}}{M} \\ -\arctan(\frac{S}{C}) \end{pmatrix}. \quad (4.1)$$

The final SPR estimator vector $\hat{\mathbf{f}}$ then has the covariance matrix

$$\text{Cov}[\hat{\mathbf{f}}] \approx \frac{\partial \mathbf{f}}{\partial \mathbf{b}} \text{Cov}[\hat{\mathbf{b}}] \frac{\partial \mathbf{f}}{\partial \mathbf{b}}^T, \quad (4.2)$$

where $\hat{\mathbf{b}}$ is the least squares solution of the linear system (2.31), and

$$\frac{\partial \mathbf{f}}{\partial \mathbf{b}} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{\sqrt{C^2+S^2}}{M^2} & \frac{C}{M\sqrt{C^2+S^2}} & \frac{S}{M\sqrt{C^2+S^2}} \\ 0 & \frac{S}{C^2+S^2} & -\frac{C}{C^2+S^2} \end{pmatrix} \quad (4.3)$$

is the Jacobian of the variable transformation (2.33). For the covariance of $\hat{\mathbf{b}}$, we consider the linear system

$$\mathbf{y} = \mathbf{X}\mathbf{b}, \quad (4.4)$$

with observations $\mathbf{y}$, design matrix $\mathbf{X}$, and parameter vector $\mathbf{b}$. The least squares solution is given by

$$\hat{\mathbf{b}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}. \quad (4.5)$$

The covariance matrix of this least squares estimator $\hat{\mathbf{b}}$ is generally given by:

$$\text{Cov}[\hat{\mathbf{b}}] = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \text{Cov}[\mathbf{y}] \mathbf{X} (\mathbf{X}^T \mathbf{X})^{-1}. \quad (4.6)$$

For homoscedastic observations, i.e., uncorrelated observations with constant variance $\text{Cov}[\mathbf{y}] = \sigma^2 \mathbf{I}_{N \times N}$, this takes the simplified form:

$$\text{Cov}[\hat{\mathbf{b}}] = (\mathbf{X}^T \mathbf{X})^{-1} \sigma^2. \quad (4.7)$$

However, generally the assumption of homoscedasticity is not valid. In practice, one should thus use a weighted least squares approach. Ideally, the observation vector's heteroskedasticity is then compensated for by using a proper weighting (for uncorrelated) or precision (for correlated data) matrix that effectively performs a whitening transform on the data vector. For uncorrelated observations, which I assume throughout

this thesis, this can be achieved by using

$$\mathbf{W}_{ii} = \frac{1}{\sigma_i^2}, \quad (4.8)$$

for the weight matrix in (2.31). The weighting can be interpreted as transforming the observation vector $\mathbf{y}$ and design matrix $\mathbf{X}$ by rescaling them according to the standard deviations of the observations. This leads to

$$\tilde{\mathbf{y}} := \mathbf{y} \oslash \boldsymbol{\sigma}, \quad (4.9)$$

$$\tilde{\mathbf{X}}^\top \tilde{\mathbf{X}} := \mathbf{X}^\top \mathbf{W} \mathbf{X}, \quad (4.10)$$

where $\oslash$ denotes a Hadamard division and $\boldsymbol{\sigma}$ the vector of standard deviations. Per definition, the covariance matrix of the transformed observation vector is then equal to an identity matrix:

$$\text{Cov}[\tilde{\mathbf{y}}] = \mathbf{I}_{N \times N}. \quad (4.11)$$

This leads to

$$\text{Cov}[\hat{\mathbf{b}}] = (\tilde{\mathbf{X}}^\top \tilde{\mathbf{X}})^{-1} \quad (4.12)$$

$$= (\mathbf{X}^\top \mathbf{W} \mathbf{X})^{-1}. \quad (4.13)$$

In summary, the covariance matrix of the SPR estimator is therefore given by:

$$\text{Cov}[\hat{\mathbf{f}}] \approx \frac{\partial \mathbf{f}}{\partial \mathbf{b}} (\mathbf{X}^\top \mathbf{W} \mathbf{X})^{-1} \frac{\partial \mathbf{f}^\top}{\partial \mathbf{b}}. \quad (4.14)$$

4.1.2 Variance Minimizing Phase Sampling

From the last section, it is evident that the SPR estimator's covariance matrix depends on the choice of phase sampling points as they significantly influence the shape of the design matrix $\mathbf{X}$. In this section, minimizing the covariances of the SPR estimator through the choice of phase sampling points is discussed.

For simplicity, let's assume an idealized measurement of a stepping curve with N grating positions, constant visibility V and intensity I , which we can define without loss of generality to be $I = 1$. Furthermore, we assume heteroskedastic yet uncorrelated observations with variances σ_i^2 . This means the covariance matrix of the observation vector is an $N \times N$ diagonal matrix with variances σ_i^2 on the diagonal. Therefore, as discussed in the last section, a proper weighting of the data is assumed. For the simplified case considered here, the transformed design matrix can then be expressed as:

$$\tilde{\mathbf{X}} = \begin{pmatrix} \tilde{\mathbf{1}}, & V\tilde{\mathbf{c}}, & V\tilde{\mathbf{s}} \end{pmatrix}, \quad (4.15)$$

with

$$\tilde{\mathbf{c}} = \left(\frac{1}{\sigma_1} \cos(\phi_1), \quad \dots, \quad \frac{1}{\sigma_N} \cos(\phi_N) \right)^\top, \quad (4.16)$$

$$\tilde{\mathbf{s}} = \left(\frac{1}{\sigma_1} \sin(\phi_1), \quad \dots, \quad \frac{1}{\sigma_N} \sin(\phi_N) \right)^\top, \quad (4.17)$$

$$\tilde{\mathbf{1}} = \left(\frac{1}{\sigma_1}, \quad \dots, \quad \frac{1}{\sigma_N} \right)^\top, \quad (4.18)$$

where to shorten the notation $\phi_i := \phi(t_i)$ is defined. The solution of the least squares problem then becomes:

$$\hat{\mathbf{b}} = (\tilde{\mathbf{X}}^\top \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{X}}^\top \tilde{\mathbf{y}}. \quad (4.19)$$

It's covariance matrix is given by

$$\text{Cov}[\hat{\mathbf{b}}] = (\tilde{\mathbf{X}}^\top \tilde{\mathbf{X}})^{-1}. \quad (4.20)$$

Therefore, the covariance of the estimator depends on the employed phase sampling positions as they dictate the shape of the system matrix:

$$\tilde{\mathbf{X}}^\top \tilde{\mathbf{X}} = \begin{pmatrix} \sum_i \frac{1}{\sigma_i^2} & V\tilde{\mathbf{1}} \cdot \tilde{\mathbf{c}} & V\tilde{\mathbf{1}} \cdot \tilde{\mathbf{s}} \\ V\tilde{\mathbf{1}} \cdot \tilde{\mathbf{c}} & V^2 \tilde{\mathbf{c}} \cdot \tilde{\mathbf{c}} & V^2 \tilde{\mathbf{c}} \cdot \tilde{\mathbf{s}} \\ V\tilde{\mathbf{1}} \cdot \tilde{\mathbf{s}} & V^2 \tilde{\mathbf{c}} \cdot \tilde{\mathbf{s}} & V^2 \tilde{\mathbf{s}} \cdot \tilde{\mathbf{s}} \end{pmatrix}. \quad (4.21)$$

To find an optimal set of phase sampling positions that results in minimal covariances of the least-squares estimators, the generalized variance is minimized. It is defined as the determinant of the covariance matrix:

$$\det(\text{Cov}[\hat{\mathbf{b}}]) = \det(\tilde{\mathbf{X}}^\top \tilde{\mathbf{X}})^{-1}. \quad (4.22)$$

Therefore, the determinant of the system matrix has to be maximized. Using the rule of Sarrus, it is given by:

$$\det(\tilde{\mathbf{X}}^\top \tilde{\mathbf{X}}) = V^4 \left(\sum_i \frac{1}{\sigma_i^2} \left(|\tilde{\mathbf{c}}|^2 |\tilde{\mathbf{s}}|^2 - (\tilde{\mathbf{c}} \cdot \tilde{\mathbf{s}})^2 \right) - \left| (\tilde{\mathbf{1}} \cdot \tilde{\mathbf{s}}) \tilde{\mathbf{c}} - (\tilde{\mathbf{1}} \cdot \tilde{\mathbf{c}}) \tilde{\mathbf{s}} \right|^2 \right). \quad (4.23)$$

For the general case of heteroskedastic data, this expression can be used for numerical optimization of the phase sampling positions. However, finding an analytical solution for its maximum is quite hard in practice; especially since the observation's variances are dependent on the measured detector intensities and thus the sampled phase values themselves.

Optimal Sampling for Homoscedastic Data

For a simplified case of constant variances $\sigma_i = \sigma$, Equation (4.23) is maximized if two conditions are satisfied. First, the two strictly negative terms have to

vanish. This leads to the conditions:

$$\tilde{\mathbf{c}} \cdot \tilde{\mathbf{s}} = \sum_i \frac{1}{\sigma^2} \cos(\phi_i) \sin(\phi_i) = \sum_i \frac{1}{2\sigma^2} \sin(2\phi_i) \stackrel{!}{=} 0, \quad (4.24)$$

and

$$\left| (\tilde{\mathbf{f}} \cdot \tilde{\mathbf{s}}) \tilde{\mathbf{c}} - (\tilde{\mathbf{f}} \cdot \tilde{\mathbf{c}}) \tilde{\mathbf{s}} \right|^2 \stackrel{!}{=} 0 \quad (4.25)$$

$$\Rightarrow \tilde{\mathbf{f}} \cdot \tilde{\mathbf{s}} = \sum_i \frac{1}{\sigma^2} \sin(\phi_i) \stackrel{!}{=} 0, \quad (4.26)$$

$$\Rightarrow \tilde{\mathbf{f}} \cdot \tilde{\mathbf{c}} = \sum_i \frac{1}{\sigma^2} \cos(\phi_i) \stackrel{!}{=} 0. \quad (4.27)$$

Second, the product $|\tilde{\mathbf{c}}|^2 |\tilde{\mathbf{s}}|^2$ has to be maximal. It is trivial to show that for two variables α, β which sum to a constant γ , the following holds:

$$\alpha + \beta = \gamma, \quad (4.28)$$

$$\Rightarrow \max(\alpha\beta) = \alpha_{\max}\beta_{\max} = \frac{\gamma^2}{4}, \quad (4.29)$$

$$\Rightarrow \alpha_{\max} = \beta_{\max} = \frac{\gamma}{2}, \quad (4.30)$$

i.e., the product of α and β is maximal if both have half the value of γ . Using $\cos(x)^2 + \sin(x)^2 = 1$ it follows that

$$|\tilde{\mathbf{c}}|^2 + |\tilde{\mathbf{s}}|^2 = \frac{N}{\sigma^2}, \quad (4.31)$$

and thus $|\tilde{\mathbf{c}}|^2 |\tilde{\mathbf{s}}|^2$ is maximal for

$$|\tilde{\mathbf{c}}|^2 = |\tilde{\mathbf{s}}|^2 \stackrel{!}{=} \frac{N}{2\sigma^2}. \quad (4.32)$$

This leads to the final condition for optimality:

$$|\tilde{\mathbf{c}}|^2 = \sum_i \frac{1}{\sigma^2} \cos(\phi_i)^2 \quad (4.33)$$

$$= \sum_i \frac{1}{2\sigma^2} (1 + \cos(2\phi_i)) \stackrel{!}{=} \frac{N}{2\sigma^2}, \quad (4.34)$$

$$\Rightarrow \sum_i \frac{1}{2\sigma^2} \cos(2\phi_i) \stackrel{!}{=} 0. \quad (4.35)$$

Therefore, we can summarize the conditions for optimal phase sampling for homoscedastic observations using complex notation:

$$\sum_j \exp(i\phi_j) \stackrel{!}{=} 0, \quad (4.36)$$

$$\sum_j \exp(i2\phi_j) \stackrel{!}{=} 0. \quad (4.37)$$

For optimal phase sampling, the system matrix is thus diagonal, and the covariance matrix of the least squares

estimator $\hat{\mathbf{b}}_{\text{opt}}$ is given by:

$$\text{Cov}[\hat{\mathbf{b}}_{\text{opt}}] = \begin{pmatrix} \frac{\sigma^2}{N} & 0 & 0 \\ 0 & \frac{2\sigma^2}{NV^2} & 0 \\ 0 & 0 & \frac{2\sigma^2}{NV^2} \end{pmatrix} \quad (4.38)$$

Using Equation (4.14), the covariance matrix of the optimal SPR estimator is thus:

$$\text{Cov}[\hat{\mathbf{f}}_{\text{opt}}] \approx \begin{pmatrix} \frac{\sigma^2}{N} & -\frac{\sigma^2}{NDT} & 0 \\ -\frac{\sigma^2}{NDT} & \frac{\sigma^2}{NT^2} \left(\frac{2}{V^2} + \frac{1}{D^2} \right) & 0 \\ 0 & 0 & \frac{2\sigma^2}{NV^2T^2D^2} \end{pmatrix}. \quad (4.39)$$

A set of phase values that lead to an optimal sampling can be determined by using the Ansatz:

$$\phi_j = \Delta\phi j. \quad (4.40)$$

Inserting this into condition (4.36) results in

$$\sum_{j=0}^{N-1} \exp(i\Delta\phi j) = \frac{1 - \exp(i\Delta\phi N)}{1 - \exp(i\Delta\phi)} \stackrel{!}{=} 0, \quad (4.41)$$

$$\Rightarrow \exp(i\Delta\phi N) \stackrel{!}{=} 1 \quad \wedge \quad \exp(i\Delta\phi) \stackrel{!}{\neq} 1, \quad (4.42)$$

where the geometric sum formula was applied. Similarly, it follows from condition (4.37)

$$\Rightarrow \exp(i2\Delta\phi N) \stackrel{!}{=} 1 \quad \wedge \quad \exp(i2\Delta\phi) \stackrel{!}{\neq} 1. \quad (4.43)$$

The smallest phase increment that fulfills these requirements is $\Delta\phi = 2\pi/N$:

$$\phi_j^{\text{opt}} = \frac{2\pi j}{N}. \quad (4.44)$$

An equidistant phase stepping with phase values that are spread from 0 to 2π thus results in a variance minimizing sampling of the stepping curve for homoscedastic observations.

Optimal Sampling for Heteroscedastic Data

For the more general case of heteroscedastic and uncorrelated data, the summands in conditions (4.36) and (4.37) need to be weighted by the respective inverses of the variances σ_j^2 :

$$\sum_j \frac{\exp(i\phi_j)}{\sigma_j^2} \stackrel{!}{=} 0, \quad (4.45)$$

$$\sum_j \frac{\exp(i2\phi_j)}{\sigma_j^2} \stackrel{!}{=} 0. \quad (4.46)$$

However, these weighted exponential sums can no longer sum to zero for arbitrary variances, and the conditions (4.45) and (4.46) generally cannot be fulfilled exactly. This complicates finding an analytical solution considerably. Nevertheless, given that the exact variances are typically unknown in practice, assuming homoscedasticity is a pragmatic approach when determining a phase sampling pattern.

Sampling Quality Factor

To quantify the quality of sampling, one can further introduce a sampling quality factor q based on Equation (4.23). The sampling quality q is given by normalizing the determinant of the system matrix with its maximum value for ideal sampling (see Equation (4.38)):

$$q = \det(\tilde{\mathbf{X}}^T \tilde{\mathbf{X}}) \frac{4\sigma^6}{N^3 V^4}. \quad (4.47)$$

Therefore, $q \in [0, 1]$ describes the isolated influence of the sampling on the value of $\det(\tilde{\mathbf{X}}^T \tilde{\mathbf{X}})$, with a value of 1 being achieved by ideal sampling. Due to Equation (4.14) and since $\det(\mathbf{AB}) = \det \mathbf{A} \det \mathbf{B}$ for square matrices $\mathbf{A}$ and $\mathbf{B}$, the generalized variance of the SPR estimator $\hat{\mathbf{f}}$ is scaled by $1/q$. As a result, the effect of poor sampling can increase the generalized variance of the SPR estimator by an arbitrarily high amount, but there is an upper limit to how much the variances can be improved by improving the sampling.

4.1.3 Continuously Acquired Data

For continuously acquired data, as discussed in Section 3.4.1, a proper spectral separation of the different carrier signals is necessary to perform an unambiguous demodulation of the three sample channels. This depends on the modulation carrier $\cos(\phi)$ and thus the interferometer phase ϕ . Designing a suitable phase stepping is therefore the basis for achieving a good demodulation. In practice, this depends on the experimental setup and how the physical grating movement can be performed to achieve a change in the interferometer phase ϕ . Since the level of control over the exact grating position and orientation can vary from setup to setup, I discuss different schemes of phase sampling and how to optimize them for better spectral separation. Furthermore, since continuously acquired data requires continuously moving gratings, averaging effects of the interference pattern are discussed.

Linear Phase Stepping

The simplest change in the interferometer phase can be achieved by a uniform relative movement of one grating. This leads to a phase stepping that is linear in

time and that is described by

$$\phi(t) = 2\pi f_\phi t. \quad (4.48)$$

Optimizing the spectral separation for this case is trivial in theory: The modulation peak has to be shifted to frequencies as high as possible by changing f_ϕ . However, it should be noted that f_ϕ is limited by the spectral resolution of the setup and the bandwidth of the sample signal. Thus, if f_ϕ is too high and the modulation band extends over the Nyquist frequency, aliasing artifacts will occur. Therefore, an estimate of the sample signal's bandwidth is necessary to determine the highest f_ϕ possible. The Fourier spectrum of a qualitative example signal y with linear phase stepping is illustrated in Figure 4.2.

Vibration-induced Phase Stepping

While linear phase stepping is favorable due to its simplicity, in some settings, fast data acquisition can't be performed easily with a controlled stepping procedure. However, in this case, the necessary grating movement can be induced by physical vibrations of the gratings as demonstrated by [24]. In this case, the grating movement is less controlled and the current grating position has to be estimated for each shot in a pre-processing step. This will be discussed in Chapter 5. Additionally, a vibration-induced stepping can lead to non-uniform deformations of the gratings that have to be addressed in the model as well, which complicates processing further [27].

Considering only one vibration frequency f_{vib} , the phase is given by a sinusoidal:

$$\phi(t) = z \sin(2\pi f_{\text{vib}} t). \quad (4.49)$$

This means that the phase of the carrier signal itself is modulated by an oscillation. For the description of such signals, one can employ the so-called Jacobi-Anger expansion, which is given by [59]:

$$e^{iz \sin \theta} = \sum_{n=-\infty}^{\infty} J_n(z) e^{in\theta}, \quad (4.50)$$

where $J_n(z)$ is the n -th Bessel function of the first kind. A plot of J_n for $n = 1, 2, 3$ is shown in Figure 4.1. Using this expansion, the carrier $\cos(\phi)$ can be written as a series expansion:

$$\cos(z \sin(2\pi f_{\text{vib}} t)) = \sum_{n=-\infty}^{\infty} J_{2n}(z) \cos(2\pi(2n f_{\text{vib}})t) \quad (4.51)$$

In theory, the carrier is thus composed of infinitely many overtones of $2f_{\text{vib}}$, which leads to aliasing artifacts in measurements if the high-frequency modes

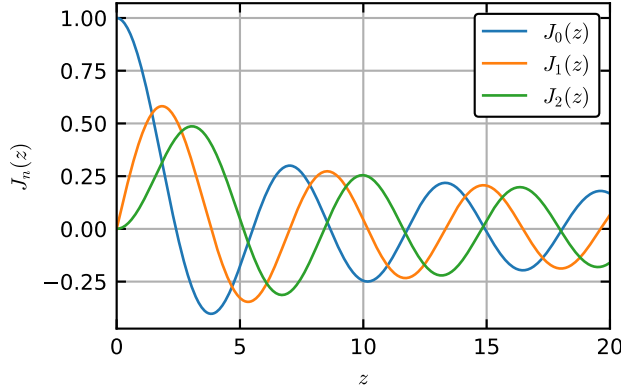


Figure 4.1: Plot of Bessel function of the first kind, $J_n(z)$, for $n = 0, 1, 2$. Higher orders decay rapidly with $J_n(z) \sim 1/n!$.

are too strong and can't be resolved. However, higher modes decrease rapidly in amplitude as $J_n(z) \sim 1/n!$ and can be neglected.

From Equation (4.51), it is evident that the oscillation frequency f_{vib} determines the possible carrier frequency modes and the oscillation amplitude z the energy distribution among the modes. Tuning these parameters can thus improve the phase sampling.

The grand goal in optimizing phase-stepping for continuous measurements is to increase the carrier's signal power in the high frequency ranges as much as possible. Therefore, a strategy for a phase given by Equation (4.49) is to tune the vibration amplitude z to be the first maximum ($z \approx 3.054$) of $J_2(z)$ and choose the vibration frequency as high as possible without introducing aliasing. Here, the bandwidth of the sample information needs to be considered as well. The dominant modulation frequency is then $2f_{\text{vib}}$.

In a more general case, the phase movement can be composed of multiple oscillations:

$$\phi(t) = \sum_k z_k \sin(2\pi f_k t + \delta_k). \quad (4.52)$$

Using complex notation, the carrier can be expressed as a generalized Cauchy product of Jacobi-Anger expansions:

$$\cos(\phi) = \text{Re} \left(e^{i\phi} \right) \quad (4.53)$$

$$= \text{Re} \left(\exp \left(i \sum_k z_k \sin(2\pi f_k t + \delta_k) \right) \right) \quad (4.54)$$

$$= \text{Re} \left(\prod_k \sum_{n_k=-\infty}^{\infty} J_{n_k}(z_k) e^{in_k(2\pi f_k t + \delta_k)} \right). \quad (4.55)$$

The resulting spectrum is therefore a superposition of integer combinations of the base vibration frequencies f_k . The Fourier spectrum of a measured signal

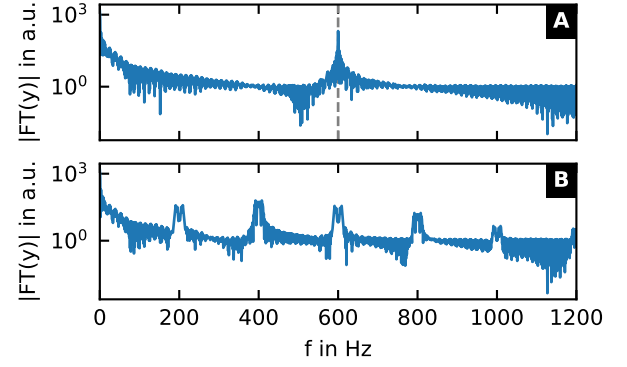


Figure 4.2: **A:** Example Fourier spectrum of a test signal acquired with linear phase stepping. The modulation channels C and S are decoded in the side bands around the modulation peak at $f_\phi = 600$ Hz. The DC modulation M causes a response in the low frequency bands. **B:** Fourier spectrum of a signal acquired with vibration induced phase stepping with vibration frequencies of 200 Hz and 1 Hz. The emerging spectrum features multiple carrier peaks whose side bands contain the information on the modulation channels S and C (see Equation (4.55)).

y where two oscillations induce phase stepping is depicted in Figure 4.2B. In this example, a high frequency oscillation of 200 Hz and a low frequency oscillation of 1 Hz are assumed. This is similar to the phase movement observed at the dark-field CT prototype [24]. The emerging spectrum features multiple carrier peaks at frequencies $n \cdot 200 \text{ Hz} + m \cdot 1 \text{ Hz}$ with integers n and m . The side bands of each peak contain the information of the sample parameters S and C . For real experimental setups, not all oscillation parameters (z_k, f_k, δ_k) can be influenced freely. Optimizing the experimentally accessible parameters to maximize the carrier's signal power for high frequencies is therefore not trivial and depends on the individual conditions of the setup.

For the dark-field CT prototype, the amplitude of the high frequency component of approximately 176 Hz, which is most important for the phase-sampling, can be tuned by changing the coupling of the G0 and G1 mount with the vibrating gantry. This makes the gratings more susceptible to the CT system's inherent vibrations and increases the fluctuation amplitude. By increasing this amplitude, the modulation signal power is shifted towards higher frequencies and the spectral separation improves. Additionally, by choosing a slower gantry rotation, a higher effective modulation frequency can be achieved, assuming a constant grating vibration speed.

Spatial Phase Sampling

In addition to the temporal modulation, the signal can also be modulated spatially by adjusting the interferometer gratings to introduce a pixel-wise change of the

phase (see also Section 3.4.2). This can be achieved by detuning the grating orientation to create Moiré fringes on the detector and generate a linear phase gradient in one direction. An example image of the necessary fringe pattern is given in Figure 3.7. The phase stepping carrier peak is then not shifted along the time direction in the Fourier space of the data sinogram but orthogonally in the spatial domain. Therefore, the signals are separated along an additional axis when combined with a temporal modulation. Similar to linear per-shot stepping, this "spatial stepping" benefits from dense fringes and thus a steep linear phase gradient over the detector. However, the steepness of the gradient is limited by the finite pixel resolution and the resulting averaging of the Moiré fringe pattern, which decreases performance and is discussed next.

Averaging Induced Visibility Loss

An important effect that has to be considered when designing the interferometer phase $\phi(x, t)$ for continuous acquisitions is the visibility loss induced by averaging of the cosine function in Equation (2.18) due to the finite pixel-size and exposure time. This effect gets stronger for faster changes of the phase and leads to a visibility loss V_{drop} and phase shift $\hat{\phi}$ relative to the ideal measurement. Consequently, there is a trade-off between improving the phase sampling by increasing the rate of phase change and decreasing the interferometer performance.

Assuming the signal is averaged over an interval of Δx (either in time or spatially), the visibility loss V_{drop} can be determined by calculating the averaging integral:

$$\begin{aligned} \frac{1}{\Delta x} \int_{x-\frac{\Delta x}{2}}^{x+\frac{\Delta x}{2}} \cos(\phi(\chi)) d\chi &= \text{Re} \left(\frac{1}{\Delta x} \int_{x-\frac{\Delta x}{2}}^{x+\frac{\Delta x}{2}} e^{i\phi(\chi)} d\chi \right) \\ &\stackrel{!}{=} \text{Re} \left(V_{\text{drop}} e^{i(\phi(x) + \hat{\phi}(x))} \right) \end{aligned} \quad (4.56)$$

The visibility drop is thus given by:

$$V_{\text{drop}} = \left| \frac{1}{\Delta x} \int_{x-\frac{\Delta x}{2}}^{x+\frac{\Delta x}{2}} e^{i\phi(\chi)} d\chi \right|. \quad (4.57)$$

While in general Equation (4.57) has no analytical solution, it can be calculated explicitly for a linear change of the phase:

$$\phi(x) = 2\pi m \cdot x. \quad (4.58)$$

Evaluating Equation (4.57) and assuming an averaging interval of $\Delta x = 1$ yields the visibility loss:

$$V_{\text{drop}} = \frac{\sin(\pi m)}{\pi m} \approx 1 - \frac{(\pi m)^2}{6}. \quad (4.59)$$

Therefore, the loss in visibility scales approximately quadratically with the rate of change of the interferometer phase.

4.2 Phase Sampling at Dark-Field CT Prototype

Based on the previously presented ideas in this chapter, the phase sampling at the dark-field CT prototype will be analyzed and optimized next. As discussed in Chapter 3, the sampling at the prototype is performed in a hybrid fashion, combining spatial and temporal phase sampling. To optimize the sampling, I will consider both influences on the total phase sampling separately. Furthermore, the general strategy is to optimize the sampling within small localized regions of the sinogram in terms of the resulting variances. The idea is that this approach also leads to a good spectral separation if the region size is chosen properly. For simplicity, constant variances of the observations $\sigma_i^2 = \sigma^2$, i.e., homoscedastic observations are assumed within these regions.

4.2.1 Spatial Phase Sampling

For the spatial sampling, the main parameters are the fringe orientation and density. For CT scans, a horizontal orientation of the fringe pattern was found to be advantageous, i.e., sampling the phase in z -direction and parallel to the rotation axis. Only a very small detector region is considered during processing to minimize the blurring in z -direction. Here, I consider a region size of $N = 5$ pixels, which has proven a good compromise between resolution and usable phase sampling. To optimize the fringe density, one has to consider both the resulting sampling quality factor q in the region and the visibility drop due to averaging V_{drop} , which also leads to an increased variance.

A linear phase-gradient $\phi(x) = 2\pi m \cdot x$ results in a regular fringe pattern on the detector. Therefore, one can define:

$$m = \frac{\text{\#fringes}}{\text{\#pixels}}, \quad (4.60)$$

where the number of pixels is the total number of detector pixels in z -direction. The expected visibility loss due to averaging in the linear case is plotted in Figure 4.3. It scales approximately quadratically with the number of fringes on the detector.

By definition, the optimal determinant of the system matrix (4.23) is decreased by the sampling quality factor $q(m)$. In addition, the visibility is lowered by a factor of $V_{\text{drop}}(m)$. The optimal value of the quantity (4.23) is thus lowered by a factor of qV_{drop}^4 due to the sampling. To find the best fringe density, one thus

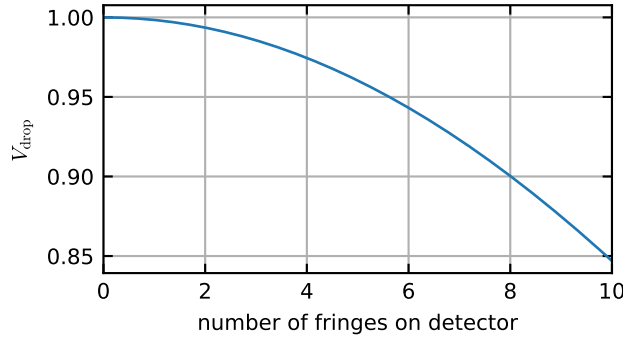


Figure 4.3: Expected visibility loss due to averaging for a horizontal fringe pattern on the dark-field CT prototype's detector.

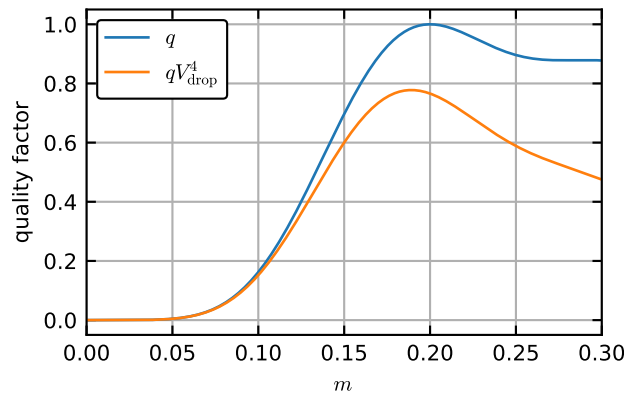


Figure 4.4: Dependency of the sampling quality on the steepness of the phase gradient over the detector for a region size of $N = 5$ and a detector with 32 pixels. To account for averaging effects, the sampling quality has to be multiplied by V_{drop}^4 to assess the sampling's effect on the generalized variance.

has to determine the maximum of $q(m)V_{\text{drop}}(m)^4$. Figure 4.4 depicts the dependency of qV_{drop}^4 and q on the phase gradient's steepness m for $N = 5$ and a linear phase change. From this it is evident that the effect of the averaging leads only to a minor shift of the position of the maximum compared to the ideal case but to a significant reduction of the value of the maximum from 1 to approximately 0.78. The ideal steepness determined via this approach is $m_{\text{opt}} \approx 0.19$. This is very close to the ideal value without considering averaging of $m_{\text{opt}} = 1/N = 0.2$ suggested by Equation (4.44). From Equation (4.60) and the fact that there are a total of 32 pixels in z -direction, the optimal fringe count is approximately 6. Therefore, a change of the interferometer phase of 12π over the total detector height is optimal.

4.2.2 Temporal Phase Sampling

In the dark-field CT prototype, optimizing the temporal phase sampling is considerably more difficult in practice than optimizing the spatial phase sampling.

Table 4.1: Median sampling quality factor for axial scans with the DFCT prototype with varying rotation times

Rotation Time in s	Median Sampling Quality q
0.3	0.11
0.4	0.18
0.5	0.34
0.7	0.48
1.0	0.72
1.5	0.88

This is because a change in the sampling characteristic can not be achieved by a simple realignment of the gratings. Instead, the coupling of the gratings themselves to the gantry has to be adapted to change the influence of the system vibrations on the gratings. Achieving a specifically designed movement of the interferometer phase, or just approximating it, requires substantial additional engineering efforts and possibly even a re-design of the grating mount. However, the influence of the vibrations on the gratings can be influenced to a limited degree by changing the tightness of the screws that are used to fixate the grating mount. A thorough analysis of the origin and characteristics of all system vibrations is given in [41]. Here, I give an analysis of the current state of the temporal sampling and compare it for different acquisition modes.

Single Rotation Scans

For the first test, an axial scan with a rotation time of 1.5 s, and 2400 acquired shots is analyzed. This corresponds to the slowest setting of the scanner and yields the best visual results. For the analysis, a window of 9 consecutive shots was considered, and the sampling quality factor was calculated for all 2400 windows based on the global change in phase ($c_{p,0}(t)$ in Equation (5.4)). Figure 4.5 depicts the global per-shot phase of the first 600 shots alongside the resulting sampling quality factors. Additionally, the distribution of quality factors for the full rotation is visualized in a histogram. From this, it is evident that while the majority of windows have a good sampling factor close to one with a median value of 0.88, there is a significant spread among the windows because the sampling quality decreases in regular intervals. Therefore, a gain in performance is still possible by improving the temporal sampling to be consistently close to a quality factor of one.

While this analysis shows an acceptable sampling for rotation times of 1.5 s, minimizing the overall data acquisition time, which is important for clinical use of the prototype, requires minimizing the rotation time of the scanner. To demonstrate the influence of the rotation time on the sampling, a test series consisting of six scans was acquired with different rotation times.

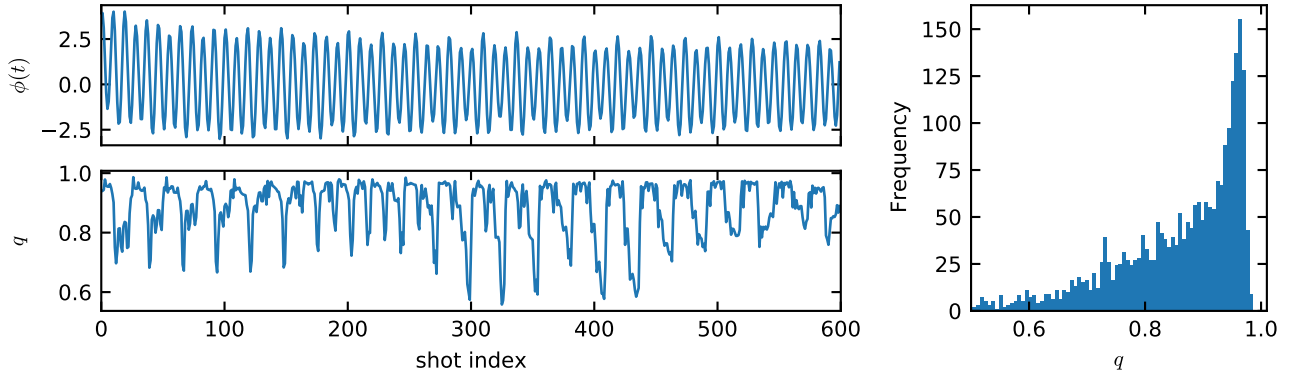


Figure 4.5: Global interferometer phase for the first 600 shots of an axial scan and the resulting sampling quality factors for a temporal window size of nine shots. The quality factor exhibits an oscillating behavior, with a wide distribution of values among windows as evident from the histogram of quality factor values of the full scan.

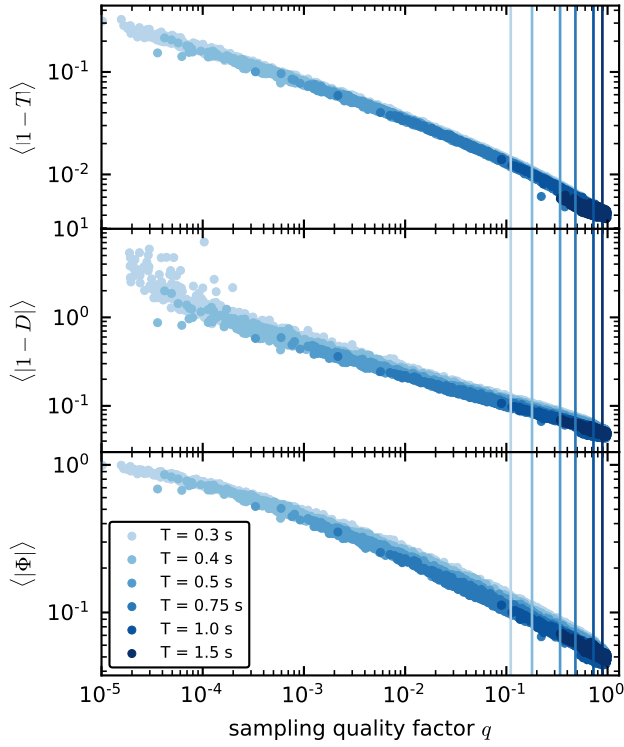


Figure 4.6: Detector mean of absolute errors of each window from the phase retrieval of test air scans acquired with different rotation speeds plotted against the sampling quality factor. The median values of the respective scans is indicated by vertical lines.

All scans use the same tube settings of 180 mAs and 80 kVp. Table 4.1 lists the examined rotation speeds alongside the resulting median sampling quality factors. The quality factors of all scans plotted against the shot index and thus the time are shown in the appendix in Figure A.1. From this, it is immediately evident that there is a substantial drop in median sampling quality when using faster rotation speeds. This is because the vibration of the gratings appears slower relative to the gantry rotation and there is less grating movement

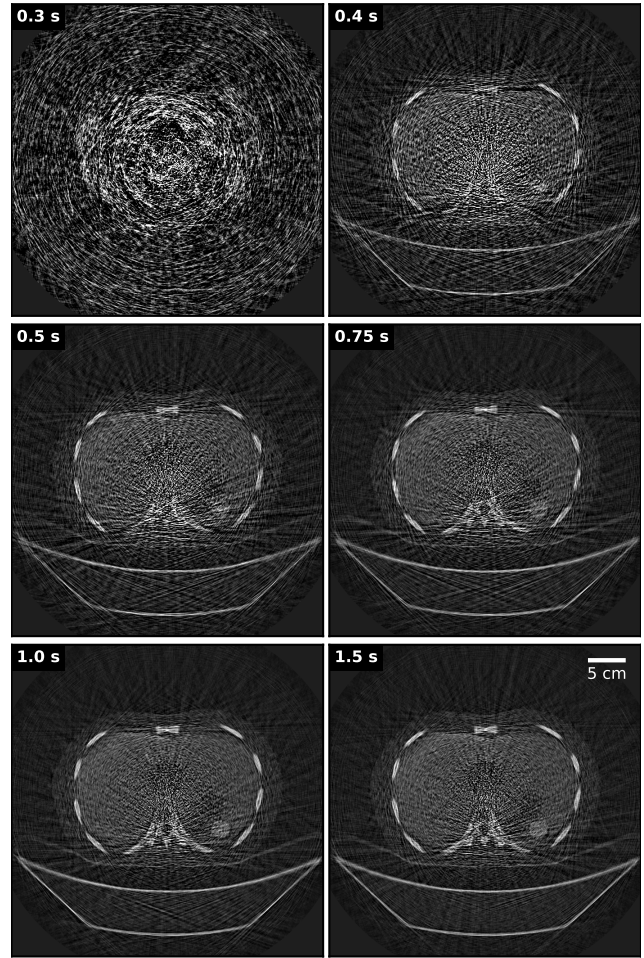


Figure 4.7: Reconstructed dark-field channels for scans varying only in their rotation times. For shorter rotation times the image quality suffers considerably from poor temporal phase sampling.

within one window for short rotation times.

To analyze the loss in performance in each window of the test air scans, phase retrieval was performed using the SPR algorithm considering only the temporal

sampling, i.e., only a pixel-wise phase retrieval was performed in each window. For air scans $T = D = 1$ and $\Phi = 0$ are the expected results for all pixels of this procedure due to the lack of a sample. Figure 4.6 depicts the detector means of the absolute deviations from the expected results for each window and all rotation speeds plotted against the respective sampling quality factor. Furthermore, the median sampling quality factors are indicated for all rotation speeds. There are two main results from this analysis. First, a strong dependency of the deviation magnitudes on the sampling quality is apparent. Second, faster scans tend to show worse sampling and significantly higher deviations.

To examine these effects on sample reconstructions, six test scans of the lung man phantom were performed using the same settings as above and processed using the full processing pipeline discussed in Chapter 5. The resulting reconstructions of the dark-field channels immediately after the final phase retrieval, i.e., without any post-processing, are depicted in Figure 4.7. Again, a significant drop in image quality is evident for faster rotation speeds, which can be attributed to a loss in phase sampling quality. This increase in noise level and streak artifacts for faster rotations has to be addressed before moving on to rotation speeds below 1 s.

Multi Rotation Scans

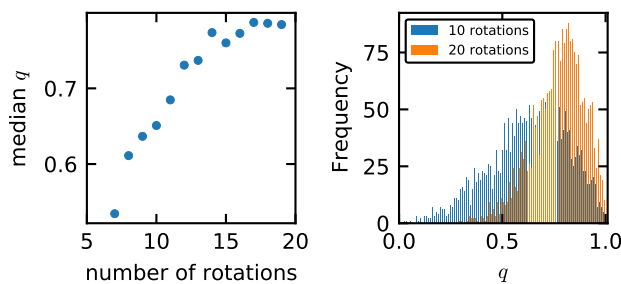


Figure 4.8: Median sampling quality for multi rotation scans using different numbers of rotations. Histogram of the distribution of sampling qualities for a 10 and 20 rotation scan.

Multi rotation scans use a special acquisition mode, where multiple subsequent axial scans are performed at the same position. This effectively enables one to acquire multiple views under the same gantry angle. Due to the intrinsic vibrations of the system, the interferometer phase may vary from shot to shot due to the gratings' movement which leads to a sampling of the stepping curve. For multi rotation scans, this therefore allows one to employ conventional phase retrieval techniques for signal extraction. While this acquisition mode is not viable for clinical applications, the results from these measurements are of interest as ground truth images free from movement-induced artifacts [60]. Furthermore, the acquisition mode allows for higher total dose levels than single rotation axial

scans, which additionally decreases noise levels and improves possible ground truth images further. Here, a brief analysis of the sampling quality of these multi rotation scans is conducted. For this, the sampling of a 20 rotation scan was analyzed and the sampling quality factors were calculated. To assess the effect of smaller numbers of rotations on the sampling, the quality factors were also calculated for subsets of the scan to provide data for rotation numbers from 7 to 20 rotations. The result of this analysis is depicted in Figure 4.8, where the median sampling quality factor is plotted against the number of rotations. Additionally, two histograms are shown depicting the distribution of sampling qualities for a 10 and 20 rotation scan.

From this, it is evident that the sampling quality increases when using more rotations, but there is a saturation effect after 15 rotations. This can be explained by the more randomized approach to sampling for the multi rotation scan, where more rotations lead to a better distribution of the sampling points spread across the stepping curve. However, due to the limited amplitude of the phase vibrations below 2π there is a limited range for the sampling points to be drawn from and a proper sampling over the full range of the stepping curve is not possible. This sets an upper limit for the sampling quality that cannot be overcome without increasing either the vibration amplitude or by using phase retrieval techniques from the continuous acquisition modes to improve the sampling.

4.3 Conclusion

This chapter underscores the role of phase sampling strategies in achieving high-quality dark-field imaging results. Both the aspect of variance minimization and cross-talk artifact mitigation by improving the spectral separation of the carrier signals were discussed. By analyzing the generalized variance of the SPR estimator and introducing the sampling quality factor as a metric for assessing sampling quality, I have demonstrated that phase sampling is a crucial determinant of image fidelity and provided a quantitative measure. Together, these insights lay the groundwork for more robust sampling schemes, highlighting specific areas where optimization is necessary to increase performance. The findings further suggest an optimal fringe orientation and density for the dark-field CT prototype, yet temporal sampling remains suboptimal and will require further engineering efforts to reach desired levels of performance by changing the grating mount's coupling with the gantry. Additionally, I find that multi rotation scans exhibit limitations due to the sampling that constrain their effectiveness as ground truth data.

Chapter 5

Data Processing

*Nullum magnum ingenium sine mixtura
dementiae fuit.*

Seneca, *De Tranquillitate Animi*

This chapter, presents the sample data processing pipeline developed for the dark-field CT prototype scanner. This includes the topics of fluctuation modeling and correction, the handling of higher-order effects, reconstruction, and post-processing. Additionally, helical and surview scanning is discussed. Parts of this chapter have been published in [61].

5.1 Introduction

The application of X-ray dark-field imaging to clinical-scale computed tomography (CT) presents unique challenges in data processing, primarily due to the susceptibility of Talbot-Lau interferometers to mechanical vibrations and system instabilities. These factors introduce significant artifacts during image reconstruction. Correcting for the occurring perturbations is particularly difficult when scanning large samples that occupy most of the detector's field-of-view (FOV). This chapter discusses the advanced data processing techniques developed to overcome these limitations, enabling artifact-free imaging of human-scale objects.

Traditional dark-field CT systems relied on step-and-shoot acquisition methods and stationary gantry designs, which allowed for precise control of grating positions and facilitated the separation of sample and system-induced fluctuations. However, such approaches were constrained by small FOVs and slow scan times, making them unsuitable for clinical workflows. The prototype discussed in this work adopts continuous gantry rotation to achieve scan times comparable to conventional CT scanners. This shift necessitates novel data acquisition and processing strategies to account for the dynamically changing interferometer state, as precise grating control is no longer feasible.

To address these challenges, this chapter introduces a robust processing pipeline tailored for human-scale dark-field CT imaging. The pipeline leverages prior

knowledge of system vibrations, obtained through principal component analysis (PCA) of air reference scans, to identify and mitigate the effects of mechanical perturbations. By correlating vibration patterns from air scans with sample scans, the pipeline effectively separates system-induced fluctuations from sample information, even in cases where the sample occupies most of the detector. This eliminates the reliance on sample-free detector regions, enabling the reconstruction of arbitrary sample dimensions without vibration-induced artifacts.

A key component of this pipeline is the enhanced phase-retrieval algorithm introduced in Chapter 3, specifically adapted for continuous data acquisition protocols. While this algorithm ensures accurate reconstruction of dark-field signals, its efficacy depends on the precise characterization of the interferometer state, which fluctuates due to vibrations and other mechanical instabilities. This chapter details the integration of PCA-based fluctuation models into the reconstruction workflow of axial, helical, and surview scans with the dark-field CT prototype to ensure a proper description of the interferometer state.

5.2 Fluctuation Modeling

The basis for modelling sample scans at the dark-field CT prototype is the general forward model for Talbot-Lau imaging introduced in Section 2.2.2:

$$y^s = TI + \sum_{n=1} TIV_n D^{n^2} \cos(n(\phi + \Phi)), \quad (5.1)$$

where all the introduced interferometer parameters I , V_n , and ϕ generally depend on the individual detector pixel x and time t as they change from shot to shot due to the data acquisition scheme utilizing a continuously rotating gantry and the presence of vibrations and other fluctuations. If the fluctuations perturbing the interferometer state are not properly accounted for, the resulting changes in the interferometer parameters I , V , and ϕ are attributed to sample information during phase retrieval. This leads to residual fringe artifacts in the extracted sample projections, as well as concentric

ring artifacts in the final reconstructions. Examples for these are depicted in [27] and in Figure 5.6E.

To describe fluctuations of the interferometer parameters due to grating deformations, changes of the source spot, and other perturbations, a data model based on PCA has been developed by Schmid et al. [27]. Here, fluctuations are expressed by linear combinations of per-pixel correction arrays R that represent the impact of the main modes of grating deformations on the measured fringe pattern and their respective per-shot strength coefficients c . These perturbations alter the main three flat-field parameters $I^{(0)}$, $V_1^{(0)}$, and $\phi^{(0)}$ as follows:

$$I(x, t) = I^{(0)}(x) \cdot \left(1 + \sum_{n=0}^{N_i} c_{i,n}(t) R_{i,n}(x) \right), \quad (5.2)$$

$$V_1(x, t) = V_1^{(0)}(x) \cdot \left(1 + \sum_{n=0}^{N_v} c_{v,n}(t) R_{v,n}(x) \right), \quad (5.3)$$

$$\phi(x, t) = \phi^{(0)}(x) + \sum_{n=0}^{N_p} c_{p,n}(t) R_{p,n}(x). \quad (5.4)$$

Here, the total number of considered correction arrays for each channel is given by $N_i + 1$, $N_v + 1$, and $N_p + 1$, respectively. Fluctuations of the higher-order visibilities are neglected. In this model, all parameters that have a sole per-pixel dependency are scan to scan consistent, while the per-shot strength coefficients are scan specific. Consequently, the flat-fields $I^{(0)}$, $V_1^{(0)}$, and $\phi^{(0)}$ and the correction arrays R_i , R_v , and R_p can be determined during the reference processing of an air scan, while the fluctuation strength coefficients of the sample scan c_i , c_v , and c_p have to be determined during sample processing.

The observed temporal fluctuations described by c_i , c_v , and c_p are caused by periodic processes happening during the measurement, which were analyzed in detail in [26]. These include, most importantly, the gantry rotation, the change of the X-ray focal spot due to induced magnetic fields, and vibrations caused by the motors on the gantry. The effects of the gantry movement lead to a reproducible low frequency change in the coefficients at speeds close to the rotation speed (≈ 1 Hz) due to centrifugal and gravitational forces acting on the gratings. The latter effects result in high frequency oscillations, which are significantly faster than the rotation frequency at above 100 Hz. It is therefore possible to extract the low frequency components of the coefficients $\text{LF}_t \left[c_n^{\text{ref}}(t) \right]$ from the reference scan by low-pass filtering the reference coefficients. Only high frequency effects are estimated separately during sample processing.

By analyzing multiple reference scans it was found that the high frequency components of the fluctuation coefficients can be approximated well by a sum of a

few sinusoidals due to the sparsity of their Fourier representation [26]. These oscillate at characteristic frequencies of the setup which can be assigned to different components on the gantry. The most important frequencies are two dominant frequencies from the X-ray tube motor, which drives the anode rotation. This asynchronous motor is operated at a frequency of 189 Hz. It induces system fluctuations through electromagnetic interference as it causes distortion of the X-ray source spot shape. It is characteristic for an asynchronous motor to exhibit a small slippage between the rotating magnetic field excited by the stator coils and the resulting rotation of the rotor, in this case, the anode. Therefore, the physical rotation of the anode has a lower frequency of approximately 176 Hz which causes vibrations on the gantry that lead to grating displacements and deformations. The induced grating displacements are the main mechanism of phase sampling in the system and are essential to the signal retrieval process. Moreover, vibrations induced by the cooling unit at 117 Hz are included in the model.

It was further found that the amplitudes of these oscillations stay constant from scan to scan and can therefore also be extracted from the respective reference coefficient. Additionally, the phase relationship of the oscillations between different coefficients stays constant. This can be explained by the fact that changes of the different coefficients at a specific frequency are caused by only one physical oscillation present in the system that has stable characteristics from scan to scan. These two facts can be illustrated by using pairs of the high frequency components of the coefficients to plot Lissajous curves, which can then be compared between scans. This is depicted for an air scan in Figure 5.1. An example for the arising correlations are the intensity fluctuation coefficients: All of their values lie on a well-defined Lissajous ellipse. This ellipse stays constant between scans, which indicates a constant amplitude and phase relation of the coefficients' oscillations. For coefficients with differing primary oscillation frequencies, no correlation pattern arises. This is the case, for example, for the coefficient pair $c_{i,0}$ and $c_{p,0}$ which correspond to global changes in intensity and phase, respectively: Since the main driver of the global intensity fluctuations is the electromagnetic interference caused by the tube stator of 189 Hz and the main driver for the phase movement is the physical movement of the anode rotor at 176 Hz, the resulting Lissajous curve is a square.

Based on this analysis, each coefficient $c_{z,n}$ is expressed as a sum of a few sinusoidals that oscillate at the characteristic system frequencies:

$$c_{z,n}(t) = \sum_{k=0}^{N_f-1} A_{z,n,k}^{\text{ref}} \sin(2\pi f_k t + \phi_{z,n,k}^{\text{ref}} + \delta_k) + \text{LF}_t \left[c_{z,n}^{\text{ref}}(t) \right], \quad (5.5)$$

where $A_{z,n,k}^{\text{ref}}$ and $\phi_{z,n,k}^{\text{ref}}$ are the reference amplitude and oscillation phase of the respective reference coefficient $c_{z,n}^{\text{ref}}$ and frequency f_k which were retrieved during processing of an air scan with $z \in \{i, v, p\}$. N_f denotes the number of considered frequencies. Employing this model has the advantage that only the global vibration phase offsets of each frequency δ_k have to be found for a sample scan, which strongly decouples sample information and the systems' fluctuations. The process of finding these vibration phase offsets will be described in Section 5.4.

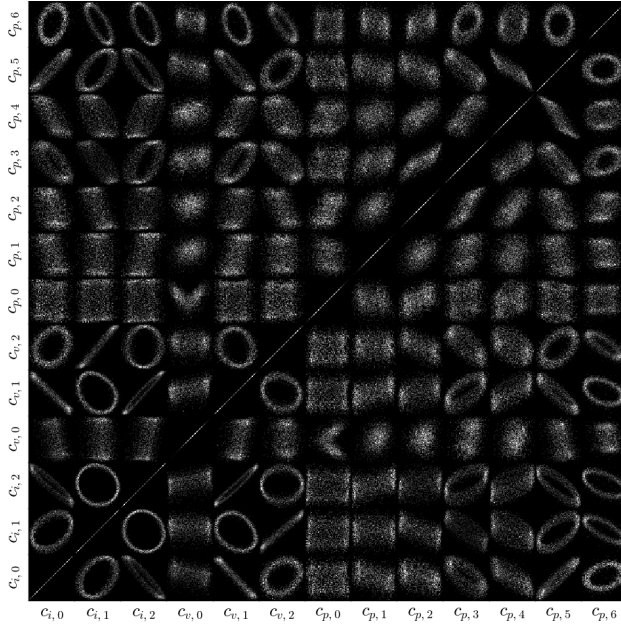


Figure 5.1: Correlation diagram of the fluctuation coefficients extracted from a reference scan. Each tile shows the two-dimensional histogram of the respective pair of coefficients. A fixed phase relationship between different fluctuation coefficients results in Lissajous curves that are constant between scans. Figure adapted from [61].

5.3 Air Scan Processing

The air scan processing is based on the algorithm outlined by Schmid et al. [27], which provides a detailed analysis of the procedure. Processing of the air scan yields the flat-fields $I^{(0)}$, $V_1^{(0)}$, and $\phi^{(0)}$ as well as the correction arrays $R_{z,n}$ alongside the respective reference fluctuation coefficients $c_{z,n}^{\text{ref}}$. However, to separate

global changes of the interferometer parameters, i.e., changes that are constant over the detector, from spatial fluctuations, the algorithm is slightly modified: As described in [27] initially a fluctuation model based on polynomial correction arrays is fitted to the data. Subsequently, the PCA-based compression is applied only to the spatial fluctuation terms, which yields the dominant spatial fluctuation modes. This is in contrast to the original description of the algorithm, where all fluctuation terms were compressed jointly. Therefore, in the air scan processing in all channels, a correction array representing global changes is used with $R_{z,0}(x) = 1$, while higher indices represent the respective spatial fluctuation modes. Two spatial correction arrays are used for each the intensity and the visibility channel R_i and R_v , respectively. For the phase, the first six principal components are used as spatial correction arrays R_p . In addition, the higher-order visibilities are determined in a last step via a trivial extension of the linearized statistical phase retrieval (SPR) algorithm presented in [27]. Here, the higher-order visibilities $V_n(x)$ are fitted to the model in Eq. (2.12) with a least-squares optimization, where the fluctuation model (5.5) was used to compute the intensity $I(x, t)$, visibility $V(x, t)$ and phase $\phi(x, t)$. Since this poses a linear problem, it can be solved with matrix factorization.

In a next step, the vibration strength coefficients are analyzed to extract prior knowledge about the vibration characteristics for the later sample processing. This step entails low pass filtering each coefficient to extract the low frequency component $\text{LF}_t \left[c_{z,n}^{\text{ref}}(t) \right]$. The low pass filter has to feature a cutoff below 100 Hz, to properly separate the low frequency influences of the gantry rotation from the high frequency vibrations. Next, the fluctuation model (5.5) is fitted to the high frequency components of the reference coefficients to extract the amplitudes $A_{z,n,k}^{\text{ref}}$, reference phases $\phi_{z,n,k}^{\text{ref}}$ and frequencies f_k . By definition, the global vibration phase offsets δ_k are zero for the reference scan. For the fitting procedure, first an initial guess is found via Fourier analysis of the coefficient data. Since the sampling frequency of the scanner is substantially faster than the fastest vibration frequency (352 Hz), this initial guess already yields a very good initial guess, which results in a robust fit. Subsequently, the parameters are optimized via first-order gradient descent by minimizing the following cost function for each channel individually:

$$C_z = \sum_n \left(\sum_{k=0}^{N_f-1} A_{z,n,k}^{\text{ref}} \sin(2\pi f_k t + \phi_{z,n,k}^{\text{ref}}) - \text{HF}_t \left[c_{z,n}^{\text{ref}} \right] \right)^2. \quad (5.6)$$

For the frequencies, f_k , the main system frequencies were considered. To allow for slight deviations in the oscillation frequencies, they are determined for each

Table 5.1: Summary of dominant fluctuation frequencies for the different channels.

Channel	Frequency / Hz	Origin
Intensity I	189	tube stator
	176	tube rotor
Visibility V	189	tube stator
	352	2nd harmonic of tube rotor
Phase ϕ	117	cooling unit
	189	tube stator
	176	tube rotor
	352	2nd harmonic of tube rotor

scan from the Fourier spectrum by finding the respective peak positions that lie in a narrow band (± 1 Hz) around the expected oscillation frequencies. A different set of frequencies is used for each of the three channels, they are summarized in Table 5.1.

For the intensity coefficients, the anode motor frequencies of approximately 189 Hz and 176 Hz are considered, as both the deformation of the source spot and the grating displacement cause intensity fluctuations. For the visibility coefficients, the stator frequency of 189 Hz and twice the frequency of the rotor 176 Hz is considered: The change of the source spot shape results in a change of its shadowing characteristics and thus in a fluctuation of the interferometers' visibility [26, 42]. Additionally, grating movement during the measurement results in a reduced visibility due to a smearing of the fringe pattern. Therefore, a visibility variation at twice the main grating vibration frequency of 176 Hz is observed [26, 27]. For the phase coefficients, four frequencies are considered: the rotor and stator frequency, the 2nd mode of the rotor frequency and the vibration frequency of the system's cooling unit of approximately 117 Hz.

5.4 Sample Scan Processing

In summary, the air scan processing delivers most system parameters necessary to describe the interferometer state. During the sample scan, only the global vibration phase offsets of the fluctuation coefficients (a total of three free parameters) as well as the sample parameters themselves have to be determined. In the following, the individual steps of the sample processing are elaborated in more detail:

1) Flux estimation

The first step in the sample scan processing pipeline is to acquire an estimate for the flux, i.e., the global change of the intensity channel $c_{i,0}$. While in conventional CT scanners this is performed with a reference flux detector, in the employed prototype system this

reference detector has been removed due to space restrictions. Furthermore, since a reference detector has to be positioned before the patient, it cannot detect all intensity fluctuations seen by the detector due to the changing interferometer state as the G_2 grating is positioned behind the patient. To find an estimate of the flux, the sample scan measurement is averaged by meaning all pixels of a shot of the normalized measurement:

$$c'_{i,0}(t) = \langle y^s(x, t) / I^0(x) \rangle_x - \langle y^s(x, t) \rangle_{x,t}. \quad (5.7)$$

Here, I assume that all spatial fluctuations are sufficiently removed by averaging over the full detector and I ignore higher-order modes. This requires a fine fringe pattern on the detector, which is achieved by tuning the interferometer gratings to feature a spatial gradient in $\phi^{(0)}$. Due to the presence of the sample, this estimate is corrupted by sample information. However, one can assume the averaged sample information to be only present in the low frequency part of $c'_{i,0}$, because it is known that the intensity fluctuation is significantly faster than the rotation speed of the gantry and thus the spatial mean of the sample projection. Therefore, since only high frequency information is needed to determine the sample coefficients, $c'_{i,0}$ is high-pass filtered and the low frequency information from the reference scan is added:

$$c_{i,0}(t) = \text{HF}_t [c'_{i,0}(t)] + \text{LF}_t [c_{i,0}^{\text{ref}}(t)]. \quad (5.8)$$

2) Acquire first transmission estimate

Next, an initial estimate of the sample transmission is calculated by filtering the normalized measurement both in time and spatially:

$$T_{\text{raw}}(x, t) = \text{LF}_{tx} \left[\frac{y_s(x, t)}{I^{(0)}(x)(1 + c_{i,0}(t))} \right]. \quad (5.9)$$

As the employed fringe pattern is tuned to produce a high frequency modulation of the sample data, low-pass filtering the normalized measurement yields a blurred guess of the sample transmission.

3) Updating fluctuation coefficients

The main driver of high frequency fluctuations in the global intensity is the anode motor drive with a frequency of $f_0 = 189$ Hz. Analyzing $c_{i,0}$ thus yields the vibration phase offset δ_0 for the sample scan. Like for the analysis of the reference coefficients, this is achieved by a least squares fitting procedure of the coefficient model (5.5) initialized by Fourier analysis. Naturally, the quality of this fit depends on $c_{i,0}$, and it is important that all influences of the sample have been removed during the averaging procedure in (5.7).

However, since the sampling rate of a scan is significantly higher than f_0 , determining the vibration phase offset has proven quite robust via Fourier analysis. Once δ_0 is known, all the fluctuation contributions at the frequency f_0 to each coefficient are synthesized with the fluctuation model (5.5).

4) Global phase estimation

The global fringe phase fluctuation coefficient $c_{p,0}(t)$ is extracted with the SPR algorithm described in [27]. For this, a simplified forward model y' is fitted to the measurement:

$$y' = T_{\text{raw}}(x, t) I'(x, t) \left(1 + V'(x, t) \times \cos \left(\phi^{(0)}(x) + c_{p,0}(t) \right) \right), \quad (5.10)$$

where higher order effects are neglected and $c_{p,0}$ is treated as the optimization parameter. The intensity I' and visibility V' are calculated with (5.2) and (5.3). For their respective coefficients, the flux estimate for $c_{i,0}$ and the synthesized coefficients from the previous step are used. The previously determined information about the interferometer intensity and sample transmission is vital to avoid corruption of the phase coefficient by sample information.

5) Updating fluctuation coefficients

As the impact of the tube stator frequency f_0 is strongest on the global intensity, δ_0 is estimated from just the flux. The remaining frequencies affect the global phase most strongly. Therefore, analysis of $c_{p,0}(t)$ yields the remaining vibration phase offsets. This is again performed by a gradient based optimization using model (5.5) initialized by Fourier analysis. For the second mode of the anode rotation frequency, the vibration phase offset is given by $\delta_2 = 2\delta_1$ with $f_2 = 2f_1 = 2 \cdot 176 \text{ Hz}$. Therefore, all the fluctuation vibration phase offsets δ_k are known and all remaining fluctuation coefficients are synthesized.

6) Initial correction for higher-order effects

Due to the use of a triangular G_1 grating, higher-order effects have to be considered during processing. This is especially important for the vibration-based sampling of the prototype, since an equal sampling of the stepping curve over a range of 2π is not guaranteed within a processing window during phase retrieval. The fit of the stepping curve is thus less robust against the systematic error of neglecting even small higher-order deviations from the pure cosine-shaped stepping curve. The resulting systematic error in the fit then leads to artifacts resembling the tuned fringe pattern on the detector. To avoid these, higher-order contributions

are removed before phase retrieval. Since the strength of the higher-order contributions is quite small, it is sufficient to do this in a two-step procedure. In the first step, an initial estimate of higher-order contributions (ignoring the phase shift and visibility reduction by the sample) is calculated based on model (2.16) and subtracted from y^s :

$$\mathcal{H}_1 = \sum_{n=2}^3 T_{\text{raw}} I V_n \cos(n(\phi)), \quad (5.11)$$

$$y_1^s = y^s - \mathcal{H}_1. \quad (5.12)$$

7) Initial phase retrieval

Using the estimate for a higher-order-free measurement y_1^s , the sliding window phase retrieval algorithm presented in [43] is used to extract a first estimate of the sample parameters T_1 , D_1 , and Φ_1 . However, since the dark-field and phase signals were not considered so far, their contributions to the higher-order modes are missing.

8) Final correction for higher-order effects

To fix this, a second estimate for a higher-order-free measurement y_2^s is calculated using the initial SPR results:

$$\mathcal{H}_2 = \sum_{n=2}^3 T_1 I V_n D_1^{n^2} \cos(n(\phi + \Phi_1)), \quad (5.13)$$

$$y_2^s = y_1^s - \mathcal{H}_2. \quad (5.14)$$

Due to the overall small contribution of the higher orders to the stepping curve, this second estimate has proven sufficient in removing higher-order-related artifacts from final reconstructions.

9) Final phase retrieval

Next, sliding-window-based phase retrieval is performed on y_2^s using the patch-wise dual-pass algorithm presented in [43]. Alternatively the global phase retrieval algorithm introduced in Section 3.5 can be used. This returns the three sample signals T_{SW} , D_{SW} , and Φ_{SW} .

10) Dark-field beam hardening correction

Due to the energy-dependency of the visibility of the Talbot-Lau interferometer, an artificial dark-field signal is produced by beam hardening effects. Therefore, a beam hardening correction for the dark-field channel is performed, which is outlined in more detail in [26]. The corrected dark-field signal is calculated by:

$$D_{\text{corr}} = D_{\text{SW}} \cdot (T_{\text{SW}})^{-h}, \quad (5.15)$$

where h is determined from a calibration scan of a non small-angle scattering sample.

11) Calculating residual based transmission

According to model (2.16), a second estimate of the transmission can be calculated using the results from the sliding window analysis:

$$T_{\text{res}} = \frac{y^s}{I} - \sum_{n=1} T_{\text{SW}} V_n D_{\text{SW}}^2 \cos(n(\phi + \Phi_{\text{SW}})). \quad (5.16)$$

Here, the fringes modulated onto the detector are viewed as a perturbation of the conventional transmission measurement and are removed by subtracting the fringe modes from the raw measurement. This second estimate of the transmission is thus unaffected by the smearing out of the sliding window phase retrieval process and expected to show sharper edges than the sliding window guess of the transmission.

12) Reconstruction

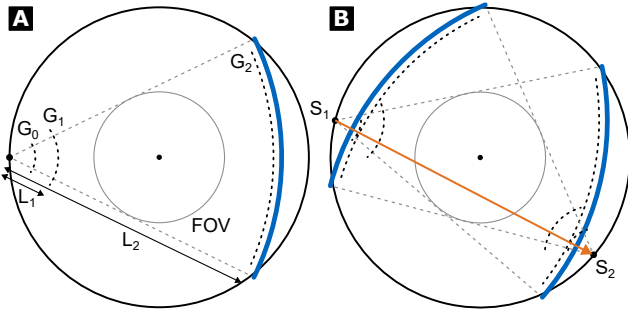


Figure 5.2: **A:** Sketch of the system geometry. **B:** An example direct ray measured from a source position S_1 is shown together with the complementary source position S_2 . Figure adapted from [61].

Once the sample information has been retrieved in the projection domain, it is reconstructed using filtered backprojection (FBP). However, in both the dark-field and the phase channel, the geometry-specific sensitivities s_D and s_ϕ have to be considered during reconstruction. Neglecting them can lead to the appearance of cupping artifacts for very large samples, as depicted in [36]. Therefore, a dedicated FBP algorithm is used here that accounts for the sensitivity gradient along the rays (see Eq. (2.20) and (2.21)). This concept has been described in [36] for axial CT and a different grating geometry where the object is located between G_0 and G_1 and the concept is adapted here to the new scenario. The basic idea is that a linear sensitivity gradient along the ray can be eliminated in 2D fan-beam geometry if a direct and a complementary ray are averaged; see Figure 5.2. Then, the only change in a 2D FBP is a pre-weighting to compensate for a fan-angle dependency of the mean sensitivity after averaging. Using this adapted FBP algorithm, the linear diffusion coefficient

ε is reconstructed, while the attenuation coefficient μ is reconstructed with a conventional FBP.

The reconstruction algorithm for the dark-field channel is therefore performed as follows. First, the measured dark-field line integrals p_0 are obtained according to Equation (2.20) by applying the negative logarithm:

$$p_0(\lambda, \beta, h) = -\log(D_{\text{corr}}), \quad (5.17)$$

$$= \int_{L_1}^{L_2} s_D(l) \varepsilon(\mathbf{S}(\lambda) + \ln(\lambda, \beta, h)) dl, \quad (5.18)$$

where β is the fan-angle, λ the gantry position and h is the height position on the 2-D focus centered detector array. Next, the dark-field sensitivity specific pre-weighting is applied to the data in native geometry with fan-angle dependent weights:

$$p_1(\lambda, \beta, h) = \frac{L_2 - L_1}{L_2 - R \cos \beta} p_0(\lambda, \beta, h). \quad (5.19)$$

Here R is the distance from the source to the isocenter. The reason for the fan-angle dependency is that S_1 and S_2 get closer to each other with increasing fan-angle, see Figure 5.2. After pre-weighting, the projections are re-binned into wedge-geometry, which is explained in more detail in [62]:

$$p_2(\phi, u, h) = p_1(\phi - \arcsin \frac{u}{R}, \arcsin \frac{u}{R}, h), \quad (5.20)$$

$$\phi = \lambda + \beta, \quad (5.21)$$

$$u = R \sin \beta. \quad (5.22)$$

Now, a weighting with the cosine of the cone-angle of each individual detector element is performed:

$$p_3(\phi, u, h) = \frac{R}{\sqrt{h^2 + R^2}} p_2(\phi, u, h). \quad (5.23)$$

Before back-projecting the data, a ramp filter is applied:

$$p_4(\phi, u, h) = \int_{-\infty}^{\infty} p_3(\phi, u, h) h_{\text{ramp}}(u - u') du', \quad (5.24)$$

$$h_{\text{ramp}}(u) = \int_{-\infty}^{\infty} |v| e^{i2\pi uv} dv. \quad (5.25)$$

For each voxel $\mathbf{x}$ to reconstruct, back-projection from all projection angles ϕ is done using an aperture weighting scheme as described by [63] with the modification that normalization of weights is done over 2π partners instead of π partners to ensure proper averaging of the sensitivities, i.e.,

$$\varepsilon(\mathbf{x}) = \frac{1}{2\pi} \int_{\phi_f(\mathbf{x})}^{\phi_l(\mathbf{x})} w_{\text{all}}(\phi, \mathbf{x}) p_4(\phi, u(\phi, \mathbf{x}), h(\phi, \mathbf{x})) d\phi, \quad (5.26)$$

$$w_{\text{all}}(\phi, \mathbf{x}) = \frac{1}{2} \frac{w_{\text{ap}}(\phi, \mathbf{x})}{\sum_n w_{\text{ap}}(\phi + 2\pi n, \mathbf{x})}, \quad (5.27)$$

where w_{ap} is the aperture weighting function described in [63] and $\phi_f(\mathbf{x})$ ($\phi_l(\mathbf{x})$) represents the first (last) parallel projection that illuminates the voxel, respectively. This is analogue to the normalization concept for high-resolution scans as described in [62].

13) Post-processing

To improve the image quality further, multiple post-processing steps are applied. These include steps standard to conventional CT imaging, like a truncation [64] and ring correction [65]. Furthermore, the multi-modal nature of the scan is exploited and a cross-channel-based bilateral filter [66, 67] is applied to the dark-field reconstruction:

$$\varepsilon_{\text{bl}}(x) = \frac{1}{W} \sum_{x_i \in \Omega} \varepsilon(x_i) f_r(\mu(x_i) - \mu(x)) f_d(x_i - x), \quad (5.28)$$

$$W = \sum_{x_i \in \Omega} f_r(\mu(x_i) - \mu(x)) f_d(x_i - x), \quad (5.29)$$

where the domain and range filter functions f_d and f_r are chosen to be Gaussian kernels and Ω represents a 3D patch centered around the voxel x . To retain edge information in the dark-field channel, information from the attenuation coefficient μ is passed to the range filter function. For the final result, the bilateral filtered image is blended with the raw image to return a mixture of both images.

5.5 Results

To demonstrate the presented processing chain, it was applied to data from the dark-field CT prototype. A detailed description of the prototype's hardware design is given in [42]. Two test phantoms were measured and reconstructed with the proposed processing pipeline. The two phantoms are identical to the phantoms used in [24] and are described there in more detail alongside an interpretation of the measurement results and an introduction to the dark-field Hounsfield unit (HUD) scale for quantification, which I also employ here. In this work, I focus on the image quality achieved with the presented processing approach. For clinical CT imaging, three acquisition modes are of interest: axial, helical and surviue scans. The data from all three modes can be processed with the presented pipeline, which is demonstrated in the following.

5.5.1 Axial Dark-Field CT Scans

To test axial acquisitions, first, a small phantom consisting of multiple filled plastic tubes and a polyoxymethylene cylinder was scanned. This phantom is positioned in the isocenter and leaves a significant portion of the detector uncovered during measurement. Two axial

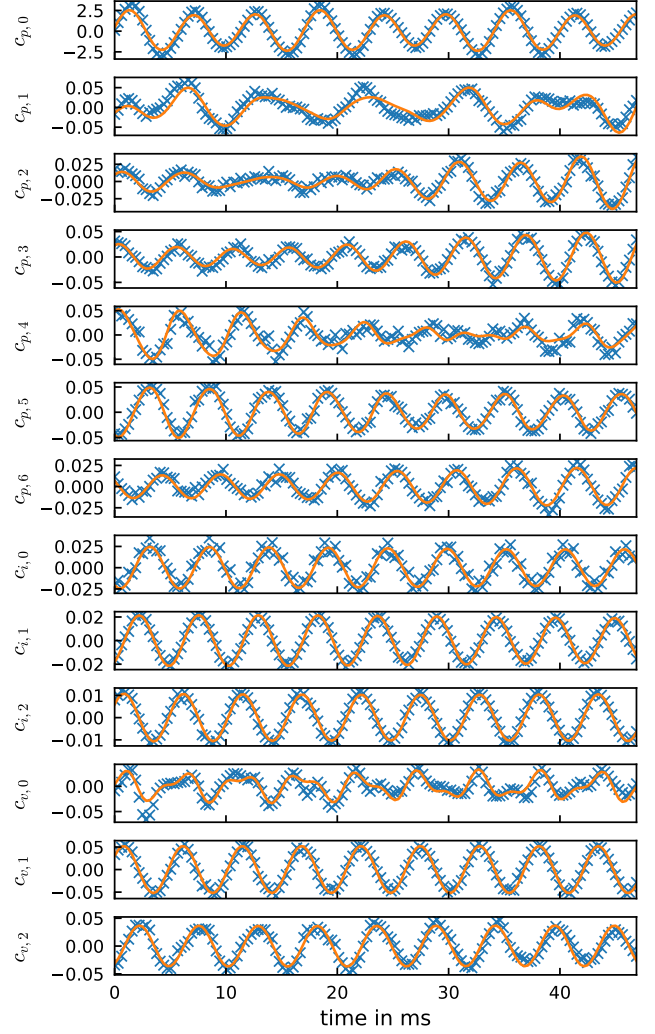


Figure 5.3: High-pass filtered fluctuation coefficients of the first 150 shots of a reference air scan (blue) alongside with the fluctuation model (5.5) fitted to the data (orange). Figure adapted from [61].

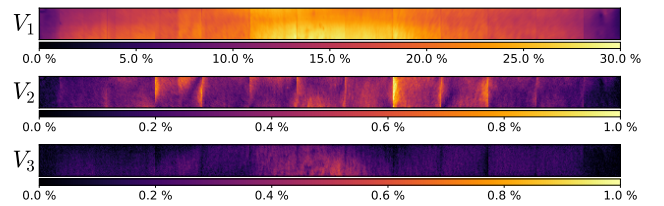


Figure 5.4: System visibility V_1 alongside higher-order visibilities V_2 and V_3 , extracted from a reference air scan. Due to the employed triangular G_1 grating, higher-order visibilities show a small, yet significant, contribution to the shape of the stepping curve. Note: scale differs among the first order and higher-order visibilities. Figure adapted from [61].

measurements were performed with a tube voltage of 80 kVp and a current setting of 550 mAs. A total of 2400 projections were acquired over a rotation period of 1 s for both the reference and the sample scan.

A detailed analysis of the air scan processing alongside

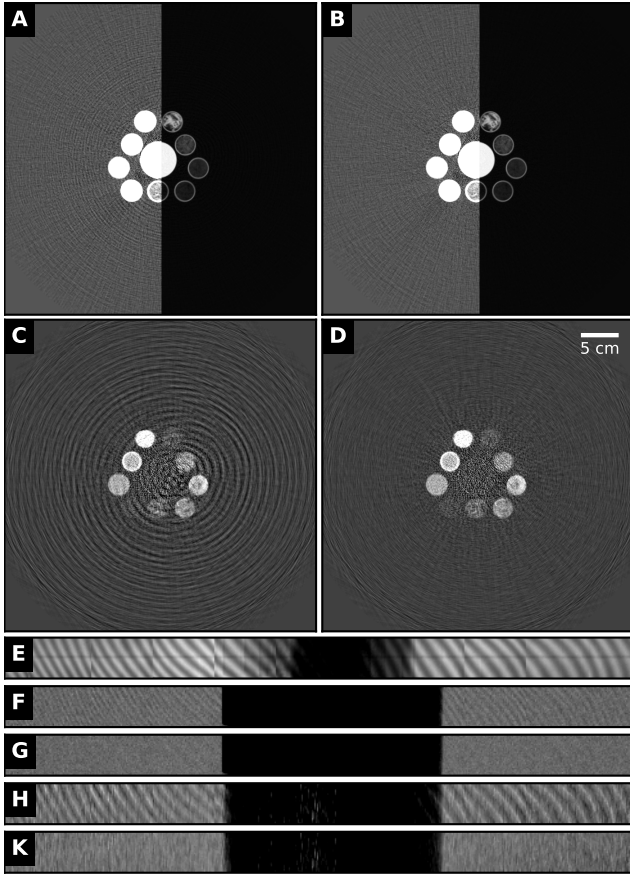


Figure 5.5: Effect of correcting for higher order effects on cylinder phantom **A:** Reconstructed sliding-window-based attenuation channel considering only first order effects. The left side depicts a windowing close to the background (level, -980 HU; width, 70 HU), while the right side is windowed around the sample (level, -310 HU; width, 740 HU) **B:** Reconstructed attenuation channel with proposed correction for higher-order terms using the same windowing as in **A** **C:** Reconstructed dark-field channel considering only first order effects, which leads to rippling artifacts **D:** Reconstructed dark-field channel with the proposed correction for higher-order terms (level, 570 HUd; width, 1140 HUd) **E:** Raw detector measurement featuring a vertical fringe pattern **F:** Extracted attenuation projection considering only first order effects **G:** Extracted attenuation projection with proposed correction for higher-order terms **H:** Extracted dark-field projection considering only first order effects **I:** Extracted dark-field projection with proposed correction for higher-order terms **J:** Extracted dark-field projection with proposed correction for higher-order terms. Figure adapted from [61].

a discussion of the results is given in [27]. Therefore, here, I only focus on the additional analysis of the fluctuation coefficients as well as the higher-order visibilities. Figure 5.3 depicts the first 150 high-pass filtered fluctuation coefficients that were extracted from the reference air scan. All of them exhibit an oscillating behavior. After the analysis of the oscillation properties, the fitted fluctuation model (5.5) describes these coefficients sufficiently well. Adapting this model to the sample scan thus upholds the correlations visualized in Figure 5.1. The three system visibilities are shown in Figure 5.4. Due to the worsening acceptance of the G_0

grating towards the left and right edges of the detector, all visibilities are reduced to the sides [26]. While both higher-order visibilities are in the sub-percent range, they affect the shape of the stepping curve significantly and cannot be neglected during processing.

To demonstrate the effect of the higher-order contributions, the scan featuring the small phantom was processed once with and once without the correction for higher-order effects, i.e., without performing steps 6) and 8) from Section 5.4. The results are depicted in Figure 5.5. Due to the decrease of higher-order terms with D^{n^2} , their effect is most obvious in sample free regions. Therefore, higher-order effects corrupt the sample free area of the sample measurement most. For the attenuation reconstruction the resulting rippling artifacts are only noticeable when choosing a narrow windowing close to the background as on the left side of Figure 5.5A and B. For the dark-field channel the artifacts are more prominent and visible even for a more sample centered windowing (Figure 5.5C and D).

Figure 5.5F – K depict a respective attenuation and dark-field projection extracted with and without considering higher-order terms in the stepping curve. In both channels, residual fringe artifacts can be observed when higher-order effects are neglected. They resemble the shape of the employed fringe pattern depicted in Figure 5.5E and shift accordingly from shot to shot: For the attenuation channel, the fringe artifacts manifest with a multiple of the frequency of the original fringe pattern (see Figure 5.5F). In the dark-field channel they appear with the same frequency as the fringe pattern but exhibit a relief like appearance due to the alternating bright and dark edges of the fringes (see Figure 5.5H). These fringe artifacts in the projection domain result in a rippling artifact in both the reconstructed channels. However, the application of the proposed higher-order correction removes these effectively.

For the second test, a scan of an anthropomorphic thorax phantom was performed. This is a large phantom that covers most of the detector and features realistic dimensions for scans of human patients. The thorax phantom was modified with a small-angle scattering neoprene insert to simulate the porosity of lung parenchyma. The axial measurement was conducted with a tube voltage of 80 kVp and a current setting of 550 mAs. A total of 4800 projections were acquired over a rotation period of 1.5 s. To improve the phase sampling for this large object, the fringe pattern was changed to a narrow and horizontal pattern as discussed in [43].

The reconstructed sample channels of the thorax phantom scan are presented in Figure 5.6. For demonstration purposes, first a naive reconstruction of the data was performed which is depicted in Figure 5.6A and E.

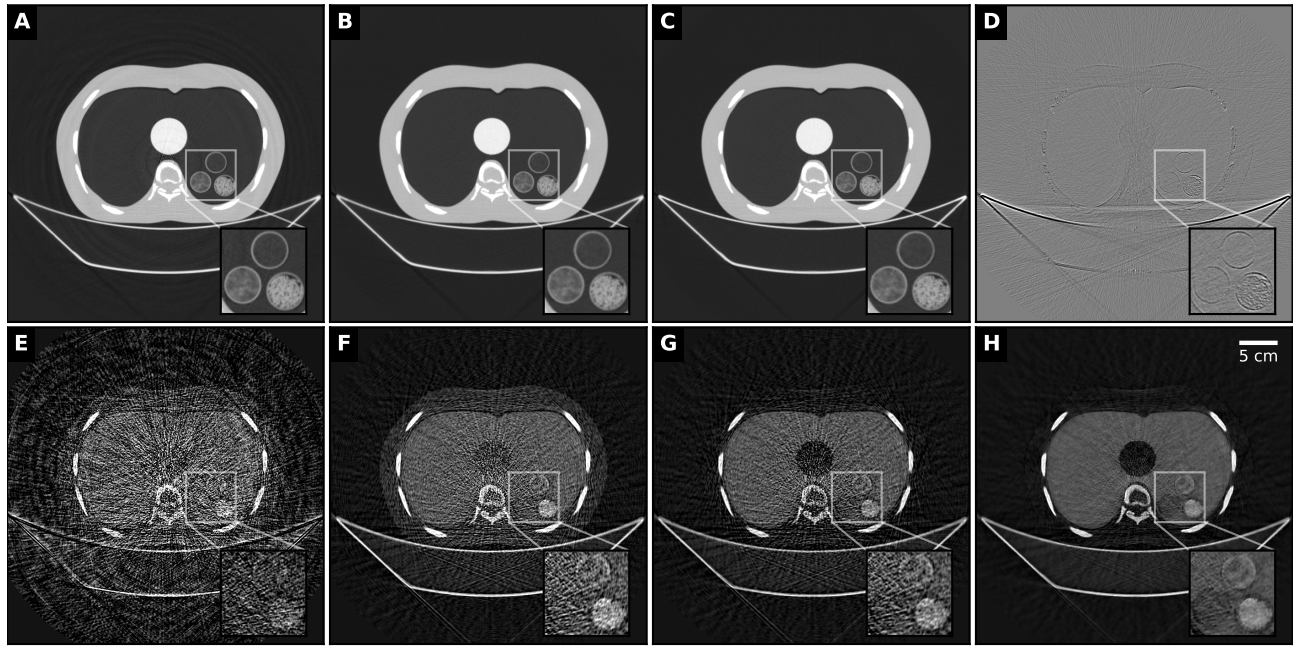


Figure 5.6: Reconstructions of the thorax chest phantom depicting different processing stages **A:** Reconstructed attenuation channel from naive processing approach (level, -350 HU; width, 675 HU) **B:** Reconstructed sliding-window-based attenuation channel with proposed processing pipeline **C:** Reconstructed residual-based attenuation channel **D:** Difference image of B and C **E:** Reconstructed dark-field channel from naive processing approach using only a global phase estimate and no advanced vibration modelling and phase retrieval (see Section 5.5 for more details) **F:** Reconstructed dark-field channel using the proposed processing pipeline without the dark-field beam hardening correction (step 10) and before post-processing (step 13) **G:** Reconstructed dark-field channel with proposed processing pipeline after the beam hardening correction proposed in [26] **H:** Reconstructed dark-field channel after post-processing (level, 1680 HUd; width, 1990 HUd). Figure adapted from [61].

Here, most steps of the processing pipeline were omitted: First, the flux and the sample transmission were estimated (steps 1 and 2). Next, a simple estimate of the global fringe phase was performed (step 4), which subsequently was used for a conventional sliding-window-based phase retrieval and conventional FBP. In both channels, rippling artifacts are visible which stem from neglecting the influence of vibrations and higher-order terms. In the dark-field channel, the conventional sliding-window phase retrieval leads to movement induced streaking artifacts due to the continuous acquisition which are especially prominent at the edges of the patient couch. This is discussed in more detail in [43].

Reconstruction results using the proposed processing pipeline are presented in the remainder of Figure 5.6. For these, both the attenuation and the dark-field channel are fringe artifact free. To demonstrate the difference of the sliding window and the residual-based attenuation reconstruction, a difference image is shown in Figure 5.6D. As expected, there is a strong difference in the representation of edge information in both channels. Figure 5.6F and G demonstrate the necessity for the beam hardening correction of the dark-field channel: After the correction, the surrounding soft-tissue of the phantom no longer exhibits a dark-field signal, since this was purely caused by beam-hardening. In

Figure 5.6H the final result of the dark-field channel is shown. Due to the cross-channel-based filtering, the visual impression has been improved substantially from the raw image Figure 5.6E and streaking artifacts are reduced.

5.5.2 Helical Dark-Field CT Scans

For helical acquisitions, the gantry is rotating while the patient couch is moved and the measurement is performed. In the prototype scanner, the gantry is supported on an air cushion, which decouples it from its surroundings. As a result, the couch movement does not introduce additional perturbations to the interferometer. Therefore, until the reconstruction step, each individual rotation of the gantry can be treated during processing like an axial scan. For the reference scan, it is thus sufficient to use an axial air scan with the same flux, exposure, and rotation time. Using this reference scan, one then determines the interferometer state of the sample scan for each rotation separately. The main difference to the axial case is the couch movement, which introduces an additional movement of the sample projections in z-direction. However, this movement can be compensated for by using the techniques described in Chapter 3.

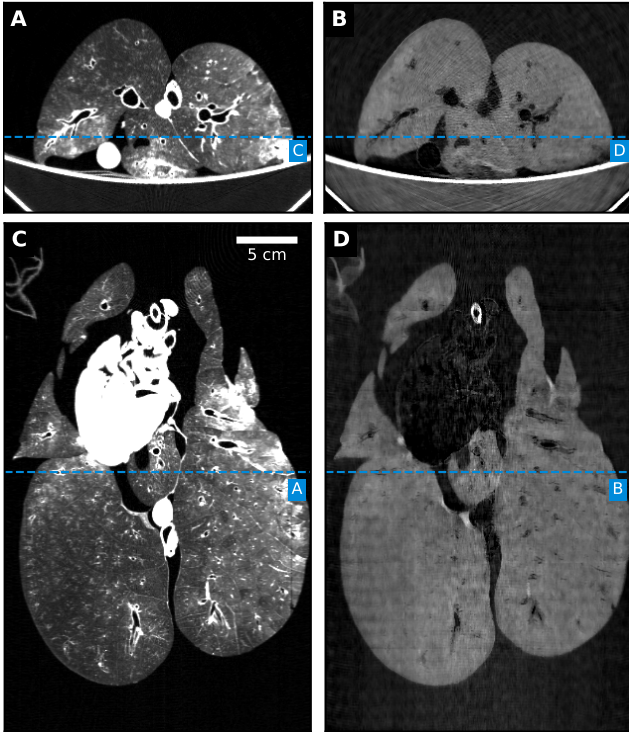


Figure 5.7: Helical CT scan of an inflated porcine lung and heart acquired with the dark-field CT prototype. **A** shows a conventional attenuation-based axial slice, while **B** depicts the dark-field modality. **C** and **D** show respective coronal slices.

Once the sample information has been retrieved in the projection domain, it is reconstructed using a dedicated helical FBP algorithm for the dark-field channel that accounts for the sensitivity gradient along the rays and is described in Section 5.4 12).

To test helical acquisitions, a helical dark-field CT measurement of an inflated porcine lung and heart was acquired. For the scan, a current of 353 mA, tube voltage of 80 kVp, and a rotation time of 1.5 s was used. To cover a length of 52 cm in z -direction with a pitch of 0.72, 86736 shots were acquired with a total acquisition time of 55.3 s. The reconstructed helical scan is depicted in Figure 5.7.

This demonstrates not only helical acquisition but also the prototype's capability to resolve the dark-field signal of real lung parenchyma. Here, a strong dark-field signal is evident in the lung parenchyma while the heart tissue vanishes in the dark-field channel due to a lack of small-angle scattering interfaces.

5.5.3 Dark-Field Surview Scans

Surview scans, also known as scanograms or scout scans, are 2D images obtained at the start of a CT examination used as localizer radiographs for further procedure planning. For surview imaging, the beam is collimated to only the four central detector rows.

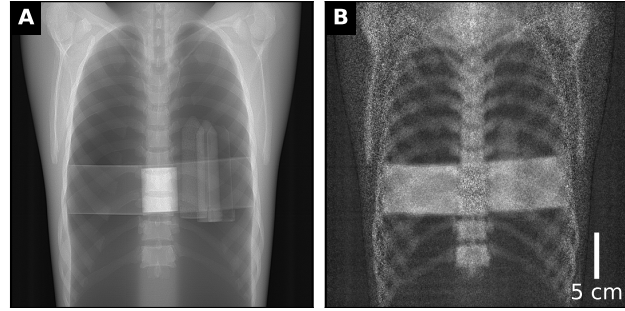


Figure 5.8: Surview scan of a human thorax phantom acquired with the dark-field CT prototype. **A** shows the conventional attenuation-based surview, while **B** depicts the dark-field modality. By the methods design, both channels are perfectly co-registered. Figure adapted from [68]

Furthermore, the detector position is fixed during the scan and only the patient couch is moved during the measurement. Detector readings are acquired roughly every 60 μm , providing a fine temporal sampling for phase retrieval. The lack of the gantry rotation removes the centrifugal force acting on the gratings that has to be considered for axial scans. Thus low frequency vibrations are excluded from the surview vibration model. For surview scans, I therefore only include the vibrations generated by components on the gantry in the vibration model, which simplifies the employed perturbation model.

In practice, surview imaging is performed by first acquiring an air surview scan that uses the same scan parameters as the sample scan and next performing the sample surview scan. Based on both scans, the simplified perturbation model is fitted to the data and the interferometer state is determined. Next, phase retrieval is performed to extract the transmission T and the dark-field D . To access the line integrals, $-\log(\cdot)$ is applied. Finally, post-processing is applied to the data. It involves binning the data in scanning direction and averaging the four detector rows. Furthermore, a beam hardening correction is applied to the dark-field channel [26]. In the attenuation channel, a Laplace-filtered version is mixed in with the original image to enhance edges.

To demonstrate the capabilities of the dark-field CT prototype for surview imaging, a dark-field surview scan was performed of the lungman phantom with a tube voltage of 80 kVp and a current of 200 mA. During scanning, 7006 shots were acquired in 3.2 s to cover a total scan length of 45 cm. The resulting surview scan is depicted in Figure 5.8.

5.6 Discussion

With the proposed processing pipeline, I present a robust procedure to find an estimate of the interferometer

state despite the highly dynamic environment. This is important to avoid the occurrence of residual fringe artifacts in the final reconstructions. Examples for such artifacts are given in [27]. The presented fluctuation model, enables one to exploit existing correlations and drastically reduces the number of free parameters that have to be determined for a sample scan: Only the vibration phase offsets δ_k have to be updated from estimates of the flux and the global phase. Therefore, one does not have to rely on sample-free detector area and can perform an unambiguous separation of sample and fluctuation information. Furthermore, this approach ensures fast processing times as it avoids costly optimization steps of per-shot parameters.

To enable rapid data acquisitions with a continuously rotating gantry, I expect that vibration corrections will be important for other future dark-field CT systems as well. This is especially the case, since vibrations of the interferometer gratings provide an effective way to perform fast phase sampling. However, the more chaotic grating movement induced by these vibrations also leads to a more uneven sampling of the stepping curve. If the fringe pattern shows significant higher-order contributions, this uneven sampling leads to fringe artifacts if the higher-orders are not considered during processing. With the proposed higher-order correction scheme, small contributions of the higher-order terms can be removed successfully.

Finally, I want to highlight the effectiveness of cross-channel-based post-processing approaches. As grating-based dark-field imaging is a multi-modal imaging technique, exploiting the information from complementary channels can improve the visual impression of the dark-field channel significantly.

and sample information, enabling vibration-artifact free sample reconstructions. This development brings dark-field CT an important step closer to clinical usage as it demonstrates as a first proof of concept the feasibility of all three standard acquisition modes of clinical CT scanners (axial, helical, and surviue) for human-sized dark-field imaging.

5.7 Conclusion

This chapter presented the sample processing pipeline of the first human-scale dark-field CT scanner for arbitrary sample sizes, i.e., without a need for sample free detector areas in the sample scan. To enable rapid data acquisitions for clinical applications, a continuous acquisition protocol using a fast rotating gantry is necessary. In this, for interferometry, hostile environment, a big challenge lies in determining the interferometer state accurately for each view in spite of vibrations and other fluctuations. This is especially difficult in sample scans with large samples, where in each view, substantial portions of the detector are covered by the sample's projection. With the proposed processing pipeline, I demonstrate that it is possible to decouple fluctuations of the interferometer state and sample information using prior knowledge gained by a thorough analysis of air scans. By exploiting the found correlations, relevant fluctuation modes are identified and adapted for the sample scan. This allows to separate fluctuation

Chapter 6

Conclusion and Outlook

Si tacuisses, philosophus mansisses.

attributed to Boethius

This work represents a step forward in advancing the technical foundations for clinical phase-contrast and dark-field CT imaging, addressing key challenges in signal retrieval, optimization of phase sampling, and data processing. By developing novel algorithms and methodologies, this research lays the groundwork for robust and efficient imaging protocols that are crucial for translating the dark-field modality into clinical practice. The efficacy of the developed algorithms was tested with data from the first human-scale dark-field CT prototype which is based on a clinical CT scanner.

Phase Retrieval The analysis of sliding window phase retrieval has yielded insights and practical solutions for mitigating motion-induced cross-talk artifacts in continuous tomographic data acquisition. By reframing phase retrieval as a demodulation problem, I demonstrated that cross-talk artifacts arise from overlapping modulation sidebands. To combat the arising artifacts, three novel extensions to the sliding window algorithm were proposed, maintaining the computational efficiency and scalability of linear optimization techniques while significantly improving image quality. Furthermore, a global phase retrieval method based on Tikhonov regularization was introduced, offering superior artifact suppression, especially at sharp edges, without compromising speed or resolution. These approaches not only advance grating-based CT but also hold promise for other scanning-based imaging systems.

Phase Sampling Investigating the effect of phase sampling quality demonstrated its importance in optimizing image fidelity in dark-field CT. Both the aspect of variance minimization and cross-talk artifact mitigation by improving the spectral separation of the carrier signals were discussed. With the introduction of the sampling quality factor, I offer a concrete metric for designing optimal sampling protocols. Applied to the dark-field CT prototype, these findings suggest an

optimal fringe orientation and density for the interferometer. Furthermore, I find that temporal sampling remains suboptimal and will require further engineering efforts to reach desired levels of performance. This will be especially important when moving to rotation speeds faster than 1 s.

Data Processing To enable clinical-scale imaging with human-sized samples, I developed a novel data processing pipeline tailored to the unique challenges of grating-based CT in continuously rotating gantries. By leveraging prior knowledge from air scans, the pipeline decouples interferometer state fluctuations from sample information, overcoming the limitations posed by large sample projections and environmental vibrations. This innovation enables vibration-artifact-free reconstructions and validates the feasibility of all standard acquisition modes, namely axial, helical, and surview scanning, for clinical dark-field CT imaging.

Outlook Together, these contributions advance the state-of-the-art in grating-based imaging, addressing critical technical hurdles that previously limited its applicability to clinical settings. By enhancing data acquisition, processing, and sampling methodologies, this work brings dark-field CT closer to routine clinical use. Still, several important challenges remain to be addressed. Key issues include acquisition speed, patient dose, and noise levels, all of which are critical for ensuring the feasibility and effectiveness of this imaging modality in clinical practice.

Improving acquisition speed is essential for integrating dark-field CT into clinical workflows. The continuous acquisition protocols developed here offer a significant step forward, but further optimization is necessary to reduce total scan time without compromising image quality. This is especially important for lung imaging, where it is required to scan along the entire length of the lung within the time span of one breath-hold. Here, faster rotation speeds of the gantry and a greater coverage with gratings of the detector in scan-direction are possible paths for improvements. However, this requires overcoming the current limitations imposed by the vibration-based phase sampling, which is not

sufficient for the fastest acquisition modes available by the scanner.

When moving to faster acquisition times, the noise level is another significant challenge. While the methods proposed in this work improve the separation of signal and noise, further refinement is needed to enhance noise suppression without losing critical structural information. Developing algorithms tailored to noise-dominated scenarios and incorporating advanced priors could enhance robustness. For example, exploring iterative reconstruction techniques such as statistical iterative reconstruction with cross-channel-based regularization might yield substantial improvements. Another interesting approach would be to test if a Poisson-based likelihood can improve the phase-retrieval for low-count regions, as it more accurately describes the counting statistics. However, this would require a departure from linear optimization techniques towards gradient-based approaches, increasing processing complexity. In turn, a gradient-based approach would also unlock more sophisticated regularization methods for the phase retrieval problem. This would allow for the inclusion of advanced prior knowledge like tomographic consistency and could further improve image quality.

Artificial intelligence (AI)-driven denoising techniques represent a further promising avenue, particularly deep-learning-based approaches trained on large datasets to differentiate noise from true structural features. Convolutional neural networks (CNNs) or transformer-based architectures could be employed for high-fidelity noise reduction while preserving diagnostically relevant details. Additionally, AI methods could extend beyond denoising to phase retrieval. Here, deep-learning models trained on simulation and experimental data might provide more robust signal retrieval for continuously acquired data, potentially outperforming conventional retrieval algorithms in low-dose and low-count conditions. Implementing AI-based methods in the processing pipeline could therefore improve image contrast and avoid movement-induced artifacts while maintaining computational efficiency. A limitation for using AI-based methods is the lack of proper ground truth training data from the prototype scanner. This problem can be solved by improving the processing of multi rotation scans, which have mitigated movement-based artifacts and increased dose-levels, making them more suitable for training.

Patient dose remains a primary concern, particularly given the higher radiation requirements of grating-based imaging systems compared to standard CT imaging. Strategies to minimize dose at the existing prototype scanner should prioritize optimizing phase sampling schemes and acquisition protocols. Adopting helical scanning as a primary mode, while standard in

clinical practice, must carefully balance dose and acquisition time. Studies aimed at dose reduction through hardware improvements, such as more efficient detectors and gratings, or through software-based approaches like dose modulation, are critical next steps for future dark-field CT scanners.

A key development in future prototypes could be the integration of spectral imaging and photon-counting detectors. Spectral imaging has the potential to enhance material differentiation and improve tissue characterization, which could provide complementary information to dark-field signals. Photon-counting detectors, with their superior energy resolution and higher detection efficiency, could improve sensitivity and allow for lower-dose acquisitions. Combining dark-field CT with spectral imaging could open new diagnostic possibilities, particularly in lung imaging, where tissue composition plays a crucial role in disease characterization.

Looking ahead, a second-generation prototype should incorporate several major design improvements. Novel grating or detector designs that eliminate the need for the G2 grating would simplify the optical setup and improve dose-efficiency. Furthermore, enhancing the interferometer sensitivity even for higher X-ray energies by reducing the grating pitch is an important step in improving the image contrast. The increased sensitivity in high energy regions enables to explore higher voltage settings to optimize penetration and contrast for different anatomical regions, further aligning dark-field CT with conventional clinical imaging standards. This requires further innovations in grating manufacturing to match these higher energies to the design energy of the interferometer. Extending the propagation distance could further improve contrast by increasing the interference effects utilized in phase retrieval. This would require moving to a big bore CT scanner. Additionally, refining the mechanisms for phase sampling, possibly incorporating dynamic or adaptive sampling strategies, could enhance robustness and efficiency in clinical applications for faster acquisitions.

To catch up with conventional clinical scanners, additional already existing technological advancements should be considered for the dark-field CT prototype and future iterations. Currently, the prototype is operated without a reference flux detector. Reintroducing this reference detector could enable better flux calibration and correction capabilities. However, due to the position of the G2 grating right in front of the primary detector, the reference calibration schemes would require novel approaches because of the attenuation induced by the analyzer grating. Implementing a flying focal spot could improve spatial resolution and reduce artifacts, especially in the attenuation channel. However, this also complicates phase-retrieval due to

the added motion of the sample's projections from shot to shot. Furthermore, the dark-field beam hardening correction should be improved to correctly account for bones in addition to water, ensuring more accurate readings in clinical scenarios.

Finally, the clinical utility of dark-field CT has to be validated through reader studies and comparisons with more established techniques, such as conventional CT imaging and even 2D dark-field imaging. In addition to testing the dark-field channel itself, also the quality of the attenuation channel has to be tested against established clinical scanners. These studies will help determine its added value in diagnostic contexts and identify areas where it outperforms existing modalities. Additionally, understanding the trade-offs between acquisition speed, dose, and noise level in a clinical setting will be crucial for optimizing protocols and achieving widespread adoption.

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List of Abbreviations

CNR	contrast-to-noise ratio
CG	conjugate gradient
CT	computed tomography
DC	0 Hz carrier signal
DFCT	dark-field computed tomography system
DP	dual pass (phase-retrieval)
FBP	filtered back-projection
FOV	field of view
FP	tomographic forward-projection
GPU	graphics processing unit
IBSIR	intensity-based statistical iterative reconstruction
NRMSE	normalized root mean square error
PCA	principal component analysis
POM	polyoxymethylene
SIR	statistical iterative reconstruction
SW	sliding window
SP	single pass (phase retrieval)
SPR	(linearized) statistical phase retrieval
SVD	singular value decomposition

Appendix A

Auxiliary Data

A.1 Temporal sampling qualities for different rotation speeds

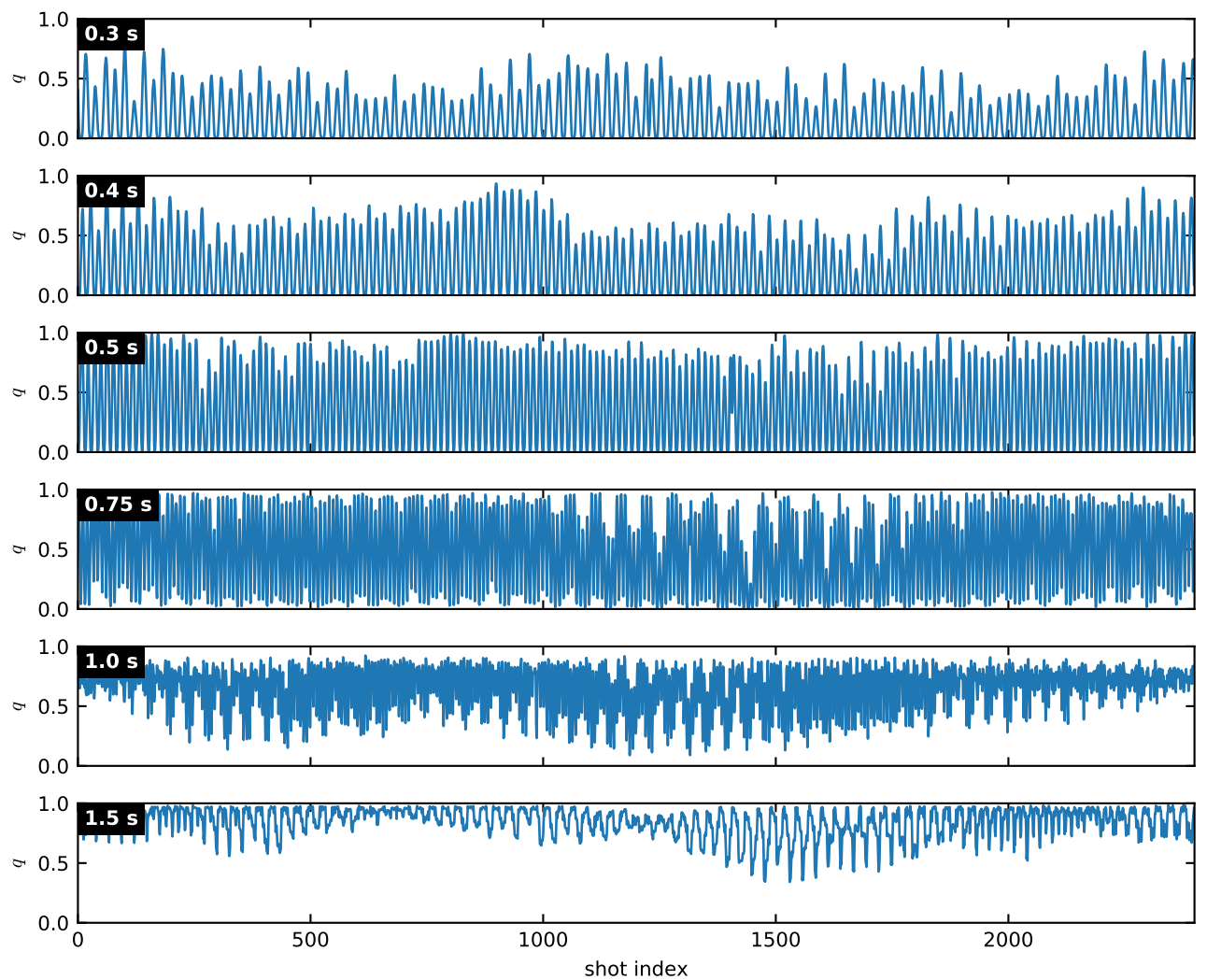


Figure A.1: Sampling quality factors for different rotation speeds. A significant decrease in sampling quality is evident for shorter rotation times.

Scientific Contributions

Vulgo enim dicitur: Iucundi acti labores.

Cicero

Publications with first authorship

- **Jakob Haeusele**, Clemens Schmid, Josepha Hilmer, Florian Schaff, Tobias Lasser, Thomas Koehler, and Franz Pfeiffer. *“Enhanced Phase Retrieval for Continuous Grating-Based Dark-Field CT via Global Tikhonov Regularization”*. TechRxiv, 2024.
- **Jakob Haeusele**, Clemens Schmid, Manuel Viermetz, Nikolai Gustschin, Tobias Lasser, Thomas Koehler, and Franz Pfeiffer. *“Robust Sample Information Retrieval in Dark-Field Computed Tomography with a Vibrating Talbot-Lau Interferometer”*. IEEE Transactions on Medical Imaging, 2024.
- **Jakob Haeusele**, Clemens Schmid, Manuel Viermetz, Nikolai Gustschin, Tobias Lasser, Thomas Koehler, and Franz Pfeiffer. *“Advanced Phase-Retrieval for Stepping-Free X-Ray Dark-Field Computed Tomography”*. IEEE Transactions on Medical Imaging, 2023.

Publications with co-authorship

- Tom Kumschier, Johannes Thalhammer, Clemens Schmid, **Jakob Haeusele**, Thomas Koehler, Franz Pfeiffer, Tobias Lasser, and Florian Schaff. *“Streak artefact removal in x-ray dark-field computed tomography using a convolutional neural network”*. Medical Physics, 2024.
- Manuel Viermetz*, Nikolai Gustschin*, Clemens Schmid, **Jakob Haeusele**, Maximilian von Teuffenbach, Pascal Meyer, Frank Bergner, Tobias Lasser, Roland Proksa, Thomas Koehler, and Franz Pfeiffer. *“Dark-field computed tomography reaches the human scale”*. Proceedings of the National Academy of Sciences, 2022.
- Manuel Viermetz, Nikolai Gustschin, Clemens Schmid, **Jakob Haeusele**, Peter Noël, Roland Proksa, Stefan Loescher, Thomas Koehler, and Franz Pfeiffer. *“Technical design considerations of a human-scale Talbot-Lau interferometer for dark-field CT”*. IEEE Transactions on Medical Imaging, 2022.
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- Clemens Schmid, Manuel Viermetz, Nikolai Gustschin, Wolfgang Noichl, **Jakob Haeusele**, Tobias Lasser, Thomas Koehler, and Franz Pfeiffer. *“Modeling vibrations of a tiled Talbot-Lau interferometer on a clinical CT system”*. IEEE Transactions on Medical Imaging, 2022.

* Shared first authorship.

Conference contributions

- **Jakob Haeusele**, Clemens Schmid, Josepha Hilmer, Florian Schaff, Tobias Lasser, Thomas Koehler, and Franz Pfeiffer. *“A Global Phase Retrieval Method for Continuously Acquired Grating-Based Dark-Field Computed Tomography”*. SPIE Medical Imaging, San Diego, 2025.

- **Jakob Haeusele**, Clemens Schmid, Josepha Hilmer, Florian Schaff, Tobias Lasser, Thomas Koehler, and Franz Pfeiffer. “A Novel Phase Retrieval Method for Continuously Acquired Grating-Based Dark-Field Computed Tomography Using Tikhonov Regularization”. SPIE Photonics West, San Francisco, 2025.
- **Jakob Haeusele**, Clemens Schmid, Manuel Viermetz, Nikolai Gustschin, Tobias Lasser, Thomas Koehler, and Franz Pfeiffer. “X-ray Dark-Field Computed Tomography at the Human-Scale: Challenges and Implementation of a First Prototype”. IMXP, Garching, 2024.
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- **Jakob Haeusele**, Clemens Schmid, Manuel Viermetz, Nikolai Gustschin, Tobias Lasser, Thomas Koehler, and Franz Pfeiffer. “Advancing X-ray Dark-Field Imaging for Human-Scale Applications: Helical CT and Surview Imaging”. SPIE Medical Imaging, San Diego, 2024.
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- **Jakob Haeusele**, Clemens Schmid, Manuel Viermetz, Nikolai Gustschin, Tobias Lasser, Frank Bergner, Thomas Koehler, and Franz Pfeiffer. “Dark-field imaging on a clinical CT system: sample data processing and reconstruction”. 7th International Conference on Image Formation in X-Ray Computed Tomography, Baltimore, 2022.

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Deforme est de se ipsum praedicare.

Cicero

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