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Enhancing Navigation Accuracy through Cooperative Satellite Networks

Simulation Study on the Influence of Satellite Architecture on GDOP

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Abstract — The rapid increase of satellites in orbit and the growth of satellite constellations has brought up the need for autonomous operations. The concept of Cooperative Satellite Networks (CSN), a decentralized network enabling satellites to collaborate and share resources, has the potential for enhancing navigation accuracy by optimizing the satellite architecture and, thus, the GDOP. This paper investigates how the satellite architecture parameters and receiver position influence the Geometirc Dilution of Precision (GDOP) and draws initial conclusions on how CSN can optimize the constellation geometry to enhance navigation accuracy. Different satellite architectures were simulated with varying numbers of satellites, orbital planes, altitudes, and inclination angles. The GDOP analysis was conducted for receivers with diverse elevation constraints located at the Equator and Poles. Throughout those GDOP simulations, key trends were identified, which demonstrate how the satellite architecture influences the GDOP values. Increasing the number of satellites generally improves the GDOP. However, the rate of improvement diminishes with an increasing number of satellites, leading to convergence of the GDOP. Satellite architectures with higher altitudes achieve better GDOP values and converge faster with increasing numbers of satellites. The receiver locations and elevation constraints significantly impact the GDOP performance with lower elevation cut-off angles, resulting in better GDOP values. However, in specific scenarios, such as three-plane Low Earth Orbit (LEO) constellations, exceptionally high GDOP results are observed due to unfavorable satellite geometries where all satellites are positioned in one plane. The inherent diversity of CSN, such as combining LEO and Medium Earth Orbit (MEO) satellites, has the potential to mitigate these challenges and achieve more favorable geometries. These findings are crucial for the development of a reward function to enable the efficient distribution of navigation tasks in a CSN. Therefore, this paper lays the foundation for autonomous satellite networks to enhance navigation accuracy.

1 Introduction

In the last decades, there has been a significant increase in the number of satellites in orbit, driven by advancements in satellite technology and the miniaturization of space electronics [1]. This growth has been most pronounced in the LEO, where small satellites such as CubeSats and large satellite constellations now have become the most represent [2]. These compact, cost-effective platforms enable a wide range of applications, such as Earth observation or communication, and have become an essential part of modern space missions. Thereby, more and more satellite systems shift from monolithic to Distributed Satellite Systems (DSS). This technological shift has also inspired the concept of Federated satellite systems (FSS), which builds up on the DSS concept by enabling independent satellites to collaborate and share their resources to perform joint tasks [3]. However, traditional centralized operational approaches for FSS face challenges in scalability, resource management, and data exchange. To address those issues, a decentralized operational methodology for coordinating FSS, known as CSN, has been proposed in [1].

At the same time, with the declining costs of artificial intelligence and enhanced connectivity, autonomous vehicles such as autonomous driving and advanced air mobility are on the rise. These technologies, which rely heavily on precise and robust navigation systems, also promise significant advancements in sustainability through their more efficient traffic management [4]. Beyond transportation, robust and highly accurate navigation systems are also critical in domains like disaster management, where they enable precise mapping and victim localization in crisis scenarios [5] [6].
The growing interest in precise positioning and reliance on navigation accuracy across various applications has become a critical focus [7]. In this context, CSN enabling satellite navigation offers a promising solution for improving navigation accuracy

and thereby satisfying future needs.

This paper presents an initial investigation of the potential of CSN to enhance navigation accuracy by optimizing the constellation architecture and making use of its inherent flexible architecture. The objective of this preliminary study is to gain insights into the following key areas:

- How do the satellite architecture parameters - namely altitude, number of planes, inclination, and receiver characteristics (such as receiver position and elevation cut-off angle) influence the GDOP?
- Which of these parameters can be leveraged in Cooperative Satellite Networks (CSN) to optimize the satellite architecture to enhance navigation accuracy?

Ongoing research into Low Earth Orbit Positioning, Navigation, and Timing (LEO-PNT) systems and alternative satellite navigation methods is challenging the dominance of traditional Global Navigation Satellite Systems (GNSS) such as GPS and Galileo. These emerging approaches can potentially complement and extend existing capabilities by leveraging more diverse satellite architectures and dynamic operational strategies [8]. In comparison to conventional Positioning, Navigation and Timing (PNT) services, LEO-PNT offers the advantage of closer proximity to Earth, which, assuming comparable carrier frequencies with Global Navigation Satellite Systems (GNSS), results in a stronger signal strength received by the user [9]. This higher signal strength increases connectivity in challenging environments such as urban canyons or indoors. Furthermore, this would also be advantageous in the event of jamming of the navigation signal, which traditional GNSS systems are particularly susceptible to [10].

Therefore, a CSN enabling satellite navigation could potentially have several advantages of the presented ones for LEO-PNT and could facilitate and extend services provided by traditional GNSS systems. Moreover, due to the autonomous structure of CSN, an implication of an autonomous Orbit Determination and Time Synchronization (ODTS) is conceivable. There has already been research on the potential for existing GNSS to implement inter-satellite links and use them for orbit and clock corrections determination [11]. Additionally, the applicability of precise orbit determination with optical two-way links (OTWL) has been explored in [12]. This would prevent the necessity of extensive and cost-intensive ground stations, which are required for GPS or Galileo. Therefore, using CSN for navigation purposes has the potential to enhance navigation accuracy while providing a more flexible and autonomous solution and thus being potentially more cost-efficient compared to traditional navigation systems.

The structure of the paper is the following: Section 2 presents an overview of related work and background of GDOP and CSN. Section 3 illustrates the methodology used in this study, including the problem and mathematical formulation, model assumptions, analyzed design variables, simulation setup, and the validation of the simulation setup. Section 4 presents the results and outcomes of the GDOP analysis and highlights key insights of the GDOP performance across different scenarios, which are further discussed in Section 5. Finally, Section 6 draws the conclusion of this work by addressing the research question and identifying potential future research related to this work.

2 Background

DSS describes satellite architectures where multiple satellites interact and communicate with each other, as commonly observed in satellite constellations [3]. Typically, these satellites are designed with similar capabilities to perform identical tasks. A specific type of DSS is FSS, which extends this concept by enabling heterogeneous spacecraft to collaborate even when having different goals and capabilities [13]. Traditional operational methods are centralized, which face limitations regarding scalability, data exchange, and onboard satellite resources. Therefore, there is a need for decentralized operation approaches that autonomously coordinate the satellite network to facilitate further advancements in the field of FSS.

One such decentralized operational methodology for coordinating FSS is introduced in the Paper of V. Messina and A.Golkar [1] as Cooperative Satellite Networks (CSN). This methodology incorporates a reward function to simply coordinate the satellite framework without requiring high computational power for operations. The reward function enables satellites to independently evaluate their suitability for performing assigned tasks based on several factors, such as availability, power availability, and data storage. In [1], such a reward function is developed for object detection in space. The present study builds upon this concept of CSN and aims to further develop the CSN concept and its applicability in enhancing navigation accuracy.

In traditional GNSS such as Global Positioning System (GPS), Galileo, or Russian Global Navigation Satellite System (GLONASS), each satellite transmits a navigation signal that contains the satellite's position and a timestamp indicating when the signal was sent. As the signal is an electromagnetic wave traveling at the speed of light, the distance between the receiver and the satellite can be calculated once the travel time of the navigation signal from the transmitter to the receiver is known. Therefore, the receiver must at least receive four distinct navigation signals to determine its position in a three-dimensional space.

Several factors can impact the navigation signal quality and thus, the navigation accuracy. These include atmospheric errors such as ionospheric and tropospheric delays, as well as satellite clock and ephemeris inaccuracies, receiver noise, and signal delays caused by multipath effects [14] [15]. The cumulative impact of these error sources is referred to as the total User Equivalent Range Error (UERE). However, the accuracy of the position determination is not solely dependent on the quality of individual pseudorange measurements; the satellite-receiver geometry also influences it. This geometric relationship is typically quantified by a scalar value known as Dilution of Precision (DOP), as commonly discussed in navigation literature [16]. The DOP value can be interpreted as the reciprocal of the volume of the polyhedron formed by the tips of the unit satellite-receiver vectors. Consequently, the DOP value increases when the satellites used for the positioning are positioned closely together, leading to a decreased navigation accuracy. The navigation error of the navigation measurement is proportional to the DOP value. This relationship can be expressed by the following rule-of-thumb formula [15]

$$\text{Navigation error} = \text{GDOP} \times \text{UERE}. \tag{1}$$

There are several DOP values commonly used in satellite navigation, including GDOP, Position-DOP (PDOP), Horizontal-DOP (HDOP), Vertical-DOP (VDOP) and Time-DOP (TDOP). In this research, the satellite architectures were evaluated based on their GDOP values, as GDOP provides the most comprehensive measure, combining the effects of the different DOP values. Table 1 provides an overview of the GDOP ratings typically used in literature [15] [17].

Table 1 GDOP accepted levels [15]

GDOP Value	Rating
1	Ideal
2 - 4	Excellent
3 - 6	Good
6 - 8	Moderate
8 – 20	Fair
20 – 50	Poor

3 Methodology

To understand how a CSN could potentially be used to enhance the navigation accuracy for specific areas on Earth, the factors that influence the navigation accuracy need to be investigated. The components affecting the navigation accuracy were already mentioned in Section 2. In this study, only the effect due to the satellite geometry expressed as the GDOP values is investigated. Therefore, the GDOP values are analyzed through simulations in different scenarios and satellite architectures to understand how the architectural parameters affect the GDOP values and, consequently, the navigation accuracy. The goal is to examine generalized trends and rules of satellite architecture parameters impacting the GDOP.

3.1 Problem Formulation

The main objective is to select the satellite architecture with four satellites to achieve the lowest possible GDOP value and, thus, the highest navigation accuracy. The theoretical optimum satellite architecture is first investigated to obtain the lowest geometric possible GDOP. Then, the design variables of different satellite architectures are presented in section 3.1.3 and analyzed using a simulation model. This model propagates different Walker constellations and calculates the GDOP for a specified receiver. This enables the comparison of different satellite architectures in terms of their GDOP values from which the influence of the architectural parameters on the GDOP can be identified.

3.1.1 Mathematical Formulation

Given a surface point on Earth defined as a cartesian vector: $(X_{user}, Y_{user}, Z_{user})$ and four visible satellites: $(X_{sat,i}, Y_{sat,i}, Z_{sat,i})$ with $i = [1,4]$ the GDOP value can be calculated by [16]

$$P = \begin{bmatrix} \frac{X_{user}-X_{sat,1}}{R_1} & \frac{Y_{user}-Y_{sat,1}}{R_1} & \frac{Z_{user}-Z_{sat,1}}{R_1} & 1 \\ \frac{X_{user}-X_{sat,2}}{R_2} & \frac{Y_{user}-Y_{sat,2}}{R_2} & \frac{Z_{user}-Z_{sat,2}}{R_2} & 1 \\ \frac{X_{user}-X_{sat,3}}{R_3} & \frac{Y_{user}-Y_{sat,3}}{R_3} & \frac{Z_{user}-Z_{sat,3}}{R_3} & 1 \\ \frac{X_{user}-X_{sat,4}}{R_4} & \frac{Y_{user}-Y_{sat,4}}{R_4} & \frac{Z_{user}-Z_{sat,4}}{R_4} & 1 \end{bmatrix} \quad (2)$$

where R_i is the distance to from the user point to the satellite,

$$D = [[P]^T \times [P]]^{-1}, \quad (3)$$

$$GDOP = \sqrt{D_{11} + D_{22} + D_{33} + D_{44}}. \quad (4)$$

The visibility of satellites to a receiver is constrained by an elevation cut-off angle, referred to as θ_{cutoff}. This angle describes the minimum elevation above the local horizon of the receiver, where it can detect a satellite. The local horizon is the tangential plane to the Earth's surface at the receiver position. Therefore, the satellite is considered visible for a receiver only if $\theta_e \geq \theta_{cutoff}$, with θ_e being the elevation angle of the satellite in respect to the receiver.

3.1.2 Assumptions

First, it is assumed that all satellites can transmit a navigation signal and that these navigation signals are all of the same quality. Secondly, the receiver can receive the transmitted signals if the satellites are above the receiver's elevation constraint of θ_{cutoff}. In addition, orbit perturbations by orbital anomalies or external gravitational influences are neglected so that the satellite's trajectory remains steady. The Earth is considered as a regular sphere with a constant radius of $r_{Earth} = 6371km$.

3.1.3 Design Variables

To analyze which architectural parameters exert an influence on the GDOP value and to evaluate the extent to which the performance is affected, a series of design variables are defined and listed in Table 2. For the configurations having a semi-major axis of $700km$ and $2000km$ the analyzes were conducted with two additional numbers of satellites, which are [264, 321].

Table 2 Design variables

Design Variable	Description	Range	Unit
n_{sats}	Number of satellites	[24, 48, 72, 96 120, 168, 216]	-
Pos_{rec}	Receiver position	Equator, Poles	-
n_{plane}	Number of orbital planes	[3, 6, 8]	-
SMA	Semi-major axis	[700, 2000 26571]	km
θ_{cutoff}	Elevation cut-off angle	[0, 10, 15]	degree
INC	Inclination	[56, 90]	degree

This adaptation was made because, for LEO configurations with low numbers of satellites, the receiver's field of view is insufficient to detect four or more satellites. Thus, in those cases, a GDOP computation was not possible. For higher numbers of satellites, GDOP calculation became possible for LEO constellations. However, the rate of mean GDOP change from 168 to 216 was still notably high, indicating that for even higher satellite numbers significant GDOP improvements might be achieved. Therefore, additional numbers of satellites were simulated for the LEO scenarios to examine their GDOP trends.

3.2 Simulation Model

The two simulation models used in this study are resented in the following subsection. At first, the model used to investigate the theoretic optimal GDOP value is presented. Subsequently, the model used to analyze the GDOP performance for different Walker constellations is introduced. The two simulation models can be accessed via under this link: https://gitlab.lrz.de/VinMes/collaborative-satellite-networks-for-navigation.git

3.2.1 Simulation of Optimal GDOP Configurations

To find the lowest possible GDOP value for different elevation cut-off angles, a two-stage optimization approach was applied. First, the particle swarm opti-

mization algorithm, described in [18], from the MATLAB Global Optimization Toolbox [19], was used to broadly explore the solution space and provide an initial estimate of four points that result in a low GDOP. Subsequently, the MATLAB fmincon function was employed to refine this estimate and minimize the GDOP until convergence.
That approach was initially used without elevation restrictions to find an overall minimum GDOP value and validated against solutions from literature [20] [14]. Subsequently, an elevation cut-off angle was implemented to simulate the visibility constraints of a potential receiver, thereby limiting the set of possible points that could be considered. The algorithm then identified the configuration of four points that achieved the lowest possible GDOP for each specified elevation cut-off angle θ. A more comprehensive study on the mathematical GDOP minimization can be found in [21][22].

3.2.2 Simulation of Walker Constellations for GDOP Analysis

For this simulations, in contrast to the setup described in 3.2.1, satellite orbits were modeled to evaluate the GDOP performance of various Walker constellations. The individual satellites were propagated along their orbits, and the receiver was positioned on the Earth's surface, with visibility limited to satellites above a certain elevation cut-off angle starting at the local horizon.

The simulations were conducted using an existing model based on the simulation model described in reference [23] [24], which was extended with a GDOP analysis module. The GDOP module calculates the best GDOP value achievable based on the visible satellites for a given set of satellite and receiver positions. Therefore, the module evaluates the GDOP for all potential combinations of four satellites within the receiver's field of view. The combination that yields the lowest GDOP value is identified and, along with the GDOP value, returned. Consequently, the returned combination represents the most favorable selection of given satellites to generate the lowest GDOP for the specific receiver position.

This GDOP analysis is performed at each timestep throughout the simulation. The simulations were conducted on the Linux cluster of the Leibniz-Rechenzentrum (LRZ) [25].

3.3 Validation of the Simulation Model

To validate the simulation model, different scenarios from the literature were reproduced, and the resulting GDOPs were compared. The study by Güngör Y. and Taflan G.Y. [26] was used as a reference, where the GDOP was analyzed for a low earth orbit Walker constellation and GPS using their custom solver and one provided by Systems Tool Kit (STK) [27]. In their study, the receiver position in the simulation was at 39.8911° latitude and 32.7786° longitude with an elevation constraint of 10°. For the GPS scenario, the average GDOP results were 2.59 using their solver and 2.51 using STK. In comparison, the simulation model used in this study achieved a mean GDOP of 2.54, showing a difference of 1.93% relative to the result of their custom solver and 1.18% difference relative to the STK simulation. For the LEO Walker constellation, the two solvers used in [26] resulted both to an average GDOP of 2.96. In contrast, the simulation model used in this study achieved a mean GDOP of 3.01, which corresponds to a difference of 1.66%. Figure 1 presents the GDOP results over time using the simulation setup of this study for the low earth orbit Walker constellation scenario. To compare, the GDOP vs time results using both simulation models from the study by Güngör Y. and Taflan G.Y can be found in [26].

As these results from the simulation model only

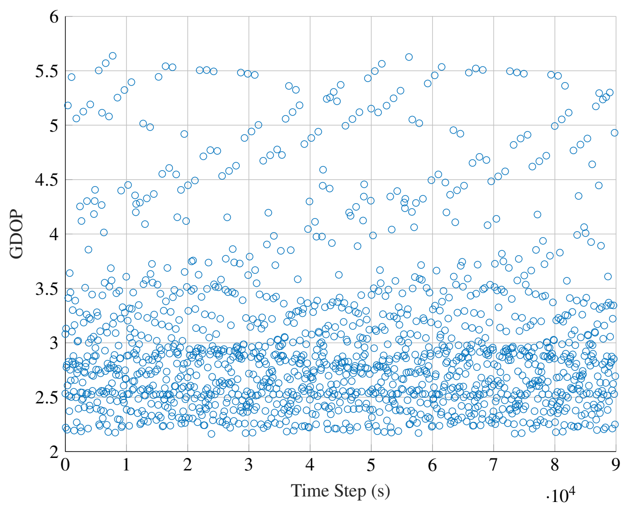

Figure 1 GDOP over Time

slightly deviate from validated solvers in literature(mean GDOP difference for the different simulation model results < 2%), it is assumed that the simulation model is suitable for analyzing satellite architectures and their GDOP performances.

4 Results

This section presents the simulation results. At first, the results for the optimal GDOP values are presented using the optimization approach described in Section 3.2.1. Then, the simulation results showing the influence of the satellite architecture on the GDOP values are presented. Therefore, the simulation model described in 3.2.2 was used to simulate various Walker constellations with the design variables presented in Section 3.1.3 for the GDOP analysis.

4.1 Optimal GDOP Value for Different Elevation Cut-Off Angles

At first, the overall optimal GDOP value was examined without any receiver elevation cut-off angle restrictions using the optimization approach described in Subsection 3.2.1. Therefore, a global minimal GDOP value of 1.58 was found, for which the satellites are distributed in a regular tetrahedron around the receiver. This theoretical GDOP minimum aligns with the results of previous studies on optimal GDOP values [14][20]. However, this value represents an idealized scenario as the receiver located on the Earth's surface cannot receive signals below its local horizon due to the obstruction of the Earth. When taking the obstruction of the Earth into account but otherwise setting a clear, unobstructed field of view above the local horizon of the receiver (elevation cut-off angle of 0°), the optimal GDOP becomes 1.73.

In a realistic scenario, typical elevation cut-off angles are between 5° to 15° [28]. The minimal achievable GDOP with an elevation cut-off angle of 10° is determined to be 1.96. For an elevation cut-off angle of 15°, the minimal (optimal) GDOP is determined to be 2.14, which corresponds to a 23.70% increase relative to the 0° scenario. Thus, these results show that without any orbital constraints, there is a significant dependence on the receiver's field of view influencing the optimal GDOP achievable in that scenario. In urban areas or for unfavorable receiver positioning, the elevation cut-off angle might increase further, resulting in higher optimal GDOP values. Figure 2 presents the obtained optimal GDOP values for different elevation cut-off angles. It becomes evident that in scenarios where the receiver exhibits an elevation cut-off angle of > 60°, the theoretical optimal GDOP is already > 10. Consequently, even with perfect satellite placement the navigation accuracy can at most be rated as "fair".

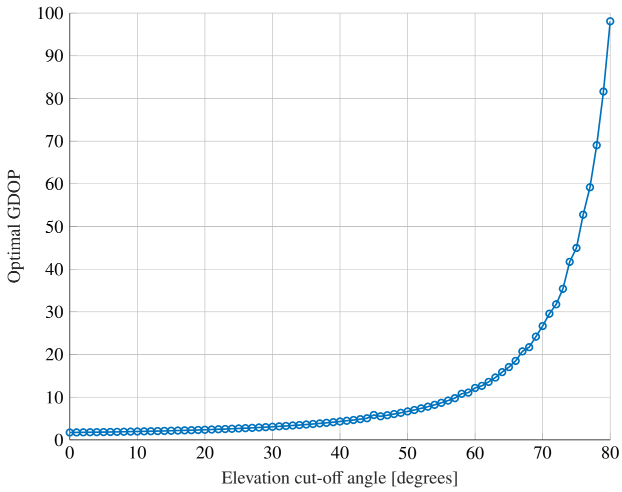

Figure 2 Optimal GDOP over Elevation Cut-Off Anlges

4.2 Influence of the Constellation Architecture on GDOP

The following section analyzes the influence of the variables described in Section 3.1.3. For each scenario presented, the GDOP at a specific position is evaluated at one-second intervals. The simulations are conducted over the duration of one orbit. The mean GDOP is then calculated for each scenario. To ensure that the mean GDOP is not disproportionately affected by brief periods of unfavorable satellite architecture, GDOP values exceeding 20 are capped at 20. Additionally, if fewer than four satellites are visible at any given timestep, making GDOP calculation impossible, no value is assigned.

4.2.1 Comparison of the Receiver Elevation Cut-Off Angles

First, the influence of the receiver elevation cut-off angle is analyzed. Figure 3 compares the mean GDOP values for different elevation cut-off angles, numbers of satellites, and numbers of orbital planes. The simulations were conducted for a receiver located at the equator, with satellites arranged in a Walker constellation having an inclination of $i = 56°$ and a semi-major axis of $26571 km$.

The results indicate that the mean GDOP decreases with increasing number of satellites and higher view angles. Furthermore, the curves show a convergence behavior with an increasing number of satellites, indicating that the GDOP improvement may diminish beyond a certain number of satellite thresholds. Therefore, Table 3 summarizes the mean GDOP values for the different configurations of orbital

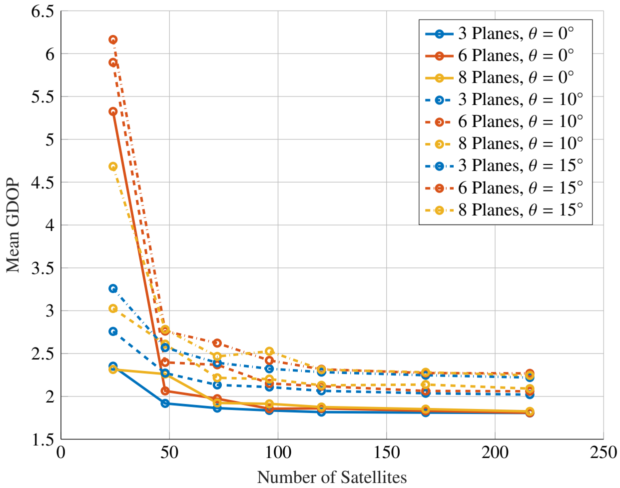

Figure 3 Comparison of θ_{cutoff}

planes, satellite numbers, and view angles, which are already shown in Figure 3.

Considering the mean GDOP values for three orbital planes with a 15° elevation cut-off angle, a 21.25% change of the mean GDOP is observed between 24 and 48 satellites, decreasing from 3.26 to 2.57. This GDOP improvement diminishes significantly with higher satellite numbers and reduces to 1.19%(0.02 absolute change) from 168 satellites to 216 satellites. This confirms a convergence pattern with diminishing GDOP improvements (<3%) beyond 120 satellites for all scenarios presented in Table 3. Similar behavior is observed for the other configurations with varying numbers of orbital planes and elevation cut-off angles, though the convergence rate varies slightly. The mean GDOP values achieved by the scenarios presented in Figure 3 for more than 48 satellites are all categorized as "excellent" according to the GDOP rating Table 1. Therefore, each of these configurations inherent a satellite architecture capable of providing high navigation accuracy based on their arrangement.

Analyzing Figure 3 and Table 3, the results show that the six plane configurations with 24 satellites have significantly higher mean GDOPs values than configurations with three or eight planes. The biggest GDOP difference between orbital planes is observed at the 0° elevation cut-off angle, where the six-plane configuration has a 3.02 higher GDOP compared to the eight-plane configuration, corresponding to a 56.66% mean GDOP decrease between the orbital planes.

Further analysis of the GDOP values over time for the six-plane configurations reveals a periodic behavior of fluctuating values increasing from low GDOP values of about ~ 2 to extreme peaks up to ~ 75. Figure 4 presents this phenomenon by showing a 450-second snipped of the GDOP values over time for the six-plane configuration with 24 satellites and a receiver elevation cut-off angle at the equator of 0°. A GDOP spike of 74.28 is present at $t = 255s$.

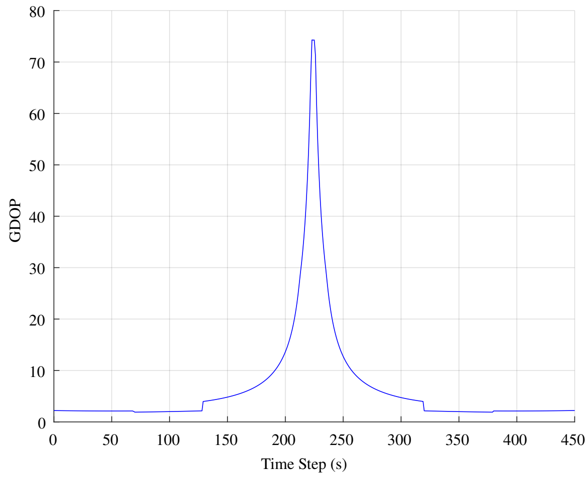

Figure 4 GDOP Peak: 6 Planes, $\theta = 0°$

Figure 5 presents a polar plot of the visible satellites from the receiver's perspective at the 225-second timestep, corresponding to the GDOP peak. The polar plot depicts the azimuth distribution of the satellites, with the radial axis representing the radius. Additionally, the elevation of the satellites is also included. While the satellites appear to be evenly distributed in elevation and azimuth, which would typically lead to low GDOP values, their similar distances to the receiver, however, result in a nearly planar geometric arrangement. Thus, this leads to a nearly singular position matrix (see equation 2), which results in a low determinant of the position matrix and, consequently, an exceptionally high GDOP value. Those geometrical planar arrangements of satellites in the six-plane scenarios for 24 satellites lead to an overall high mean GDOP, whereas the GDOP between those peaks varies between similar values observed for the other plane configurations. This phenomenon is observed in all the six-plane configurations with 24 satellites. However, with increasing numbers of satellites, this high GDOP peak effect does not occur. Therefore, adjusting the phasing of the Walker constellation potentially mitigates this phenomenon as this could prevent the satellites from aligning in a single geometrical plane

Table 3 Mean GDOP Values for Different Satellite Numbers, Planes, and View Angles

n_{sats}	3 Planes			6 Planes			8 Planes		
	$\theta = 15°$	$\theta = 10°$	$\theta = 0°$	$\theta = 15°$	$\theta = 10°$	$\theta = 0°$	$\theta = 15°$	$\theta = 10°$	$\theta = 0°$
24	3.26	2.76	2.35	6.16	5.90	5.33	4.68	3.03	2.31
48	2.57	2.27	1.92	2.76	2.40	2.06	2.78	2.61	2.26
72	2.39	2.13	1.86	2.62	2.37	1.97	2.47	2.22	1.92
96	2.32	2.11	1.84	2.42	2.15	1.86	2.53	2.20	1.91
120	2.28	2.06	1.82	2.32	2.12	1.86	2.31	2.13	1.88
168	2.25	2.04	1.81	2.27	2.06	1.83	2.28	2.14	1.85
216	2.22	2.02	1.81	2.27	2.06	1.81	2.24	2.09	1.83

and would thus, avoid the near-singular position matrix causing the extreme GDOP spikes.

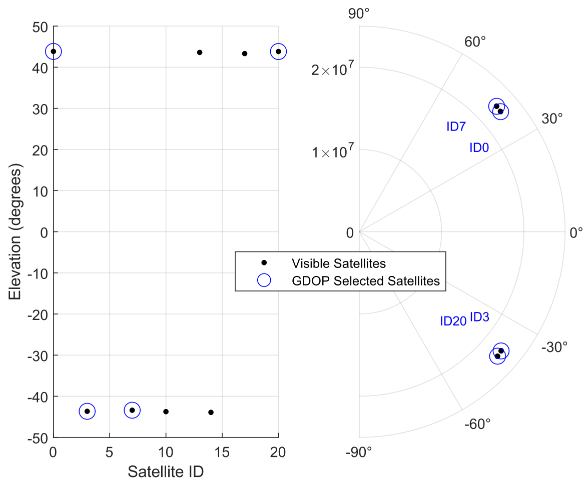

Figure 5 Receiver Plot of Visible Satellites for Timestep: 225s

When comparing the converged GDOP values across different numbers of orbital planes for the same elevation cut-off angle, the maximal difference for a 15° cut-off angle is 0.05, corresponding to a 2.14% variation. For the 10° elevation cut-off angle, the maximum difference is slightly larger at approximately 0.07, or 3.34%. For a 0° cut-off angle, the difference is 0.02, corresponding to 1.01%. In contrast, when comparing the best GDOP value for each elevation cut-off angle across all configurations, the largest difference is between the 0° and 15° elevation cut-offs. This difference is 0.41, corresponding to a 18.44% variation. Thus, this indicates that the variation in converged mean GDOP values due to changes in elevation cut-off angles is significantly more prominent than the variation observed across different numbers of orbital planes for a given cut-off angle for large numbers of satellites.

When comparing the best-achieved mean GDOP values for each elevation cut-off angle across all orbital planes to the theoretical optimal GDOP value presented in subsection 4.1, the differences are relatively small. For the 15° cut-off angle, the difference is 0.08, corresponding to a 3.55% deviation from the theoretical optimum. For the 10° cut-off angle, the difference is slightly smaller at 0.06, or 2.79%, while for the 0° cut-off angle, the difference is 0.07, corresponding to 4.12%. Thus, these results show that the difference between the theoretical optimal GDOP values and the best-achieved ones per elevation cut-off angle is less than 0.1 GDOP, which indicates a close alignment with the theoretical optimum across all evaluated elevation cut-off angles.

4.2.2 Comparison of the Semi-Major Axis

To identify the influence of the semi-major axis for a given satellite constellation, Figure 6 presents the mean GDOP results received by a user positioned at the equator. The satellites are configured in a Walker constellation with varying numbers of satellites and orbital planes. For all simulations, the elevation cut-off angle is fixed at 15°, and the inclination is set to $i = 56°$. In the following, configurations with a semi-major axis of $26571km$ are referred to as MEO constellations, those with a semi-major axis of $2000km$ as high LEO, and configurations with a semi-major axis of $700km$ as low LEO constellations.

The results show, that for satellite architectures with lower numbers of satellites, the average GDOP generally is higher in LEO compared to MEO. In the simulated LEO scenarios with altitudes of $700km$ and $2000km$, having 24 satellites fail to provide a navigation solution. This occurs as fewer than four

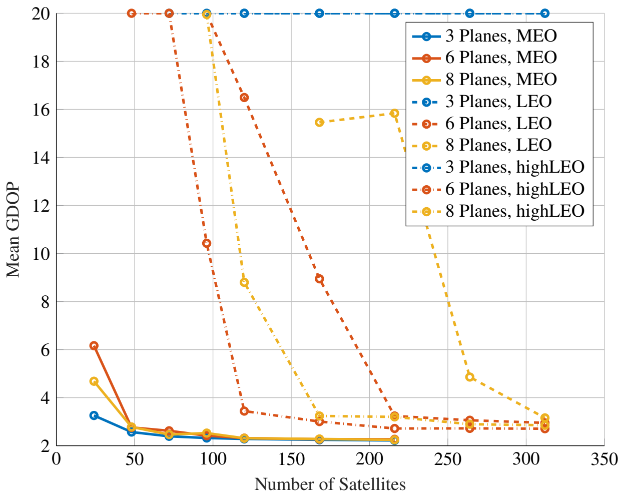

Figure 6 Comparison Semi-major axis for $\theta_{cutoff} = 15°$

satellites are visible, making it impossible to calculate a GDOP value for the given arrangement. As the number of satellites increases, enough satellites come into view, enabling the GDOP calculation. In both LEO scenarios, configurations with three orbital planes are not able to provide a useful navigation solution. When GDOP values are calculable, the values are consistently high, reaching the capped maximum value of 20 in each timestep. Even as the number of satellites increases to 316, the GDOP does not improve and remains at the maximum threshold. To confirm that this trend persists with even larger numbers of satellites, the three-plane LEO scenario was further simulated with 1000 satellites. The results showed again that the GDOP exceeded 20 in every timestep, reinforcing that these configurations with altitudes of $700km$ and $2000km$ and three orbital planes are unsuitable for navigation purposes independent of the number of satellites.

In the following, the achieved mean GDOP values across different altitude configurations for a fixed number of orbital planes are compared. For the scenario with six planes and a 15° elevation cut-off angle, the MEO constellation achieves with 2.27 a lower mean GDOP than the LEO configuration with 3.24 (42.73% higher value) and the high LEO configuration with 2.72 (19.82% higher value) for 216 satellites. A similar trend is observed for the eight-plane scenario, where the MEO configuration achieves a mean GDOP of 2.24 with 216 satellites. This is lower than the mean GDOP of the high LEO with 3.20 and substantially lower than the 15.84, which is the GDOP achieved with the low LEO constellation for the same number of satellites. Except for GDOP value of 15.84, achieved with the LEO eight-plane configuration, the just mentioned GDOP values are all < 4 and thus categorized as "excellent" (according to the GDOP rating Tabel 1). Thus, the differences between these GDOP values, even though the percentage difference seems significant, the effects on the navigation accuracy are considerably low as GDOP values achieved are all within the same acceptance level.
However, considering the number of satellites for which the mean GDOP values drop below 4 and are therefore suitable to provide high navigation accuracy, significant differences are observed. While the MEO constellation with a 15° elevation cut-off angel requires only 48 satellites to achieve a better mean GDOP than 4, the high LEO configuration need 120 satellites, and the low LEO configuration 216 satellites. A similar behavior is observed with the eight-plane configurations where the MEO constellation needs 48 satellites again for a mean GDOP under 4. In contrast, the high LEO requires 168 and the LEO configuration even 312 satellites.

These results indicate that for a given number of satellites, the MEO configuration provides the lowest GDOP, followed by the high LEO configuration, with the low LEO configuration yielding the highest GDOP values. In addition, it is observed that significantly higher numbers of satellites are required for LEO configurations to achieve GDOP values < 4, which is required for high-accuracy navigation.

The same analysis is performed using an elevation cut-off angle of 0°, thus taking all satellites above the local horizon of the receiver into account. All other parameters remain consistent with those described in the semi-major axis comparison for the elevation 15° cut-off angle. Figure 7 presents the results for the 0° cut-off angle. As observed with the 15° elevation cut-off angle, the LEO scenarios with three orbital planes fail to provide suitable GDOP results. Also, for 24 satellites, most scenarios -except for the eight-plane configuration at $2000km$ altitude- do not have enough satellites in the receiver's field of view as seen with the 15° cut-off angle. However, compared to the 15° cut-off angle scenario, the LEO configuration with 0° cut-off can provide valid navigation solutions with fewer satellites. Specifically, the required number of satellites to provide a mean GDOP value classified as "excellent" (according to the rating Table 1) is significantly lower. The highest difference in the

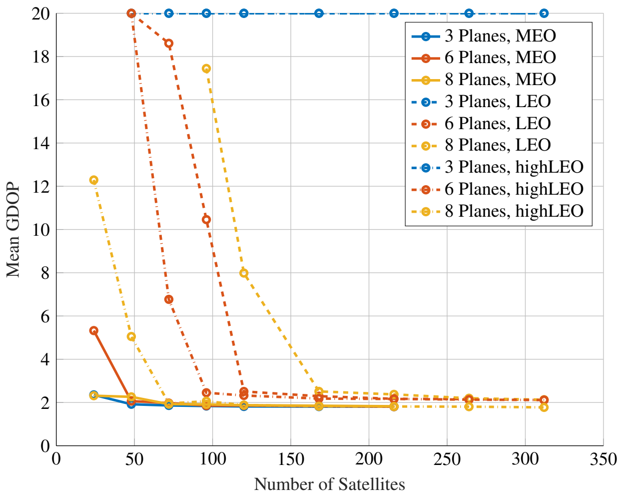

Figure 7 Comparison Semi-Major Axis for $\theta_{cutoff} = 0°$

When analyzing the 0° elevation cut-off scenario, the mean GDOP of the six and eight-plane configurations converges faster than the 15° cut-off. For instance, in the high LEO, eight-plane scenario, the most significant GDOP change of 58.6% occurs between 24 to 48 satellites, where no valid GDOP solution was found for the same scenario under the 15° elevation cut-off. At the step from 96 to 120 satellites, the mean GDOP change drops to 8.66% for the 0° cut-off angle, which is significantly lower than the 55.9% observed for the corresponding 15° cut-off scenario. Thus, for the LEO and high LEO scenarios, the elevation cut-off angle has a considerable effect on the rate of GDOP convergence, with 0° cut-off allowing for faster improvement in navigation geometry as the number of satellites numbers increases.

number of satellites required to achieve a mean GDOP < 4 arises for the eight-plane low LEO configurations comparing the elevation cut-off angles. In the scenario of a 15°, the configuration required more than 264 satellites whereas the mean GDOP value in the 0° elevation cut-off angle scenario with 168 satellites already is classified as "excellent". Similar behavior comparing the elevation cut-off angles is observed for in the high LEO configurations and for eight-orbital planes.

Table 4 summarizes the mean GDOP values for both scenarios. Data for the three-plane configurations are excluded, as when a valid GDOP is calculable, it reaches the maximum threshold of 20. The mean GDOP values for the MEO configurations included in Figure 6 and Figure 7 are not listed in the Table 4 as they are already discussed in Table 3.
The curves in Figure 6 and Figure 7 show for the configuration of six and eight planes a consistent tendency for the mean GDOP values to converge as the number of satellites increases. For the high LEO configuration (altitude: 2000km), eight orbital planes, and an elevation cut-off angle of 15°, the mean GDOP decreases by 55.9% when the number of satellites increases from 96 to 120. This rate of change gets smaller, reaching 1.26% from 264 to 312 satellites.
Even though one can observe a decline in the mean GDOP for the low LEO (altitude: 700km), eight planes configuration as well, the GDOP change from 264 to 312 is with 35.0% considerably higher and thus indicating a slower convergence rate compared to the high LEO scenario.

4.2.3 Comparison of the Receiver Position

In this subsection, the influence of the receiver's position on the GDOP using different satellite configurations is analyzed. Therefore, Figure 8 compares the mean GDOP values received by a receiver located at the equator and at the poles. The satellites are arranged in a Walker constellation with an inclination of 56° and a semi-major axis of $26571km$. The elevation cut-off angle of the receiver's field of view is set to 15°. The simulations were performed for different numbers of satellites spread on three, six, and eight orbital planes.

The results presented in Figure 8 show that, for all configurations, the mean GDOP values are consistently lower for a user located at the equator than one positioned at the poles, given the same number of satellites. For instance, with 216 satellites, the mean GDOP ranges for a receiver at the poles between 3.16 to 2.97 for the different orbital planes, while for the equatorial receiver, it varies between 2.27 to 2.22, representing an approximate 25% reduction.

The curves in Figure 8 further show convergence behavior for both receiver positions as the number of satellites increases, consistent with observations in subsections 4.2.1 and 4.2.2. For the receiver positioned at the poles, the three-plane configuration exhibits a mean GDOP change of 1.11% (0.04 absolute change) from 168 to 216 satellites and thus indicates that convergence might be achieved. However, for the six and eight-plane configurations, the mean GDOP change over the same range is

Table 4 Mean GDOP Values for Different Satellite Numbers, Altitudes, Planes, and View Angles

n_{sats}	Altitude: 700 km				Altitude: 2000 km			
	6 Planes		8 Planes		6 Planes		8 Planes	
	$\theta = 15°$	$\theta = 0°$	$\theta = 15°$	$\theta = 0°$	$\theta = 15°$	$\theta = 0°$	$\theta = 15°$	$\theta = 0°$
24	–	–	–	–	–	–	–	12.29
48	–	20.00	–	–	20.00	20.00	–	5.05
72	–	18.61	–	–	20.00	6.77	–	1.95
96	19.98	10.46	–	17.44	10.43	2.45	19.95	2.06
120	16.49	2.51	–	7.99	3.44	2.32	8.80	1.88
168	8.95	2.29	15.46	2.51	3.01	2.18	3.24	1.83
216	3.24	2.17	15.84	2.37	2.72	2.16	3.20	1.81
264	3.06	2.16	4.86	2.20	2.72	2.13	2.90	1.81
321	2.95	2.11	3.16	2.13	2.71	2.12	2.86	1.77

notably higher: 6.77% (0.22 absolute change) for the six and 6.55% (0.22 absolute change) for the eight-plane configuration. Thus, this indicates that a higher number of satellites could further reduce the mean GDOP significantly for the scenario with a receiver positioned at the poles for the six- and eight-plane configuration.

Additionally, Figure 9 presents the results for differ-

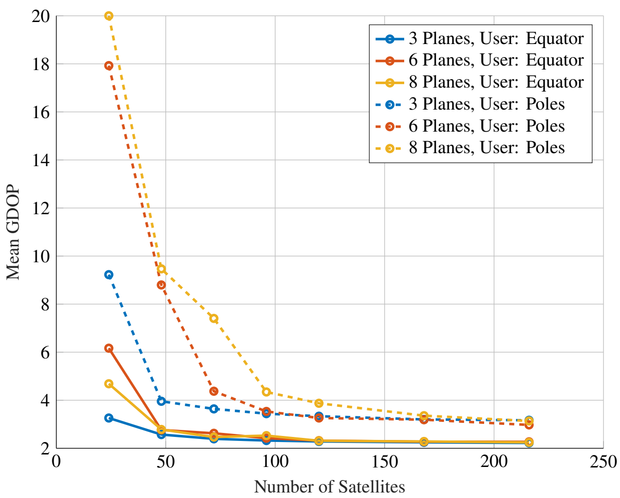

Figure 8 Comparison Receiver Position for $i = 56°$

ent receiver positions for polar Walker constellations having an inclination of 90°. All other constellation parameters remain the same as those discussed in Figure 8. The results indicate that the mean GDOP values for the receiver positioned at the equator are still better than those for the receiver at the poles when comparing the same number of orbital planes. In Figure 8, the mean GDOP was consistently lower for the equatorial receiver across all configurations, regardless of the number of orbital planes. In Figure 9, however, this trend only holds when directly comparing the same number of orbital planes. Table 5

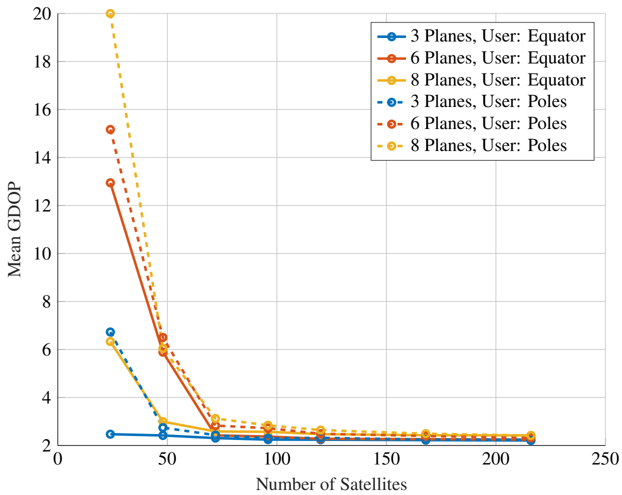

Figure 9 Comparison Receiver Position for $i = 90°$

summarizes the mean GDOP values for the configurations presented in Figure 8 and Figure 9. The data for the configuration with an inclination of 56° and a receiver positioned at the equator, which is presented in Figure 8, is excluded in Table 5 because it is already included in Table 3 under the 15° elevation cut-off angle.

Comparing the mean GDOP values for the receiver positioned at the equator under different inclinations, only minor differences are observed between the configuration with 56° inclination (listed in Table 3 for an elevation cut-off angle of 15°) and 90° (polar orbit)(listed in Table 5). For the 216 satellites, the mean GDOP values for the 56° inclination range between 2.22 and 2.27 across the orbital planes, whereas for 90°, the mean GDOP ranges from

Table 5 Mean GDOP Values for Different Satellite Numbers, Planes, Receiver Positions, and Inclinations (with $\theta_{cutoff} = 15°$)

n_{sats}	Poles (56° INC)			Equator (90° INC)			Poles (90° INC)		
	3 Planes	6 Planes	8 Planes	3 Planes	6 Planes	8 Planes	3 Planes	6 Planes	8 Planes
24	9.23	17.93	20.00	2.47	12.94	6.33	6.73	15.16	19.99
48	3.96	8.80	9.47	2.42	5.89	2.99	2.74	6.50	6.04
72	3.64	4.37	7.40	2.31	2.42	2.58	2.43	2.83	3.12
96	3.44	3.53	4.34	2.24	2.37	2.57	2.30	2.72	2.84
120	3.33	3.26	3.87	2.24	2.28	2.47	2.32	2.49	2.64
168	3.20	3.19	3.36	2.22	2.26	2.44	2.24	2.40	2.49
216	3.16	2.97	3.14	2.21	2.24	2.42	2.24	2.31	2.41

2.21, and 2.42. Although the range for the 90° inclination is slightly broader, the absolute difference between the two configurations is considerably low, between 0.01 to 0.2. In contrast, the differences are more prominent when the receiver is located at the poles. For the 56° inclination configurations, the mean GDOP ranges for the 216 satellites between 3.16 and 2.97, whereas for the 90° inclination configuration, the mean GDOP value is notably smaller between 2.23 and 2.41. This represents a maximum reduction of 78.49% in GDOP for the polar orbit configuration compared to the 56° inclination.

This highlights that while the equatorial receivers show only marginal differences in the mean GDOP for 56° and 90° inclinations, polar receivers benefit from a 90° inclination, resulting in lower mean GDOP values.

5 Discussion

The results presented in this study show the significant impact of satellite architecture on the achieved GDOP value. Thereby, also different receiver locations and fields of view are analyzed, highlighting the dependency of the GDOP on the relative geometry between satellites and receivers. This analysis identifies key findings and trends of the GDOP performance for different satellite configurations applicable to the design and optimization of using CSN for navigation purposes and to enhance navigation accuracy.

Influences of the Satellite Architecture on GDOP and Convergence Behaviour: From the results of this study, a clear trend is apparent that increasing the number of satellites in a constellation improves the GDOP. This, thereby, reduces the navigation error as they are directly coupled (see equation 1). However, the GDOP improvement decreases as the number of satellites increases, showing a convergence behavior. The rate of the mean GDOP convergence varies significantly depending on the altitude of the satellite constellation. Constellations with higher altitudes as MEO constellations achieve faster GDOP convergence than lower ones as constellations in LEO. This highlights that the orbital altitude plays a significant role in enhancing navigation accuracy. Higher orbital altitudes are useful for efficiently achieving satisfactory GDOP values with fewer satellites.

Influence of Receiver Characteristics: Receiver characteristics, such as location and elevation cut-off angle of the receiver's field of view, also influence the GDOP convergence. For constellations having an inclination of 56° with receivers positioned at the equator the mean GDOP is converging faster for increasing numbers of satellites than those located at the poles. This trend diminishes for polar constellations with 90° inclination, where the uniform global coverage reduces positional biases. The cause of this behavior likely relates to the relative satellite geometry and distribution of visible satellites but requires further investigation to clarify the exact mechanisms.
Smaller elevation cut-off angles achieve significantly better optimal GDOPs - with an increase of up to 23.70% from 0° to 15° elevation cut-off angle. This is due to having a more extensive solution space where points can theoretically be set for an optimized geometry, reducing the GDOP. Thus, it is unsurprising that receivers with broader fields of view also result in notably lower mean GDOPs than configurations with

reduced receiver fields of view.

LEO Challenges and Opportunities: The study reveals that for LEO satellites in Walker constellations with three, six, or eight planes, a high number of satellites is required (>264 for 15° receiver elevation cut-off angles) to achieve mean a GDOP below 4. This is further constrained by the three-plane LEO configuration as it fails to provide viable navigation solutions for an equatorial user position. This issue arises from the geometry of the visible satellites in the three-plane LEO scenario; due to the proximity of the orbit, the field of view of the receiver only contains satellites from one orbital plane. As those satellites are all in the same geometrical plane, it results in a nearly singular position matrix (see equation 2) and thus to an extremely high GDOP value, unsuitable for navigation purposes.

Potential of Cooperative Satellite Networks to enhance navigation accuracy: The inherent diversity of CSNs might offer a promising solution to these challenges. Integrating and combining diverse satellite architectures such as LEO with MEO satellites, CSN can achieve lower GDOP values and reduce navigation errors while benefiting from the advantages of LEO-PNT. As a result, the LEO-constellations provide high-density coverage and low latency. Their geometrical limitations can be mitigated by including satellites in higher altitudes or at different orbital planes. Such hybrid approaches align with the inherently diverse structure of CSNs and enable more robust navigation solutions.

Moreover, CSN could also optimize its satellite architecture for specific regions or scenarios to enhance navigation accuracy locally. For instance, selecting favorable satellite inclinations for a specific user position can significantly decrease the GDOP and thus enhance navigation accuracy. This adaptability is particularly relevant for applications such as in polar regions or areas with challenging receiver conditions.

While receiver specifications such as location and field of view are typically determined by external constraints and treated as fixed parameters, their influence remains critical for CSN providing navigation. In scenarios where a satellite position offers potential for GDOP improvement, but the receiver field of view is too narrow to receive the navigation signal, trade-offs must be assessed. For example, small satellite maneuvers to position satellites with favorable geometry might enhance overall navigation accuracy. However, this must balance potential operational demands. In such cases, the potential GDOP reduction has to be assessed with the chances of being able to be detected by the receiver with other limitations on site of the satellite as it probably needs to stop tasks.

6 Future Work and Conclusion

6.1 Future Work

This study provides a preliminary investigation on the potential of improving navigation accuracy through the use of CSN. However, additional research is required to expand and refine the insights gained in this research.

First, additional work should broaden the range of the investigated design variables for the GDOP analysis. Investigating a bigger set of receiver positions, including diverse constellation architectures and orbit types. Simulations with current operational LEO satellites would be beneficial to evaluate a realistic scenario of the received GDOP. Moreover, the proposed hybrid solutions combining LEO and MEO satellites must be investigated and analyzed to assess their expected GDOP improvement. Further, to provide navigation using a decentralized Cooperative Satellite Networks (CSN), it is crucial that the CSN is capable of autonomously assigning tasks to satellites to provide navigation for a specific user. This necessitates the development of a reward function that enables satellites to evaluate, based on their status and positioning with respect to the user, their potential to contribute to an enhancement of navigation accuracy. While this study has already laid the groundwork for such a reward function, further analysis is needed to complement the results with further simulations with more receiver positions and constellation types.

The simulation model used and extended in this study could be enhanced by incorporating a communication model to evaluate latency times between a user's navigation request and the received navigation signal. Moreover, the computing time and computational load could be decreased by implementing optimization algorithms, such as the heuristic local search algorithm called Simulated Annealing(SA), which was suggested in [15]. The current simulation model evaluates all satellite combinations at each time step to attain the lowest GDOP value. This approach is computationally intensive, particularly when dealing with high numbers of satellites, such as simulating all operational LEO satellites.

Future work should also extend the simulations to account for additional error sources. In this study, the navigation error is only assessed based on the geometrical distribution of the satellite. However, the quality of the pseudorange measurements expressed as UERE, must be incorporated into the analysis as well to fully evaluate the navigation error and thus the accuracy. Understanding how high the UERE is for a CSN based navigation is essential for developing a more comprehensive evaluation of the accuracy enhancement through CSN.

In summary, addressing those topics and research gaps will be crucial to properly evaluate the potential of utilizing CSN to enhance navigation accuracy.

6.2 Conclusion

This research highlights the influence of satellite architecture parameters on the GDOP. It became evident through multiple GDOP simulations that the number of satellites highly influences the mean GDOP with a maximum decrease of 13.95 (corresponding to 69.78%) while adding 24 satellites. This GDOP enhancement with an increasing number of satellites follows a convergence pattern, and thus, beyond a certain threshold, increasing the number of satellites results in only minor GDOP improvements. This convergence rate, however, is significantly influenced by the satellite altitude. Constellation configurations with higher altitudes converge faster than lower ones.

The results demonstrate only a minor impact on the GDOP for the different investigated three, six, and eight orbital plane configurations with a maximum difference of 3.34% in the converged MEO cases. In contrast, for the LEO configurations, a substantial influence of the number of orbital planes on the GDOP is observed. For configurations with six- and eight-planes, mean GDOP values < 3 are achieved with 120 satellites in 0° elevation cut-off angle scenarios. However, for the three-plane configurations, no suitable navigation solution is found, even with a satellite number of 1000. This limitation is attributed to the satellite arrangement of the visible satellites for a receiver in only one geometrical plane, which causes the position matrix between the receiver and the satellites to become nearly singular, resulting in exceptionally high GDOP values.

In terms of receiver characteristics and their influence on the GDOP value, it has been shown that low elevation cut-off angles of the receiver's field of view can significantly improve GDOP results with up 18.44% of the converged mean GDOP values between 15° and 0° cut-off angle. In addition, a dependence of the receiver position and the inclination of the satellite architecture is observed for the achieved mean GDOP. While an inclination change for a receiver located at the equator showed only a minor impact in the resulting GDOP values (maximal difference of 0.2 GDOP change), an inclination change from 56° to 90° for a receiver at the poles resulted in a reduction of 78.49% in GDOP. Thus, the impact of the inclination on the GDOP is significantly dependent on the receiver position.

With these findings on the influences of the architecture parameters on the GDOP, key parameters can be selected for CSN to optimize the satellite architecture and enhance navigation accuracy. As highlighted, the satellite altitude significantly influences the GDOP in two aspects: it determines the required number of satellites to provide suitable GDOP values and influences the impact of the orbital planes on the GDOP. By carefully selecting satellite altitudes, the required number of satellites can be minimized, thus reducing the resource demand while maintaining low GDOP values. Using the dynamic structure of Cooperative Satellite Networks (CSN) could integrate satellites from different altitudes and avoid scenarios where satellites are positioned in one geometrical plane. Therefore, combining LEO with MEO satellites in CSN could incorporate the advantages of LEO-PNT while efficiently optimizing the satellite architecture.

Moreover, as a dependency of the inclination in combination with the receiver position on the GDOP was observed, those parameters can also be leveraged to enhance navigation accuracy through CSN. By aligning the selection of satellites based on their inclination with the receiver's position, CSN can dynamically optimize the satellite architecture for specific regions or scenarios.

In conclusion, the findings present key insights into the influence of satellite architecture parameters and their impact on the GDOP value in dependence on the receiver characteristics. These insights are crucial for developing a reward function for CSN to enable the satellites to autonomously distribute navigation tasks. Furthermore, this study presented preliminary ideas and their potential of CSN to enhance navigation accuracy.

References

[1] Vincenzo Messina and Alessandro Golkar. Advancing federated satellite systems performance: A collaborative method for imporved object detection in space. AIAA Scitech Forum 2025, 2025.

[2] Carmen Pardini and Luciano Anselmo. Evaluating the impact of space activities in low earth orbit. *Acta Astronautica*, 184:11–22, 7 2021.

[3] Carles Araguz, Elisenda Bou-Balust, and Eduard Alarcón. Applying autonomy to distributed satellite systems: Trends, challenges, and future prospects. *Systems Engineering*, 21:401–416, 9 2018.

[4] European Space Agency. Navigation tech for the future of mobility, 10 2024. Available at: https://www.esa.int/Applications/Satellite_navigation/Navigation_tech_for_the_future_of_mobility?utm_source=chatgpt.com. Accessed: December 19, 2024.

[5] Anil Jangral. Multi-gnss assisted navigation for disaster management: A comprehensive review. *International Journal for Research in Applied Science and Engineering Technology*, 12:5752–5758, 5 2024.

[6] E.D. Kaplan and C.J. Hegarty. *Understanding GPS: Principles and Applications*. Artech House mobile communications series. Artech House, 2006.

[7] Mohamed Elsheikh, Umar Iqbal, Aboelmagd Noureldin, and Michael Korenberg. The implementation of precise point positioning (ppp): A comprehensive review. *Sensors*, 23:8874, 10 2023.

[8] Lionel Ries, Miguel Cordero Limon, Florin-Catalin Grec, Marco Anghileri, Roberto Prieto-Cerdeira, Felix Abel, Javier Miguez, Jose Vicente Perello-Gisbert, Salvatore D'Addio, Rigas Ioannidis, Alessandra Ostillio, Manuela Rapisarda, Rui Sarnadas, and Paride Testani. Leo-pnt for augmenting europe's space-based pnt capabilities. In *2023 IEEE/ION Position, Location and Navigation Symposium (PLANS)*, pages 329–337. IEEE, 4 2023.

[9] Kaan Çelikbilek and Elena Simona Lohan. A performance study on the combination of available gnss and potential leo-pnt constellations. *IEEE Access*, 12:162909–162917, 2024.

[10] Tyler G.R. Reid, Andrew M. Neish, Todd Walter, and Per K. Enge. Broadband leo constellations for navigation. *Navigation*, 65:205–220, 6 2018.

[11] Tomasz Kur, Tomasz Liwosz, and Maciej Kalarus. The application of inter-satellite links connectivity schemes in various satellite navigation systems for orbit and clock corrections determination: simulation study. *Acta Geodaetica et Geophysica*, 56:1–28, 3 2021.

[12] Stefan Marz, Anja Schlicht, and Urs Hugentobler. Galileo precise orbit determination with optical two-way links (otwl): a continuous wave laser ranging and time transfer concept. *Journal of Geodesy*, 95:85, 7 2021.

[13] Alessandro Golkar and Ignasi Lluch i Cruz. The federated satellite systems paradigm: Concept and business case evaluation. *Acta Astronautica*, 111:230–248, 6 2015.

[14] Peter J.G. Teunissen and Oliver Montenbruck, editors. *Springer Handbook of Global Navigation Satellite Systems*. Springer International Publishing, 2017.

[15] Ch. Raja Sekhar, V.B.S. Srilatha Indira Dutt, and G. Sasibhushana Rao. Gdop estimation using simulated annealing for gps and irnss combined constellation. *Engineering Science and Technology, an International Journal*, 19:1881–1886, 12 2016.

[16] Jean-Marie Zogg. Gps und gnss: Grundlagen der ortung und navigation mit satelliten, 10 2011.

[17] V B S Srilatha Dutt, Indira Dutt, G Sasi, Bhushana Rao, S Swapna, Swarna Ravindra Babu, Rajkumar Goswami, and Usha Kumari. Investigation of gdop for precise user position computation with all satellites in view and optimum four satellite configurations. 13, 01 2009.

[18] J. Kennedy and R. Eberhart. Particle swarm optimization. In *Proceedings of ICNN'95 - International Conference on Neural Networks*, pages 1942–1948. IEEE.

[19] MathWorks. Global optimization toolbox. Available at: https://de.mathworks.com/products/global-optimization.html. Accessed: December 15, 2024.

[20] Bertrand T. Fang. The minimum for geometric dilution of precision in global positioning system navigation. *Journal of Guidance, Control, and Dynamics*, 10:116–116, 1 1987.

[21] Yunlong Teng, Jinling Wang, and Qi Huang. Mathematical minimum of geometric dilution of precision (gdop) for dual-gnss constellations. *Advances in Space Research*, 57:183–188, 1 2016.

[22] Shuqiang Xue and Yuanxi Yang. Positioning configurations with the lowest gdop and their classification. *Journal of Geodesy*, 89:49–71, 1 2015.

[23] Vincenzo Messina and Alessandro Golkar. Initial formulation of a time varying dynamic graph decentralized optimization framework for scaled satellite network infrastructure operations. *Proceedings of the International Astronautical Congress, IAC*, 2023-October, 2023.

[24] Vincenzo Messina and Alessandro Golkar. Latency optimization in centralized and decentralized coordination of time-varying evolutionary satellite networks: The impact of packet size. In *75th International Astronautical Congress (IAC)*. International Astronautical Federation (IAF), Oct 2024.

[25] Leibniz-Rechenzentrum (LRZ). High performance computing - linux cluster. Available at: https://doku.lrz.de/linux-cluster-10745672.html. Accessed: December 15, 2024.

[26] Yasin Güngör and Gaye Taflan. Design and performance analysis of a low earth orbit walker constellation as a regional navigation satellite system for turkiye. 09 2023.

[27] Karynna Tuan. Tracking satellites based on the dilution of precision, 4 2016. Available at: https://www.agi.com/articles/tracking-satellites-based-on-the-dilution-of-preci. Accessed: December 15, 2024.

[28] Jiexian Wang, Wolfgang Keller, and Mohammad Sharifi. Comparison of availability of galileo, gps and a combined galileo/gps navigation systems. *Artificial Satellites*, 41, 1 2006.