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The effect of random field parameter uncertainty on the response variability of composite structures

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ABSTRACT

The accurate quantification of the random spatial variation of material properties at different scales is crucial for the systematic propagation of uncertainties through engineering models. In a previous work, the spatial variability of the apparent material properties of two-phase composites has been quantified in a Bayesian framework. This framework enables a consistent modeling of the statistical uncertainty in the parameters of the respective mesoscale random fields and also allows selecting the most plausible correlation model among different models belonging to the Matérn family. In this work, the most plausible random field model is employed in the context of uncertainty propagation of composite structures. Sample functions of the mesoscale random fields are generated using a covariance decomposition approach and the response variability of various composite structures is computed through Monte Carlo simulation. Parametric investigations are conducted to highlight the effect of the identified parameter uncertainty on structural response variability.

1. Introduction

Composite materials display a random spatial variation of mechanical properties, attributed to the mismatch of the properties of their constituent materials. Structural analysis using the traditional Finite Element Method (FEM) in this case cannot account for all possible output scenarios. The Stochastic Finite Element Method (SFEM) tackles this issue by taking the input uncertainty into account, propagating it through the system and assessing its stochastic response. The challenges therein lie in modeling uncertain input when little experimental data is available, as well as deciding on the method of obtaining uncertain structure output. This paper focuses on the response variability of composite structures as a continuation of the authors' previous work, since the parameters of the random input (material property fields) have been derived in [1].

Regarding SFEM, it is considered as an extension of the deterministic FEM for the solution of stochastic problems and typically involves the use of finite elements with random properties [2]. Uncertain system input is assigned to the elements by discretizing the respective random processes/fields and obtaining a finite number of random variables either pointwise or in an average sense. Three SFEM variants are commonly used: the perturbation approach, utilizing a Taylor series approximation, spectral SFEM, which involves Polynomial Chaos Expansion (PCE) of the response and Monte Carlo simulation (MCS). In the present paper, MCS is preferred, due to its robustness and simplicity. By generating a number of input samples and performing the respective analyses, MCS allows for assessing the response variability by examining the statistics of the system output.

In the case of composite materials, the macroscopic response of the structure is directly linked to the microscopic configuration, which is often plagued by uncertainty. Establishing this link is a challenging task and often accomplished through homogenization methods [3–5], whereas direct numerical simulations of the entire microstructure can be performed when the required computational power is available [6]. Depending on the scale considered, the mechanical properties of a composite can display various degrees of spatial randomness, as shown in [7,8]. Mesoscale random fields can be used to model this spatial variability [9], by applying homogenization in conjunction with the moving window technique [10,11]. These fields will then serve as input in applying the SFEM and therefore identifying them is of paramount importance. A large amount of microstructure data is required to completely determine these fields, which is often unavailable. As a result, assumptions are often made regarding the statistical characteristics of the random mechanical properties and methods to accurately predict their parameters are sought. When insufficient input data is available, data driven approaches, such as transfer learning, or PCE surrogate models can be used to predict the material properties of random microstructures [12,13].

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In [1], the authors applied Bayesian analysis to identify the parameters defining random property fields, given available composite microstructure data. More specifically, random fields are considered for the elasticity tensor components, which according to previous findings [8] can be modeled by a lognormal marginal distribution for a high stiffness ratio and hence can be obtained from the transformation of an underlying Gaussian field. In the Bayesian framework, field parameters are modeled as random variables and their full posterior distribution is obtained instead of a point estimate, while potential dependencies between these parameters are also revealed. In this particular case, the model parameters include the mean and standard deviation of the underlying field, as well as the correlation lengths in the x- and y-direction. Additionally, using the Bayesian approach, the most plausible correlation model belonging to the Matérn class was determined to be the exponential one, for the given microstructure data.

Having defined the random property fields, the next step would be to study their effect on the response of the structure. Several recent papers have dealt with the structural response variability when material uncertainty is present. In [14] for example, the generalized variability response function (GVRF) methodology is employed to compute the displacement response and the effective compliance of linear plane stress systems. Geißendörfer et al. [15] proposed a stochastic multiscale method for the computation of the natural frequencies of metal foams, analyzing random field data derived from CT images. In [16, 17], random fields of the mesoscale elasticity tensor of polycrystalline materials are generated using Stochastic Volume Elements (SVEs) and SFE is carried out, obtaining the resonance frequency of MEMS microbeams using MCS while establishing a link between the random field correlation and the SFE mesh size. In [18] random eigenvalue analysis is performed using MCS with an optimally selected start vector, while studying the effect of the material property field correlation length on the Coefficient Of Variation (COV) of the eigenvalue output. Naskar et al. [19] computed the stochastic natural frequencies of laminated composite beams with the Radial Basis Function (RBF) approach, considering randomness in the material properties and matrix cracking damage. A Spectral Stochastic Isogeometric Analysis method is proposed in [20] for free vibration analysis, obtaining the statistics of the eigenvalues and eigenvectors and showing its efficiency compared to MCS. In [21], a multilevel Monte Carlo approach was applied to assess the influence of microstructural variability on the homogenized properties of periodic metal lattice structures. Lastly, in [22], the inelastic static and dynamic response of frames with spatially varying properties is investigated by applying MCS and utilizing force-based elements.

As an extension of previous publications by the authors [8,23], the present work aims to compute the response variability of composite structures, whose mechanical properties are modeled by random fields and the uncertainty in their parameters has been quantified through Bayesian analysis for a given microstructure. In order to assess the effect of the uncertain mechanical properties on the response of the composite structure, Stochastic Finite Element Analysis is carried out herein, studying the variability of the system output in static as well as dynamic problems. The output variability is determined through MCS and the convergence of the response statistics (mean, standard deviation) is investigated. Since the model parameters derived from Bayesian analysis are random variables, several combinations are tested, including fixing the parameters at their posterior mean values as well as accounting for their full uncertainty in assessing the posterior predictive random field. The effect of these random field parameter combinations on the response variability of composite structures is investigated through several numerical examples. Random fields conditional on the model parameters are generated with spectral covariance decomposition, which is a discrete form of the Karhunen–Loève (KL) expansion. In the static case, the statistics of the nodal displacement of a plate under uniaxial tension and a cantilever beam under a concentrated load are studied, while in the dynamic case, the random eigenvalues (eigenfrequencies) of a cantilever and a simply supported beam are computed. Results are also given at different scales, depending on the size of the moving window, which reflects the amount of microstructure data taken into account.

The contents of this paper are as follows: In Section 2, the Bayesian approach for identification of random field parameters is explained and previous findings are reported. Section 3 contains a thorough description of the method adopted for the calculation of the structural response variability. Numerical results are presented and analyzed in Section 4, while useful conclusions are drawn in Section 5.

2. Bayesian identification of random material property fields

Before conducting a response analysis, it is vital to obtain accurate estimates of the random material property fields serving as model input. In the case of limited data, a Bayesian approach is well suited for the identification of the parameters of these random fields. In [8], starting with a computer simulated image of a two-phase composite, which can be seen in Fig. 1, realizations of random fields of the elasticity tensor components are obtained, through homogenization and application of the moving window technique. Through this procedure, the composite is divided into Stochastic Volume Elements (SVEs) which are smaller than the Representative Volume Element (RVE) and possess random homogenized mechanical properties. The random field is then constructed from considering these properties at the center of the SVEs.

Different mesoscale random fields are obtained from the same composite image, depending on the moving window size. The non-dimensional scale factor $\delta = L/d$ is used to characterize each mesoscale model, where L is the moving window size, d is the inclusion diameter and $\delta \in [1, \infty]$. By adjusting the moving window size, random fields at two different scale factors are obtained, $\delta_1 = 11.21$ and $\delta_2 = 22.42$, leading to 3249 and 625 SVEs, respectively [9].

The examined composite contains a volume fraction $v_f = 40\%$ of randomly dispersed circular inclusions with a diameter $d = 7.14$ μm, while it has a stiffness ratio $E_{incl}/E_m = 1000$. According to [8], a lognormal marginal distribution is well suited for a property field with such a high stiffness ratio. The elasticity components investigated are the axial stiffness C_{11} and the shear stiffness C_{33} of the 2-D elasticity tensor under the isotropy assumption.

Figs. 2, 3 show the computed realization of the random field, as well as its empirical marginal distributions and 2-D autocorrelation functions for the C_{11} and C_{33} components of the apparent elasticity tensor. As the scale factor increases, i.e., less microstructure data is taken into account, the correlation length is increased, leading to reduced spatial variability. In the limit case of $\delta \to \infty$, the random fields become fully correlated, as the moving window reaches the RVE size. Having obtained one realization of the random property fields and with no additional microstructure information available, Bayesian inference can be applied to learn the parameters of an adopted model of the mesoscale random fields.

2.1. Bayesian identification of model parameters and model selection

In the Bayesian approach, model parameters are regarded as random variables and using Bayes' rule, their full posterior distribution is determined given available microstructure data, instead of a single point estimate. Consider a homogeneous random field $A(\omega, \mathbf{x})$, with $\mathbf{x} \in B$, defined in terms of a model M with parameter vector $\theta \in \mathbb{R}^m$. B is the spatial domain of the composite, B_δ is a SVE (see Fig. 1) and ω denotes the randomness of a quantity. $A(\omega, \mathbf{x})$ models a component of the apparent elasticity tensor, for some mesoscale size δ. The vector θ can be learned using direct measurements of the random field $\mathbf{d} = [a^1; ...; a^{n_d}]$ at locations $\mathbf{x}^1, \dots, \mathbf{x}^{n_d}$. The measurement locations refer to the midpoint positions of the moving window and the data $\mathbf{d}$ refer to the corresponding homogenized property values. Bayesian analysis is employed to learn the vector θ. That is, a prior density $f(\theta|M)$

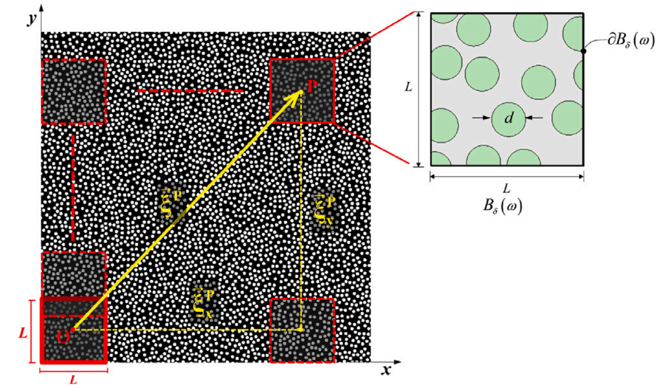

Fig. 1. Illustration of the composite material and the moving window technique.

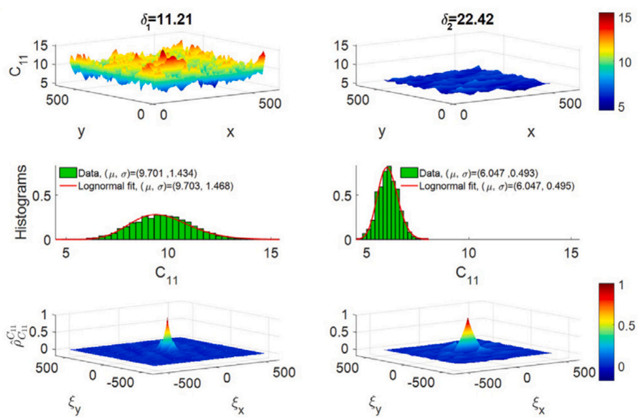

Fig. 2. Mesoscale random fields of elasticity component C_{11}.

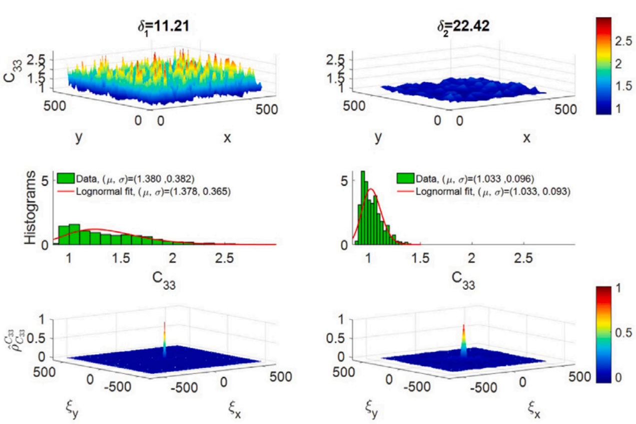

Fig. 3. Mesoscale random fields of elasticity component C_{33}.

given model M is imposed, describing prior knowledge on the model parameters, i.e. before measurements become available, and Bayes' rule is applied to update the prior density given the data. Bayes' rule states:

$$f(\boldsymbol{\theta}|\boldsymbol{d}, M) = c_{\mathrm{E}|M}^{-1} L(\boldsymbol{\theta}|\boldsymbol{d}, M) f(\boldsymbol{\theta}|M) \tag{1}$$

where $f(\boldsymbol{\theta}|d, M)$ is the posterior density of the parameters given the data $\mathbf{d}$ and model M, $L(\boldsymbol{\theta}|\mathbf{d}, M)$ is the likelihood function, describing the measurement information and $c_{E|M}$ is the model evidence.

Consider first the case where the random field $A(\omega, \mathbf{x})$ is Gaussian. Then, $\boldsymbol{\theta}$ includes the mean μ, standard deviation σ and parameters of the correlation kernel of the field $\rho(\boldsymbol{\xi}|\boldsymbol{\theta}_\rho)$, with $\boldsymbol{\xi} = (\xi_x, \xi_y)$ being the space lag; i.e., $\boldsymbol{\theta} = [\mu; \sigma; \boldsymbol{\theta}_\rho]$. If $A(\omega, \mathbf{x})$ is a non-Gaussian homogeneous translation field, then it is given by:

$$A(\omega, \mathbf{x}) = F^{-1} \cdot \Phi[U(\omega, \mathbf{x})] \tag{2}$$

where F^{-1} is the inverse of the marginal cumulative distribution function (CDF) of $A(\omega, \mathbf{x})$, $U(\omega, \mathbf{x})$ is a standard Gaussian field and Φ is the standard normal CDF. In such case, $\boldsymbol{\theta}$ includes the parameters $\boldsymbol{\theta}_F$ of the marginal distribution of $A(\omega, \mathbf{x})$ and the parameters $\boldsymbol{\theta}_\rho$ of the correlation kernel of $U(\omega, \mathbf{x})$, i.e., $\boldsymbol{\theta} = [\boldsymbol{\theta}_F; \boldsymbol{\theta}_\rho]$. For the case where the marginal distribution of a translation field is of the lognormal type, Eq. (2) reads:

$$A(\omega, \mathbf{x}) = \exp[\mu_G + \sigma_G U(\omega, \mathbf{x})] \tag{3}$$

where $U(\omega, \mathbf{x})$ is the underlying Gaussian field and μ_G, σ_G are auxiliary parameters of the transform.

The posterior distribution is often obtained numerically, due to the difficulty in evaluating the model evidence $c_{\mathrm{E}|M}$. An adaptive version of the BUS approach (Bayesian Updating with Structural reliability methods) combined with Subset Simulation (SuS) is adopted in [1], yielding a sample approximation of the posterior distribution along with an estimate of the model evidence [24]. It must be noted that the Bayesian approach is not limited to parameter identification but can also be used for the selection of the most appropriate (plausible) model by computing the posterior probabilities $\Pr(M_i|\mathbf{d}, \mathbf{M})$ of different candidate random field models M_i (with various correlation kernels or marginal distributions) given the data.

2.2. Bayesian analysis results

Bayesian analysis was first performed to obtain the most plausible correlation model, belonging to the Matérn class, by varying the smoothness parameter ν. The anisotropic version of the Matérn auto-correlation function adopted herein is given by the following equation [25]:

$$\rho_\nu(r) = \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\sqrt{2\nu} r\right)^\nu K_\nu \left(\sqrt{2\nu} r\right) \tag{4}$$

where Γ is the gamma function, K_ν is the modified Bessel function of the second kind, ν is a non-negative smoothness parameter and r is defined as:

$$r = \sqrt{\left(\frac{\xi_x}{b_x}\right)^2 + \left(\frac{\xi_y}{b_y}\right)^2} \tag{5}$$

with b_x, b_y being the respective non-negative correlation length parameters. The exponential ($\nu = 1/2$), modified exponential ($\nu = 3/2$) and squared exponential ($\nu \to \infty$) models belong to the Matérn family of autocorrelation functions [26]. Results showed the most plausible model to be the exponential one ($\nu = 1/2$) for the C_{11} component of the elasticity tensor. In that case, the autocorrelation function is reduced to the following equation:

$$\rho_{1/2}(r) = \exp(-r) \tag{6}$$

For the shear stiffness C_{33}, all correlation models were nearly equally matching, with a slight preference for the exponential one. As a result, this model is adopted for both tensor components in the present paper.

Following the correlation model selection, the posterior distributions of all model parameters are obtained, which are depicted in Fig. 4 for stiffness components C_{11}, C_{33} and scale factors δ_1 and δ_2. Note that Bayesian analysis revealed a positive correlation between the correlation lengths b_x, b_y and the standard deviation of the underlying Gaussian field σ_G (see [1]). It is also worth noting that Bayesian analysis results are affected by the moving window size and a smaller window (smaller scale factor) will lead to less variable model parameters, since more microstructure data is considered.

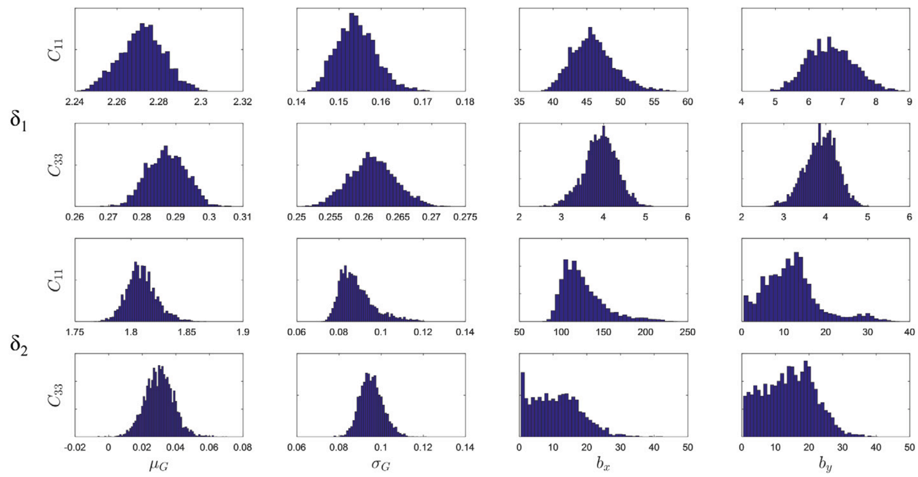

Fig. 4. Random field parameter posterior distributions for elasticity components C_{11} and C_{33} at mesoscale sizes $\delta_1 = 11.21$ and $\delta_2 = 22.42$.

3. Computation of structural response variability

Following the random field parameter identification, response variability analysis of the composite structure is the next step in the SFEM approach. As the use of finite elements with random properties is involved, a technique for assigning discrete values of the random property fields is needed. In this paper the discrete form of the KL-expansion is applied to discretize the random fields, which is also called the covariance decomposition method. SFEM is conducted via MCS, meaning a large number of samples are generated and analyzed. The response variability is studied and the effect of the input random field parameters derived through Bayesian analysis is investigated across different scales.

3.1. Random field discretization using covariance decomposition

Uncertainty propagation and computation of the response requires not only identifying the parameters of the input random fields, but also their discretization and generation of respective sample functions. Simulation of a lognormal random field conditional on the identified parameters $\boldsymbol{\theta}$ can be performed through generating random realizations of the underlying Gaussian random field $U(\omega, \mathbf{x})$ and applying the transformation of Eq. (3). There are several well established methods for simulating Gaussian random fields, e.g., see [27]. Among the existing methods, a discrete form of the KL expansion is adopted in this work, also known as spectral or modal decomposition method [28–30]. From this point on, ω, which denotes the randomness of a quantity will be omitted for simplicity.

Consider an approximation of a Gaussian random field $U(\mathbf{x})$, expressed in terms of the finite random vector $\mathbf{U} = [U_1, \dots, U_n]^T$, whose values are given, for instance, at the midpoints of a finite set of subdomains, also called stochastic elements. This discretization is consistent with the finite element formulation of structural mechanics problems, whereby the spatial domain is discretized into a set of finite elements. This random field discretization approach is known as the midpoint method [31]. The size of the subdomains should be small enough such that the value of the random field over the subdomain can be considered to be constant.

The random vector $\mathbf{U}$ has zero mean, while its covariance matrix, $\boldsymbol{\Sigma}_{UU}$, can be obtained from the autocovariance of the random field $U(\mathbf{x})$ evaluated at the coordinates of the element midpoints. Since the covariance matrix is $n \times n$ bounded, symmetric and positive semi-definite, it has n real non-negative eigenvalues $\widetilde{\lambda}_i$ and corresponding eigenvectors $\mathbf{v}_i$, satisfying

$$\boldsymbol{\Sigma}_{UU}\mathbf{v}_i = \widetilde{\lambda}_i \mathbf{v}_i \tag{7}$$

Thus, the covariance matrix $\boldsymbol{\Sigma}_{UU}$ can be decomposed as follows:

$$\boldsymbol{\Sigma}_{UU} = \sum_{i=1}^{n} \widetilde{\lambda}_i \mathbf{v}_i \mathbf{v}_i^T \tag{8}$$

The eigenvectors are orthogonal and it is further assumed that they have been normalized. Therefore, they form a basis in $\mathbb{R}^n$, meaning every element of $\mathbb{R}^n$ can be expressed as a linear combination of the eigenvectors $\mathbf{v}_i$. Hence, realizations of the random vector $\mathbf{U}$ can be represented as a linear combination of the eigenvectors $\mathbf{v}_i$ multiplied by random amplitudes. As a result, the random vector can be expressed as follows:

$$\mathbf{U} = \sum_{i=1}^{n} \sqrt{\widetilde{\lambda}_i}\, \mathbf{v}_i \zeta_i \tag{9}$$

where $\zeta_i, i = 1, \dots, n$ are random variables, which due to the orthonormality of the eigenvectors are given by:

$$\zeta_i = \frac{1}{\sqrt{\widetilde{\lambda}_i}} \mathbf{v}_i^T \mathbf{U} \tag{10}$$

From Eq. (10) it becomes apparent that the variables ζ_i have zero mean and are orthonormal, i.e., it holds:

$$E[\zeta_i] = 0\,, \;\; E[\zeta_i \zeta_j] = \delta_{ij} \tag{11}$$

Since $\mathbf{U}$ is Gaussian, the random variables ζ_i are independent standard normal random variables. Thus, simulation of the Gaussian random field can be achieved by drawing realizations of ζ_i and applying Eq. (9).

3.2. Structural response variability

Stochastic finite element analysis of 2-D composite structures is carried out herein. The input uncertainty is limited to the elastic tensor components, which are described by random fields. These fields are homogeneous, have lognormal distributions and their correlation structure is of the Matérn class. Through Bayesian analysis, the parameters defining these fields have been identified in [1] given computer generated microstructure data.

The response variability of the composite structure is computed using the following procedure. The first step requires selection of the random property field parameters (μ_G, σ_G and correlation lengths b_x, b_y), which have been obtained as random variables through Bayesian analysis (see Table 1 and Fig. 4 for their posterior distributions). After selecting a characteristic parameter set, e.g. the means of all parameters, the random property field is fully defined.

In the second step, MCS is employed to obtain the stochastic response and in each simulation, sample functions of the material property fields are generated through covariance decomposition, yielding

FE models with different spatial variation of mechanical properties and thus different response. The convergence of the estimated response statistics is observed through plotting the mean and COV of the response quantity against the number of MCS samples. In order to account for the full uncertainty in the random field model parameters, a predictive approach is also examined, where the above procedure is followed but instead of fixing the parameter set, different parameters for the random field are drawn from their posterior joint distribution at each MC simulation.

In the final step, the effect of the random field parameter variability is investigated by observing the response statistics. The advantage of following the Bayesian approach to describe the uncertain input in the structural variability problem is that, not only is the random spatial variation of the mechanical properties taken into account, but also the uncertainty in selecting the parameters to model this spatial variation. Additionally, since the random field parameters are also affected by the scale factor used to analyze the microstructure data, results are reported at two different scales, corresponding to scale factors δ_1 and δ_2. It is worth noting that the selection of the appropriate length scale (or, equivalently, the moving window size) depends on the specific problem and should be based on experimental data of the material microstructure used e.g., to quantify the correlation length of the random material property fields, which is related to the length scale [32,33].

According to [31],[2], care must be taken, in order to match the stochastic element mesh size with the spatial variability of the random fields and as a result, the element length should lie between $b/4$ and $b/2$, where b is the correlation length. In the present work, since the property fields are statistically anisotropic, the minimum of both correlation lengths (x- and y-direction) is used to calculate the element length.

The response variability is studied for static and dynamic problems. In all FE models analyzed, the plane stress assumption is made while using a unit thickness. In the static case, the quantity of interest is the displacement at a specific node of the structure. In the dynamic problem, eigenvalue analysis is carried out and the resulting first 15 eigenvalues are obtained, sorted in increasing order. In that case the statistics of these eigenvalues are examined. Eigenvalues are vital in the calculation of the dynamic response of structures in the framework of modal analysis and it is crucial to investigate how they are affected by input uncertainty. It should be noted that this paper only focuses on uncertain mechanical properties and thus the composite density affecting the mass matrix is not considered spatially varying. However, such an extension is possible through the authors' proposed multiscale framework and can be implemented in future works.

4. Results and discussion

This section contains the results of the SFE analysis. The examples have been selected such that the effect of input random field parameter uncertainty on the response variability can be thoroughly studied in both static and dynamic cases. Moreover, in the first example, the membrane behavior is dominant whereas in the other examples the influence of the shear stiffness is substantial. For each structural problem, 1000 Monte Carlo simulations are performed for different samples of the mesoscale random property fields. As shown in Figs. 6, 7, 9 and 10, statistical convergence is achieved within this number of simulations. The parameters defining these fields, given by the random vector $\theta = [\mu_G, \sigma_G, b_x, b_y]$, are the mean and standard deviation of the marginal distribution of the underlying Gaussian field, as well as the correlation lengths in the x- and y-directions. Four different types of random field parameter combinations are examined: the $mean$, $mean - COV \cdot mean$, $mean + COV \cdot mean$ (see Table 1) as well as a predictive case, for which a random field realization is generated for posterior samples from the parameter vector θ. The latter analysis requires significant overhead computational cost for the random field simulation, as the spectral decomposition in Eq. (8) needs to be performed for each sample of

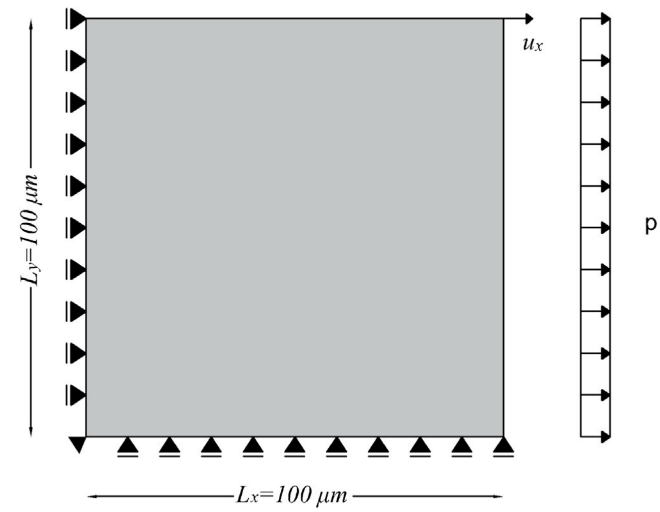

Fig. 5. Thin plate in tension.

the correlation length parameters. An alternative technique for efficient sampling of random fields with uncertain correlation kernels is discussed in [34]. The difference in analyzing the microstructure in more detail (smaller scale factor δ_1) versus less detail (larger scale factor δ_2) is also visualized. It should be noted that random field parameters in the predictive case for δ_2 have been selected such that correlation lengths lower than 2 μm are excluded from MCS in order to achieve a reasonable finite element size and computational cost.

4.1. Static case – Plate in tension

A thin square plate is examined and subjected to a tensile side load. The response quantity of interest in this case is the horizontal displacement of the top right corner node, as can be seen in Fig. 5.

As shown in Figs. 6 and 7, regardless of the field parameters chosen, the mean response is nearly identical and is increased by approximately 33% in the mesoscale size δ_2. In the higher mesoscale size however, the response COV is decreased. This can be explained by observing Fig. 4, where it is shown that the standard deviation of the marginal distribution (σ_G) for δ_2 takes lower values compared to δ_1. The posterior mean correlation lengths for δ_1 are lower than for δ_2, and, hence, we expect a higher spatial averaging for δ_1. However, this does not compensate for the difference in the point-wise standard deviation, which has a stronger effect on the response COV.

For the case of δ_1, there is little difference in the response corresponding to different field parameters and as a result, the response histograms seem to coincide. This is not true for δ_2, however, where the differences in the response COV and histograms become more pronounced for the examined random field parameter combinations. In particular, in both mesoscale sizes, results obtained for parameter case ($mean + COV \cdot mean$) show the largest COV compared to the rest of the parameter combinations tested. The above observations demonstrate the benefit of the Bayesian approach, which is able to account for this additional uncertainty in response predictions.

It becomes evident that the variability of the response is significantly affected by the input field parameters chosen, while such effect is diminished as the scale decreases and microstructure data is examined in more detail. This is reflected by the Bayesian analysis yielding random field parameters with less variability, as the moving window size decreases. Nonetheless, as explained above, the output COV for δ_2 remains lower than that of δ_1 for the examined parameter cases.

Table 1
Random field parameter combinations for tensor components C_{11} and C_{33} at different mesoscale sizes δ_1, δ_2.

	C_{11}				C_{33}			
δ_1	μ_G	σ_G	b_x	b_y	μ_G	σ_G	b_x	b_y
Mean−COV·mean	2.261	0.149	42.532	5.922	0.281	0.258	3.515	3.487
Mean	2.271	0.154	45.671	6.601	0.287	0.261	3.901	3.863
Mean+COV·mean	2.281	0.158	48.810	7.280	0.292	0.264	4.287	4.242
δ_2	μ_G	σ_G	b_x	b_y	μ_G	σ_G	b_x	b_y
Mean−COV·mean	1.795	0.080	97.622	4.752	0.021	0.090	3.652	6.151
mean	1.808	0.088	122.829	11.270	0.030	0.095	10.679	13.605
Mean+COV·mean	1.822	0.096	148.035	17.788	0.038	0.100	17.706	21.059

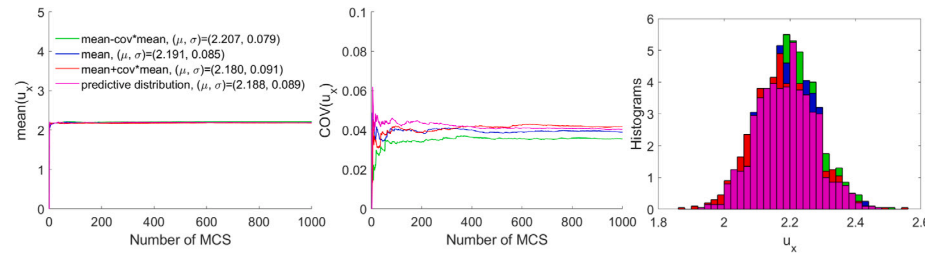

Fig. 6. Response statistics for thin plate in tension (scale factor δ_1).

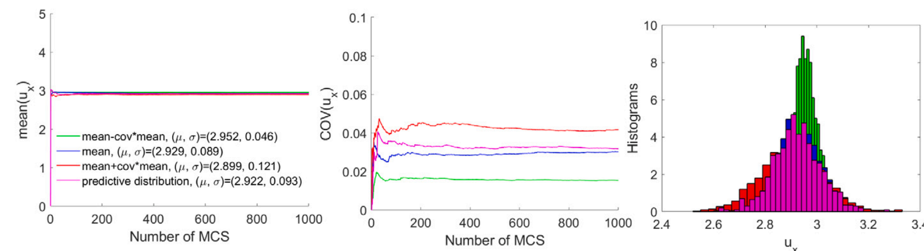

Fig. 7. Response statistics for thin plate in tension (scale factor δ_2).

4.2. Static case – Cantilever bending

The second static model is that of a cantilever beam in bending, which is also modeled with plane stress elements. The beam is under a concentrated load at its free end and the vertical displacement of the bottom right corner node is studied, as can be seen in Fig. 8.

Similarly to the previous model, the mean of the response is increased while the COV is decreased at the higher scale (δ_2). For this problem, we observe a larger decrease of the COV for the higher scale as compared to the plate in tension. This is likely due to the influence of the shear stiffness C_{33}, which is higher for the bending problem compared to the plate in tension. As can be observed in Fig. 4 and Table 1, the difference in the mean of σ_G is higher for C_{33} than for C_{11}. The difference in the response variability for δ_2 and the various considered parameter choices becomes slightly more pronounced in the bending problem, as can be observed when comparing Figs. 7 and 10. Again, the $(mean + COV \cdot mean)$ parameter case results in the largest response variability for both scale factors, with the variability of the higher scale factor (δ_2) remaining lower than the variability of the smaller scale factor (δ_1).

4.3. Dynamic case – Simply supported beam

In this dynamic problem, the first 15 eigenvalues (eigenfrequencies) of a simply supported beam are calculated. Results are shown in Figs. 11 and 12.

In all cases, the means of the eigenvalues do not appear to depend on the different considered choices of the input field parameters. As also shown in the static problem, increasing the values of these parameters will also lead to an increased eigenvalue COV and thus a more variable response, for the same scale factor. As previously shown, this effect is more prominent in mesoscale size δ_2 due to larger parameter uncertainty caused by limited data in this case, which, however, exhibits again less variability than the smaller size δ_1. It is also worth noting that the difference between the response COV values for the two scale factors is again considerable due to the combined influence of the stiffness factors C_{11} and C_{33} on the eigenvalues of the structure.

4.4. Dynamic case – Cantilever beam

In the last example, the eigenvalues of a cantilever beam are examined. Results are shown in Figs. 13 and 14. Again, the mean does not depend on the input field parameters chosen and the eigenvalues for δ_1 display a nearly identical COV of around 5–6%. For δ_2 different field parameters will lead to substantial differences in the eigenvalue COV, with the $(mean{+}COV \cdot mean)$ parameter case leading to the largest response variability, reflecting the statistical uncertainty due to limited data.

5. Conclusions

In this paper, a complete link is established between composite microstructure and macroscopic response. The uncertain parameters of the mesoscale random fields describing the spatially variable material properties serve as input in the SFE analysis of composite structures and their effect on the statistical characteristics of the response is examined.

The novel contribution of this paper lies on the investigation of the influence of random field parameter uncertainty (estimated with Bayesian analysis) on the response variability, which is computed for

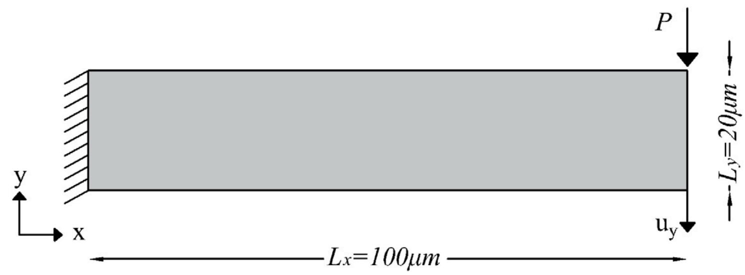

Fig. 8. Cantilever beam with concentrated load.

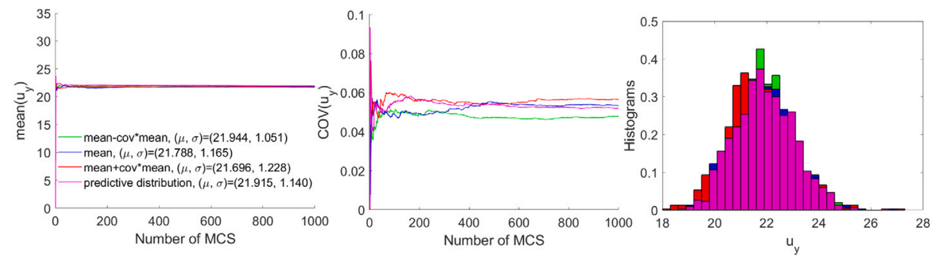

Fig. 9. Response statistics for cantilever beam in bending (scale factor δ_1).

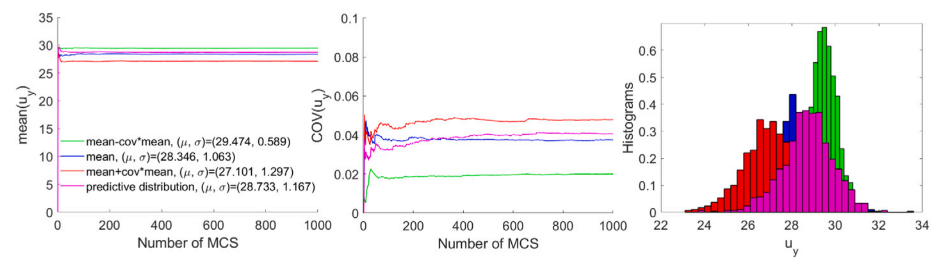

Fig. 10. Response statistics for cantilever beam in bending (scale factor δ_2).

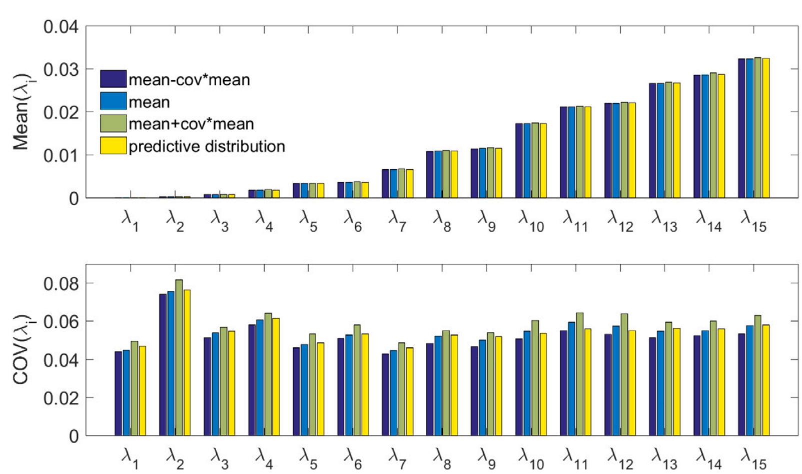

Fig. 11. Mean, COV of eigenvalues for δ_1, simply supported beam.

different parameter choices. These correspond to the posterior mean, the values obtained by adding and subtracting the posterior standard deviation from the mean and the full predictive random field, which accounts for the entire range of the parameter posterior distribution.

In all cases studied, the mean response appears unaffected by the chosen random field parameters. For the lower mesoscale size, the output COV does not particularly depend on the random field parameters. In the higher mesoscale size, however, different parameter choices have significant effect on the response COV. The substantial effect of uncertain random field parameters on the response variability observed in higher mesoscales can be explained by the fact that Bayesian analysis results are more variable in this case due to the smaller amount of microstructure data. Finally, for all examples considered, the lower mesoscale size resulted in higher response variability compared to the higher mesoscale size, while this difference is more pronounced in problems where the shear modulus has stronger effect on the structural response. The selection of the appropriate length scale is problem dependent and should be based on experimental data of the material

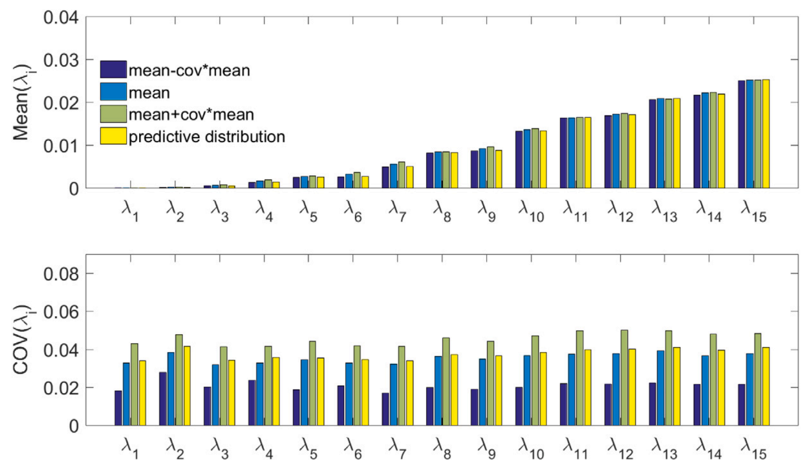

Fig. 12. Mean, COV of eigenvalues for δ_2, simply supported beam.

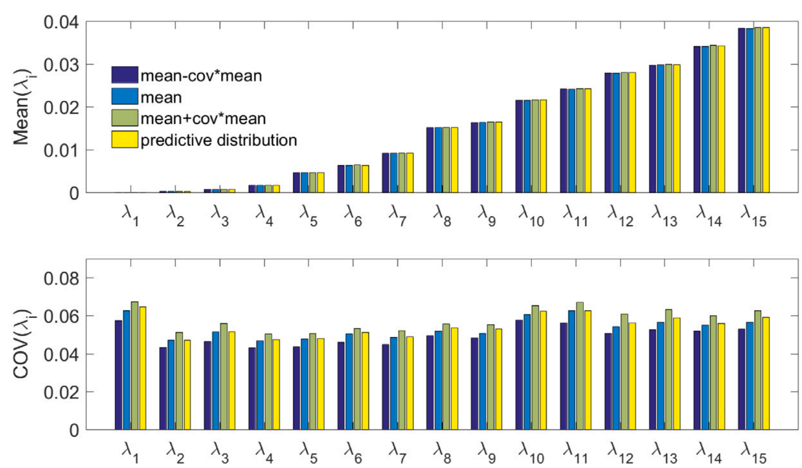

Fig. 13. Mean, COV of eigenvalues for δ_1, cantilever beam.

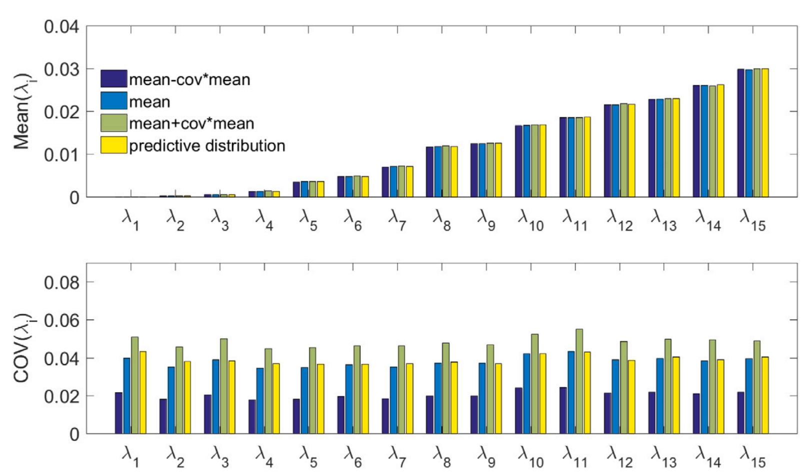

Fig. 14. Mean, COV of eigenvalues for δ_2, cantilever beam.

microstructure used e.g., to quantify the correlation length of the random material property fields, which is related to the length scale.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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References

[1] D. Savvas, I. Papaioannou, G. Stefanou, Bayesian identification and model comparison for random property fields derived from material microstructure, Comput. Methods Appl. Mech. Engrg. 365 (2020) 113026.
[2] G. Stefanou, The stochastic finite element method: past, present and future, Comput. Methods Appl. Mech. Engrg. 198 (9) (2009) 1031–1051.
[3] T. Zohdi, P. Wriggers, An introduction to computational micromechanics, Lect. Notes Appl. Comput. Mech. 20 (2005).
[4] N. Charalambakis, Homogenization techniques and micromechanics, a survey and perspectives, Appl. Mech. Rev. 63 (2010) 030803.
[5] S. Sakata, Y. Yamauchi, Stochastic elastic property evaluation with stochastic homogenization analysis of a resin structure made using the fused deposition modeling method, ASCE-ASME J. Risk Uncert. Engrg. Syst. B Mech. Engrg. 5 (3) (2019).
[6] J.E. Bishop, J.M. Emery, R.V. Field, C.R. Weinberger, D.J. Littlewood, Direct numerical simulations in solid mechanics for understanding the macroscale effects of microscale material variability, Comput. Methods Appl. Mech. Engrg. 287 (2015) 262–289.
[7] M. Grigoriu, Material responses at micro- and macro-scales, Comput. Mater. Sci. 107 (2015) 190–203.
[8] G. Stefanou, D. Savvas, M. Papadrakakis, Stochastic finite element analysis of composite structures based on mesoscale random fields of material properties, Comput. Methods Appl. Mech. Engrg. 326 (2017) 319–337.
[9] D. Savvas, G. Stefanou, M. Papadrakakis, Determination of RVE size for random composites with local volume fraction variation, Comput. Methods Appl. Mech. Engrg. 305 (2016) 340–358.
[10] C. Miehe, A. Koch, Computational micro-to-macro transitions of discretized microstructures undergoing small strains, Arch. Appl. Mech. 72 (4–5) (2002) 300–317.
[11] S.C. Baxter, L.L. Graham, Characterization of random composites using moving-window technique, J. Eng. Mech. 126 (4) (2000) 389–397.
[12] Y. Xu, H. Weng, X. Ju, H. Ruan, J. Chen, C. Nan, J. Guo, L. Liang, A method for predicting mechanical properties of composite microstructure with reduced dataset based on transfer learning, Compos. Struct. 275 (2021) 114444.
[13] X. Peng, D. Li, H. Wu, Z. Liu, J. Li, S. Jiang, J. Tan, Uncertainty analysis of composite laminated plate with data-driven polynomial chaos expansion method under insufficient input data of uncertain parameters, Compos. Struct. 209 (2019) 625–633.
[14] K. Teferra, S.R. Arwade, G. Deodatis, Generalized variability response functions for two-dimensional elasticity problems, Comput. Methods Appl. Mech. Engrg. 272 (2014) 121–137.
[15] M. Geißendörfer, A. Liebscher, C. Proppe, C. Redenbach, D. Schwarzer, Stochastic multiscale modeling of metal foams, Probab. Eng. Mech. 37 (2014) 132–137.
[16] V. Lucas, J.-C. Golinval, S. Paquay, V.-D. Nguyen, L. Noels, L. Wu, A stochastic computational multiscale approach; application to MEMS resonators, Comput. Methods Appl. Mech. Engrg. 294 (2015) 141–167.
[17] V. Lucas, J.-C. Golinval, R.C. Voicu, M. Danila, R. Gavrila, R. Müller, A. Dinescu, L. Noels, L. Wu, Propagation of material and surface profile uncertainties on MEMS micro-resonators using a stochastic second-order computational multi-scale approach, Internat. J. Numer. Methods Engrg. 111 (1) (2017) 26–68.
[18] G.S. Székely, G.I. Schuëller, Computational procedure for a fast calculation of eigenvectors and eigenvalues of structures with random properties, Comput. Methods Appl. Mech. Engrg. 191 (8) (2001) 799–816.
[19] S. Naskar, T. Mukhopadhyay, S. Sriramula, S. Adhikari, Stochastic natural frequency analysis of damaged thin-walled laminated composite beams with uncertainty in micromechanical properties, Compos. Struct. 160 (2017) 312–334.
[20] K. Li, D. Wu, W. Gao, C. Song, Spectral stochastic isogeometric analysis of free vibration, Comput. Methods Appl. Mech. Engrg. 350 (2019) 1–27.
[21] N. Korshunova, I. Papaioannou, S. Kollmannsberger, D. Straub, E. Rank, Uncertainty quantification of microstructure variability and mechanical behavior of additively manufactured lattice structures, Comput. Methods Appl. Mech. Engrg. 385 (2021) 114049.
[22] G. Balokas, M. Fragiadakis, Inelastic static and dynamic seismic response assessment of frames with stochastic properties, Struct. Infrastruct. Eng. 17 (3) (2021) 413–424.
[23] G. Stefanou, D. Savvas, M. Papadrakakis, Stochastic finite element analysis of composite structures based on material microstructure, Compos. Struct. 132 (2015) 384–392.
[24] W. Betz, I. Papaioannou, J.L. Beck, D. Straub, Bayesian inference with subset simulation: Strategies and improvements, Comput. Methods Appl. Mech. Engrg. 331 (2018) 72–93.
[25] M.S. Handcock, M.L. Stein, A bayesian analysis of kriging, Technometrics 35 (4) (1993) 403–410.
[26] M. Faes, M. Broggi, P.D. Spanos, M. Beer, Elucidating appealing features of differentiable auto-correlation functions: A study on the modified exponential kernel, Probab. Eng. Mech. (2022) 103269.
[27] Y. Liu, J. Li, S. Sun, B. Yu, Advances in Gaussian random field generation: A review, Comput. Geosci. (2019).
[28] W.K. Liu, A. Mani, T. Belytschko, Finite element methods in probabilistic mechanics, Probab. Eng. Mech. 2 (4) (1987) 201–213.
[29] F. Yamazaki, M. Shinozuka, Simulation of stochastic fields by statistical preconditioning, J. Eng. Mech.-Asce - J. ENG. MECH.-ASCE 116 (1990).
[30] B. Sudret, A. Kiureghian, Stochastic finite elements and reliability - A state-of-the-art report, 2000, Rep. No. UCB/SEMM-2000/08, University of California At Berkeley, USA.
[31] A. Der Kiureghian, J.-B. Ke, The stochastic finite element method in structural reliability, Probab. Eng. Mech. 3 (2) (1988) 83–91.
[32] J. Guilleminot, C. Soize, D. Kondo, C. Binetruy, Theoretical framework and experimental procedure for modelling mesoscopic volume fraction stochastic fluctuations in fiber reinforced composites, Int. J. Solids Struct. 45 (21) (2008) 5567–5583.
[33] N. Rauter, Numerical simulation of the elastic-ideal plastic material behavior of short fiber-reinforced composites including its spatial distribution with an experimental validation, Appl. Sci. 12 (20) (2022) 10483.
[34] J. Latz, M. Eisenberger, E. Ullmann, Fast sampling of parameterised Gaussian random fields, Comput. Methods Appl. Mech. Engrg. 348 (2019) 978–1012.