Time Reversal of Elastic Waves

Wave Physics and Imaging Applications

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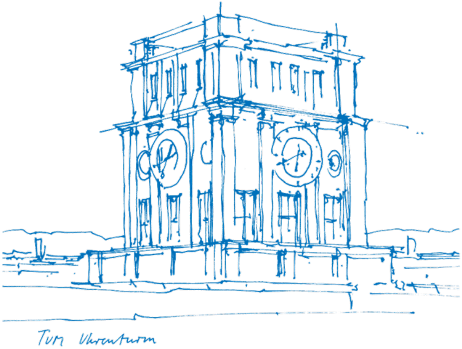


Key Features of SeisSol

- Elastic materials with viscoelastic attenuation and plastic deformation
- Anisotropic (Wolf, Gabriel, and Bader, 2020) and Poroelastic materials (Wolf, Galis, et al., 2022)
- Elastic-Acoustic coupling (Krenz et al., 2021)
- Dynamic Rupture, Kinematic Rupture (SRF) or Point Sources
- Tetrahedral meshes for arbitrary geometry in 3D, e.g. real topography
- Heterogeneous materials, e.g. depth dependent or unstructured
- Arbitrary order of accuracy, optimized for supercomputers
- Open Source: `https://github.com/SeisSol/SeisSol`

Current developments

- More physics
  - Elastic-Acoustic coupling (Lukas Krenz)
  - Rupture in poroelastic materials (Sebastian Wolf)
- Inverse problems
  - Time Reversal (Wendland, 2021)
  - Bayesian inversion (Sperling, 2022)

Projects:

Application Example: Dynamic Rupture Simulation for Pohang

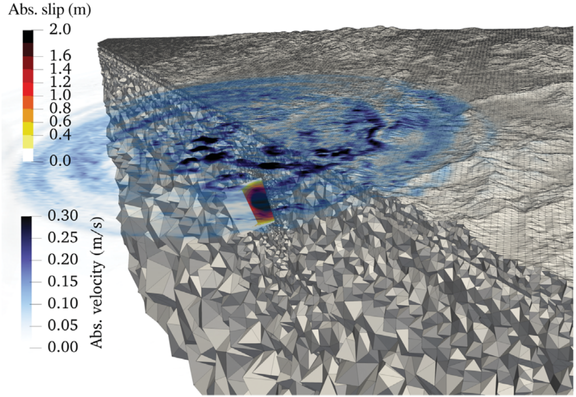


Figure: Velocity at the free surface and slip on the fault. Taken from (Palgunadi et al., 2020)

How to build a time machine

- Observation: The wave equation is symmetric in time. If $u(t, x)$ is a solution, then also $u(-t, x)$ is a solution.

$$\partial_{tt} u = \Delta u$$

- Time Reversal Mirror: Impose Boundary conditions to impose a time reversed wave field.
- Instantaneous Time Mirror: Sudden change in material parameters reverses the outward propagating wave.
- In both cases: The wave collapses into its origin.
- For an overview, see (Fink and Fort, 2017).

Time Reversal Mirror

1. Observation / Forward simulation: Store wave field at a set of receivers.
2. Play recorded data backwards to impose time-reversed wavefield.

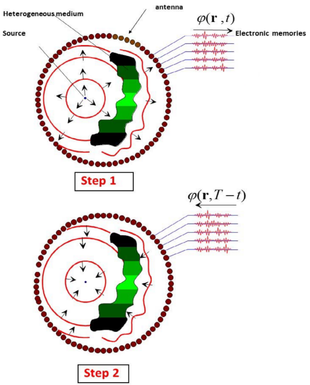


Figure: Time Reversal Mirror, adapted from (Fink and Fort, 2017)

Forward wave field

We use the velocity–stress formulation of the elastic wave equation:

$$\partial_t q - A\partial_x q - B\partial_y q - C\partial_z q = 0$$

with

$$q = (\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, u, v, w)\,.$$

- Already implemented.
- Place a grid of n receivers at x_i on the boundary.
- Record $\hat{q}(x_i, t_i)$.

Time Reversal Boundary Conditions

- Idea: Impose inflow boundary conditions on the stress.
- Stress is the analogue of pressure from acoustics.
- Initially: Read all receivers $\hat{q}(x_i, t_i)$ from storage.
- Loop over time and boundary Gauss points:
  - Interpolate $\hat{q}(x, T - t)$.
  - Set $q^+ = 2\hat{q} - q^-$.
  - Compute flux $F(q^-, q^+)$.

More details: (Wendland, 2021)

Verification

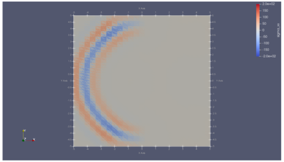
- Cuboidal domain $[-5, 5]^3$
- Centered point source with Ricker time history
- Record forward simulation.
- Use time-reversal boundary conditions with recorded data.

Expectation:

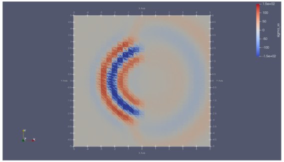
Focus on a point $\tilde{x}$ in the interior

- Observe a converging wave field until time t^*: $p(\tilde{x}, t) = q(\tilde{x}, t^* - t)$.
- At time t^*, the wave field has collapsed.
- After t^* the wave field diverges again $p(\tilde{x}, t) = q(\tilde{x}, t - t^*)$.

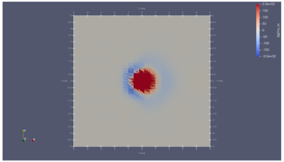
Contact of two acoustic half-spaces



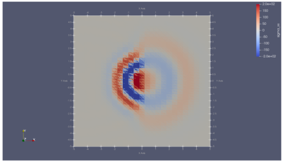
(a) σ_{xx} at $t = 13$ s.



(b) σ_{xx} at $t = 14.3$ s.



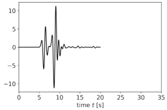
(c) σ_{xx} at $t = 16$ s.



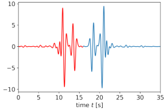
(d) σ_{xx} at $t = 17$ s.

Figure: (Wendland, 2021), `https://www.youtube.com/watch?v=uq6Eetvf4EE`

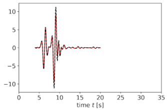
Elastic full-space



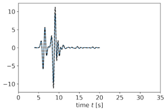
(a) σ_{xx} as recorded by the receiver located at $(4.0, -4.0, -1.0)$ in the forward direction.



(b) σ_{xx} as recorded by the receiver located at $(4.0, -4.0, -1.0)$ in the time-reversed direction. The red part marks the converging and the blue part the diverging wave.



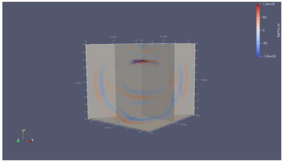
(c) The convergent wave, i.e. the first wavefield in fig. 5.15b, is time-reversed in the interval $[0, t_{\text{conv}}] = [0, 16]$, which corresponds to the interval $[4, 20]$ in the forward direction.



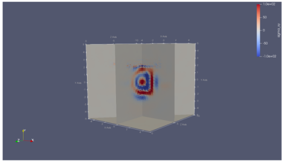
(d) The divergent wave, i.e. the second wavefield in fig. 5.15b, is translated in time, such that it coincides with the original forward propagating wavefield, i.e. $t = 16$ from fig. 5.15b is translated to $t = 4$.

Figure: (Wendland, 2021), `https://www.youtube.com/watch?v=G5QZm7ZvAPk`

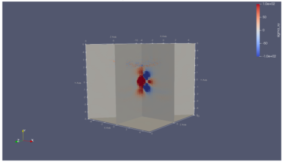
Stiff inclusion



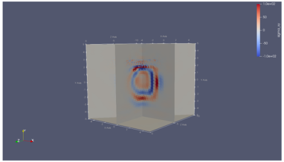
(a) σ_{xy} at $t = 12\,\text{s}$.



(b) σ_{xy} at $t = 15\,\text{s}$.



(c) σ_{xy} at $t = 16\,\text{s}$.



(d) σ_{xy} at $t = 17.5\,\text{s}$.

Figure: (Wendland, 2021), `https://www.youtube.com/watch?v=c-5geHHmugo`

Resolution of the focal spot

- Elastic full space
- Original source: $(2, 0, 0)$
- Snapshot at convergence time $16s$
- Upper: Receiver spacing $\Delta x = 0.5m$
- Lower: Receiver spacing $\Delta x = 2.0m$

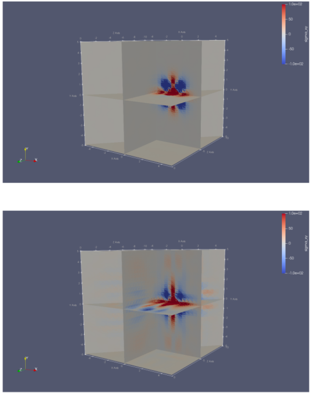


Figure: (Wendland, 2021)

Instantaneous Time Mirror

- Simulate the forward wave field.
- Suddenly change the material parameters everywhere, wavespeeds $\times 1000$.
- Waves are (partly) reflected at space-time material interface.

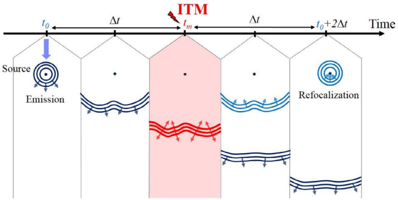


Figure: Instantaneous Time Mirror, adapted from (Fink and Fort, 2017)

ITM example

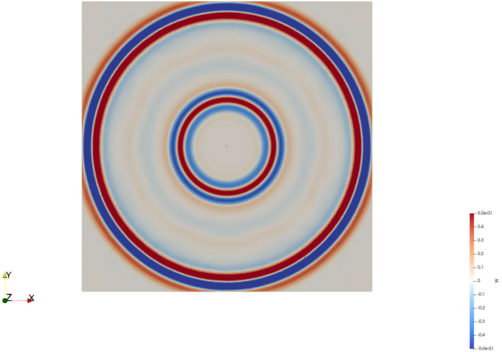


Figure: joint work with A.-A. Gabriel and K. Sager (LMU)

Conclusion

- Successfully included time reversal boundaries and instantaneous time mirrors for elastic waves into SeisSol.
- Verification of correct treatment for elastic materials with heterogeneous materials for TRM

Upcoming work

- Verification of ITM implementation
- Parameter study: How do we have to alter the material parameters to reverse the waves?
- Can we only invert one of the waves (e.g. P or S wave)??
- Receivers only on one side (e.g. free surface)
- Anisotropy studies with Bruno Giammarinaro

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